FLOOR RESPONSE SPECTRA BY THE USE OF A MODIFIED WHITE NOISE TECHNIQUE

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SUMMARY

The problem of computing the floor response spectra at various levels of a nuclear power building is studied.

A standard modal analysis of the building is assumed to be performed in advance and the peak accelerations at any point of the structure for any significant mode are assumed to be known.

It is proposed to specify the earthquake by means of a power spectral density distribution of the type $\phi = \phi_0$ [function (frequency)]; its value ϕ_0 and frequency dependence can be given to reproduce the specified response spectra in the region of relevance for structural response.

The so specified power spectral density is filtered through two consequent filters, the first one representing the structure (while vibrating in one of its significant modes) and the second one the equipment (in one of its significant modes).

The average quadratic accelerations both of the equipment and of the structure in the point of connection are consequently evaluated and the ratio of the two values computed. The floor response spectrum is consequently evaluated by means of a standard square root of the squares of responses for significant modes.

The case of a stationary white noise power spectral density is considered as of special importance as the amplification factor (i.e. the ratio of the accelerations of the equipment and of the structure at the point of connection) can be derived in a closed form independent from the frequencies of the structure and equipment and dependent only on their ratio. The importance of the stationary hypothesis is discussed and finite length correction factors introduced.

All this procedure is available on a computer by means of the ad hoc program CFRS.

Typical cases of response spectra have been considered and discussed; the applicability of the white stationary noise analysis has been studied and the validity range for such a simplified and compact analysis derived. A comparison with other standard methods (time history, etc.) is provided.

The author considers this method as having some advantages due to its quickness as compared with other standard methods, which permits a fast (and generally conservative) evaluation of the peak accelerations in the equipments during seismic excitation.

1) Introduction

The problem of computing floor responsa spectra, due to seismic excitation, has become quite relevant in order to correctly design equipment placed at various locations inside the buildings of a nuclear power station.

These pieces of equipment are generally quite light in comparison with the massive buildings to which they are attached, consequently they can be neglected in the dynamic analysis of the building without making a large error.

Besides the dynamic analysis of a building can be performed by the use of a spectral analysis tecnique, by which maximum accelerations can be derived, but no information is generally obtained about the earthquake spectral deformation due to the filtering of the various building modes. So it has become customary to perform a "time history" analysis too in order to compute spectra at various locations in the buildings, anyhow some difficulty may occur in order to have proper normalization of the selected earthquake (either registered or artificial) [1,2].

Some other methods [3] have been proposed in order to avoid this time-consuming job and to derive floor response spectra from the standard spectral analysis by means of "universal" amplification curves.

The purpose of this paper is to try a simplificative analysis in order to avoid a time history analysis consequent to a standard building dynamic analysis.

2) Analysis by means of a white noise tecnique

The simplest way of representing an earthquake is to assume it is a stationary white process, indefinite in length and with a power density constant all over the spectrum, this idea is quite old and has been used and modified extensively [4,5].

The analytical advantages of this assumption are quite obvious, with reference to fig.1, it is a simple matter to define the transfer function for the building (subscript S), and for the equipment (subscript E):

$$(1a,b) \quad \begin{cases} H_S(w) = \dfrac{w_S^2\,(w_S^2 + 4\beta_S^2 w_S^2)}{w^4 + 2w^2 w_S^2 (2\beta_S^2 - 1) + w_S^4} \\[2ex] H_E(w) = \dfrac{w_E^2\,(w_E^2 + 4\beta_E^2 w_E^2)}{w^4 + 2w^2 w_E^2 (2\beta_E^2 - 1) + w_E^4} \end{cases}$$

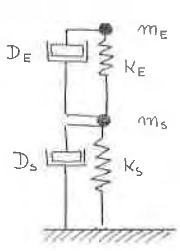

fig.1

where

w = eigenfrequency

w_S, w_E = building, equipment eigenfrequency

β_S, β_E = critical damping ratio

it has been assumed $\quad w_S^2 = K_S/m_S \qquad w_E^2 = K_E/m_E$

If ϕ is the power density the value of the quadratic average absolute acceleration will be:

$$(2a,b)\begin{cases} a_s^2 = 2\int_0^\infty H_s(w)\,\phi(w)\,dw \\ a_e^2 = 2\int_0^\infty H_s(w)\,H_e(w)\,\phi(w)\,dw \end{cases}$$

