INSTABILITY ANALYSIS OF ELBOWS IN THE PLASTIC RANGE

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SUMMARY

The elbow components of a LMFBR piping system have a high degree of flexibility in comparison with that of the straight components. This flexibility may result in severe strain concentrations and instability of the elbow. Thus, it is not surprising that the literature is presently replete with investigations devoted to the important problem of the structural behavior of elbows. However, the overwhelming majority of the investigations is devoted to the case of elastic behavior, and there is a relative paucity of experimental and analytical results for the important problem of the inelastic instability of elbows made from a strain hardening material.

A complete instability analysis of the plastic behavior of an elbow is very expensive, as both geometric and material nonlinearities must be included, and the response variables (stresses, displacements, etc.) vary in both the axial and circumferential directions. Therefore, it is desirable to use a simplified method of analysis that is economical, but which leads to results that are reasonably satisfactory for engineering purposes.

This work presents some basic considerations underlying the instability in the plastic range of typical LMFBR elbows (radius-to-thickness ratio $r/t = 20\text{-}50$), and the impact of these considerations on the practical aspects of solution by computer methods. The elbows are made from a strain hardening material and they are loaded by in-plane closing or opening bending moments. A simplified method of computer analysis is used and the particular type of instability considered is that of geometric buckling into a mode that is characterized by significant ovalization of the cross section and by a relatively gradual variation in shape along the length of the elbow. Emphasis is placed on the effects of including geometric nonlinearities (large displacement terms) in the inelastic (time-independent) analysis. Numerical results are presented for a typical LMFBR elbow.

A general discussion of the basic concepts and definition of the instability of structures made from strain hardening or from elastic materials is given. Such a discussion is needed to facilitate a complete understanding of the present numerical results and to avoid possible misinterpretations in a comparison of these and previous results.

The major conclusions of this investigation are as follows: (1) Geometric nonlinear effects are significant, (2) The inclusion of such effects in an analysis of an elbow subjected to either a closing or an opening moment reveals that the buckling load is lower for the case of a closing moment. The buckling load for this case corresponds to a maximum point (zero slope) in a typical load-displacement curve, and (3) The omission of geometric nonlinear effects in the inelastic analysis leads to a load-displacement curve in which the load monotonically increases with displacement. Hence, such an analysis does not provide a definitive buckling load for an elbow made from a strain hardening material.

1. INTRODUCTION

The pipelines used in LMFBR (Liquid Metal Fast Breeder Reactor) plants are of major importance with regard to considerations of safety and cost. Therefore, in recent years, increasing attention has been focused on the methods of structural analysis and design of such pipelines. The major stresses experienced by the pipelines are due to seismic and dead weight loads, and thermal expansions. Due to the thermal expansions, forces are induced on pressure vessels and supporting structures that connect to the pipelines. It is highly desirable to keep these forces as small as possible or, equivalently, to allow more of the free thermal expansion of the straight portions of the piping system to take place. This expansion is accommodated by the curved pipe components (elbows), which, for thin-walled (r/t = 20-50) systems, have a high degree of flexibility in comparison with that of the straight components. Thus, it is not surprising that the literature is presently replete with investigations devoted to the important problem of the structural behavior of curved pipes or elbows.† However, the overwhelming majority of these investigations is devoted to the case of elastic behavior, and there is a relative paucity of experimental and analytical results for the important problem of the inelastic instability of elbows made from a strain hardening material.

Experimental instability results are given by Bolt and Greenstreet [5] and by Vrillon, Montfort, and Befre [6]. Reference 5 presents load-displacement curves for 14 commercial 6-in. carbon steel elbows and one 6-in. stainless steel elbow. The elbows are subjected to either in-plane or out-of-plane bending moments, with or without internal pressure. Unfortunately, only one of the elbows tested in [5] is made from a material that is typical of the type that is used in LMFBR main piping components. The two test results reported in [6] are for elbows made from a stainless steel material. One test is for the case of an in-plane closing moment, and the other is for an opening moment, with the buckling load being about 25% lower in the former case. Very little information is given in [6] that would help shed light on the basic nature of the instability phenomenon of elbows. Such information is also useful in the selection of methods to be employed in the instability analysis of elbows.

