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Some Practical Aspects of Stability of Cylindrical Shells

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Abstract

With the help of the appropriate calculation and measurement procedures we managed to develop such a construction of doublewalled heatexchanger which - according to the conducted though here not detailed economic calculations - results in considerable savings. Because of the above mentioned reason and its technical solution it proved to be suitable for being patented.

As an auxiliary result we acquired practical experience completing the stability theory.

1. Introduction

One essential task of the stability examination is the definition of the maximal loading at which the structure does not loose its stability yet for sure. However at the mechanical examination of the doublewalled heatexchanger the reverse task was needed. To define the minimal loading at which a cylindrical thinwalled vessel looses its stability already for sure. The task became more interesting because the loosing of stability had to be general and the material was not allowed to get into plastic state anywhere. The theoretical part of the stability examinations in the technical literature is rather straight, however because of the difficulties in the solution approximate relations ara used in practice, which - with no exception - depart towards safety. In our case it meant that the vessels designed with the usual equations did not loose their stability. Examining the task more concretely,according to the operating circumstances the developing of a structure preventing not the loosing of stability but its harmful consequences - for example with the help of a support - seemed to be evident. The advantage of this - besides the saving of materials as a result of the thinner inner vessel - is that after removing the support the incrustation scaled onto the wall of the vessel can be cleared away, by which the heat exchange so the utilization of energy can be improved significantly. The final thought is just one step from here: to limit the strain following the loosing of stability with the suitably chosen support so that the vessel should remain within the elastic range, still the incrustation chip off.

On the basis of these a more general examination of stability questions of cylindrical shells was needed. First we looked over the approximate equations available in the technical literature, then we worked out a method for the calculation of the inner vessel loaded for outer pressure of the doublewalled heatexchanger considering the conditions mentioned in the exposition.

On the basis of these we constructed the heatexchanger and then made measurements on the ready prototype. Based on the measurement experiences the final construction was completed of which standardized production and patent process has begun.

2. Theoretical examination

The equations worked out by the theory of stability for cylindrical vessels is seldom used because of its complexity, instead simplifying equations have spread which mean a relatively rough approaching and bend towards safety. This suits the original aim - though in some cases results in considerable oversizedness - however the task we examine needs the exact connections.

These relations besides the radial loading count with the axial forces as well, and the shape of the vessel at the loosing of stability. Only in the case of unlimited length of the vessel is the number of the waves in directions of axis and of tangent one. Otherwise the loading which causes the loosing of stability is a definite function of relative numbers formed from the length, diameter and thickness of the wall.

According to the technical literature [1],[2] stability task of long, cylindrical, thin shells can be examined on the basis of the

$$\frac{\alpha^4}{R^2}(1-\mu^2)+\frac{h^2}{12R^2}\left[R^2(\alpha^2+\beta^2)^4-2\beta^6+\frac{\beta^4}{R^2}\right]+(1-\mu^2)\frac{pR}{Eh}\left\{-\beta^2(\alpha^2+\beta^2)^2+\frac{1}{R^2}\left[\beta^4+(2+\mu)\alpha^2\beta^2-\mu\alpha^4\right]\right\}-\frac{F(1-\mu^2)}{2\pi REh}\alpha^2\left[(\alpha^2+\beta^2)^2+\frac{\beta^2}{R^2}\right]=0,\quad \alpha=\frac{n\pi}{l},\quad \beta=\frac{m}{R},\qquad (1)$$

equation, where p and F stand for the outer pressure and axial force R, h and l mean the geometrical sizes, E and μ the characteristics of the material, n and m stand for the number of longitudinal and transversal waves arising on the shell's surface from the loosing of stability.

If $F = 0$ and $l > 8R$, then at $n = 1$, $m = 2$

$$p_{kr}=\frac{Eh^3}{4(1-\mu^2)R^3}\qquad (2)$$

approaching relation derives. We get the same result at $n > 1$ as well. If $F = 0$ and $l < 8R$, then applying

$$m_{kr} = \frac{\pi \cdot R}{l} \sqrt{\frac{R}{h}} \sqrt[4]{36(1-\mu^2)} \tag{3}$$

the approaching relation is

$$p_{kr} = \frac{2{,}6E\left(\frac{h}{2R}\right)^{\frac{5}{2}}}{\frac{l}{2R} - 0{,}45\left(\frac{h}{2R}\right)^{\frac{1}{2}}} . \tag{4}$$

If $p = 0$, then

$$F_{kr} = \frac{2\pi E h^2}{[3(1-\mu^2)]^{\frac{1}{2}}} \tag{5}$$

is the approaching value of the critical axial force. If $F \neq 0$ and $p \neq 0$, then /1/ relation must be applied. However we have to consider that in the examined real case there is a concrete connection between F and p determined by the geometrical relations. On the other hand the axial force - depending on its sign - can influence the relations favourably as well.

3. Strength calculation

The operating conditions of the doublewalled heatexchanger are such, that the p_2 inner pressure periodically, for a short time /daily for a few minutes/ decreases to p_2^* where $p_2^* << p_2$. It is praticable to dimension the inner vessel so, that as a result of $p_1 - p_2$ pressure general loosing of stability should occur. The big strain following this would be limited by the suitably dimensioned support, so that the steel vessel itself would remain within the elastic range but the incrustation significantly brittler would chip off the inner wall.

According to this the steps of dimensioning are the following. First the incrustation's characteristics of the material have to be defined. After this has to be dimensioned on the one hand the inner vessel so that the loosing of stability should occur, on the other hand the support so that the strain should be limited. The limit here is from one side the vessel, from the other side the elasticity of the incrustation: the incrustation should chip off the vessel remaining within the elastic limit from as big part as possible. A further task is the strength calculation of the support.

The thickness of wall by which the loosing of stability occurs can be determined to the given pressures and geometrical relations with equations similar to that of the previous chapters. After this e.g. considering the equivalent detail of the vessel as a curved bar knowing the sizes of the support the strain relations and the domain of the incrustation's separation can be determined.

The calculation can be considerably simplified and it has slight influence on the accuracy if we follow the behaviour of the curved bar with geometrical consideration. With the procedure outlined above we managed to design a structure adequate for the set aim.

4. Measurements

Our measurements cover three territories: the definition of the incrustation's characteristics of the material, how the material behaviour of steel is influenced by the incrustation incrusted on it, the measurements of loosing the stability made on the experimental vessels and the material tasting of the strained vessel.

The first group of question was answered by bend-test. Because of the character of the experiment and of the incrustation the dispersion of the results was big, but we got the required characteristics and we could state, that the incrustation does not influence measurably the characteristics of steel.

The measurements about the loosing of stability completed on the experimental vessels proved the correctness of the primary assumption the support limits the strain but does not prevent the incrustation's chipping off.

The subsequent material test served as appropriate control.

References

[1] Timoshenko, S.: Theory of Elastic Stability. Mc Graw Hill, New York, 1961.

[2] Ponomarjov, Sz.D.: Strength Calculation in Machine Design. Vol.7. Müszaki Könyvkiadó, Budapest, 1966.