if ϕ is a constant it is a simple matter to compute a_s, which it is:

$$(3a)\quad a_s = \sqrt{\pi\phi w_s/2\beta_s}$$

The peak acceleration a_{sp} will be some multiple of a_s, it is a common hypothesis to assume:

$$(3b)\quad a_{sp} = 3a_s = 3\sqrt{\pi\phi w_s/2\beta_s}$$

It is possible to resolve the integrals in equation (2b) and to compute a_E, it will be an expression of the type:

$$a_E = \delta(\beta_e, \beta_s, R)\cdot\phi^{1/2}$$

where

$$R = w_s/w_e$$

The ratio of equipment quadratic average acceleration to the building one will consequently be:

$$(4)\quad A = \frac{a_E}{a_s} = \left\{\frac{\int_0^\infty H_s(w)\,H_e(w)\,dw}{\int_0^\infty H_s(w)\,dw}\right\} = \delta(\beta_s, \beta_e, R)$$

The function $\delta(\beta_s, \beta_e, R)$ is given by

$$(5)\quad \delta(\beta_s,\beta_e,R) = \sqrt{\frac{1}{R}\left\{\frac{AR^2+(1-B)R^4}{R^3} + \frac{\beta_s}{\beta_e}(B-A)\right\}}$$

where

$$\alpha_1 = 2\left\{2\beta_e^2 - 1 - R^2(2\beta_s^2-1)\right\}$$

$$\alpha_{12} = 1 - R^4$$

$$\alpha_2 = 2\left\{(2\beta_e^2-1)R^4 - (2\beta_s^2-1)R^2\right\}$$

$$\gamma_1 = 16\beta_s^2\beta_e^2 R^4 - R^4$$

$$\gamma_2 = 2R^4 + 4\beta_s^2 R^2$$

$$A = \frac{\alpha_2\gamma_1 + \alpha_{12}\gamma_2}{\alpha_1\alpha_2 + \alpha_{12}^2}$$

$$B = -\frac{\alpha_1\gamma_2 - \alpha_{12}\gamma_1}{\alpha_1\alpha_2 + \alpha_{12}^2}$$

The expression given by (4) will be considered as a measure of the amplification of the equipment response due to the building filtering of the original signal; in this sense it is inmaterial which is the value of the ratio a_{sp}/a_s and a_{EP}/a_E (peak to average), provided it is the same for the two cases.

The value of A has been computed for a number of cases (different couples) and it is given in figs. 2-3.

3) Influence of not white spectrum

Let us now assume that ϕ is not a constant, but is a function of w, as generally it is, if such is the case (4) is modified into (6)

$$(6) \quad A = \left\{ \frac{\int_0^\infty H_s(w)\, H_e(w)\, \phi(w)\, dw}{\int_0^\infty H_s(w)\, dw\, \phi(w)} \right\}^{1/2}$$

while the value of a_s is again given by (2a).

The computation involved in expression (6) can be performed either analytically or numerically and has been performed for a number of cases by means of the standard CFRS code [7] . The absolute value of ϕ is irrelevant as only amplification is computed here, anyhow in the computation of a_s the code normalizes the value of ϕ so that $a(w = \infty) = 1$.

Two cases were examined, the first one, spectrum A, is characterized by a ϕ (w) expression given as follows:

$$w \leq 15 \text{ rad/sec} \qquad \phi = \phi_0$$
$$15 \leq w \leq 51 \qquad \phi = \phi_0 (51 - w)/(51-15)$$
$$w > 51 \qquad \phi = 0$$

The spectrum of ϕ, normalized as specified, is given in fig. 4-5 for various β_s values. As one can see there is a close resemblance to the standard Newmark type spectrum, so that this spectrum can be considered as representative for the regular type spectrum like Newmark or similar.

A number of cases were examined by means of the CFRS code and are given in table 1,2; a comparison is provided with the results of the use of the white noise tecnique; as can be seen from the results given in the tables 1,2, there is a wide area for which the conventional white noise analysis is accurate enough.

The spectrum B is characterized as follows:

$$w \leq 10 \text{ rad/sec} \qquad \phi = \phi_0$$
$$10 \leq w \leq 20 \qquad \phi = \phi_0 \left\{ -0.18\, w^2 + 5.3\, w - 34 \right\}$$
$$w \geq 20 \qquad \phi = 0$$

The spectrum of ϕ_B is given in fig. 4-5 for β_s=0.10, 0.07, it is a spectrum with a peak around a period of 0.3-0.4 secs; its amplification factor is given in table 3 and compared with the results of the white noise analysis. In this case too there is a wide zone for which the white noise tecnique is valid enough.

