STATISTICAL ANALYSES OF EARTHQUAKE RESPONSE SPECTRA

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SUMMARY

This work was undertaken to develop statistically valid seismic response spectra for both horizontal and vertical ground motion. The first study made included 25 earthquake recording stations, of which 14 had records for all three components of motion, and was published as a joint work by the two groups making the study. This was adopted as a Regulatory Guide by the U.S. Atomic Energy Commission. Following that publication, additional studies of over 100 earthquake motion records from 56 stations were made by the authors. The results of the more recent studies indicate that some simplification, as well as a reduction in conservatism, could be made in the earlier recommendations.

The data presented give statistical parameters for median response spectra and log-normal standard deviations, from which response spectra for any probability of exceedance can be determined. The data also include studies of the effect of intensity of motion on spectral amplification factors, and other statistical parameters defining ground motion, such as the median and the standard deviation of (1) the ratio of maximum ground acceleration multiplied by maximum ground displacement divided by the square of the maximum ground velocity; (2) the ratios of maximum to minimum values of motion in the horizontal directions; and (3) the relative values of maximum ground displacement, velocity, and acceleration.

Considerable attention is focussed on the relative values of vertical to horizontal motion, and the implications on design of the simultaneous motions in various directions.

1. Introduction and Acknowledgment

Statistical studies were made of a number of earthquakes to gain an insight into the factors affecting the horizontal and vertical ground motions and resulting structural responses. The first study included 25 earthquake recording stations, of which 14 had records for all three components of motion, and was published as a joint work by the two groups making the study [1,2,3]. Subsequently, an additional study of earthquake motion records from 56 seismic events [4] was undertaken by the authors. The purpose of this paper is to summarize the most important of the observations arising out of the statistical studies conducted as a part of their two investigations [1,4]. These studies tend to suggest that the response spectra used for present seismic design are conservative. The statistical studies demonstrate the variability in observed effects and suggest that future studies include, in addition to consideration of acceleration levels, consideration of site dependence and distance from the source.

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2. Earthquake Considered

The earthquake events considered are tabulated in Table I. The numbers in parentheses following the earthquake designation indicate the number of sets of records used in each of the two studies. The first number indicates the number of sets of records used in Study 1 [1], and the second number of sets of records used in Study 2 [4]. Hereafter these studies will be designated as the First Study and the Second Study. It will be noted that the first set of numbers in Table I totals 14, indicating that 14 earthquakes were considered with two components of horizontal motion and one component of vertical motion being employed for each earthquake. The maximum ground acceleration for these 14 earthquakes varied from 0.016g to 0.718g in the vertical direction, and from 0.036g to 1.25g in the horizontal direction. In the Second Study, 56 sets of records were considered, again with two components of horizontal motion and one component of vertical motion. In this case the maximum ground acceleration varied from 0.004g to 0.709g in the vertical direction, and from 0.012g to 1.17g in the horizontal direction.

All of the earthquake events studied in the First Study were included in the Second Study. In the First Study, the basic data adopted were uncorrected strong motion earthquake accelerograms processed by the Earthquake Engineering Research Laboratory, California Institute of Technology [5]. The authors balanced these acceleration data using a parabolic baseline correction and segmental adjustment as described in Ref. [1]. In the Second Study, corrected earthquake accelerogram data from the same source were employed as well as response spectra based upon the corrected accelerograms [6,7]. The basis of the accelerogram balancing technique employed by the CalTech group (employing a filtering technique) is considerably different from that employed by the authors in Study 1, and as a result the peak values are slightly different. The various balancing techniques normally result in slight changes in peak ground acceleration, small changes in peak ground velocity, but significant changes in the peak ground displacement. The authors' technique of balancing led to larger peak displacements than those of the CalTech group. It is rare in practice that one is interested in the displacement affected portion of the response spectrum; for this reason, and because of the variations arising from different balancing techniques, only limited attention is given herein to statistical variations of displacement parameters.

