INDICES OF FERTILITY DERIVED FROM DATA ON THE LENGTH OF BIRTH INTERVALS, USING DIFFERENT ASCERTAINMENT PLANS

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Institute of Statistics Mimeo Series No. 768

September 1971

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A dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Biostatistics in the School of Public Health

Chapel Hill

1971

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JAG MOHAN SEHGAL. Indices of Fertility Derived From Data on The Length of Birth Intervals, Using Different Ascertainment Plans. (Under the direction of H. BRADLEY WELLS.)

Birth intervals have recently attracted considerable attention because of their possible use

(i) as sensitive indices of fertility which could reflect the early effects of contraception in a society, or otherwise measure changes in the level of fertility, and

(ii) to understand fertility patterns and differentials.

Birth intervals, however, have different distributions under different ascertainment plans and the present study recommends the ascertainment plans which yield fertility indices which are

(a) sensitive to changes in fertility level and

(b) robust if the only variables changing, if any are those not directly affecting fertility.

A simulation approach, using the computer programs, POPSIM and SURVEY, is used to generate events to both cohort and cross-sectional populations, and to measure birth intervals for different methods of ascertainment.

The analysis showed that the immediately previous closed intervals and straddling intervals meet the above criteria. Open intervals also are good indicators of fertility performance, if these are measured by parity and age of the mother.

ACKNOWLEDGEMENTS

I wish to thank Dr. Mindel C. Sheps for suggesting the problem and guidance throughout the duration of the study. I deeply appreciate the assistance of my committee chairman, Dr. Bradley Wells, and of the members of my committee, Drs. Peter Lachenbruch, R. C. Elandt-Johnson, A. Hawley and O. D. Williams.

Thanks are due to Dr. Wells, who kindly agreed to take over the chairmanship and to Dr. Williams who agreed to serve on the committee in the latter part of the study.

I gratefully acknowledge the assistance of Dr. Roy Kuebler. I am thankful to Mrs. Alexa J. M. Sorant for her continuous help in the use of computer programs. I thank Mr. C. M. Suchindran for discussion at various stages of study.

I sincerely thank the Carolina Population Centre for their financial support during my stay in the United States.

I wish to thank my wife, Pragati, and others who assisted me in various stages of work in this study. I express my appreciation to Mrs. Faye Koonce for typing the dissertation in an expeditious manner.

TABLE CONTENTS

LIST OF TABLES

LIST OF CHARTS

CHAPTER I. INTRODUCTION

1.1. Introduction

1.1.1. Background

In most developing countries, demographic data for measuring fertility may either be totally lacking, or, if available, be very unreliable. Stable or quasi-stable population methods can usually be applied to available census data to obtain estimates of fertility and mortality under various assumptions. The censuses, however, are usually conducted at 10-year intervals, and to get estimates of levels of fertility and mortality at more frequent intervals, sample surveys must often be the sources for demographic data. Inevitably, questions arise as to what data should be collected.

Implementation of a family planning program is usually followed by an assessment of the success of the program in relation to fertility changes, and it is desirable to have this information as soon as possible after the program has been in force. Most common fertility measures such as birth rates and age-specific fertility rates usually reflect only large changes in fertility within short periods of time, and the question arises, therefore, as to whether more sensitive fertility measures which could be used for evaluating the family planning program can be found.

Birth intervals, in the recent past, have attracted considerable attention because of their possible use as sensitive indices[1] of fertility (Srinivasan [1966], Sheps [1964], Venkatacharya [1969], Sheps and Menken [1970d], etc.). Many questions about the properties of birth intervals, however, must be resolved before recommendations can be made for the data to be collected and for survey design, e.g., longitudinal versus one-time surveys.

Conventional fertility measures such as birth rates and age-specific fertility rates relate to the number of births in a given time period (relative to the appropriate base), and indicate how often women have children. The parity progression ratio, which is defined as the proportion of women achieving next parity, tells us how many women of a given order ever proceed to the next parity. Birth intervals include information about both quantity and timing and therefore might be more sensitive in the short run. In a general sense, birth intervals are simply the inverse of birth rates, but the rationale for investigation of their properties as fertility indices is that one of these properties may be an earlier reflection of change.

Birth intervals may be defined and measured under different ascertainment plans. An ascertainment plan for a birth interval

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[1] In a restricted sense, the term 'index' is used as a ratio showing the value of a given quantity relatively to a base, and may, therefore, be misleading in describing duration variables such as lengths of birth intervals. In its more general sense, however, index may be employed for any number measuring a given quantity, and it is in this context that the term fertility index would be used to refer to measures of fertility, including birth intervals.

refers to the manner in which a birth interval is determined. For example, if a survey is taken at time T, and all births occurring between (T-y,y), where y is predetermined, are recorded, the birth intervals can be determined prospectively. If, however, births are recorded for only those women who are alive at survey date T, we would miss those births which occurred to women who died before the survey date T, and the resulting birth intervals would be according to retrospective ascertainment. These ascertainment plans will be discussed in greater detail in the next section.

A brief review of the literature in the field of birth intervals and related topics is presented next. In order to comprehend the described material better, some definitions are given now.

Fertility is defined as the actual reproductive performance of an individual or group. Fecundity, on the other hand, is the capacity of a man, a woman, or a couple to participate in reproduction (*i.e.*, the production of a live child). The lack of that capacity is called infecundity, sterility, or physiological infertility.

Couples who do not practice contraception during the period studied are called non-contracepting couples. The fecundability of these couples is defined as the probability, p, of conception in a menstrual cycle. (More often, one month is preferred as the duration of a menstrual cycle, because of ease in mathematical considerations.) Non-susceptible period is the time from termination of a pregnancy to the next ovulatory cycle.

We shall always mean by an i^{th} closed birth interval, the time interval between the i^{th} and $(i+1)^{th}$ live birth; $i \geq 0$. The time elapsed since i^{th} birth, if no $(i+1)^{th}$ birth has occurred is called open birth interval. The zero-th closed birth interval is the interval is the interval between marriage and first live birth.