Mello and Griffin [7] present an analysis of the in-plane bending of elbows made from strain hardening materials. The analysis is based on the use of the MARC special pipe-bend element. Geometric nonlinear (i.e. large displacement) effects were not included in the analysis. Load-displacement curves are given in [7] for a variety of elbow geometries and materials. These curves show that the load increases monotonically with the displacement. Plastic (material) collapse loads were arbitrarily defined according to a number of methods that are presently in use. Due to the omission of geometric nonlinear effects, the analysis presented in [7] could not address the possibility of geometric buckling taking place at a well defined buckling load.

In view of the above discussion, it seems desirable to perform a more complete analysis of the instability of elbows in the plastic region. Such an analysis is presented herein for a typical LMFBR elbow made from a strain hardening material. Specific attention is devoted to the effects of including geometric nonlinearities in the nonlinear analysis of the inelastic (time-independent) behavior of elbows under in-plane closing and opening bending

† It is not the intention herein to present an extensive list of references, or to review the literature, as this has already been done in [1-4] and elsewhere.

moments. To keep the analysis tractable, it was decided to use the MARC special pipe-bend element (library element 17) in the present investigation. It is basically an axisymmetric isoparametric shell element that has been modified through the addition of "beam-type" deformation modes. A description of the theoretical basis and properties of the MARC library element 17 is presented in [8, 9].

It is noted that the US-ERDA is sponsoring the testing of full size elbows and other piping components in the Multi-Load Test Facility (MLTF) at the Westinghouse Advanced Reactors Division. The objectives of these tests are to provide a better understanding of the complex behavior of these piping components and to provide experimental data to be used in an assessment of current design and analysis methods. The work reported herein was performed for use in the design of the MLTF test fixture and in the selection and location of the instrumentation. It will also be used in a comparison with the MLTF test results.

This paper is divided into a number of sections. The remainder of the current section contains a description of the specific elbow configuration considered herein, and it establishes the notation. Section 2 presents a general discussion of the basic concepts and definition of the instability of structures made from strain hardening or from elastic materials. The rationale behind the method of analysis employed in the present work is given in Section 3 and results obtained therefrom are found in Section 4. Section 5 concludes the investigation with a summary of major conclusions and observations.

1.1 Specific Elbow Configuration

The specific elbow configuration analyzed herein is the curved portion of the pipe structure shown in Fig. 1, which also shows the finite element model to be discussed later. This elbow is representative of a typical LMFBR elbow and it is similar to one that will be tested in the Multi-Load Test Facility. The straight and elbow portions of the structure are of the same uniform thickness t, mean pipe radius r, and material properties E = Young's modulus and ν = Poisson's ratio. The material is 304 Stainless Steel at room temperature. The multilinear representation of the average properties stress-strain curve employed herein is given in Fig. 5 of [7]. Figure 1 shows that one end of the elbow structure is fixed, and the other end is subjected to an in-plane bending moment M. Numerical results are presented for the case in which M is either a closing or an opening moment.

2. INSTABILITY

To facilitate an understanding of the present results, and to interpret them properly relative to Code design limits on buckling and instability, it is necessary to define the instability phenomenon with greater precision than that offered in the ASME Code.

2.1 Definition of Instability

For a general structure made either from an elastic or from a strain hardening material, it seems reasonable to define instability as being the phenomenon in which a small increase in load causes a disproportionately large increase in deformation. The load at which the instability takes place is called the critical load. From this definition of instability it follows that load and deformation are not proportional and, hence, instability is intrinsically a nonlinear phenomenon. In general, there are two types of nonlinearities; namely, material nonlinearity and geometric nonlinearity. Material nonlinearity obviously accounts for the nonlinear relationship between stress and strain at a generic point of the structure. Geometric nonlinearity accounts for geometric distortions of the structure, primarily in the form of rotations, in the sense that such distortions must be accounted for in equilibrium considerations of the structure. For the special case of linear elastic behavior, instability is clearly due to the effects of geometric nonlinearities. For the more general case of inelastic behavior of a structure made from a strain hardening material, both geometric and material nonlinearities interact to precipitate instability. Therefore, in the analysis of such a structure, it is absolutely necessary to include geometric nonlinear effects if the possibility of instability of the structure is to be investigated†, as will be illustrated herein for the particular problem of an elbow subjected to a pure in-plane bending moment.

