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PROPOSAL OF A FORMULA FOR STRESS INTENSITY FACTOR EVALUATION OF RECTANGULAR SURFACE CRACK

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ABSTRACT

In recent years, Stress Corrosion Crackings (SCCs) in welded joints of pressurized water reactors of nuclear power plants have been reported. The rules on fitness-for-service of the Japan Society of Mechanical Engineers (JSME) for SCC assumes semi-elliptical-shaped flaws. The SCC propagation often stops at the fusion line between the weld and base metal. SCC propagates only in depth direction. The assumption of rectangular flaw may give more appropriate approximation than that of semi-elliptical flaw. Hence, formulae of the stress intensity factor for rectangular surface flaws have been developed in this investigation. The developed formulae are valid upto ratio of flaw depth to plate thickness to be 0.8.

BACKGROUND AND OBJECTIVES

Evaluation methodologies for stress corrosion cracking, the so-called PWSCC, in nickel alloy welds of primary piping system of nuclear power plants were discussed by Hojo and Kamaya (2012). The rules on fitness-for-service for nuclear power plants of the Japan Society of Mechanical Engineers (JSME) provides procedures for modelling and evaluating flaws (Japan Society of Mechanical Engineers (2020)). The flaws were assumed to be semi-elliptic in their shapes. On the other hand, it has been seen that flaws propagated in the weld metal. They did not penetrate the fusion lines and the flows were contained in the weld metal. A possible way to reproduce the behaviour of such flaw propagation may be to treat the flaw to be rectangular in its shape, as proposed by Kamaya et al. (2023).

In this study, we developed formulae of the stress intensity factor for rectangular surface flaw. The stress intensity factor was calculated using finite element method with quadratic tetrahedral elements. An automatic finite element analysis model generator and three-dimensional J-integral for rectangular surface cracks based on domain integral method were employed. They were developed in the authors' research group. This paper is organized as follows. Following background and objectives, the J-integral and the finite element model preparation methodologies are discussed. The results of the stress intensity factor evaluations by the J-integral method are briefly presented. Then, the developed stress intensity factor formulae are proposed, followed by conclusions.

COMPUTATIONS OF STRESS INTENSITY FACTOR BY J-INTEGRAL METHOD

The stress intensity factor was computed by the J-integral method. Figure 1 and Equation 1 show the integral domain and the formulation of the three-dimensional J-integral by the domain integral method. The first term in the right-hand side of Equation 1 is the domain integral. The second and third integrals are related to the crack face loads. W is the strain energy density, σ_{ij} are the stresses, and u_j are the displacements. ΔA is the area of virtual crack extension. $q(x)$ represents the virtual crack extension along the crack front and is extended to be a continuous and piecewise differentiable function in the integral domain (see, for example, Okada and Ohata (2013)).

$$J = -\frac{1}{\Delta A}\int_{V^0}\left(\delta_{i1}W - \sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)\frac{\partial q(x)}{\partial x_1}dV + \frac{1}{\Delta A}\int_{S^+_{Crack}}\left(-n_i\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)q(x)dS^+_{Crack} + \frac{1}{\Delta A}\int_{S^-_{Crack}}\left(-n_i\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)q(x)dS^-_{Crack} \quad (1)$$

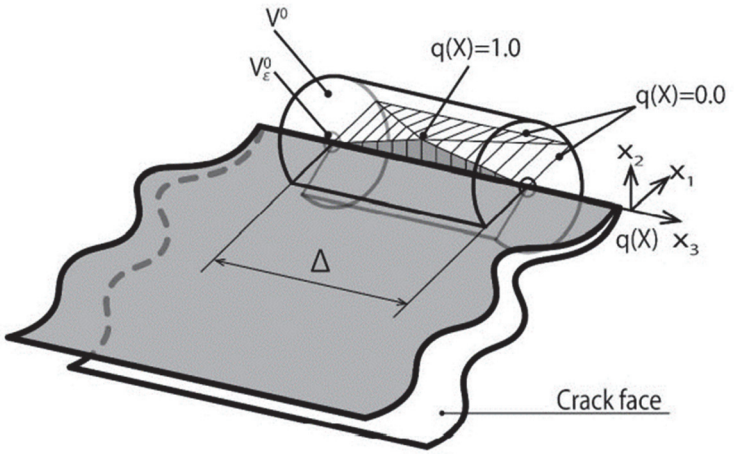


Figure 1. Schematics of the integral domain for the three-dimensional J-integral and variation of the function $q(x)$.