A closed birth interval consists of the following:

a) a post-partum infecund period due to last live birth, during which a woman cannot conceive;

b) a waiting time from the end of infecund period to the pregnancy leading to live birth; this includes two parts:

  (i) ovulatory cycles,

  (ii) time added by pregnancy wastage.

c) gestation period of the next live birth.

The asymptotic fertility rate, p', has been defined as the inverse of mean birth interval.

A woman is susceptible to the risk of having a birth, but this risk is competitive with risks of several other events. To be able to bear a child at age A, she must remain fecund until the time of becoming pregnant, and must survive until age A. She is, therefore, subjected to the risks of mortality and sterility. In most societies, marriage, divorce, or widowhood greatly affect the probability of conceiving and consequently affect birth intervals.

1.1.2. Review of the Pertinent Literature

The distribution of birth intervals and related functions were initially derived by Henry under various models. Henry [1953] put forward a simple mathematical model to estimate natural fertility, i.e., fertility in the absence of deliberate measures to limit births. The inter-live birth interval depends upon:

(i) duration of pregnancy,

(ii) non-susceptible period following confinement,

(iii) fecundability of woman.

Under the assumptions of no fetal wastage, constant fecundability, and constant duration of non-susceptible period associated with a live birth, he showed that the instantaneous birth rate, $B(x)$, approaches the asymptotic fertility rate, p', with increasing duration of marriage, where p' is the inverse of the mean birth interval. This result is the same as the one obtained in renewal theory. Expression for the expectation of number of births to a woman in x years of marriage were also derived. Similar expressions for a population, where fecundability and non-susceptible periods were allowed to vary, were also obtained.

An extension of the above model included fetal losses, and non-susceptible periods associated with a pregnancy varying with the outcome of pregnancy (Henry [1957]). Fecundability, p, was taken as a function of age in non-contraceptive society.

Perrin and Sheps [1964] introduced the notion of treating reproductive processes as Markov renewal processes under certain limiting conditions, and derived the distribution and moments of birth intervals under these models.

Potter [1969], using the basic model of Perrin and Sheps, obtained expressions for number of births averted when contraception is used right after a birth and practiced without interruption. Using two groups of women, identical in all respects, except that one group used contraceptives, while the other did not, he concluded that if a contraceptive user using Y_1 switches to Y_2 within the same pregnancy interval, rather than stopping use of Y_1 and replacing it with Y_2 after the next pregnancy, the resultant number of births averted is greater. Also, a single abortion has small impact, unless it is coupled with contraception.

Potter, in association with Parker [1964], presented a waiting time model to study sterility and conception delays. They assumed constant fecundability for a couple, a Beta distribution for fecundability, and a large number of couples. They also considered the validity of the assumption that the next conception delay would be as long as the last one, and the manner in which a mother's history of previous abortions would modify the wait for a live birth after discontinuance of contraception. Using Princeton Fertility Study data, the model for predicting the delay of second conception (S) as a function of first conception delay (F) gave the following expression using a linear regression analysis

$$S = 3.5 + 0.36F$$

Next, the assumption that 100 pregnancies yield 10 third-month miscarriages, 2 stillbirths, and 88 live births led to the conclusion that the average time added by reported pregnancy wastage was surprisingly small (between 1 and 3.5 months).

Sheps [1964], writing in same issue of Population Studies which contained Potter and Parker's [1964] article about waiting time, tackled the questions of variations in the monthly conception incidence, expected distributions of waiting time to conception and the underlying distribution of fecundability. She used the same assumptions as Potter and Parker, except that instead of a Beta distribution for fecundability, she used an unspecified density function f(p). She concluded that in a heterogeneous population with constant p for any couple, the proportion of first conceptions tends to fall during each successive month of exposure. For assessing contraceptive effectiveness, therefore, a life table approach is better than the number of conceptions per 100 years of use, since the latter may have very different expectations of conception rates depending upon the mixture of long and short period of observations.

Wolfers [1968] differentiated between the concepts of birth intervals (births) and birth intervals (women). He defined intervals (births) as the weighted mean interval, where the weights are number of births to a woman, and the women are represented according to the number of births to them. This is the usual way any arithmetic mean is computed. On the other hand, in birth intervals (women), each woman in the defined reproducing population is represented equally, regardless of number of births to them.

Let i = length of birth interval

and f_i = frequency of birth intervals of length i.

Then,

$$\text{mean birth interval (births)} = \sum_i i f_i / \sum_i f_i$$

Mean birth interval (women) lies somewhere between

$$\frac{\sum_i i f_i}{\sum_i f_i} \quad \text{and} \quad \frac{\sum_i i^2 f_i}{\sum_i i f_i}$$

Wolfers contended that while the concept of mean birth intervals (births) was appropriate for studies of some aspect of fertility and birth prevention, the estimation of physiological determinants of birth intervals requires use of mean birth intervals (women).

Venkatacharya [1969a] investigated the effects of short marital durations on live birth intervals. The bias, which is the difference between the mean live birth interval estimated retrospectively from birth histories of a group of women observed at a point of time and the mean live birth interval in the population at the end of reproductive period for the i^{th} parity, i = 0,1,2,..., is important in the context of changing fertility. He showed that these biases are not necessarily small, and are serious in low fertility situations.

Srinivasan [1966] assumed a Beta distribution for fecundability, constant fecundability for a woman over time, and no sterility. Using the Gandhigram data, he observed that expected frequencies under given models for parities 2 and 3 were consistent with observed frequencies.

Later, he extended the above model to include open intervals [1967], assuming the open interval (until survey point) to be a random part of the closed interval. Based on this assumption, he obtained expressions for the first two moments of open intervals.

Leridon [1969] debated the above assumption that distribution of a closed interval which includes a fixed point (survey point) is same as that of any closed interval. On the contrary, he showed that the longer the closed interval, more likely it is to be interrupted by the survey point. Therefore, the mean length of i^{th} interval, which include the survey point, is greater than the mean length of all i^{th} intervals.

