The Optimum Structural Design for a Doubly-Layered Perforated Plate with Stiffeners

Y.C. Wang

East China Institute of Chemical Technology, Dept. of Mechanical Engineering, 130 Meilong Road, Shangai 201107, P.R. of China

Abstract

The doubly-layered perforated plate is an important component in nuclear reactors. In this paper the structure of the improved plate with stiffeners is discussed. To optimize it in view of a specific use, a study of structural optimization with the gradient projection method for nonlinear programming is proceeded.

In this problem the design variables are the height of the surrounding shell and the thickness of both upper and lower plates, and the objective function is the weight of the total structure. The aim of the optimization is to search for the optimal design parameters at minimum objective function value under some inequality constraints. Among those constraints the most essential one is the restriction of the central deflection due to the pressure loading, which is solved by means of the finite element analysis.

1. Introduction

The doubly-layered perforated plate is known as an important component in nuclear reactor. In this paper its structure is mainly composed of two circular perforated plates with diameter 820 mm and a cylindrical surrounding shell. In addition, there are 257 cup-ended short tubes arranged in 40X40 mm square grid between the upper and lower perforated plates. In order to improve its stiffness a nontraditional design with #-type stiffeners welded beneath the upper plate is accepted. The ratio of stiffener height and upper plate thickness is strictly regulated in 5:1. How to affirm the geometric parameters would then be the difficult problem of such structure. This paper will suggest a rather simple approach according to the optimal theory coorperated with the finite element method.

2. The application of nonlinear programming with inequality constraints

The thickness of the upper and lower perforated plates t_1 and t_2 respectively, combined with the height of the surrounding shell h are preferred as design variables. They constitute a 3-dimensional space R_3 within which an arbitrary point denotes a certain design case. Now the structural weight W is assumed to be the objective function of this problem. Hence the aim of the optimization is obviously to minimize the objective function subject to some inequality constraints. Finally, a set of optimal parameters t_1^*, t_2^* and h^* will be searched out thereby. The mathematical expressions of the previous idea are as follows:

Design variables vector $\quad x = (t_1 \ t_2 \ h)^T$

Objective function $\min_{x \in R_3} W(x)$

Constrained condition $\psi(x) \leqslant 0$ (1)

The lay-out of the doubly-layered perforated plate is shown in Fig. 1, and the initial values of the structural parameters are listed in Table 1.

The structural weight can be written in four parts, i.e.

$W = W_a + W_b + W_c + W_d$ (2)

in which

$W_a = [(\frac{\pi D^2}{4} - \sum_{i=1}^{257} A_i)t_1 + 5t_1 bl]\gamma$, represents the weight of the upper plate together with the stiffeners; where A_i is the area of ith tubular hole.

$W_b = (\frac{\pi D^2}{4} - \sum_{i=1}^{257} A_i)t_2\gamma$, represents the weight of the lower plate.

$W_c = \pi D h t_3 \gamma$, represents the weight of the surrounding shell.

$W_d = (\sum_{i=1}^{257} C_i)h\gamma$, represents the total weight of the short tubes; where C_i is the cross-sectional area of ith tube.

After substituting the corresponding values from Table 1 into eq. (2), we get

$W = 4.7372t_1 + 2.9715t_2 + 0.5157h$ (3)

Apparently, the objective function is a linear function of the design variables. In other words, the geometric significance of the equal weight planes within the space R_3 implies a group of parallel planes (see Fig. 2).

In regard to the constraint conditions there are four terms all in inequality, among which three terms are explicit, i.e.

$\psi_2(t_1) = t_1 - 20 \leqslant 0$ and

$\psi_3(t_2) = t_2 - 20 \leqslant 0$ (4)

They mean that the thickness of either plate not exceeds 20 mm. Meanwhile, the gap between the lower face of the stiffener and the top plane of the lower plate must be greater than 5 mm, i.e. $\psi_4(t_1, t_2, h) = (11t_1 + t_2)/2 + 5 - h \leqslant 0$ (5)

All of the previous constraints are also linear function. Hence the feasible region due to such constraints is bound to constitute a convex set (see Fig. 2).