In this study, we proposed that the J-integral for rectangular flaws be computed with virtual crack extension spanning for entire length of each straight edge of the crack, as shown in Figure 2. Hence, the computed J-integral represents the energy release rate when each straight edge of the crack extends parallelly at once. In other words, the present J-integral method computes the energy release rate along each straight edge in an average sense. The mode I stress intensity factor is evaluated by the relationship of Equation 2 with assuming the plane strain condition.

$$J = \frac{1-\nu^2}{E}K_I^2 \quad (2)$$

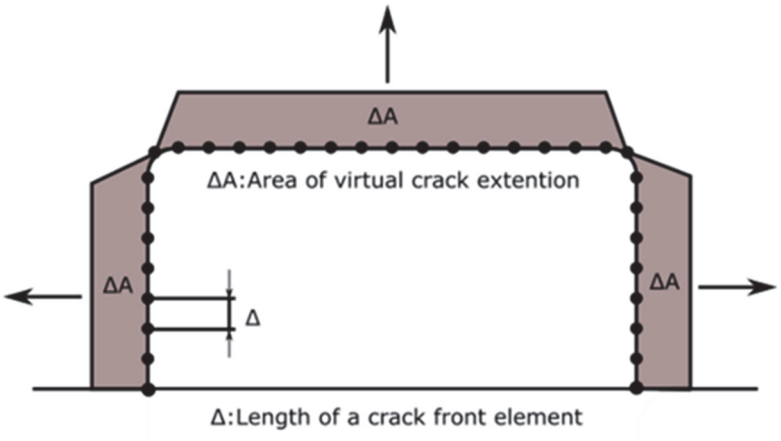


Figure 2. Virtual crack extension for J integral evaluation assuming the entire edges of the deepest and the surface locations of the rectangular crack.

FINITE ELEMENT ANALYSES AND STRESS INTENSITY FACTOR EVALUATIONS

The problem of a plate with a rectangular surface flaw as shown in Figure 3 is considered. A uniformly distributed load (σ) is applied to the crack surface. Then, the stress intensity factor is computed by the J-integral method.

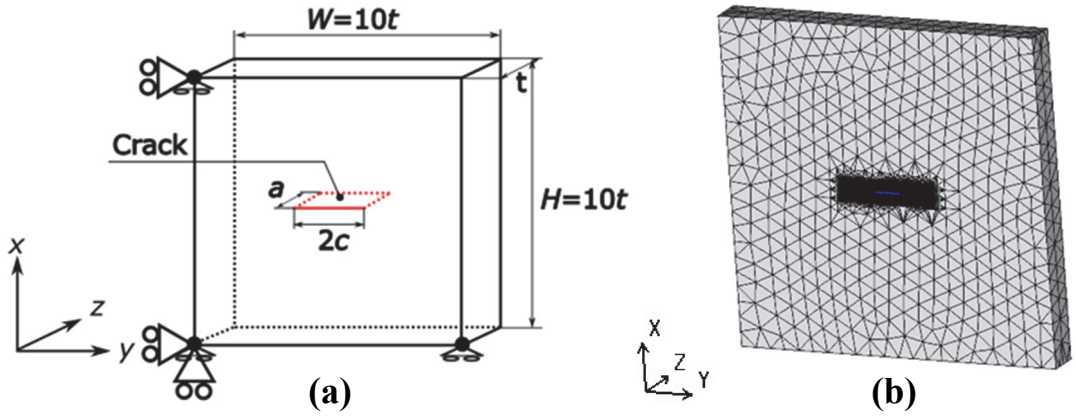


Figure 3. Finite element model for the problem of the rectangular surface crack [(a) Plate with a rectangular surface flaw subject to a uniformly distributed load and boundary conditions and (b) whole view of the finite element model].

In order to develop a stress intensity factor equation for rectangular crack, a number of finite element analyses and stress intensity factor computations were carried out. We introduced two parameters s and u for specyfing the depth and aspect ratio of the flaw. s specifies the aspect ratio of the rectangular flaw and u is the ratio of the square root of area of the rectangular flaw to the plate thickness. They are defined as shown in Equation 3 and 4. Table 1 summarizes the ranges of u and s for which finite element analysis was performed. The parameter s was set to be from -0.90 to 0.90 with an increment of 0.05. However, the larger the crack area, the deeper the crack, even for the same aspect ratio. In this study, the ratio a/t of the crack depth to the plate thickness was limited to be equal or less than 0.6. It was because the stress concentration due to the narrower ligament width affected the stress intensity factor value. The stress intensity factor calculation was performed in the range of $s \leq 0.85$ for $u \geq 0.28$.