2.2 Analysis of Instability

The critical load at which the above instability phenomenon takes place is usually determined from either (1) an eigenvalue analysis, or (2) a geometrically nonlinear boundary-value analysis. It is convenient to illustrate the type of structural behavior represented by each of these analysis methods in a so-called load-displacement curve in which the load P is plotted against an appropriate displacement.

The eigenvalue method of analysis determines the load $P = P_{bb}$ at which the load-displacement curve emanating from the origin is intersected by another curve that represents a different equilibrium configuration (see Fig. 2a). The point of intersection of these curves is called the "bifurcation point" and it signifies the existence of two infinitesimally close equilibrium configurations at the same value of the load $P = P_{bb}$, which is called the "bifurcation buckling load". The two equilibrium configurations corresponding to this load are orthogonal in a certain sense [10]. Actually, the bifurcation buckling (eigenvalue) method of analysis is a mathematical abstraction or idealization of the real instability behavior of structures, and it is used primarily for reasons of economy (see the excellent book by Brush and Almroth [10]).

The real instability behavior of most structures is more properly represented by the solution obtained from a geometrically nonlinear boundary-value analysis. In this case, the load-displacement curve emanating from the origin is not intersected by another curve. The load-displacement behavior is given by a smooth curve whose slope (for values of P that are not small) will continually decrease with increasing load until a relative maximum is reached at a point ("limit point") where the slope vanishes (see Fig. 2b). The stiffness of a structure is obviously related to the slope of its representative load-displacement curve. Therefore, under steady increasing load, the structure loses its stiffness (and hence its stability) at the limit point, as it seeks to move ("snap-through") to a possibly distant stable state of equilibrium that may or may not exist. As an alternate way of describing this instability behavior, imagine that an attempt is made to apply a load larger than that required (P_b in Fig. 2b) for equilibrium of the structure at the limit point. This attempt would result in an unbalanced force, and an attendant acceleration, which would cause the structure to move. Such a movement involves large displacements and a considerable exchange of energy. It can be quite violent for load-controlled problems. This type of instability is described by a number of names in the literature; to wit, "snapping", "limit point buckling", "(geometric) collapse", and "nonlinear buckling". It is called "nonlinear buckling" here to dis-

† This statement doesn't hold for structures made from elastic, perfectly-plastic materials. The present work is not concerned with such structures.

tinguish it from bifurcation buckling[†], or it will simply be referred to as "buckling", for brevity, when such a distinction is not being made. Thus, the structure that is being considered in a geometrically nonlinear boundary-value analysis is said to buckle at the limit point where the load-displacement curve has its first relative maximum. The load corresponding to the limit point is called the buckling load P_b. The corresponding buckling mode is characterized by the presence of a dominant component that grows very rapidly as the buckling load is approached. This component is present at all load levels, although it is generally not dominant at low load levels. In contrast, it is noted that bifurcation buckling problems are characterized by the development of a new mode of deformation (at the bifurcation buckling load) that it is not present in the prebuckled shape of the structure.

The present work is based on the use of geometrically nonlinear boundary-value analysis to determine the buckling load M_b appropriate to the inelastic behavior of the specific elbow structure depicted in Fig. 1. This load will be compared with plastic (material) collapse loads that are arbitrarily defined according to a number of methods that are presently in use [7] such as, for example, the ASME Code method.

3. METHOD OF ANALYSIS

A complete buckling analysis of the plastic behavior of an elbow is very expensive, as both geometric and material nonlinearities must be included, and the response variables (stresses, displacements, etc.) vary in both the axial and circumferential (hoop) directions. Therefore, it is desirable to use a simplified method of analysis in the present investigation that is economical, but which leads to results that are reasonably satisfactory for engineering purposes. The MARC pipe-bend element (library element 17) appears to be well suited for this purpose. It is noted that this element cannot account for the axial variations of the response variables or for the axial variation of the ovalization of the cross section of the elbow. However, this is not a serious limitation in view of the nature of the buckling mode that is expected for the critical case of an elbow under an in-plane closing moment, and which is discussed below. Physical reasoning as well as the results of [6] indicate that the buckling load is lower for the case of a closing moment as opposed to an opening one. From a practical point of view, a closing moment is just as likely to be acting on the elbow as is an opening moment; thus, the loading condition of a closing moment is critical for an elbow under in-plane bending. The expected buckling mode for the case of a closing moment is discussed next.

