The Viability of the K_{Ia} Procedure for Predicting the Arrest of a Deep Crack During a Hypothetical Thermal Shock Event Involving a Water-Cooled Nuclear Reactor Pressure Vessel

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ABSTRACT

For a hypothetical thermal shock event involving a water-cooled nuclear reactor steel pressure vessel, earlier theoretical analyses have shown that, provided certain conditions are satisfied, it is possible for a crack to propagate deep into a reactor vessel thickness by a series of run-arrest-reinitiation events. Model vessel tests confirm this prediction. Against this background, this paper examines the viability of the K_{Ia} procedure for predicting the arrest of a deep crack. The results from a theoretical analysis of a simulation model suggest that the K_{Ia} procedure gives conservative predictions of the crack jump length for the deep crack situation.

INTRODUCTION

Interest in the development of a crack arrest methodology has been stimulated by the problem of the propagation and arrest of a crack when a water-cooled nuclear reactor is subjected to a hypothetical thermal transient, say due to a loss-of-coolant accident (LOCA) or a pressurized thermal shock event (PTSE). This situation could occur when the emergency core cooling system (ECCS) injects water into the vessel. The resulting thermal stresses, possibly with the assistance of primary stresses due to the system pressure, might conceivably enable a pre-existing defect at the vessel inner surface to propagate into the vessel thickness. The ASME Code procedure (1975) for predicting crack arrest is based on static linear elastic fracture mechanics (LEFM) principles: crack arrest is presumed to occur when the crack tip stress intensity factor K_I^{ST} falls below a value K_{Ia}. This is assumed to be a material property, and is referred to as the arrest toughness.

In the view of many workers in the field, the theoretical justification for using the K_{Ia} procedure stems from the behaviour of a semi-infinite crack propagating in an unbounded solid due to the application of time-independent loads. In this case it has been demonstrated (Freund, 1972) by a comprehensive dynamic analysis that Mode I crack crack propagation is governed by the relation $K_I^{DYN}(v) = f_I(v)K_I^{ST} = K_{ID}(v)$ where $K_I^{DYN}(v)$ is the dynamic crack tip stress intensity factor, in general a function of the crack tip velocity v, K_I^{ST} is the static crack tip stress intensity factor, and $K_{ID}(v)$ is the dynamic fracture toughness, which again can be a function of the crack tip velocity; $f_I(v)$ is a known function of the crack tip velocity. Since $f_I(v) \rightarrow 1$ as $v \rightarrow 0$, a crack arrests when $K_I^{ST} = K_{ID}(v \rightarrow 0) = K_{ID}$ say; hence if K_{Ia} is equated with K_{ID}, crack arrest occurs when $K_I^{ST} = K_{Ia}$. This particular result provides some measure of physical justification for the K_{Ia} procedure. For other more general situations, the usefulness of the K_{Ia}

procedure depends on its ability to make arrest conservative predictions that are sufficiently accurate for practical purposes. Against this background, the present paper examines the viability of the K_{Ia} procedure for predicting the arrest of a deep crack. The results from a theoretical analysis of a simulation model suggest that the procedure gives predictions of the crack jump length. This conclusion is relevant to a hypothetical thermal shock event, since it has been shown (Cheverton et al, 1982), by use of the K_{Ia} procedure, that provided certain conditions are satisfied, it is possible for a crack to propagate deep into the thickness of a reactor pressure vessel by a series of run-arrest- reinitiation events.

THEORETICAL ANALYSIS

When a crack has a finite size and/or the solid has finite dimensions, and wave reflection effects are neglected, the governing equation for Mode I crack propagation is (Freund, 1972; Rose, 1976; Melville, 1977; Eshelby, 1969)

$$K_I^{DYN}(v) = f_I(v)K_I^* = K_{ID}(v) \qquad (1)$$

when $K_I^{DYN}(v)$, $K_{ID}(v)$ and $f_I(v)$ have already been defined in the Introduction. K_I^* is referred to as the reflectionless stress intensity factor. If the initial position of a crack tip is $x = 0$ and the position at time t is $x = \epsilon$, K_I^* at time t is given by the expression (Freund, 1972; Rose, 1976; Melville, 1977; Eshelby, 1969)

$$K_I^* = \sqrt{\frac{2}{\pi}} \int_0^{\epsilon} \frac{p(\lambda)d\lambda}{\sqrt{\epsilon-\lambda}} \qquad (2)$$

where $p(\lambda)$ is the tensile stress ahead of the crack tip when it is in its original position at time $t = 0$. It is important to emphasize that the theory which leads to relations (1) and (2) is based on an exact dynamic analysis, even though the reflectionless stress intensity factor K_I^* itself is obtained by a purely static analysis. Since $f_I(v) \rightarrow 1$ as $v \rightarrow 0$, relation (1) shows that crack arrest occurs when $K_I^* = K_{ID}(v\rightarrow 0) = K_{ID}$, which may of course be a function of position.