Srinivasan [1966a] proposed that the open birth interval be considered as a valid, sensitive and easily applicable index of fertility. Venkatacharya [1969a], however, stressed that the open birth intervals should be standardized for marital duration and parity distribution.

Since the time for reproduction is finite, the length of closed birth intervals X_i, which hopefully provide information about the probability density function of χ_i, the length of the corresponding random interval if the reproductive period was infinite, form a truncated sample of χ_i's. Sheps _et al_. [1970c] discussed the truncation effect in detail, and concluded that birth intervals may be misleading as sensitive birth indices, and that whether the analyses of open intervals would be more informative than comparison of parity distribution is questionable.

A closed interval _straddles_ the survey date T if that interval includes T. All intervals beginning and ending between T and T + y, where y is predetermined are called _interior_ intervals. Henry [1961] used the concept of straddling and interior intervals to estimate the fertility of "subsequently fecund" women of a particular age group, where the age group is so chosen that a woman had at least one birth before the start of the interval, and the width of the age group is greater than longest birth interval, _i.e._, at least 5 years.

Potter _et al_. [1965] utilized the basic formula of Sheps and Perrin model [1964] to analyze data from Khanna Study, Punjab, India. They observed that the relative long mean birth interval of 31 months among Khanna Study women is attributable to long post-partum amenorrhoea, since birth control practices among them are negligible. Here, birth intervals were classified by age of the mother. Since birth intervals span a period of time, it is essential to know whether the age of mother is measured at beginning of an interval or at the end of the interval, because the resulting distributions in these two cases are likely to be different.

Hoem [1968], in a series of working papers, advanced probabilistic models using competing risks to estimate marital fertility using registration data. Retrospective fertility investigations give rise to another kind of function, which he denoted by "purged" measures. The model formulation was in terms of transition probabilities, and not in terms of survivorship functions, which, according to Hoem, are superfluous and even potentially harmful in that these could lead to misleading conclusions.

In a comprehensive review, Sheps, Menken, and Radick [1969] described the available literature of probability models for family building, and proposed a classification system according to the assumptions involved in them.

Sheps and Menken [1970d] discussed the duration variables under different methods of ascertainment and using very general forms of distributions for competing risks, derived their distributions for cohorts as well as stable and stationary populations. This paper is discussed in greater detail in Section 2.2.

1.2. Specification of the Problem

As explained in Section 1.1, an ascertainment plan refers to the manner of determining a birth interval. Following are some of the ways in which an i^{th} birth interval ($i=0,1,2,3,\ldots$) may be defined in a cohort population, where all the women are of the same age:

1. i^{th} intervals by age A

   a) i^{th} intervals ending before age A, and the open interval, U_i.

Let

X_i = random variable defining length of $(i-1)^{th}$ closed birth interval.

In this case, X_i depends upon the probability of survival until the $(i+1)^{th}$ birth, and also on the probability of survival from that birth to age A. Intervals such as these, which do not consider those who died before reaching age A, were named purged data by Hoem [1968].

   b) i^{th} intervals ending at age A. Here, the open interval, $U_i = 0$.

2. i^{th} intervals ending after age A.

   c) i^{th} intervals starting at age A.

   d) i^{th} birth occurring before age A and $(i+1)^{th}$ birth occurring after age A--straddling intervals (Henry [1961]).

   e) All intervals beginning or ending in age group (A,A+y), where y is predetermined--interior intervals.

Chart 1.1 illustrates the above intervals. The letters signify the ascertainment plans described above.