As is well known, the magnitude and spatial variation of the stresses in an elbow undergoing pure in-plane bending due to an applied moment M are different from those experienced by the corresponding straight pipe, with the same cross sectional dimensions as those of the elbow and subjected to the same bending moment M [4]. This difference in behavior is associated with the ovalization of the cross section of the elbow. This phenomenon is illustrated in Fig. 3, which shows the elbow loaded by an in-plane closing moment. Due to the axial curvature, the axial stress in an arbitrary axial fiber will have a component (resultant) p directed towards the centroidal axis of bending C-C (which passes through the crowns) of the cross section, i.e., p acts in a plane that is parallel to the plane of the bend. The sketch shows two such fibers; those at the extrados and the intrados, and the attendant

[†]Thus, it is to be understood that the terms "buckling" (Fig. 2b) and the previously discussed "bifurcation buckling" (Fig. 2a) correspond to different types of instability analyses.

components p. These components cause the elbow to deform into an oval shape (Brazier effect), as is indicated by the dotted line in Fig. 3b. The ovalization of the cross section results in a reduction of the cross sectional moment of inertia I. Therefore, as the moment is further increased, there will be a proportionately larger reduction in the axial curvature, $\kappa = 1/\rho$, due to geometric nonlinear effects, which in turn reduces I even more, and so on. Due to the reduction of bending stiffness with increasing M, the slope of the M-κ curve will continually decrease until a maximum value is reached ($dM/d\kappa = 0$) where buckling occurs, as was discussed previously in connection with Fig. 2b. The expected shape of the buckling mode in the hoop direction consists of an ovalization or flattening of the cross section, as is shown in Fig. 3b. This ovalization varies in the axial direction due to the stiffening effects afforded by the straight pipe portions of the elbow structure. However, for elbows with radius-to-thickness ratios in the range of 20 to 50, with reasonable angular extent (90° here), and with lengths of straight pipes that are typical of those normally encountered in practice, it seems reasonable to expect that the axial variation of the ovalization will be rather gradual and that buckling for the case of a closing moment will be initiated at the central section of the elbow, where the stresses are the highest. The present analysis effectively concentrates on the behavior of that section. Hence, the use of the special pipe-bend element is appropriate for such an analysis, in addition to being economical. The possibility of buckling into an "axial wavy mode" cannot be explored with the present approach. Test results to date indicate that buckling into an axial wavy mode is not critical for the case of a closing moment.

3.1 Finite Element Model

Figure 1 also shows the finite element model of the elbow structure. The straight pipes are each modeled by one beam element (MARC library element 14). For the finite element analysis of the elbow, it is noted that the MARC pipe-bend element is based on an analysis which assumes that the axial variations of both the ovalization of the cross section and the bending moment M are constant over the axial length of the element, with the amount of ovalization being dependent on the magnitude of M. Hence, for the present case of pure bending, only one pipe-bend interval is required in the axial direction. With regard to the hoop direction, it is seen from Fig. 1 that 18 hoop intervals are employed. This number of hoop intervals was previously used in [7] in the inelastic, but geometrically linear, analysis of the elbow depicted in Fig. 1. However, for the same degree of accuracy, a geometrically nonlinear instability problem generally requires a finer mesh than that appropriate to the corresponding geometrically linear problem. It is anticipated that refinements in the finite element modelling will be considered in a subsequent investigation. In the meanwhile, it is the intention of the present investigation to reveal the relative effects of including geometric nonlinearities in the analysis and of changing the direction of the moment from closing to opening, and 18 hoop intervals should be satisfactory for that purpose.

The loading history used in the MARC analysis is shown in the following table:

M (in. - lb.)	ΔM (in. - lb.)
0 → 600,000	200,000
600,000 → 800,000	50,000
800,000 and above	10,000

The load step used in the regions of the load-displacement curves where displacements grow quite rapidly is $\Delta M = 10{,}000$ in.-lb. The moment, M_0, which causes the highest stressed fiber to first reach the proportional limit stress (26,250 psi)† is $M_0 \approx .3 \times 10^6$ in.-lb. Hence, the smallest load step is $\Delta M \approx M_0/30$, which should lead to reasonably accurate results for present purposes.

