

## STRESS ANALYSIS OF PRESSURE VESSELS AND HEAT EXCHANGERS, USING FINITE ELEMENT TECHNIQUES

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### SUMMARY

The purpose of the paper is to illustrate the finite element technique when applied to the elastic stress analysis of steel pressure vessels and heat exchangers. The method yields more realistic predictions for the stress distribution than the conventional methods do (as for example the ASME Boiler and Pressure Vessel Code).

The problem which arises when applying the finite element technique to the stress analysis of reactor pressure vessels and heat exchangers is, that the inclusion of the individual tubes and perforations in the supporting tube sheets in the idealisation results in an extremely high computational effort.

The way in which this problem is solved consists in performing the analysis in two steps. The first step is to analyse the entire reactor pressure vessel or heat exchanger by replacing the perforated plates by solid plates with the same stiffness behaviour and to idealize the tubing by a continuous elastic support. This approach is based on a solid plate idealization of the tube sheet with modified elastic constants and the same dimensions as the solid plate. This involves the idealization of the structure with shell elements, allowing for branching, where it may be necessary to use transition elements and thick shells elements rather than thin shell elements, with additional solid elements, connecting both tube sheets. They are constructed compatible with the bending modes of the equivalent plate elements.

The second step is then to compute stresses in the individual tubes and the stress distribution in the perforated plates from the gross deformation of the equivalent solid plates.

Concerning the local stress distribution in the tube sheets, it will suffice to idealize only special regions of interest by using finite element idealizations which take into account the individual penetrations, tubes, etc., and using as boundary conditions the gross deformation or the membrane forces and moments of the equivalent solid plate solution.

In this paper several finite element models, for both, the first and the second step, are investigated. Among others, the following questions are considered:

- (i) When can we assume that the deformation of the tube sheets (which seem to be the most critical parts of the structure) are symmetrical about the midheight of the reactor?
- (ii) When can we assume rotational symmetry even though the reactor is only sector-symmetrical, or there are irregular penetrations and loading conditions which are not axisymmetric?
- (iii) With respect to the second calculation step, where we are interested in the stress concentration around the plate perforations, the main question is whether a two-dimensional idealization can be used instead of a three-dimensional idealization to obtain sufficiently accurate peak stresses.

These considerations are finally illustrated with the finite element analysis of a 1 : 1 scale chemical reactor vessel for which experimental results have been made available during proof testing. A comparison of the test data with the computational results of the finite element analysis indicates that the finite element method yields more realistic solutions and by far more comprehensive insight into the response behaviour than conventional methods.

## 1. Introduction

Heat exchangers and chemical reactors have an important place in the construction of chemical plants and apparatus. A chemical reactor is a heat exchanger in which, in normal use, a chemical reaction is constantly taking place. Heat exchangers consist essentially of a bundle of steel tubes, supported by one or two tube sheets, which is enclosed in a cylindrical steel casing (Fig. 1). The reactor is usually subjected to pressure and thermal loads. Nowadays the size of a chemical reactor varies enormously. The largest, which have a total height of nearly 20 metres, a diameter of 5 metres and consist of over 4000 tubes (40mm  $\varnothing$ ), are constructed to sustain a pressure of 25 bar (Fig. 2 and 3).

Since the chemical industry is constantly demanding larger and more durable plants, even bigger reactors, which can sustain even higher pressures, may be expected in the near future.

In Germany the layout of such a reactor normally follows the guidelines put forward by the "Arbeitskreis Druckbehälterbau" in the "AD-Merkblätter". These guidelines are equivalent to the ASME Boiler and Pressure Vessel Code [1] and lie within the safety limits set by the German Government. However, it has been found that, for certain vital structural parts of the larger reactors, these guidelines lead to overdimensioning (e.g. tube sheet thickness: 500%). This is by no means the result of technical demands but rather the expression of the lack of theoretical understanding of the special conditions in the structural section in question.