4) Effect of not stationary response.

We have assumed till now that both the inlet signal and the response are stationary, what is often not verified; the not-stationary response of an oscillator to a stationary signal was studied by Caughey and Stumpf [8] and their results have been used.

Let us consider first the case of the motion of m_s, we could assume that the inlet signal is regular (no sharp peaks) around the building period, if that is so the transitory response will be

$$(7) \quad a_{ps} = a_{s\infty} \left(1 - \exp(-2\, w_s\, \beta_s\, s) \right)^{1/2}$$

where $a_{s\infty}$ is the stationary response, and s is the time, as β_s in generally quite high, say 7-10%, it is clear that few seconds are enough to have the stationary value if ω_s is of the order of 10 as it is quite often.

However if the equipement is considered, the damping ratio is generally lower (of the order of 1-2%) so that even if the inlet signal is stationary, the response will be transitory for a large time. The case in which the motion of the building is stationary and the equipment one is transitory can be studied by means of the general expression given by ref.8.

It is obvious that the earthquake duration, which is introduced has to be related to the period for which its intensity is nearly constant, i.e. after the development phase and before the die out one, which is generally of the order of few seconds. In this way the earthquake will be characterized by both its spectrum and its "significative duration"; it is an approximation and a crude one, but the author thinks it is a comparatively simple way to account for the "finite duration" effects.

Some calculations have been performed by means of the CFRS code, assuming that the spectrum is the A type, typical results are given in fig. 6; as it could be expected the "finite duration" effects can be characterized in a reduction of the amplification, which increase as the equipment or structure frequency is increased.

5) Typical examples

Let us now assume that a standard spectral analysis has been performed for a building and that modal acceleration, damping and frequency are specified at each point in the building. Let us assume that amplification factors can be derived by means of one of the described methods, let a (i) be the building acceleration and A(i) be the correspondent amplification factors, the equipment acceleration could be derived as follows:

$$a = \left(\sum_i a^2(i)\, A^2(i) \right)^{1/2}$$

This can be done automatically by means of the CFRS code and has been done for a typical case [10], the results are shown in fig.7; a comparison is provided with the amplification curves given by [3] and the result of a standard time history analysis [9].

6) Conclusions

A simplified tecnique to compute floor response spectra is proposed; it involves the use of the results of a standard modal analysis tecnique and properly computed amplification factors in order to compute peak accelerations at various levels in the building (floor response spectra).

The amplification factors may be computed on the basis of a conventional earthquake, given by means of its power density distribution and duration. The "conventional earthquake" should have a spectrum resembling the one, which was employed in the standard building analysis. Anyhow no close resemblance should seem to be necessary, so that the simple "white noise tecnique" is often precise enough in order to compute the floor responsa amplification factors.

References

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TABLE 1

Spectrum A

Amplification factor-structural frequency as indicated

$\beta_s = 10\%$ $\beta_e = 1\%$

freq equip / f strutt	1cps	2cps	4cps	5cps	10cps	Indef. white noise	Conventional validity limit (cps) 10%	20%
0.10	0.68	0.84	1.09	1.25	3.21	1.01		
0.20	1.22	1.39	1.66	1.86	4.69	1.47	2	4
0.30	1.72	1.85	2.15	2.41	5.74	1.91	4	4
0.40	2.24	2.34	2.69	3.01	6.47	2.38	4	4
0.50	2.86	2.94	3.36	3.73	7.07	2.99	4	4
0.60	3.70	3.76	4.29	4.56	7.52	3.82	4	5
0.70	4.97	5.03	5.55	5.80	7.49	5.11	4	5
0.80	7.17	7.22	7.69	7.89	5.05	7.34	5	5
0.90	11.30	11.30	11.60	11.70	1.63	11.50	5	5
1.00	15.10	15.20	14.90	14.60	1.37	15.40	5	5
1.10	11.10	11.10	10.50	9.97	1.26	11.30	5	5
1.20	7.37	7.40	6.67	6.15	1.19	7.52	4	5
1.30	5.39	5.35	4.67	4.14	1.15	5.51	2	4
1.40	4.26	4.17	3.53	2.98	1.13	4.34	2	4
1.50	3.53	3.41	2.81	2.25	1.11	3.60	2	2
1.60	3.04	2.90	2.32	1.74	1.09	3.10	2	2
1.70	2.68	2.54	1.97	1.47	1.08	2.74	2	2
1.80	2.41	2.26	1.72	1.38	1.07	2.46	2	2
1.90	2.21	2.05	1.52	1.32	1.06	2.25	2	2
2.00	2.04	1.89	1.37	1.28	1.05	2.09	2	2