One further comment is necessary with regard to Table I. All except one of the earthquake events are West Coast seismic events. Moreover, there is a heavy biasing toward the San Fernando earthquake in the Second Study, some 18 sets of records being used in that study, a higher percentage of the total than in the First Study. These points are made because the selection of the earthquakes employed in statistical studies can have a significant influence on the observed results.

3. Grouping of Data

In the First Study, an attempt was made to treat the data in terms of geologic conditions and intensity of motion. Valid statistical inferences about the nature of these differences could not be made because of the small number of samples involved. It was realized that the site descriptions for the various earthquakes such as rock, alluvium, or otherwise are not completely dependable due to a lack of satisfactory information about the geologic conditions at most of the sites where strong motion instruments have been located. Nevertheless, the trends reported in Ref. [1] seem to be borne out by more recent studies [8], wherein much greater attention has been given the site characteristics.

In the Second Study, no particular attention was paid to site characteristics but instead the grouping method depended upon the acceleration level that was employed. The first of the two grouping methods employed in the Second Study consisted of placing each component of the set of records in an acceleration group (based on peak acceleration level), slightly different grouping being used for the horizontal and vertical components reflecting the fact that the vertical ground accelerations are normally lower than the horizontal ground accelerations. A second grouping method was used also in the Second Study for investigating ground motion parameters only. In this grouping, if the maximum of the two horizontal ground accelerations fell within a particular acceleration domain, then all three components (two horizontal and one vertical) were included in the statistical computation in that domain (meaning the comparable vertical domain as well).

4. Results of Ground Motion Studies

In the First Study, a normal probability distribution was used, while in the Second Study, both normal and log-normal distributions were employed. The studies indicated that a log-normal distribution is more appropriate; all of the parameters summarized in this paper for the Second Study are for log-normal probability distributions. Hence the mean and the median values are not the same, and the 50 percentile values given for the Second Study are the median values of the distribution.

From the statistical studies and from other studies of seismic response it is the authors' recommendation that response spectra for use in design be constructed through the use of controlling ground motion parameters (peak acceleration, peak velocity, and peak displacement) with individual amplification factors for each of these. The 50 percentile or median values of the controlling peak ground motion parameters should be employed in the procedure. The conservatism desired in the response spectrum should be included in the amplification factors that can then be chosen to be representative of whatever probability level is desired, for the appropriate levels of damping and/or ductility. For these reasons then, the summaries of the ground motion data presented in Table II are restricted to 50 percentile values. The data at the top of Table II arise from the studies conducted in Study 1, and the data at the bottom from Study 2.

In the case of the v/a ratio, the results of the First Study indicate that the ratios for rock are considerably lower than for alluvium (solid firm ground) and the same observations have been borne out in a study by Seed, et al. [8]. There is such a limited amount of data for rock available that until further studies of this subject are made it is suggested that these values be treated with caution. The vertical v/a values are noted to be slightly less than those for the horizontal records. It should be noted that, in Study 1, if one of the extreme ratios was excluded for the alluvium and rock, the v/a value would change in the first line from 45 for horizontal motion to 48. And, similarly, from 37 to 40 for vertical motion.

The results of the Second Study show values slightly lower than those of the First Study, and values are shown there for both grouping methods 1 and 2, as described earlier. The values in the Second Study show a decrease for acceleration greater than 0.20g.

As noted earlier, all of the data contained in the First Study are contained in the Second Study, and these statistical values then clearly indicate the trends that occur as a result of the number of earthquakes included, and to some degree the record balancing techniques employed, site characteristics, etc. Clearly then, these factors need to be taken into consideration in evaluating such statistical data. Further, it should be realized that in both Studies 1 and 2 the number of record sets employed for vertical motion is almost precisely 1/2 the number employed for horizontal motion; thus the statistical variations for vertical motion are based on fewer observations.

In the absence of other more definitive data, for firm ground the authors recommend a v/a value for horizontal motion of 48 in./sec/g. A slightly lower value may be applicable for the vertical spectra, although it is our belief that this needs further study before definitive recommendations can be offered.