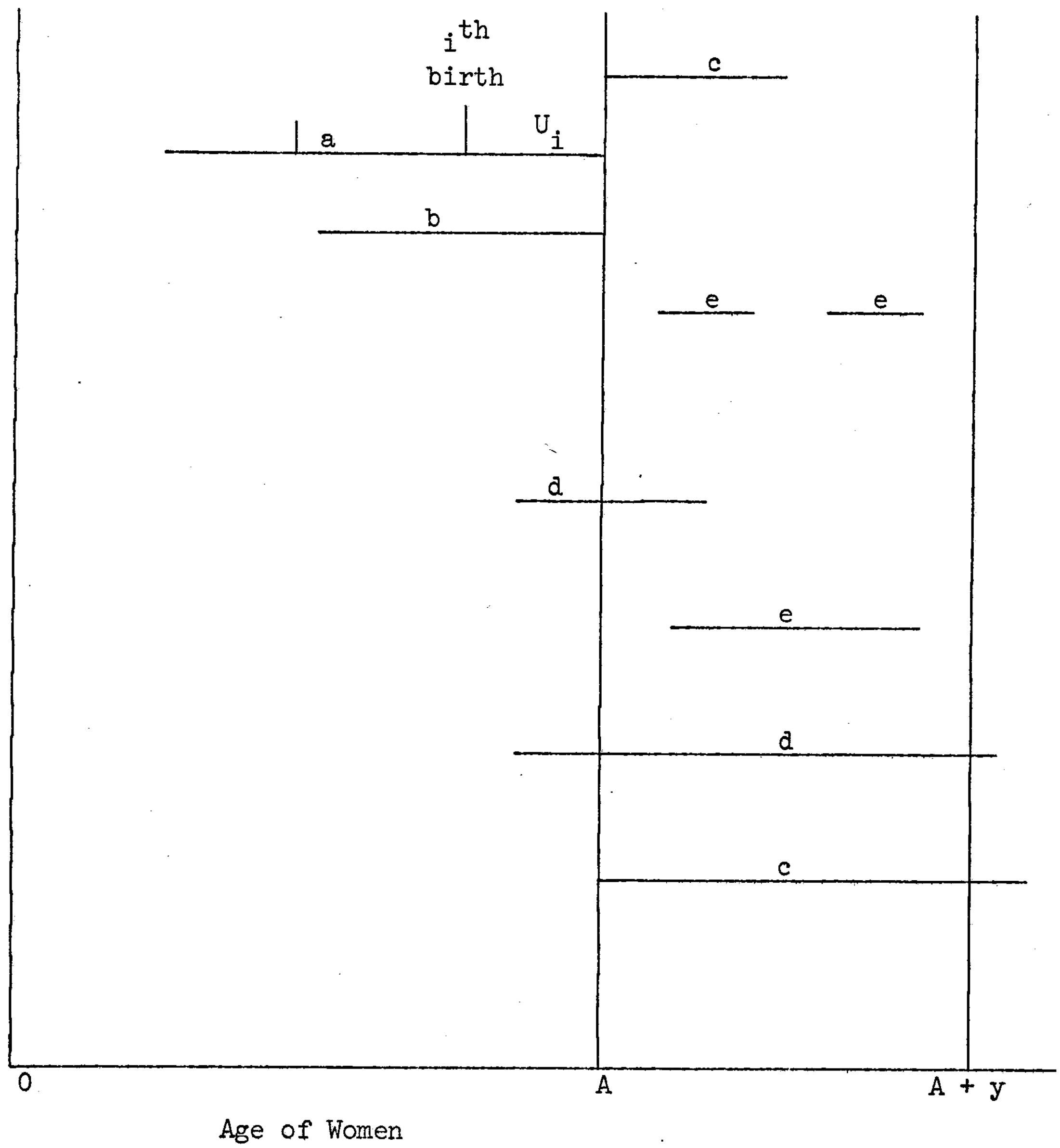

Chart 1.1. Birth Intervals in Different Ascertainment Plans

For cross-sectional populations, the term 'age A' can be replaced by the term 'time T' to define different ascertainment plans.

Since birth intervals will have different distributions under different ascertainment plans, the problem is to determine the "best" methods of ascertainment to produce indices which are

(i) sensitive to changes in fertility, and

(ii) robust to changes in other risks which do not directly affect fertility.

The term 'robust', here, is used to mean that a birth interval should not change if there are no changes in any risks which do not affect fertility. One may argue with the choice of this term, since 'robustness' has different meanings in different disciplines, but the author could not settle on a more suitable word.

Birth intervals should be compared with the more conventional indices to determine which are the most sensitive. This study, however, will be restricted to studying only the birth intervals.

In the present study, a fertility index will be called sensitive, if a change in fertility parameters is reflected by an early proportional change in the index. Since most indices would show changes over a sufficiently long period of time, sensitive indices should reflect these changes in the shortest period of time. A less than proportional change reflected by an index would not necessarily make that index insensitive, but we preferred an index with at least proportional change. Any index which showed consistent change (proportional or not), was examined as a potentially desirable index.

If, in a particular ascertainment of birth intervals,

μ_1 = mean length of any birth interval index without fertility change, and

μ_2 = mean length of that birth interval index when the theoretical input parameters are changed to increase the birth intervals by 25%, say,

then H_0: $\mu_2 \leq 1.25\ \mu_1$ was the null hypothesis tested against H_1: $\mu_2 > 1.25\ \mu_1$ in order to determine whether the fertility index under consideration was sensitive. A rejection of the null hypothesis lends credence to the sensitivity of the index.

A fertility index would be considered specific (robust), if it remained unchanged when the only risks changing, if any, were those not directly affecting fertility. If μ_a is the mean birth interval in a particular ascertainment under a certain set of parameters describing various competing risks (e.g., fertility, mortality, marriage, etc.), and μ_b is the mean birth interval under a different set of parameters, where the only differing parameters are those variables not directly affecting fertility, then H_0: $\mu_a = \mu_b$ was tested against the alternative H_1: $\mu_a \neq \mu_b$ to determine specificity of the index. Rejecting the null hypothesis H_0 would amount to the index being non-robust.

On the basis of these criteria, the 'best' ascertainment plans for tenth intervals could be recommended to be used for gauging the level of fertility.

1.3. Study Design

Initially in this research, investigation of the theoretical distributions of birth intervals under various ascertainment plans was attempted. These distributions were based upon certain assumed distributions for fertility, mortality and marriage. Finding that these distributions for birth intervals would involve numerous integrations as shown in Chapter II, the theoretical approach was abandoned. Alternatively, a simulation approach was adopted.

Samples of women from computer populations were subjected to different risks of mortality, fertility and marriage. Mean birth intervals resulting from the different ascertainment procedures, simulating both cohort and cross-sectional survey procedures, were then analyzed both for sensitivity and robustness.

A computer program, POPSIM, was used in tackling the problem. POPSIM requires an initial population for which vital events are generated for a specified period of time. Stable populations were used as the initial populations, here. A stable population, which is stable with respect to age, marital status, parity and duration since last birth, is defined and derived in Chapter III. One method to compute such a stable population is also described.

Chapter IV briefly describes the simulation program, POPSIM, used in this study, along with the program, SURVEY, which was utilized for analyzing the intervals generated in different simulated populations. POPSIM was modified in order to enable us to use different subroutines for generation of vital events. This chapter also deals with the estimation of fertility, mortality, marriage, and sterility parameters, which were necessary for use in POPSIM.

All sample populations consisted solely of females. In the subroutine generating date of marriage of a woman, we assumed that marriage could not occur before age 15 years. The first initial cohort population, therefore, was a population of 500 women which had all members aged 15 and married. Table 1.1 summarizes the cohort runs.

The second cohort of 500 women was identical to the first cohort except for the mortality schedule. The expectation of life at birth, e_o^o, was 60 years in this case as opposed to $e_o^o = 40$ years in first cohort in order to give an indication of the effect of mortality on birth intervals.

The time used by IBM computer model 360/75 for generating events to 500 women was not appreciable, and coupled with the fact that some of the ascertainment plans yielded only a handful of birth intervals, it was decided to increase the initial sample population size to 1,000 women. Cohort 3 consisted of 1,000 women, married and aged 15 years at initial time. This cohort was subjected to the same risks of vital events as cohort 1 until age 30. Next, the parameters for input birth intervals were increased by 25%, and the population of 872 women, who survived until age 30, was subjected to this reduced fertility through age 45 years. Comparing changes after age 30 with those in cohort 1 permits comparison of changes in fertility on birth intervals.