4. NUMERICAL RESULTS

This section presents a discussion of numerical results for the nonlinear behavior of the elbow structure depicted in Fig. 1. These results are conveniently displayed in the form of load-displacement curves in which the in-plane bending moment M is plotted against the corresponding rotation θ at the loaded end section, which is constrained to remain plane in the analysis. These load-displacement results were generated through the specification of M. Therefore, buckling is indicated by the inability of the finite element computations to converge for a reasonably small load step. In the MARC program, a message is printed out that the "system is no longer positive definite". This isn't completely satisfactory as a criterion for buckling, as the convergence difficulties conceivably could have been caused by numerical problems. Fortunately, this doesn't appear to be the case for the results obtained herein. These results are presented in Fig. 4.

The three load-displacement curves in Fig. 4 pertain to the following analysis cases: (1) Curve A; closing or opening moment, geometric nonlinear effects omitted in the analysis, (2) Curve B; closing moment, geometric nonlinear effects are included, and (3) Curve C; opening moment, geometric nonlinear effects are included. Observations made from an examination of these curves are discussed next.

As expected, results obtained from an analysis that omits the effects of geometric nonlinearities are independent of the direction of loading, apart from sign. It is seen from Curve A of Fig. 4 that the omission of such effects leads to a load-displacement relation in which the load monotonically increases with displacement at a rate that is governed by the shape of the stress-strain curve for the strain hardening material. The slope of the load-displacement curve never vanishes and, consequently, an analysis which excludes the effects of geometric nonlinearities cannot provide a definitive buckling load for the elbow.

Curve B of Fig. 4 tends to approach a horizontal asymptote and thus it seems reasonable to assume that buckling was about to occur. The buckling load M_b is taken to be the value of M for which the system ceases to be positive definite. From Fig. 4, it is seen that $M_b = .92 \times 10^6$ in.-lb. $\approx 3M_0$ for the selected number of finite elements and the load step used herein.

The effects of including geometric nonlinearities in the analysis are apparent through comparison of the curves depicted in Fig. 4. For the case of a closing moment, these effects are more readily apparent from the particular load-displacement results shown in Fig. 5. In this figure, M is plotted against that portion of θ that is due to the geometric nonlinearity; namely $\theta_{gnl} = \theta_B - \theta_A$, which is the difference in Curves B and A in Fig. 4 for a fixed value of M. From Fig. 5 it is seen that θ_{gnl} is quite small, but it grows very rapidly as $M \to M_b$. The load-displacement behavior displayed in Fig. 5 is typical of that often observed in nonlinear buckling problems for other structures. For such problems, the buck-

†This is the stress at the end of the first linear segment of the multilinear representation of the stress-strain curve.

ling mode is characterized by the presence of a dominant component that grows very rapidly as the buckling load is approached, but which is generally small at low load levels.

For the case of an opening moment, and geometric nonlinearities included in the analysis, Curve C of Fig. 4 reveals that buckling did not occur for values of M up to M_{max} = 1.3×10^6 in.-lb. = 1.4 M_b = 4.3 M_0. From a comparison of Curves A, B, and C in Fig. 4, it follows that geometric nonlinearities provide a stiffening effect for the case of an opening moment. This is because the cross section ovalizes in such a way that the moment of inertia about the neutral axis of bending increases with increasing M. Hence, it is clear that if the elbow under an opening moment buckles into the mode shape investigated herein, then the corresponding buckling load would have to be greater than that appropriate to the case of a closing moment.

For the opening moment loading, a photograph of a tested elbow [6] suggests that buckling of that elbow might have occurred in an axial wavy mode accompanied by ovalization of the cross section. As mentioned earlier, an axial wavy mode is not considered in the present analysis. However, in view of the way an elbow ovalizes under an opening moment, as opposed to a closing one, it appears that the value of M corresponding to buckling in an axial wavy mode, for the case of an opening moment, is greater than the previously discussed value of M_b, for the case of a closing moment, for which the buckling mode is characterized by significant ovalization of the cross section and by a relatively gradual variation of shape in the axial direction. From a practical point of view, therefore, the case of a closing moment is critical, although further work is required for a more thorough understanding of the buckling phenomenon for an elbow under an opening moment.