In a research project, initiated by large and medium sized chemical reactor construction companies, and sponsored by the "Bundesminister für Forschung und Technologie", investigations are being carried out with a new computational method

- a) to deduce the restrictions which must be imposed on the dimensions of the constructions when the guidelines in the 'AD-Merkblätter' are to be applied.
- b) to obtain a new, reliable, and engineer-oriented algorithm for the stress analysis of large chemical reactors and heat exchangers.

Up to now all the known algorithms are based on analytical methods, (e.g. [2]), that have often been refined and improved in the last few years. In general these methods involve crude assumptions of the properties of the construction and hence do not yield very reliable results for a particular construction.

A new approach is suggested by the finite element method which has been applied, with considerable success, in other branches of modern engineering. It allows the computer-aided calculation of a structural model which corresponds to the real construction in all its important components.

## 2. Method of Approach

The problem, which arises when applying the finite element technique to the stress analysis of reactor pressure vessels or heat exchangers, is that the inclusion of all the individual tubes and perforations in the supporting tube sheets results in extremely high computational effort.

The way in which this problem can be solved consists of performing the analysis in two steps:

Step 1:

The entire reactor or heat exchanger is analysed as a shell structure, taking into account all important peculiarities of the structure. Here the perforated plates are replaced by solid plates with the same stiffness behaviour as the perforated plates, and the tubing is idealised by a continuous elastic support [2, 3]. The approach is based on a solid plate idealisation of the tube sheets with modified elastic constants but the same dimensions as the perforated plates [4]. The tubing is replaced by special solid elements connecting both tube sheets in such a way that the deformation modes are compatible with these of the equivalent plate elements. The results of this calculation step are the displacements and stresses of the entire reactor, with the restriction that for the tube sheets the results describe only the gross deformation and the corresponding membrane forces and moments.

Step 2:

- a) The stresses in the individual tubes are calculated from the gross deformation of the tube sheets.
- b) for the local stress distribution within the tube sheets, it is sufficient to idealise the special regions of interest only by using finite element idealisations which take into account the individual penetrations, tubes, etc., and using as boundary conditions the gross deformation of the equivalent solid plate solution, or the corresponding membrane forces and moments.

3. Idealisation of the Shell

Most of the shell elements, available in finite element programs, are based on the classical thin shell theory, whose governing equations are very complicated. In consequence, for a compatible displacement model, the displacements as well as their first and second order derivatives with respect to the Gaussian shell parameters are required as nodal point degrees of freedom [5]. These elements yield excellent results, but they also require expertise in their application. Moreover, for arbitrary shell structures containing folds and branches, difficulties arise due to the requirement of compatibility in those regions. Thus it is quite usual to drop the requirement of compatibility by restricting oneself to simple flat triangular elements. Such an element, which has only engineering degrees of freedom, i.e. translations and rotations, is presented in [6].

A different method proposes the concept of mixed and hybrid formulations. For those models, the difficulty of inter-element compatibility does not exist, however other difficulties arise in the formulation itself, as well as in the solution of the system of equations [7].

An alternative approach starts with three-dimensional isoparametric elements and introduces the shell assumptions, thereby developing a very simple curved shell element without using any difficult shell theory. This so-called "degeneration" technique was first presented in [8]. The shell elements obtained have only engineering degrees of freedom and are, therefore, very easy to apply. Moreover, folds and bifurcations are also simply to handle. Although originally developed for fairly thick shells, it has been found that these elements can also be applied to thin shells depending on the accuracy of the computer word length which is used [9]. On the other hand, the tube sheets are rather thick plates, and inclusion of the shear deformation would be preferable. To be sure that no numerical difficulties arise when applying

this type of element, also to those structural component with very small wall thicknesses, the behaviour of those parts, when idealized with "Thick Shell Elements", has been investigated. The results show, that the "Thick Shell Element" is very recommendable for the analysis according to computational step 1 of Section 2.

4. Idealisation of the Tube Bundle

The method by which the tubing is idealized will be explained in the following for the case of an axisymmetric idealisation of the reactor.