TABLE 1.1. DESCRIPTION OF COHORT POPULATIONS

POPN. NO.	INITIAL POPN. SIZE	AGE OF WOMEN AT START OF SIMULATION (in years)	MARITAL STATUS AT START OF SIMULATION	e_o^o (in years)	SIMULATION PERIOD (in years)	INPUT BIRTH INTERVALS	EVENTS GENERATED
1	500	15	Married	40	30	As estimated in Chapter IV	Birth, Death
2	500	15	Married	60	30	"	"
3	1000	15	Married	40	15	"	"
4	872	30 (The initial population in this run is one obtained in run 3, after 15 years of simulation.)	Married	40	15	Increased by 25%	"
5	1000	15	Unmarried	40	30	Same as in Population No. 1	Birth, Death, Marriage

TABLE 1.2. COMPUTER CROSS-SECTIONAL POPULATIONS

RUN NO.	INITIAL POPN. SIZE	STARTING SURVEY DATE	SIMULATION PERIOD (in years)	INPUT BIRTH INTERVALS	POPN. SIZE AT END OF SIMULATION	BIRTHS GENERATED IN YRS. 30-45	e_o^o
1	501	0	30	As estimated in Chapter IV	1605	--	40
2	1605	30	15	"	2764	4046	40
3	1605	30	15	All input intervals increased by 25%	2200	2620	40
4	1605	30	15	All input intervals of order 5+ increased by 50%	2387	3244	40
5	1605	30	15	As in Run 1	3243	4118	60

CHAPTER II. DISTRIBUTIONS OF BIRTH INTERVALS

2.1. Introduction

This chapter deals with the theoretical distributions of birth intervals in a cohort, under different methods of ascertainment. Section 2.2 incorporates the discussion contained in the article by Sheps and [1970d]. It was necessary to provide specific distributions for forces of happenings for various vital events, and these distributions are developed in Section 2.3. Distributions of birth intervals are derived in Section 2.4.

The <u>forces of happenings</u> are explained now. Let the event E_j not occur to an eligible woman by time t, and let $\rho_j(t)$ be force of happening of event E_j at time t and let

$$P\,[E_j \text{ occurs between } t \text{ and } t+\Delta t] = \rho_j(t)\,\Delta t + o(\Delta t)$$

$$P\,[E_j \text{ and } E_j',\ j \neq j' \text{ occur in } (t,t+\Delta t)] = o(\Delta t)$$

$$P\,[\text{none of the events } E_j,\ j = 1,2,..k, \text{ occurs in } (t,t+\Delta t)]$$
$$= 1 - \sum \rho_j(t)\,\Delta t + o(\Delta t)$$

where $\quad o(\Delta t)\,/\,\Delta t \to o$ as $\Delta t \to o$

Let $\quad \gamma_j(t)$ = unconditional probability that E_j occurs between t and t+dt

then,

$$\gamma_j(t) = \rho_j(t) \int_t^\infty \gamma_j(x)dx$$

Let $$P_j(t) = \int_t^\infty \gamma_j(x)dx$$

then, $$\rho_j(t) = -\frac{d}{dt}\log P_j(t)$$

2.2. Distributions in a Cohort

Notation

X_i = random variable defining length of $(i-1)^{th}$ birth interval.

$\phi_i(x,a)$ = risk of an i^{th} birth at age $(a+x)$ to a married woman who had an $(i-1)^{th}$ birth at age a.

$F_i(a,b)$ = probability that no birth occurs to a woman between age (a,b) given $(i-1)^{th}$ birth at age a

$$= \exp\left[-\int_0^{b-a}\phi_i(x,a)dx\right]$$

$\rho_L(a)$ = conditional risk of death at age a, given that death has not occurred before age a.

$\rho_W(a)$ = conditional risk of widowhood or divorce to a married woman at age a.

$P_L(a,b)$ = probability of survival from age a to b

$$= \exp\left[-\int_a^b \rho_L(\tau)d\tau\right]$$

$P_W(a,b)$ = probability that a woman does not get divorced or widowed in age group (a,b)

$$= \exp\left[-\int_a^b \rho_W(\tau)d\tau\right]$$

Let

$$g_i(x,a) = \text{conditional density of an } i^{th} \text{ birth at age } (a+x), \text{ given } (i-1)^{th} \text{ birth at age } a$$

$$= \phi_i(x,a)\, F_i(a,a+x)\, P_W(a,a+x)\, P_L(a,a+x) \qquad (2.1)$$

then, $w_i(a)$, density of an i^{th} birth at age a is given by

$$w_i(a) = \int_0^{a-15} g_i(x,a-x)\, w_{i-1}(a-x)dx \qquad (2.2)$$

where $w_o(a)$ = density of marriage at age a.

The unconditional density of an i^{th} birth at age (a+x) after an interval $X_i = x$ is, therefore given by

$$h_i(x,a) = g_i(x,a)\, w_{i-1}(a) \qquad (2.3)$$

which yields, from (2.2),

$$w_i(a) = \int_0^{a-15} h_i(x,a-x)dx \qquad (2.4)$$

The unconditional density of X_i is given by

$$h_i(x) = \int_{15}^{50-x} h_i(x,a)da = \int_{15+x}^{50} h_i(x,a-x)da \qquad (2.5)$$

Equations (2.4) and (2.5) assume no births occurring before age 15 or after age 50 years. Also, these densities relate to improper random variables, since the integrals of $h_i(x,a)$ and $h_i(x)$ do not equal 1. The probability density function of X_i for women, whose $(i-1)^{th}$ birth occurred at age a, and who have an i^{th} birth is, therefore, given by

$$\beta_i(x,a) = \frac{h_i(x,a)}{\int_0^{50-a} h_i(x,a)dx} \qquad (2.6)$$

while the p.d.f. for all women in the cohort who have an i^{th} birth is

$$\beta_i(x) = \frac{h_i(x)}{\int_0^{35} h_i(x)dx} = \frac{h_i(x)}{\int_{15}^{50} w_i(a)da} = \frac{h_i(x)}{w_i} \qquad (2.7)$$

where $w_i = \displaystyle\int_{15}^{50} w_i(a)da$

Since this considers all the intervals occurring to all women in the cohort, this ascertainment is prospective.