TABLE 2

$\beta_s = 10\%$ $\beta_e = 2\%$ Spectrum B

Amplification factor-structural frequency as indicated

freq equip / freq strut	1cps	2cps	4cps	5cps	10cps	White noise	Conventional validity limit (Hz) 10%	20%
0.10	0.59	0.67	0.81	0.90	2.28	0.72		
0.20	0.98	1.02	1.18	1.32	3.32	1.04	4	4
0.30	1.31	1.33	1.53	1.71	4.07	1.35	4	5
0.40	1.65	1.67	1.91	2.13	4.58	1.69	4	5
0.50	2.08	2.09	2.39	2.65	5.01	2.12	4	10
0.60	2.67	2.69	3.05	3.25	5.32	2.72	4	5
0.70	3.57	3.60	3.96	4.13	5.28	3.65	4	5
0.80	5.14	5.16	5.47	5.61	3.65	5.23	5	5
0.90	7.91	7.94	8.11	8.14	1.63	8.06	5	5
1.00	10.30	10.30	10.10	9.94	1.37	10.50	5	5
1.10	7.89	7.92	7.50	7.18	1.26	8.04	5	5
1.20	5.46	5.48	5.01	4.67	1.19	5.57	5	5
1.30	4.10	4.07	3.63	3.29	1.15	4.17	2	4
1.40	3.29	3.24	2.83	2.49	1.13	3.35	2	4
1.50	2.78	2.71	2.33	1.99	1.10	2.83	2	4
1.60	2.43	2.35	1.99	1.66	1.09	2.47	2	4
1.70	2.17	2.09	1.75	1.47	1.08	2.21	2	2
1.80	1.98	1.89	1.57	1.38	1.07	2.02	2	2
1.90	1.84	1.75	1.44	1.32	1.06	1.87	2	2
2.00	1.72	1.63	1.34	1.28	1.28	1.76	2	2

TABLE 3

Spectrum B

Amplification factor-structural frequency as indicated

βs = 10%

βe = 2%

f equip / f strutt	1 cps	3 cps	Indefinite white noise
0.10	0.69		0.72
0.20	1.01		1.04
0.30	1.30		1.35
0.40	1.63		1.69
0.50	2.05		2.12
0.60	2.63		2.72
0.70	3.52		3.65
0.80	5.04		5.23
0.90	7.77		8.06
1.00	10.10		10.50
1.10	7.75		8.04
1.20	5.37		5.57
1.30	4.03		4.17
1.40	3.24		3.35
1.50	2.75		2.83
1.60	2.46		2.47
1.70	2.52		2.21
1.80	2.56		2.02
1.90	2.53		1.87
2.00	2.47		1.76