In order that the 50 percentile values not be misleading, an indication of the spread in the values can be gained by noting the values of the median and the median plus one standard deviation value. These were calculated. For example, in the Second Study, for accelerations falling in the range of 0.12 to 0.20g, the median plus one standard deviation value for both groupings of v/a was about 68 in./sec/g as compared to the median value of 45 shown in Table II. For acceleration levels greater than 0.2g, the median plus one standard deviation value was 57 as contrasted to 37 to 40 shown in Table II. These numbers indicate a geometric standard deviation of a factor of about 1/2 the median.

The ad/v^2 values are shown in the next column of Table II, the variation being from 5.3 to 5.7 for horizontal motion and considerably greater for vertical motion in the case of Study 1. In the Second Study, the values for horizontal motion were slightly lower, and the values for vertical motion were considerably lower. The variations are quite large also. For example, for values of ad/v^2 for acceleration levels greater than 0.2g in the second grouping, a value of 8 is found for the median plus one standard deviation value as contrasted to 4.4 in Table II. This wide variance would be expected because of the variations in the displacement values, which had received comment previously. On the basis of the study of the data to date, the authors recommend that a value of ad/v^2 of 5 or 6 be used for both horizontal and vertical ground motions.

The ratios of vertical to horizontal acceleration and velocity are presented in the next columns in Table II. In the First Study, these values are noted to range from 0.4 to 0.7, with the values centering around 0.6. In the Second Study, the values are noted to be somewhat less ranging from 0.4 to less than 0.6 for both acceleration and velocity values. The variation in these ratios is of interest also. For accelerations greater than 0.2g in the Second Study, the median plus one standard deviation value is 0.53 in contrast to the median value of 0.38 (Grouping 1). In general, the median plus one standard deviation values appear to be about 50 percent greater than the median values. These studies suggest that a value of even 2/3 for a_v/a_h or v_v/v_h may be slightly high and that a more conservative value may be in the range of 0.5 to 0.5. Nonetheless, until further studies bear out some of these trends definitely for other than West Coast earthquakes, it is our recommendation that a value of 2/3 for these ratios be employed to account for variations slightly greater than the median value and to account for the higher amplifications that often are found for low intensity earthquakes.

Often there is interest shown in the ratio of the minimum to maximum horizontal accelerations and velocities, and these are shown in the last two columns of Table II. It will be noted here that these ratios of accelerations and velocities are in the range of 0.7 to 0.8 and the value of 0.8 would appear to be a good median value. However, the median plus one standard deviation values approach 0.95 to 1.0.

5. Response Amplification Ratios

There are presented in Tables III and IV the results of statistical studies of horizontal and vertical spectrum amplification factors. In the Second Study, values were not computed directly for 0.5 percent of critical damping, although they can be determined quite easily from interpolation on a semi-log plot. It will be observed, in comparing the amplification values from the First and Second Studies, that the acceleration amplification values are quite similar, as would be expected. The velocity amplification values differed somewhat, and the displacement amplification values differed significantly, for reasons discussed earlier in this paper. Values are tabulated for both 50 percentile (median) and 84.1 percentile (median plus one standard deviation). At present the authors feel that the amplification values reported in Ref. [4], the second set of values shown in Tables III and IV, are adequately conservative values for general use. We do not mean to imply, however, that only the 50 percentile or 84.1 percentile values are necessarily those to be employed in constructing spectra. Indeed, the appropriate probability level must be selected in the light of the structural requirements. Other percentile values can be obtained easily from those given.

Comparison of the vertical and horizontal amplification values will show that they are quite similar, as pointed out in the studies reported [1,2]. It is recommended that the same amplification value be used in constructing vertical response spectra as is used for horizontal response spectra.