The displacement functions of a rotational "Thick Plate Element", which is placed perpendicular to the axis of symmetry (Fig. 4), due to the displacements  $u_N, v_N, \alpha_N$  of node  $N$  are:

$$\begin{bmatrix} u(\xi, \eta) \\ v(\xi, \eta) \end{bmatrix} = \begin{bmatrix} \psi_N(\xi) & 0 \\ 0 & \psi_N(\xi) \end{bmatrix} \begin{bmatrix} u_N \\ v_N \end{bmatrix} + \frac{1}{2} t \eta \begin{bmatrix} \psi_N(\xi) \\ 0 \end{bmatrix} \alpha_N \quad (1)$$

where the shape functions  $\psi_N(\xi)$  are parabolic functions of  $\xi$  only.

The axial strains which result in a tube bundle element connected to this plate, caused by a displacement of node  $N$  are:

$$\epsilon_{zz} = \pm \frac{1}{h} \psi_N(\xi) v_N \quad (2)$$

For the tube bundle element, the following "material law" is used:

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{tt} \\ \sigma_{rz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{tt} \\ \epsilon_{rz} \end{bmatrix} \quad (3)$$

This means that the bending stiffness of the tubes is neglected. Thus, nonzero stiffnesses result for the tube bundle element only for displacement modes which yield nonzero axial strains.

Hence, the stiffness matrix of the tube bundle element is given by

$$k_{ij} = \int_V E/h^2 (\psi_i \psi_j) dv \quad (4)$$

5. Example

A typical example of a large chemical reactor is given in Fig. 5. Due to symmetry it is sufficient to idealize the upper half of the reactor only as an axisymmetric shell structure. Fig. 5 illustrates the deformation as one of the results obtained by such an idealisation. Due to internal pressure in the space around the tubes the cylindrical part expands radially but contracts in the axial direction. The outermost tubes become shorter, the other tubes are elongated. The resulting deformation of the tube sheet, which corresponds very well to measured values, is mainly caused by the stiffness behaviour of the tube bundle and by the layout of the region in which the tube sheet is connected to the casing. From the

axial displacements of the tube sheet, the stresses in the tubes can be deduced. Concerning the stresses in the tube sheets itself, high peak stresses in the ligaments have to be expected, which are not obtained by this idealisation, because the tube sheet is idealised as an equivalent solid plate.

Using the gross deformation of the tube sheet or the stress resultants, the local stress distribution can be obtained for any region of interest as indicated in section 2.