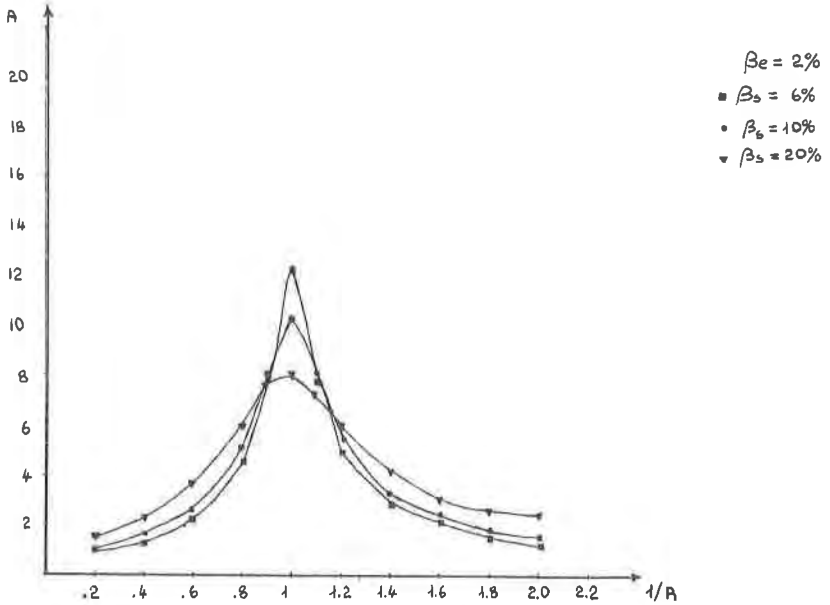

fig.2 - Amplification ratio for $\beta_e = 2\%$ βs as given

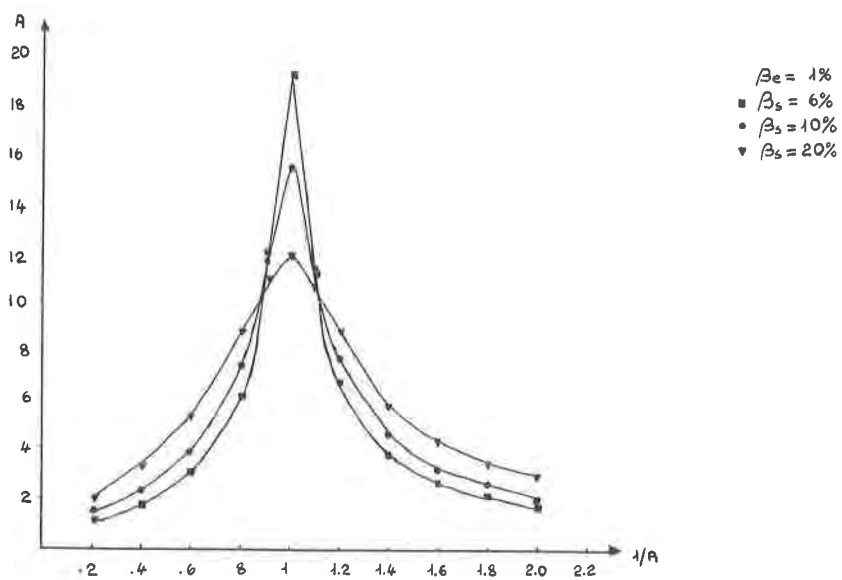

fig.3-Amplification curves for β_e = 1% β_s as given

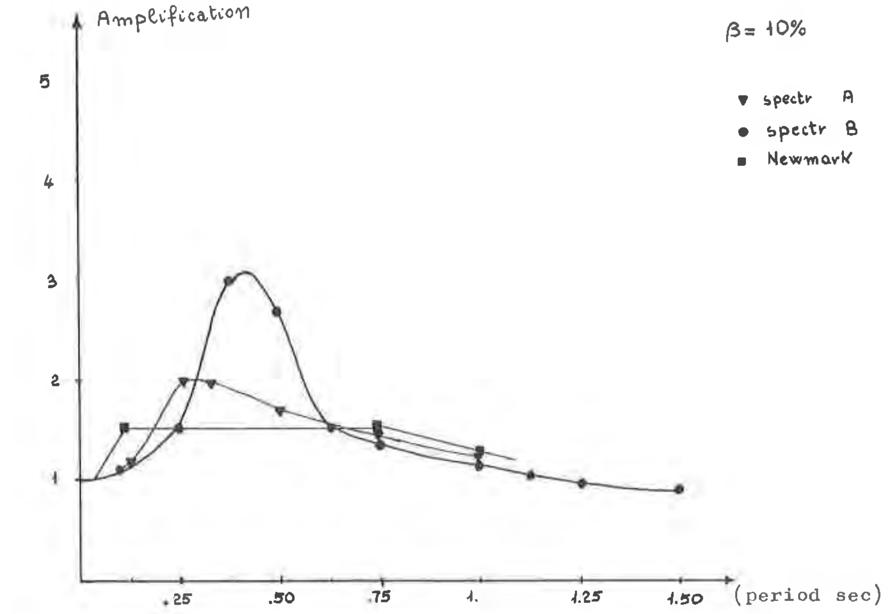