6. Response Spectra

Actual construction of the response spectrum for a particular case will depend upon the location and nature of the site, the nature of the structure or facility under consideration, and other factors. In the absence of other information, the authors' general recommendation is to construct the horizontal response spectrum based upon a v/a ground motion ratio of 48 in./sec/g with the displacement value, if required, to be determined from an ad/v^2 value of 6.

The amplification values in general should be chosen for the 84.1 percentile level, unless a greater or lesser degree of conservatism can be justified for the particular case.

In general, the elastic response spectrum is constructed by selecting the maximum ground motion parameters (acceleration, velocity, and displacement) and multiplying these by the amplification factors in Tables III (or IV) to obtain the portion of the spectrum from low frequencies to a frequency of 8 hertz. For high frequencies, greater than 33 hertz, the response spectrum is obtained by multiplying the maximum ground acceleration by an amplification factor of unity. Between 8 and 33 hertz, the response spectrum is obtained by drawing a straight line on the logarithmic plot of response vs. frequency between the values at those two frequencies.

In general, however, the vertical response spectrum can be taken as 2/3 the horizontal response spectrum over the entire range of frequencies.

7. Combination of Motions in Various Directions

Usually the stresses or strains at a particular point are affected primarily by the earthquake motions in only one direction and the second direction has little if any influence. However, this is not always the case, and it is certainly not the case if one has a simple square building supported on four columns where, in general, the stress in a corner column is affected equally by the earthquakes in the two horizontal directions and may be affected also by the vertical earthquake motions. Although this is generally not included in many design codes, more recent codes and specifications require that effects from various directions be considered when it appears that there would be additive effects.

For those parts of structures or components that are affected by motions from various directions, in general, the net response may be computed by either one of two methods. The first method involves computing the responses for each of the directions independently and then taking the square root of the sums of the squares of the resulting stresses in a particular direction at a particular point as the combined response. Alternatively, one can use the procedure of taking the seismic forces corresponding to 100 percent of the motion in one direction combined with 40 percent of the motions in the other orthogonal directions, and then adding the absolute values of the effects due to these forces to obtain the maximum resultant stresses or effects in a member or at a point in a particular direction. In general, this alternative method is slightly conservative for most cases and is quite adequate. Both techniques reflect the fact that the excitations in the three directions are not necessarily of the same magnitude, and their maxima do not occur at the same time.

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Table I. Earthquake Events and Accelerograms Considered

Long Beach Earthquake, 3-10-33, 1754 PST (0-1)
Southern California Earthquake, 10-2-33, 0110 PST (0-1)
Lower California Earthquake, 12-30-34, 0552 PST (0-1)
Helena, Montana Earthquake, 10-31-35, 1138 MST (0-1)
1st Northwest California Earthquake, 9-11-38, 2210 PST (0-1)
Imperial Valley Earthquake, 5-18-40, 2037 PST (1-1)
2nd Northwest California Earthquake, 2-9-41, 0145 PST (0-1)
Western Washington Earthquake, 4-13-49, 1156 PST (0-2)
Northwest California Earthquake, 10-7-51, 2011 PST (1-1)
Kern County, California Earthquake, 7-21-52, 0453 PST (2-5)
Northern California Earthquake, 9-22-52, 0441 PDT (0-1)
Wheeler Ridge, California Earthquake, 1-12-54, 1534 PST (0-1)
Eureka Earthquake, 12-21-54, 1156 PST (2-2)
San Jose Earthquake, 9-4-55, 1801 PST (0-1)
El Alamo, Baja, California Earthquake, 2-9-56, 0633 PST (1-1)
El Alamo, Baja, California Earthquake, 2-9-56, 0725 PST (0-1)
San Francisco Earthquake, 3-22-57, 1144 PST (1-5)
Hollister Earthquake, 4-8-61, 2323 PST (1-1)
Puget Sound, Washington Earthquake, 4-29-65, 0728 PST (0-1)
Parkfield, California Earthquake, 6-27-66, 2026 PST (0-5)
Second Northern California Earthquake, 12-10-67, 0406 PST (0-1)
Borrego Mountain Earthquake, 4-8-68, 1830 PST (1-3)
San Fernando Earthquake, 2-9-71, 0600 PST (4-18)