$$s = \begin{cases} \frac{a}{c} - 1 & \left(\frac{a}{c} \leq 1.0\right) \\ 1 - \frac{c}{a} & \left(\frac{a}{c} > 1.0\right) \end{cases} \tag{3}$$

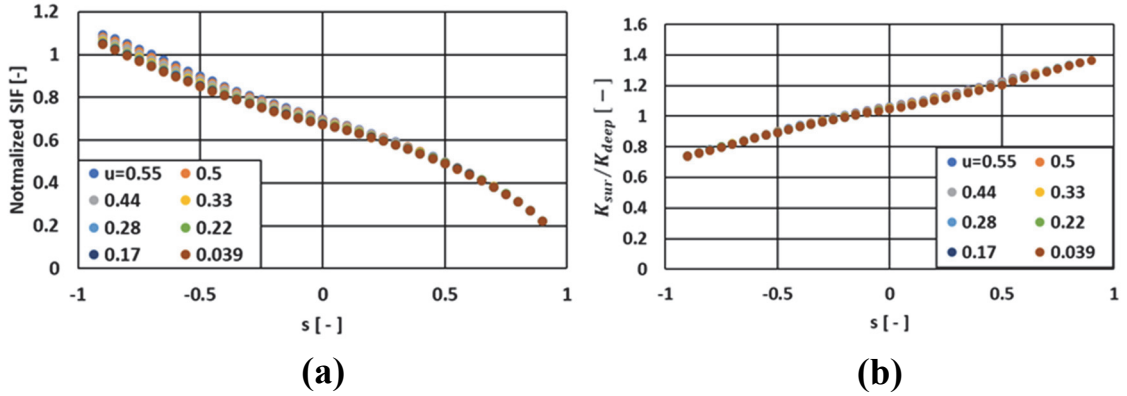
$$u = \frac{\sqrt{2ac}}{t} \tag{4}$$

Table 1: Ranges of s concerning the value of u.

u **[-]**	***s*** **[-]**
0.55	-0.90 ~ 0.60
0.50	-0.90 ~ 0.60
0.44	-0.90 ~ 0.70
0.33	-0.90 ~ 0.80
0.28	-0.90 ~ 0.85
0.22	-0.90 ~ 0.90
0.17	-0.90 ~ 0.90
0.039	-0.90 ~ 0.90

Figure 4(a) shows the relationship between the stress intensity factor at the deepest point normalized by $\sigma\sqrt{\pi a}$ and s which is the parameter representing the aspect ratio of the rectangular flaw. s approaches to unity, the stress intensity factor values converged to the same value regardless of the value of u. However, s approaches to "-1", the stress intensity factors somewhat depend on the value of u.

Figure 4(b) shows the relationship between s and the ratio of the stress intensity factor at the surface point to that at the deepest point. The ratio monotonically increases. The values do not exhibit a strong dependency on the aspect ratio of the flaw.



(a) **(b)**

Figure 4. Stress intensity factor at the deepest and the surface point [(a) Stress intensity factor at the deepest point vs. s and (b) Ratio of stress intensity factors at the deepest to that at the surface point].

DEVELOPMENT OF STRESS INTENSITY FACTOR EQUATION FOR THE DEEPEST POINT OF THE RECTANGULAR FLAW

Based on the results that are presented in previous section, we developed a stress intensity factor equation for the rectangular surface flaw. The equation is composed of two functions $A(s)$ and $B(s,u)$, as shown in Equation 5. $A(s)$ approximates the stress intensity factor at the deepest point as a function of s for infinite plate ($u = 0.039$). It was found that the case of u=0.039 could represent the flaw in infinity thick plate. $B(s,u)$ is a correction term that takes into account the effect of the finite plate, as a function of s and u.

$$\frac{K_{deep}}{\sigma\sqrt{\pi a}} = A(s) + B(s,u) \tag{5}$$

DEVELOPMENT OF FUNCTION $A(s)$

Figure 5 shows the relationship between the stress intensity factor at the deepest point and s for $u = 0.039$. For $s = -1$, the stress intensity factor value can be considered to be the same as that of edge crack problem (Brown and Strawley (1966)), because the crack length is infinite in a limiting sense.