5. CONCLUDING REMARKS

The present work is based on the use of a simplified method of inelastic analysis in an investigation of the buckling of elbows with radius-to-thickness ratios in the range of 20 to 50. The elbows are subjected to in-plane closing or opening bending moments. The particular buckling mode considered is one characterized by significant ovalization of the cross section and by a relatively gradual variation in shape along the length of the elbow. Emphasis is placed on the effects of including geometric nonlinearities in the inelastic analysis. The major conclusions and observations of this investigation are as follows:

1. Geometric nonlinear effects (large displacements) are significant.

2. If such effects are included in the analysis, buckling into a circumferential ovalization mode occurs at a maximum point (zero slope) in the load-displacement curve and a definitive buckling load can be established. Such a buckling load, M_b, was obtained herein for the case of a closing moment. Results obtained for the case of an opening moment are inconclusive as buckling did not occur for the range of load values considered, M_{max} = 1.4 M_b. A closing moment is just as likely to be acting on an elbow as is an opening moment. From a practical point of view, therefore, the loading condition of a closing moment is critical for an elbow under in-plane loading.

3. The omission of geometric nonlinear effects in the analysis leads to a load-displacement curve in which the load monotonically increases with displacement. Hence, such an analysis does not provide a definitive buckling load for an elbow made from a strain hardening material.

4. For the preceding case of a monotonically increasing load-displacement curve, [7]

presents procedures for the computation of plastic collapse loads based on the ASME Code method and on other methods that are sometimes used. Figure 6 illustrates the application of these methods to the geometrically linear load-displacement curve appropriate to the specific elbow configuration considered herein. Also shown in this figure is the load-displacement curve for the case in which geometric nonlinear effects are included in the analysis, and the attendant, more definitive, buckling load, M_b, for the case of a closing moment. The following table shows a comparison of this load with the plastic collapse loads determined by the methods of [7]:

	Geometric Nonlinear Effects Included (Closing Moment)	Geometric Nonlinear Effects Omitted		
		Tangent Method	2x Angle of Linear Response Method	ASME Code Method
$M \times 10^{-6}$	.92	1.02	.95	.58
%	-	10.9	3.3	-37.0

For the specific elbow depicted in Fig. 1, the ASME Code method† is seen to be quite conservative whereas the other methods give loads that are in reasonable agreement with M_b.

5. Since computer programs are available that account for both material and geometric nonlinearities, and since the additional computer cost of including the latter nonlinearity in an inelastic analysis is not excessive, there doesn't appear to be a strong justification in the use of a computer solution that omits geometric nonlinear effects.

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†It is noted that [7] points out that the plastic collapse load obtained from this method is highly dependent on the multilinear approximation selected for the representation of the stress-strain curve.

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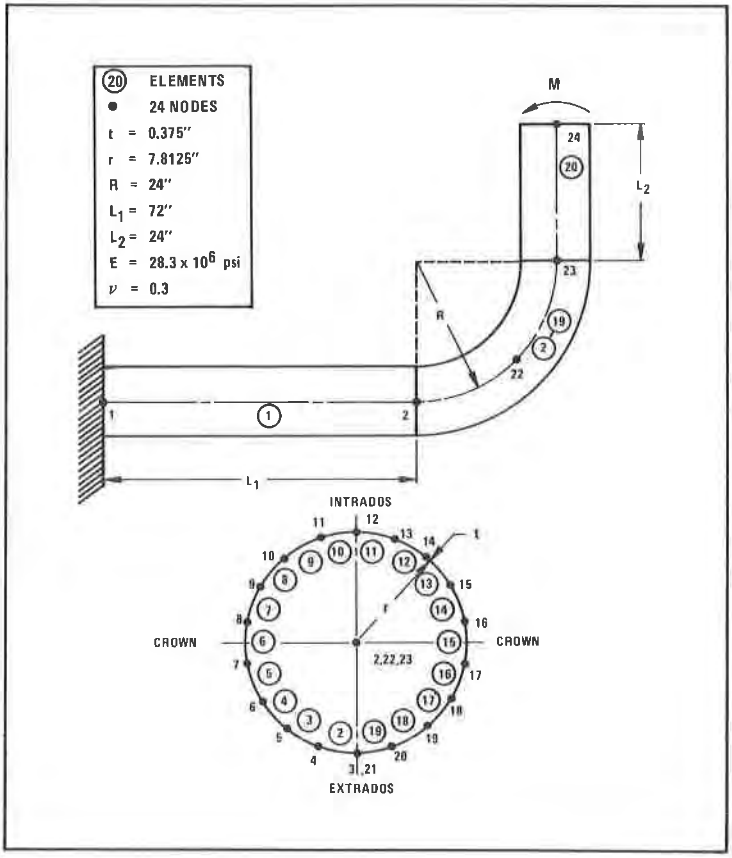