However in order to assure the normal operation of the control mechanism in reactor the dominant constraint of the structure will undoubtedly be the displacements (or deflection) of both perforated plates. This constraint condition can be expressed as

$$\psi_1(t_1, t_2, h) = \delta_{max}(t_1, t_2, h) - 0.5 \leqslant 0 \quad (6)$$

where δ_{max} is the maximum deflection in the center point of the perforated plate, and the allowable deflection is assumed to be 0.5 mm. Here δ_{max} is well known as an implicit non-linear function of design variables, and will be solved by virtue of finite element method in this paper. Therefore the whole problem becomes a nonlinear convex programming due to the nonlinear inequality constraint, eq. (6). The draft of the feasible region bounded by the previous constraints is shown in Fig. 2. Evidently it still belongs to the convex set. This paper selects the gradient projection approach accompanied with the conjugate direction method step by step to search for optimal parameters t_1^*, t_2^* and h^*.

3. The expositions on finite element analysis for δ_{max}

Owing to the symmetric character only one eighth doubly-layered perforated plate (i.e. 45° typical region) is needed for FEM analysis. Non-conforming triangular plate bending element, rectangular planar shell element, 3-D beam element and 2-D eccentric beam element are accepted as mesh elements for perforated plates, surrounding shell, short tubes and stiffeners respectively. The whole model is discretised into 178 plate/shell elements and 65 beam elements with 116 nodes and 696 d.o.f. in all. In regard to boundary constraints the upper plate is treated as clamped edge, while the lower plate as free edge. The plates and short tubes are considered as rigid connections each other.

Since the holes on plates are arranged in square grid with the ratio of diameter/pitch, $\lambda = 0.68$, the perforated plates can be looked as solid plates by the replacement of the elastic constants E and ν, i.e. using effective Young's modulus $E' = 1.16\times10^6$ kgf/cm² and effective Poisson's ratio $\nu' = 0.4$ which are obtained from experimental charts. The entire procedure of FEM analysis for calculating the maximum deflection δ_{max} has been proceeded as usual and will not be informed here in details.

4. The process in search of optimal design point

The relevant geometric parameters of the original design point are assumed as follows:

$$t_1^{(0)} = 8.0 \text{ mm}, \quad t_2^{(0)} = 10.0 \text{ mm}, \quad h^{(0)} = 102.0 \text{ mm}$$

Thus we may get $\delta_{max}^{(0)} = 0.3734$ mm through FEM and the structural weight will be $W^{(0)} =$ 120.34 kg at the time. Obviously that point locates within the feasible region, or say, represents a feasible but not optimal case shown in Fig. 2 with label a.

Hence the first step (1) in search of optimal design point would be taken along the negative gradient direction of the equality weight plane. The relevant weight gradient vector is listed in Table 2.

Now the iterative process of the design variables t_1, t_2 and h under case a will be proceeded in accordance with the following formulae:

$$t_1^{(i)} = t_1^{(i-1)} - i\varepsilon_i V_{t_1}$$
$$t_2^{(i)} = t_2^{(i-1)} - i\varepsilon_i V_{t_2}$$
$$h^{(i)} = h^{(i-1)} - i\varepsilon_i V_h \qquad (7)$$

where i is the number of steps, ε_i is the adjusting factor of the ith step.

Substituting the corresponding values of V_{t_1}, V_{t_2} and V_h in Table 2 into eq. (7), we obtain:

$$t_1^{(i)} = t_1^{(i-1)} - 0.8435\, i\varepsilon_i$$
$$t_2^{(i)} = t_2^{(i-1)} - 0.5291\, i\varepsilon_i$$
$$h^{(i)} = h^{(i-1)} - 0.0925\, i\varepsilon_i$$

Assume $\varepsilon_1 = 2.5$, then point a will move to point b, where

$$t_1^{(1)} = 5.891 \text{ mm}, \quad t_2^{(1)} = 8.677 \text{ mm}, \quad h^{(1)} = 101.769 \text{ mm}$$

and $\delta_{max}^{(0)}$ will be transformed into $\delta_{max}^{(1)} = 0.5649$ mm, exceeding the allowable deflection. In other words, point b has been fallen into unfeasible region and is not allowed by constraint.