Against this background consider the Mode I plane strain deformation of a solid with two symmetrically situated deep cracks, and with tension of the small remaining ligament due to the application of a fixed load P at a point a great distance from the ligament (Figure 1). In a crude manner this model simulates the situation where there is a deep crack in a vessel and the remaining ligament width is L. For the model in Figure 1, the static crack tip stress intensity $K_I^{ST} = P/\sqrt{\pi L}$ when the ligament width is 2L (Tada et al, 1973). If the initial ligament width at the onset of crack propagation is $2L_o$, the tensile stress along the crack plane at a distance λ ahead of a tip is $p(\lambda)$ given by the expression

$$p(\lambda) = \frac{P}{\pi\sqrt{L_o^2 - \left[L_o - \lambda\right]^2}} \qquad (3)$$

whereupon relations (2) and (3) give the reflectionless stress intensity factor K_I^* as

$$K_I^* = \sqrt{\frac{2}{\pi}} \int_0^{\epsilon} \frac{Pd\lambda}{\pi\sqrt{\epsilon-\lambda}\sqrt{L_o^2 - \left[L_o-\lambda\right]^2}} \qquad (4)$$

when the crack extension at each tip is ϵ. Thus, if $K_Q = P/\sqrt{\pi L_o}$ is the initial crack tip stress intensity, it follows from relation (4) that

$$\frac{K_I^*}{K_Q} = \frac{2}{\pi} K(m) \qquad (5)$$

where $K(m)$ is the complete elliptic integral of the first kind with $m = \epsilon/2L_o$. Furthermore since the static stress intensity for an extension ϵ

is $K_I^{ST} = P/\sqrt{\pi(L_o-\epsilon)}$, then

$$\frac{K_I^{ST}}{K_Q} = \sqrt{\frac{1}{1-\frac{\epsilon}{L_o}}}$$

Plots of both K_I^*/K_Q and K_I^{ST}/K_Q against the crack extension ϵ are shown in Figure 2.

DISCUSSION

Figure 2 clearly shows that the K_I^*/K_Q versus ϵ/L_o plot lies below the K_I^{ST}/K_Q versus ϵ/L_o plot. Assuming that a crack arrests when $K_I^* = K_{ID}$ (see equation (1)), and that arrest occurs in an increasing toughness gradient (see Figure 2), then it is clear that the static K_I^{ST}-K_{Ia} procedure is conservative, in the sense that it over-estimates the crack jump length. This result is similar to that for the case (Smith, 1985) where there is a shallow crack in the surface of a semi-infinite solid, when an analysis conducted on the same basis as that in the present paper has shown that the K_{Ia} procedure over-predicts the crack extension at arrest in an increasing toughness gradient.

The present paper's results refer to the situation where there is a pre-existing deep crack, which propagates and then arrests. However, if these results are coupled with those (Smith, 1985) for a shallow crack in the surface of a semi-infinite solid, they do suggest (though not prove) that the K_{Ia} procedure is conservative when it is applied to the propgation of a shallow crack when it arrests deep into the vessel thickness. This conclusion has to be seen against the background of the demonstration (Cheverton et al,1982) that it is possible, provided certain conditions are satisfied, for a crack to propagate deep into the thickness of a reactor pressure vessel by a series of run-arrest-reinitiation events, as a consequence of a hypothetical thermal shock event.

In conclusion it should be emphasized that the present paper's results have been obtained on the basis of an analysis, which although it is dynamic, nevertheless disregards wave reflection effects. In practice there will be wave reflections from the vessel outer surface towards which the crack is propagating, but such reflections are not expected to have a significant effect on the propagation and arrest process. It should also be emphasised that the dynamic effects associated with the present paper's model, as manifested by the difference between K_I^{ST} and K_I^*, are dynamic effects that are unconnected with wave reflections. Similar significant effects have been predicted (E. Smith, unpublished work) for a model (Mode III) where there are free surfaces parallel to the direction of crack propagation, which is a feature of many laboratory crack arrest tests.

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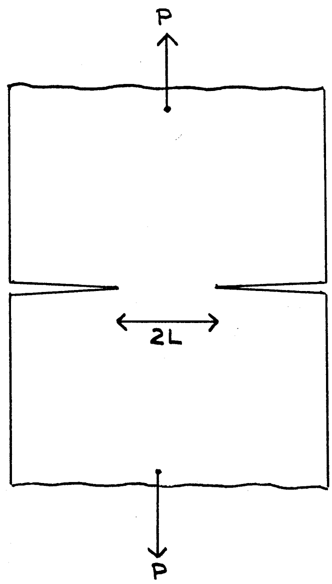

FIGURE 1 The plane strain model of a solid containing two symmetrically situated deep cracks. A constant load P is applied to the solid at a great distance from the ligament.

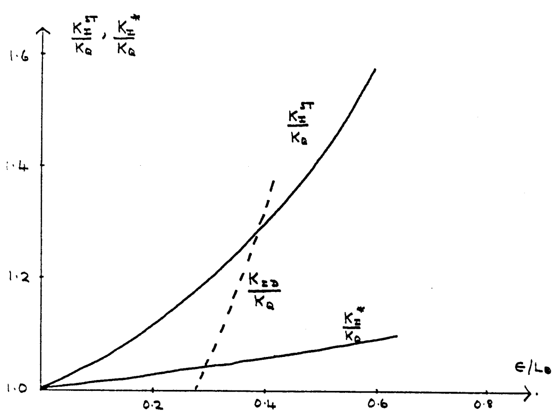

FIGURE 2 K_I^{ST} and K_I^* plotted in terms of the crack extension ϵ. The dotted curve represents the arrest toughness profile.