(i) Intervals that end at exact age b have the distribution given by

$$\bar{\beta}_i(x,b) = \frac{h_i(x,b-x)}{\int_0^{b-15} h_i(x,b-x)dx} = \frac{h_i(x,b-x)}{w_i(b)} \qquad (2.8)$$

which is different from $\beta_i(x,a)$ or $\beta_i(x)$ in eqs. (2.6) and (2.7).

(ii) Birth intervals can also be ascertained at the end of the intervals. The p.d.f. of birth intervals for such ascertainment can be given by

$$\bar{\beta}_i(x) = \frac{\int_{15+x}^{50} h_i(x,b-x)db}{\int_0^{35}\int_{15+x}^{50} h_i(x,b-x)dbdx}$$

$$= \frac{\int_{15}^{50-x} h_i(x,a)da}{w_i}$$

$$= \frac{h_i(x)}{w_i} = \beta_i(x) \qquad (2.9)$$

The reason $\overline{\beta}_i(x)$ and $\beta_i(x)$ in (2.7) are same is that both ascertainments measure the same intervals, and there is no loss of any women due to any competing risks. The same cannot be said about $\overline{\beta}_i(x,b)$ and $\beta_i(x,a)$, since $\overline{\beta}_i(x,b)$ gives the intervals which end at age b and $\beta_i(x,a)$ is the p.d.f. of those intervals which <u>begin</u> at age a, and, therefore are different. In case of $\overline{\beta}_i(x,b)$, too, there is no loss of women due to any other competing risks.

(iii) Interval X_i for surviving married women who had an i^{th} birth at age $a + x < b$ has the p.d.f.

$$\overline{\Theta}_i(x,b) = \frac{\int_{15}^{b-x} h_i(x,a)\, P_L(a+x,b)\, P_W(a+x,b)da}{\int_{15}^{b} w_i(a)\, P_L(a,b)\, P_W(a,b)da} \qquad (2.10)$$

The data governed by above are purged data, because the women have suffered attrition through death and dissolution of marriage between age (a+x) and b. The probability of survival decreases with increase in b - (a+x).

(iv) In above case, $U_{i+1} = b - (a+X_i)$ will be described as open interval measured at age b. The distribution of U_i at age b is given by

$$\psi_i(u,b) = \frac{w_{i-1}(b-u)\ P_L(b-u,b)\ P_W(b-u,b)\ F_i(b-u,b)}{{}_0\int^{b-15} w_{i-1}(b-u)\ P_L(b-u,b)\ P_W(b-u,b)\ F_i(b-u,b)du} \tag{2.11}$$

$$= \frac{R_i(u,b)}{\pi_{i-1}(b)} \quad , \quad i \geq 1 \tag{2.12}$$

where $R_i(u,b)$ and $\pi_{i-1}(b)$ are given by the numerator and denominator of $\psi_i(u,b)$, respectively.

(v) The p.d.f. of all open intervals at age b, regardless of order, is a weighted average of $\psi_i(u,b)$, and is given by

$$\psi(u,b) = \frac{\sum_{i=1}^{\infty} \pi_{i-1}(b)\ \psi_i(u,b)}{\sum_{i=0}^{\infty} \pi_i(b)} \tag{2.13}$$

$$= \frac{\sum_{i=1}^{\infty} R_i(u,b)}{{}_{15}\int^{b} w_o(a)\ P_L(a,b)\ P_W(a,b)da}$$

$$= \frac{\sum_{i=1}^{\infty} R_i(u,b)}{\Delta_A(b)} \tag{2.14}$$

where $\Delta_A(b) = \int_{15}^{b} w_o(a)\, P_L(a,b)\, P_W(a,b)da$

= unconditional risk of marrying before or at age b and surviving in the married state until age b.

(vi) The most recent interval X_i for women of exact parity i at age b has the density of an $(i-1)^{th}$ birth at age a, and i^{th} birth at age $a + x < b$, multiplied by the probability of no births, no widowhood and no death between ages $(a+x,b)$. The p.d.f. is, therefore, given by

$$\bar{\varepsilon}_i(x,b) = \frac{\int_{15}^{b-x} h_i(x,a)\, P_L(a+x,b)\, P_W(a+x,b)\, F_{i+1}(a+x,b)da}{\pi_i(b)} \tag{2.15}$$

(vii) An interval which includes a certain point of time is said to straddle that point (Henry [1961]). An i^{th} interval, which straddles age b and is of length x, has the p.d.f.

$$\Omega_i(x,b) = \frac{\int_{b-x}^{b} h_i(x,a)da}{\int_{0}^{b+y-15} \int_{b-x}^{b} h_i(x,a)da} \tag{2.16}$$

In a cohort, a point of time and age of the cohort are interchangeable.

If $h_i(x) = h(x)$ for all i and if all women survive, remain married and fertile, and if the assumptions of a renewal process hold, the p.d.f. of intervals that straddle age b, regardless of order, is given by

$$\Omega(x) = \frac{h(x) \; {}_{b-x}\int^{b} da}{{}_0\int^{b+y-15} h(x) \; [{}_{b-x}\int^{b} da]dx}$$

$$= \frac{x\, h(x)}{E(x)} \qquad (2.17)$$

an expression obtained earlier by Henry [1961].