The second step (2), hereafter, would be to move point b back from unfeasible region with half of the front step so as to approach the constraint curved plane gradually. Now the iterative process will be proceeded in accordance with the following different formulae:

$$t_1^{(i+1)} = t_1^{(i)} + \tfrac{1}{2}\varepsilon_i V_{t_1}$$
$$t_2^{(i+1)} = t_2^{(i)} + \tfrac{1}{2}\varepsilon_i V_{t_2}$$
$$h^{(i+1)} = h^{(i)} + \tfrac{1}{2}\varepsilon_i V_h \qquad (8)$$

After calculation, point b will turn into point c, where the design variables become

$$t_1^{(2)} = 6.946 \text{ mm}, \quad t_2^{(2)} = 9.339 \text{ mm}, \quad h^{(2)} = 101.885 \text{ mm}$$

while the maximum deflection $\delta_{max}^{(2)} = 0.4957$ mm lies within the constraint tolerance band.

If not so, the step has to be half cut continuously and the process should be proceeded backward and forward until the convergency is attained.

Now the third step (3) would start from point c with sidewise movement toward feasible region in the equality weight plane. Since the weight gradient vector V and the normalized direction vector S are orthogonal each other, i.e.

$$\{V\}^T\{S\} = 0 \tag{9}$$

We may prefer

$$S = \begin{Bmatrix} S_{t_1} \\ S_{t_2} \\ S_h \end{Bmatrix} = \begin{Bmatrix} -V_h \\ -V_h \\ V_{t_1} + V_{t_2} \end{Bmatrix} = \begin{Bmatrix} -0.0925 \\ -0.0925 \\ 1.3726 \end{Bmatrix} \tag{10}$$

Therefore the eq. (7) can still be utilized only with the replacement of V by S. Assume $\varepsilon_3 = -6.5$, then point c will move to point d, where

$t_1^{(3)} = 6.345$ mm, $t_2^{(3)} = 8.738$ mm, $h^{(3)} = 110.807$ mm

and $\delta_{max}^{(3)} = 0.5579$ mm, exceeding the allowable deflection once more.

In order to revise it, the fourth step (4) would be executed according to eq. (8) but with the replacement of V by S as previously mentioned. Thus the point d will move back from unfeasible region with half of the front step, and finally fall into constraint tolerance band again named point e.

So far we have completed one typical cycle of both gradient and sidewise movement, and obtain two points c and e coincided with the constraint conditions. Take the mid-point f of distance c-e as a new starting point for next searching cycle. Hence an iterative process by repeating the step (1) to (4) would be executed continuously until the optimal design point has ultimately been searched out. The conditions restrained for optimization at that time are defined as follows:

$$\begin{aligned} \Delta t_1 &= | t_1^{(n)} - t_1^{(n-1)} | \leqslant 0.1 \\ \Delta t_2 &= | t_2^{(n)} - t_2^{(n-1)} | \leqslant 0.1 \\ \Delta h &= | h^{(n)} - h^{(n-1)} | \leqslant 0.5 \end{aligned} \tag{11}$$

where n is the last iterative step.

As for the definite problem in this paper, there are three gradient and two sidewise movements in all with the total amount of 11 steps. The final point l may be defined as the optimal case with the round values of the relevant parameters as follows:

$t_1^* = 6.5$ mm, $t_2^* = 9.0$ mm, $h^* = 105$ mm, $W^* = 110.5$ kg

The corresponding values on each step are listed in details as Table 3.