Table II. Summary of Median Values from Statistical Studies of Ground Motion Parameters

Reference [1]	No. Horiz. Records	v/a, in./sec/g H	v/a, in./sec/g V	ad/v^2 H	ad/v^2 V	$\frac{a_v}{a_h}$	$\frac{v_v}{v_h}$	$\frac{a_{h\,max}}{a_{h\,max}}$	$\frac{v_{h\,min}}{v_{h\,max}}$
Alluvium and Rock	28	45	37	5.6	10.7	0.53	-	-	-
Alluvium	22	52	43	5.7	10.0	0.53	-	-	-
Rock	6	22	18	5.4	13.0	0.54	-	-	-
Alluvium and Rock > 0.1g*	18	42	33	5.7	12.4	0.65	-	-	-
Alluvium > 0.1g*	14	47	37	5.9	12.0	0.72	-	-	-
Alluvium < 0.1g*	8	60	47	5.3	8.4	0.40	-	-	-

Reference [4]	No. of Events	v/a H Gr I	v/a H Gr II	v/a V	ad/v^2 H Gr I	ad/v^2 H Gr II	ad/v^2 V	$\frac{a_v}{a_h}$	$\frac{v_v}{v_h}$	$\frac{a_{h\,max}}{a_{h\,max}}$	$\frac{v_{h\,min}}{v_{h\,max}}$
$a_h < 0.05$*	10	-	35	45	-	6.2	6.4	0.47	0.57	0.86	0.76
$0.05 \leq a_h \leq 0.12$*	15	41	40	36	4.0	4.3	7.1	0.42	0.37	0.79	0.74
$0.12 \leq a_h \leq 0.20$*	18	45	45	41	3.9	4.0	5.1	0.40	0.35	0.76	0.71
> 0.20*	13	29	34	29	5.1	4.4	6.4	0.38	0.33	0.76	0.76
> 0.12*	31	37	40	35	4.4	4.2	5.6	0.39	0.34	0.76	0.73
All data*	56	-	39	37	-	4.5	6.1	0.41	0.38	0.78	0.74

* For vertical motions, the acceleration levels and no. of events considered are approximately one half those for horizontal motions.

Table III. Horizontal Amplification Values for Design

Quantity	Probability Level, as a percentage	Damping, percentage of critical 0.5	2.0	5.0	10.0
Reference [1 and 3]					
D	50	1.97	1.68	1.40	1.15
	84.1	2.99	2.51	2.04	1.62
V	50	2.58	2.06	1.66	1.34
	84.1	3.81	2.98	2.32	1.81
A	50	3.67	2.76	2.11	1.65
	84.1	5.12	3.65	2.67	2.01
Reference [4]					
D	50	-	2.23	1.91	1.61
	84.1	-	3.15	2.60	2.10
V	50	-	1.75	1.38	1.11
	84.1	-	2.65	2.02	1.56
A	50	-	2.66	2.04	1.65
	84.1	-	3.70	2.70	2.10

Table IV. Vertical Amplification Values for Design

Quantity	Probability Level, as a percentage	Damping. percentage of critical 0.5	2.0	5.0	10.0
Reference [1 and 3]					
D	50	1.86	1.65	1.40	1.16
	84.1	2.78	2.41	2.01	1.62
V	50	2.52	1.97	1.51	1.17
	84.1	3.81	2.91	2.18	1.64
A	50	4.02	2.80	2.05	1.59
	84.1	6.15	4.13	2.82	2.08
Reference [4]					
D	50	-	2.27	1.92	1.61
	84.1	-	3.17	2.60	2.13
V	50	-	1.72	1.34	1.06
	84.1	-	2.63	1.96	1.43
A	50	-	2.76	2.08	1.63
	84.1	-	3.86	2.81	2.13