fig. 4 - Spectra for β_s = 10%

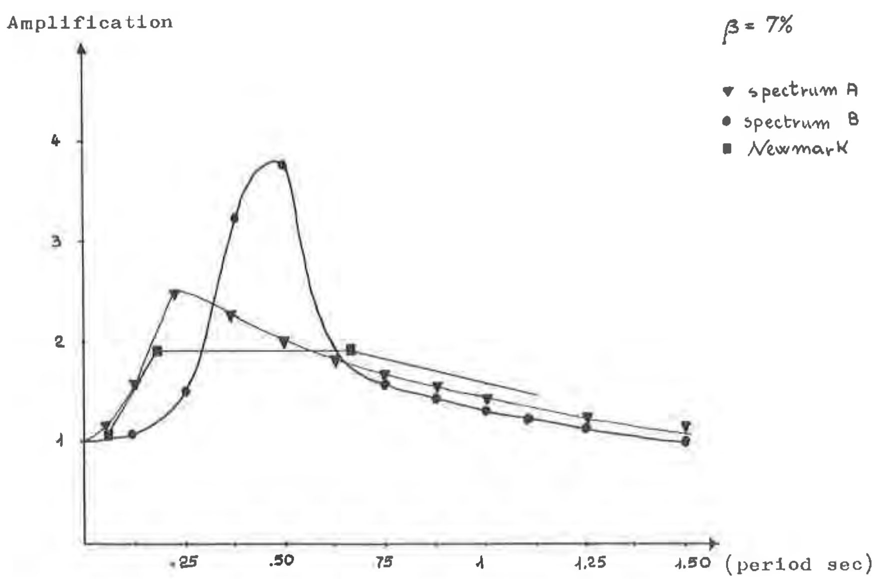

fig. 5 - Spectra for β_s = 7%

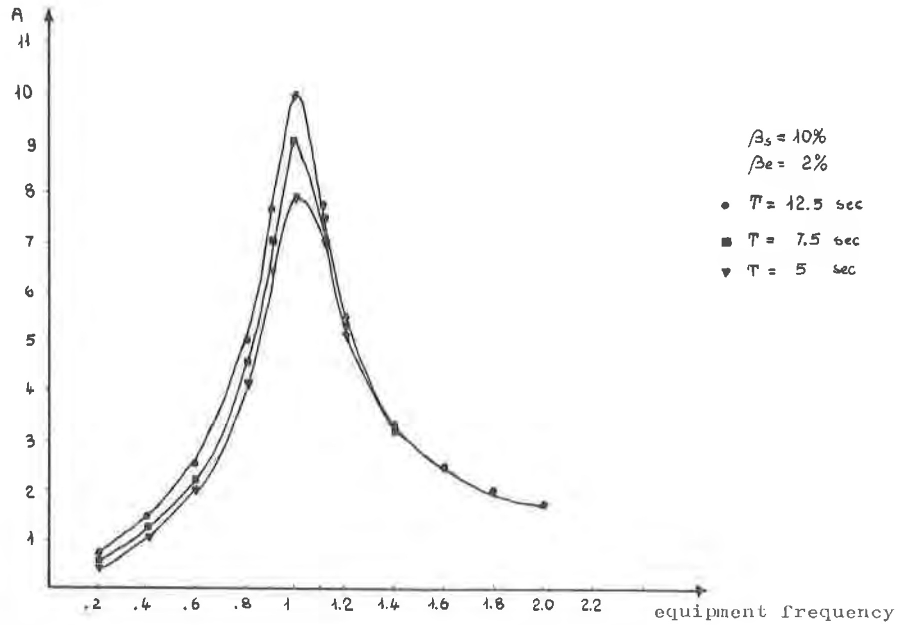

fig. 6a - Influence of duration on amplification for structural frequency = 1 cps