These results were approximated using the polynomial approximation function of Microsoft Excel (least-squares method). The developed functions $A(s)$ are presented in Equation 6 and 7. They are the cubic and quartic polynomials, respectively.

$$A3(s) = -0.178s^3 - 0.0417s^2 - 0.314s + 0.678 \tag{6}$$

$$A4(s) = -0.0598s^4 - 0.188s^3 + 0.00540s^2 - 0.310s + 0.674 \tag{7}$$

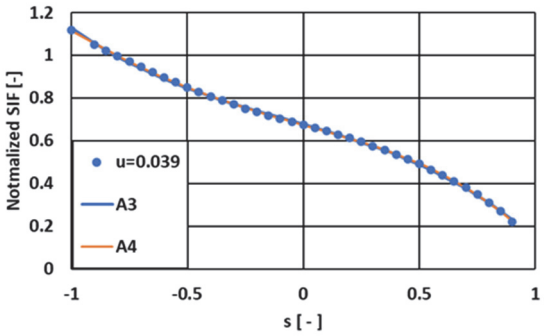


Figure 5. Stress intensity factor at the deepest point vs. s at u=0.039.

DEVELOPMENT OF FUNCTION $B(s,u)$

Figure 6 shows the relationship between s and the difference between the stress intensity factors at the deepest point with $u = 0.039$ and those with $u = 0.17 \sim 0.55$. They are shown by colored solid circles. The difference decreases as s approaches -1 because a/c is zero at $s = -1$ regardless of the value of u. Figure 6 also shows approximate lines created by the polynomial approximation function of Microsoft Excel.

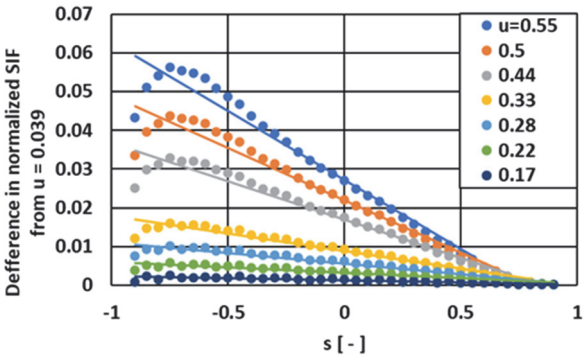


Figure 6. Difference from Normalized SIF at u=0.039 vs. s and approximate lines.

We express the approximate line, as presented in Equation 8.

$$B(s,u) = a(u)s + b(u) \tag{8}$$

Powers of u can approximate the coefficients a and b, and the function $B(s,u)$ can be expressed as Equation 9.

$$B(s,u) = -0.182u^{2.74}s + 0.110u^{2.32} \tag{9}$$

COMPARISON OF THE PROPOSED EVALUATION FORMULA AND STRESS INTENSITY FACTOR CALCULATED BY FEM

From Equations 5, 6, 7, and 9, the developed equations for the stress intensity factor at the deepest point of a rectangular surface flaw can be expressed in two ways, as presented in Equations 10 and 11.

$$A3(s) + B(s,u) = -0.178s^3 - 0.0417s^2 - (0.182u^{2.74} + 0.314)s + 0.110u^{2.32} + 0.678 \tag{10}$$

$$A4(s) + B(s,u) = -0.0598s^4 - 0.188s^3 + 0.00540s^2$$

$$-(0.182u^{2.74}+0.310)s+0.110u^{2.32}+0.674 \quad (11)$$

Table 2 summarizes the maximum difference between the stress intensity factors by the developed formulae and by the J-integral method in their absolute values. For $s = -0.9$ to 0.9 and $a/t \leq 0.6$, the maximum difference when Equation 10 is employed is about 5%. When Equation 11 is employed, it is about 2%. According to the rules of fitness-for-service (Japan Society of Mechanical Engineers, (2020)) evaluation formulas should be valid for $\frac{a}{t} \leq 0.8$. Hence, the differences were compared in the range of $s = -0.9$ to 0.9 and $a/t \leq 0.8$ also. For $s = -0.9$ to 0.9 and $0.6 < = a/t <= 0.8$, the maximum difference when Equation 10 is employed is 2.41%. When Equation 11 is employed, it is 1.97%. Thus, the developed stress intensity factor equations do not lose their accuracy even when the range of a/t is extended to $a/t \leq 0.8$.

Table 2: Maximum difference between the values of the stress intensity factors by the developed stress intensity factor equations and the J-integral method.

Ranges	$s = -0.9 \sim 0.9$ and $a/t \leq 0.6$	$s = -0.9 \sim 0.9$ and $0.6 < a/t \leq 0.8$
$A3 + B$ (Equation 10)	5.26%	2.41%
$A4 + B$ (Equation 11)	2.05%	1.97%

CONCLUSION

In this study, two formulae for evaluating the stress intensity factor for rectangular surface flaws were developed. The maximum difference between the developed formulae and the J-integral method using the results of the finite element analyses was about 5%. We believe that the accuracy of the developed formulae is sufficient.

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