(viii) An interval interior to age group $(\alpha,\alpha+\beta)$ is defined as one which is contained completely in $(\alpha,\alpha+\beta)$, i.e., which begins and ends in age group $(\alpha,\alpha+\beta)$. An i^{th} interval interior to $(b,b+y)$ has p.d.f.

$$r_i(x) = \frac{{}_0\int^{y-x} h_i(x,b+v)dv}{{}_0\int^{y} {}_0\int^{y-x} h_i(x,b+v)dv\; dx} \qquad (2.18)$$

On assumptions made to obtain (2.17), we get

$$r(x) = \frac{(y-x)\; h(x)}{y - E(x)} \qquad (2.19)$$

2.3. Specific Distributions for Risks of Events

In order to investigate the distributions of birth intervals under various ascertainment plans, it was necessary to assume some distributions for different competing risk. A combination of two Weibull density functions appeared to be an appropriate choice for risk of mortality, since the hazard rate in a Weibull distribution is monotonically increasing for $\alpha > 1$, and is unbound. Since the age-specific mortality rates are usually lowest in age group (10-14) years, it seemed reasonable to assume the cut-off point for the first Weibull distribution at age 15 years. The second part of the mortality curve may be given by another Weibull distribution truncated at age ω, where ω is the maximum age at death.

From similar considerations of hazard rates, it appeared reasonable to assume a lognormal distribution for age at marriage, and a gamma distribution for births.

2.3.1. Distributions for Mortality

As described above, mortality was assumed to have a combination of two Weibull distributions, one truncated at age 15 and the other at age ω. Later, we shall assume the age at marriage not to be less than 15 years, in which case, the first part of the mortality curve does not involve any other competing risks (i.e., marriage, births, and widowhood, etc.), and the only part we are interested in is the survival of a woman to age 15.

The density of death at age $t \geq 15$, therefore, is given by

$$f_1(t) = \frac{\lambda_1 \; \alpha_1 (t-15)^{\alpha_1 - 1} \; e^{-\lambda_1 (t-15)^{\alpha_1}}}{1 - \exp\,[-\,\lambda_1 (\omega-15)^{\lambda_1}]}, \; 15 \leq t \leq \omega; \; \alpha_1, \; \lambda_1 \geq 0$$

$$= 0 \quad \text{elsewhere} \tag{2.20}$$

For widowhood, let the mortality curve of husbands be given by

$$f_2(t) = \frac{\lambda_2 \; \alpha_2 (t-15)^{\alpha_2 - 1} \; \exp\,[-\,\lambda_2 (t-15)^{\alpha_2}]}{1 - \exp\,[-\,\lambda_2 (\omega-15)^{\alpha_2}]}, \; 15 \leq t \leq \omega; \; \alpha_2, \; \lambda_2 \geq 0 \tag{2.21}$$

Let $\rho_L(t)$ = conditional probability of death at age t, and,

$P_L(t_1, t_2)$ = probability of survival from age t_1 to age t_2; $t_2 \geq t_1$

$$= \exp \left[- \int_{t_1}^{t_2} \rho_L(t)dt \right] \tag{2.22}$$

Then,

$$f_1(t) = \rho_L(t) \; P_L(15,t) \tag{2.23}$$

$$= \rho_L(t) \int_t^{\omega} f_1(x)dx$$

$$= \rho_L(t) \; \frac{e^{-\lambda_1 (t-1t)^{\alpha_1}} - e^{-\lambda_1 (\omega-15)^{\alpha_1}}}{1 - \exp\,[-\,\lambda_1 (\omega-15)^{\alpha_1}]}$$

which yields

$$\rho_L(t) = \frac{\lambda_1 \alpha_1 (t-15)^{\alpha_1 - 1} e^{-\lambda_1 (t-15)^{\alpha_1}}}{\exp[-\lambda_1 (t-15)^{\alpha_1}] - \exp[-\lambda_1 (\omega-15)^{\alpha_1}]} \tag{2.24}$$

Let $\quad c_i = \exp[-\lambda_i(\omega-15)^{\alpha_i}], \; i = 1,2$

$\quad\quad = \text{independent of } t$

Then,

$$\int_a^b \rho_L(t)dt = -\log\left[\exp\{-\lambda_1(t-15)^{\alpha_1}\} - c_1\right] \Big|_a^b$$

$$= -\log \left[\frac{e^{-\lambda_1(b-15)^{\alpha_1}} - c_1}{e^{-\lambda_1(a-15)^{\alpha_1}} - c_1}\right] \tag{2.25}$$

Therefore,

$$P_L(a,b) = e^{-\int_a^b \rho_L(t)dt}$$

$$= \frac{\exp[-\lambda_1(b-15)^{\alpha_1}] - c_1}{\exp[-\lambda_1(a-15)^{\alpha_1}] - c_1}, \quad 15 \le a \le b \le \omega \tag{2.26}$$

2.3.2. Distribution for Marriage

As discussed earlier, a lognormal distribution was used for age at marriage. The earliest age at marriage was assumed to be 15 years, and since the fertile age is assumed to end at age 45 years, marriage is of no significant interest after age 45, in this chapter.

Age at marriage, therefore, has the density

$$g(x) = \frac{\dfrac{1}{(x-15)\sqrt{2\pi\eta}} \exp\left[-\dfrac{\{\log \theta(x-15)\}^2}{2\eta}\right]}{\Phi\left(\dfrac{\log 30\theta}{\sqrt{\eta}}\right)}, \quad 15 \leq x \leq 45, \quad \eta > 0 \qquad (2.27)$$

where

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp\left(-\frac{u^2}{2}\right)du \qquad (2.28)$$

The p.d.f. of (2.27) assumes that every woman gets married unless she dies before age of marriage.