Figure 1. Pipe Structure and MARC Finite Element Model

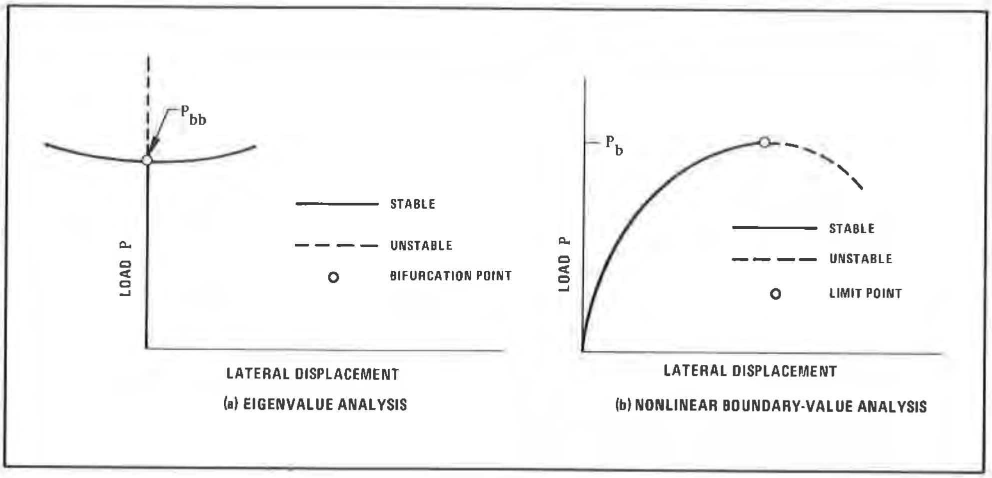

Figure 2. Typical Load-Displacement Curves

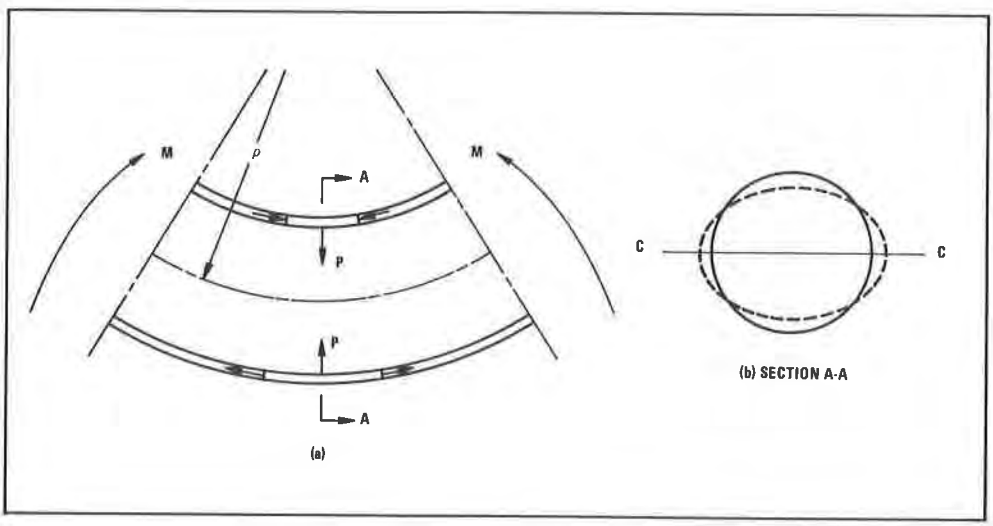

Figure 3. Ovalization of the Cross Section

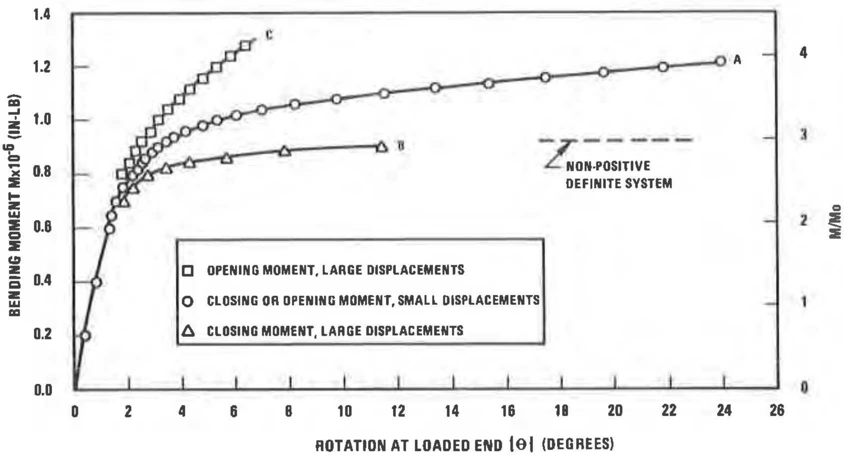