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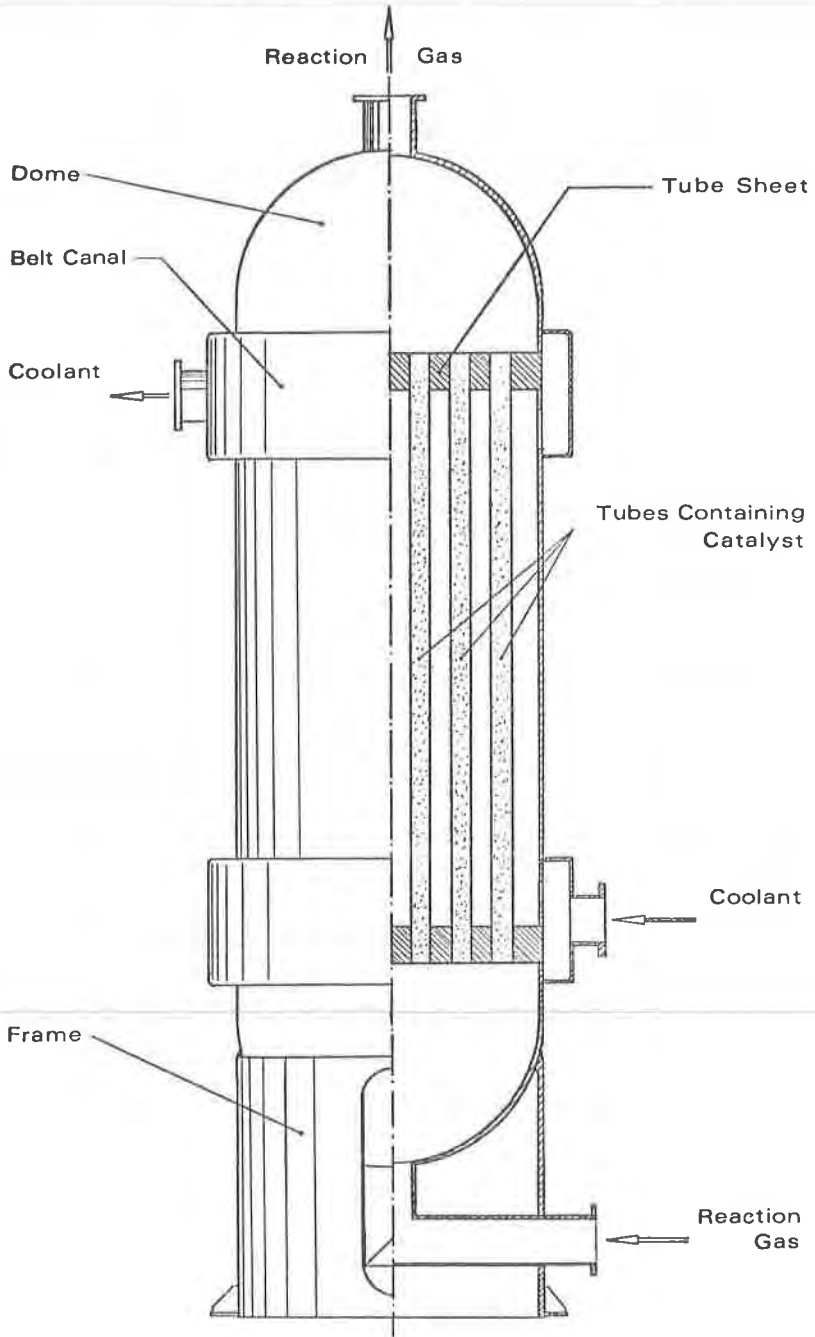


Fig. 1 Schematic Diagram of a Typical Chemical Reactor



Fig. 2 Chemical Reactor in Final Stage of Assembly

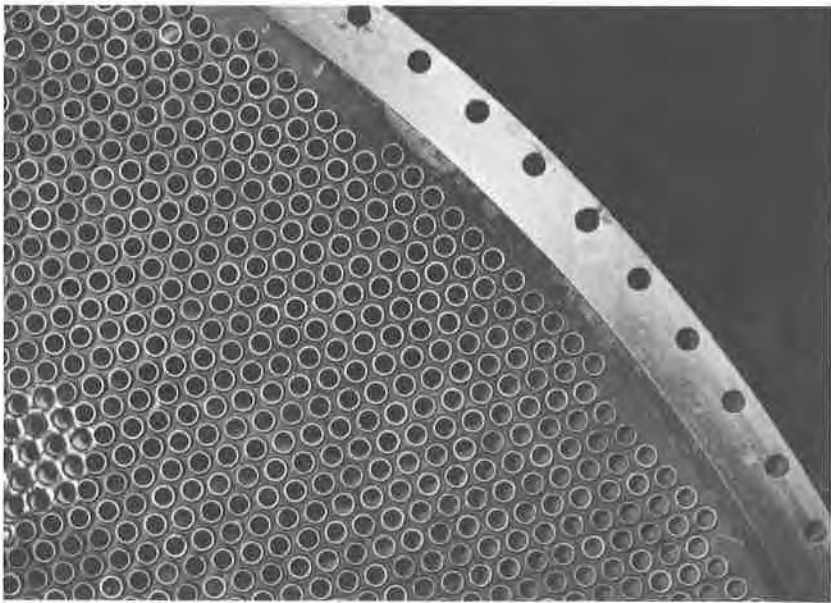


Fig. 3 View of the Circumference of the Tube Sheet