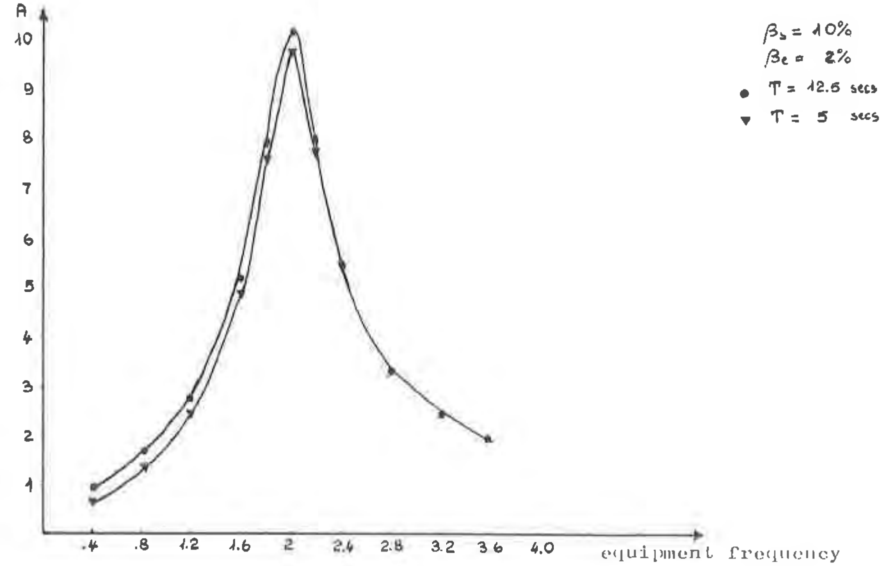

fig. 6b - Influence of duration on amplification for structural frequency = 2cps

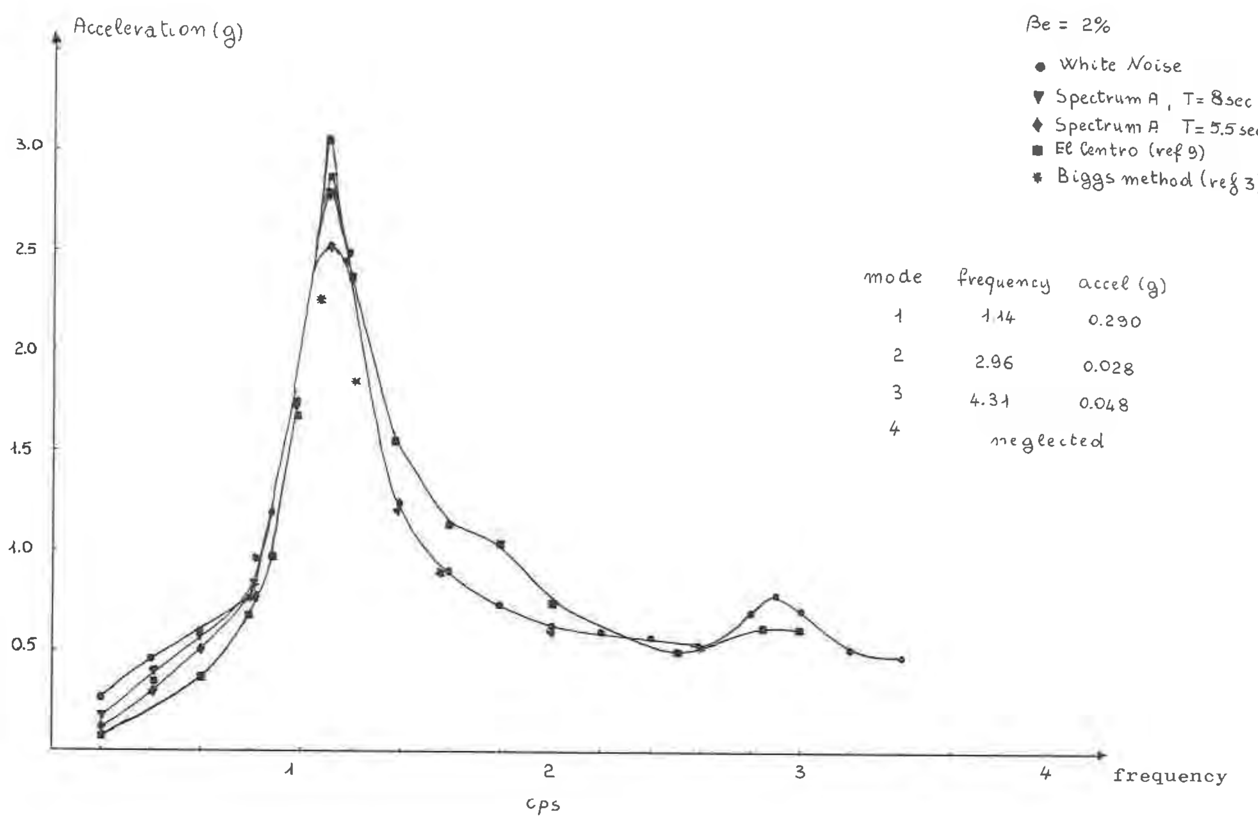

fig. 7 - Example of Floor response spectra (Node 5 ref 10) according to various methods