Figure 4. Moment-Rotation Curves, Effects of Large Displacements & Direction of Loading

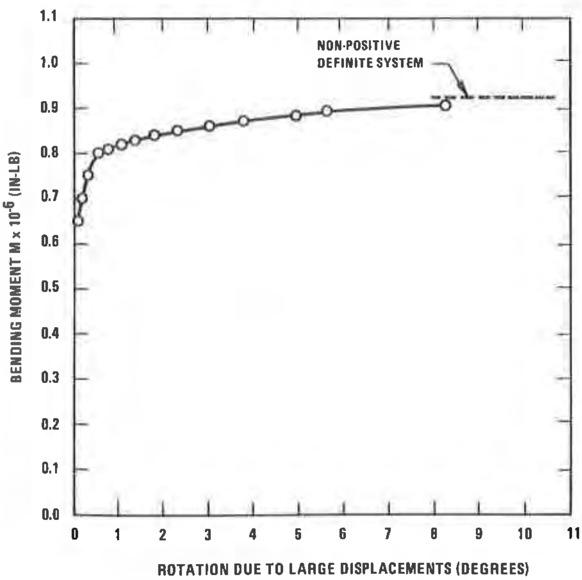

Figure 5. Deformation Due to Large Displacements

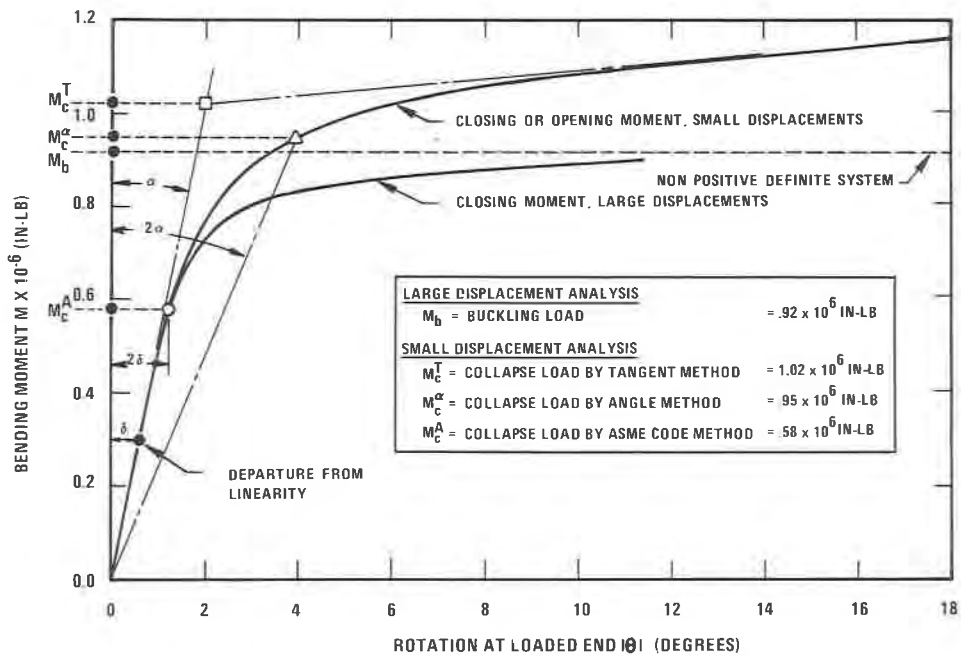

Figure 6. Computation of Instability Loads By Different Methods