5. References

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Table 1 The initial values of structural parameters

Diameter of perforated plate	D	820 mm	Material		00Cr19Ni10
Height of surrounding shell	h	120 mm	Young's modulus	E	1.94×10^{6} kgf/cm^2
Thickness of upper plate	t_1	8 mm	Poisson's ratio	μ	0.3
Thickness of lower plate	t_2	10 mm	Specific weight	γ	7.83×10^{-6} kg/mm^3
Thickness of surrounding shell	t_3	10 mm	Pressure above plate	P_1	7 kgf/cm^2(Abs.)
Specification of short tube	dXt_4	27X2 mm	Pressure within plates	P_2	10 kgf/cm^2(Abs.)
Total length of stiffeners	l	9020 mm	Pressure below plate	P_3	13 kgf/cm^2(Abs.)
Rectangular cross section of stiffeners	$5t_1$Xb	40X5 mm	Pressure outside surrounding shell	P_4	13 kgf/cm^2(Abs.)

Remarks: 1. The values of t_1, t_2 and h will be changed later.

2. Since the temperature of pressure water is 53°C, the heat stress may be negligible.

3. The upper plate is welded with pressure vessel by virtue of 20 angle-plates, and the lower plate is free from that vessel by a small circumferential gap (see Fig. 1).

Table 2 The values of weight gradient vector

$\nabla W_{t_1} = \frac{\partial W}{\partial t_1}$	$\nabla W_{t_2} = \frac{\partial W}{\partial t_2}$	$\nabla W_h = \frac{\partial W}{\partial h}$	$\|\nabla W\| = \sqrt{\Sigma \nabla W_i^2}$ ($i=t_1, t_2, h$)	$V_{t_1} = \frac{\nabla W_{t_1}}{\|\nabla W\|}$	$V_{t_2} = \frac{\nabla W_{t_2}}{\|\nabla W\|}$	$V_h = \frac{\nabla W_h}{\|\nabla W\|}$
4.7372	2.9175	0.5157	5.6158	0.8435	0.5291	0.0925

Table 3 The calculating results on each step

Step		Orientation	t_1(mm)	t_2(mm)	h(mm)	ε	δ_{max}(mm)	W(kg)
Init. case		a	8.000	10.000	102.000		0.3734	120.214
G	(1)	a→b	5.891	8.677	101.769	2.50	0.5649	106.173
	(2)	b→c	6.946	9.339	101.885	1.25	0.4957	113.182
	(3)	c→d	6.345	8.738	110.807	-6.50	0.5579	113.182
S	(4)	d→e	6.946	9.039	106.346	-3.25	0.4935	113.182
	(5)	e→f	6.946	9.189	104.116		0.4880	113.182
G	(6)	f→g	6.102	8.660	104.023	1.00	0.5565	108.284
	(7)	g→h	6.524	8.925	104.069	0.50	0.5071	111.042
	(8)	h→i	6.385	8.786	106.128	-1.50	0.5285	111.042
S	(9)	i→j	6.454	8.855	105.098	-0.75	0.5023	111.042
	(10)	j→k	6.489	8.890	104.584		0.5157	111.042
G	(11)	k→l	6.405	8.837	104.575	0.10	0.4992	110.530

Remarks: G - Gradient movement, S - Sidewise movement

$(5) = \frac{((2)+(4))}{2}$, $(10) = \frac{((7)+(9))}{2}$

Step (11) is the optimal design case

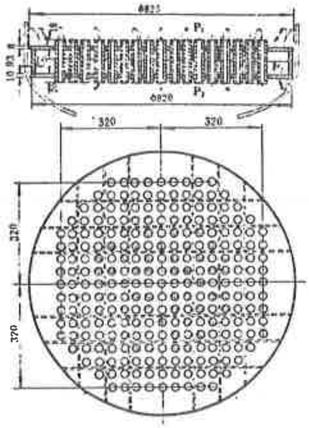

Fig. 1 Doubly-layered perforated plate

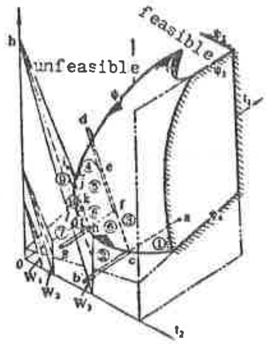

Fig. 2 3-D space with constrained surfaces