Let $\rho_S(x)$ = risk of first marriage at age x

and $P_S(a,b)$ = probability that marriage does not occur between ages a and b.

$$= \exp\left[-\int_a^b \rho_S(u)du\right] \qquad (2.29)$$

Then, we have

$$g(x) = \rho_S(x)\, P_S(x) \qquad (2.30)$$

which yields

$$\rho_S(x) = \frac{g(x)}{1 - \Phi\left[\dfrac{\log \theta(x-15)}{\sqrt{\eta}}\right]} \qquad (2.31)$$

and $$\rho_S(x) = 1 - \Phi\left[\frac{\log \theta(x-15)}{\sqrt{\eta}}\right] \qquad (2.32)$$

2.3.3. Distribution for Births

If L_i is the longest of all i^{th} intervals, the density of $X_i = x$ to an eligible married woman is given by

$$k_i(x) = \frac{1}{\Gamma(\varepsilon_i)\,\beta_i^{\varepsilon_i}} \; \frac{x^{\varepsilon_i - 1}\, e^{x/\beta_i}}{G(L_i)}, \quad \begin{array}{l} \varepsilon_i, \beta_i > 0 \\ 0 < x < L_i < 30 \end{array}$$

$$= 0 \text{ , elsewhere} \tag{2.33}$$

where,

$$G(L_i) = \frac{1}{\Gamma(\varepsilon_i)\,\beta_i^{\varepsilon}} \int_0^{L_i} x^{\varepsilon_i - 1}\, e^{x/\beta_i} \tag{2.34}$$

2.4. Derived Distributions of Birth Intervals Under Assumed Risks

We shall use the same notations used in Section 2.1.

The density of an i^{th} birth at age t is given by

$$w_i(t) = \int_0^{t-15} w_{i-1}(t-x)\, k_i(x)\, P_D(t-x,t)dx \tag{2.35}$$

where $w_o(t) = g(t)$ = density of marriage at age t.

Let,

$S_i(t)$ = proportion of i^{th} order births among all births to women at age t.

Then, the density of a birth at age t, regardless of order, is given by

$$w(t) = \sum_i w_i(t)\, S_i(t) \tag{2.36}$$

Comparing eq. (2.35) with (2.2), we find that

$$k_i(x) = \phi_i(x,t)\ F_i(t,t+x) \tag{2.37}$$

Now, we have, from eq. (2.27),

$$w_o(t) = g(t) = \frac{\dfrac{1}{(t-15)\sqrt{2\pi\eta}}\ \exp\left[-\dfrac{\{\log\,\theta(t-15)\}^2}{2\eta}\right]}{\Phi\left(\dfrac{\log\,30\theta}{\sqrt{\eta}}\right)}$$

Let

$P_D(t-x,t) = P_W(t-x,t)\ P_2(t-x,t)$ = probability of survival in married state until age t.

Then, we get, from eq. (2.26),

$$P_D(t-x,t) = \frac{\left(e^{-\lambda_1(t-15)^{\alpha_1}} - c_1\right)\left(e^{-\lambda_2(t-15)^{\alpha_2}} - c_2\right)}{\left(e^{-\lambda_1(t-x-15)^{\alpha_1}} - c_1\right)\left(e^{-\lambda_2(t-x-15)^{\alpha_2}} - c_2\right)} \tag{2.38}$$

where $c_i = e^{-\lambda_i(\ -15)^{\alpha_i}}$, independent of x (2.39)

This yields,

$$w_1(t) = A_1(t)\int_0^{t-15}\left[\frac{x^{\varepsilon_1-1}}{t-x-15}\ e^{-\frac{\{\log\,\theta(t-x-15)\}^2}{2\eta}+\frac{x}{\beta}}\right] \times\left[\frac{\left(e^{-\lambda_1(t-15)^{\alpha_1}} - c_1\right)\left(e^{-\lambda_2(t-15)^{\alpha_2}} - c_2\right)}{\left(e^{-\lambda_1(t-x-15)^{\alpha_1}} -c_1\right)\left(e^{-\lambda_2(t-x-15)^{\alpha_2}} -c_2\right)}\right]dx \tag{2.40}$$

where

$$A_1(t) = \left[\frac{\Gamma(\varepsilon_1)\ \beta_1^{\varepsilon_1}\ G(L_1)\ \Phi\left(\frac{\log\ 30\theta}{\sqrt{\eta}}\right)}{\sqrt{2\pi\eta}}\right]^{-1} \qquad (2.41)$$

The integration of eq. (2.40) is difficult. Coupled with the fact that $w_i(t)$ can be obtained only after i such integration, it appears to be a formidable job to be handled theoretically. Once the integration and the exact form of $w_i(t)$ is obtained, however, distributions of birth intervals under various ascertainment plans may be obtained as in eqs. (2.6) through (2.19).

Numerical approximation to the integral in eq. (2.40) can be obtained to estimate $w_1(t)$. To obtain expressions for $w_i(t)$ in eq. (2.35), however, more integrations and most probably more approximations would be necessary. To get the distribution of birth intervals under any ascertainment plan would require further approximations. In view of the difficulties involved in getting the theoretical distributions of birth intervals, it was decided to use a simulation approach instead. This simulation approach using a computer program, POPSIM, is described briefly in Chapter IV.

CHAPTER III. STABLE POPULATION

3.1. Introduction

Sheps and Menken [1970] have obtained theoretical distributions of birth intervals for stable populations, where the populations are stable with respect to age, marital status, parity and duration since last birth. For these results to be valid, the existence of such a stable population is essential. This chapter deals with the derivation of proof of the existence of this stability, and one method of obtaining such a stable population is outlined. Inasmuch as the cross-sectional populations in this study use as initial population an input population which has been subjected to constant schedules of fertility, mortality, marriage and sterility for a long period of time, the existence of this kind of stability will be of considerable interest.

3.2. A Special Kind of Stable Population

3.2.1. Definition

Let $c(x,t)$ = proportion of persons aged x at time t in a population.

If $c(x,t) = c(x)$ for all t, and if the age-specific mortality rates are constant over time, then such a population is called a stable population (Lotka [1939]). A stable population can be obtained by exposing any population to constant age-specific fertility and mortality rates for a sufficiently long period of time. Let $c(x)$ be the resultant proportional age distribution.

$$\int_0^{\omega} c(x)dx = 1 \tag{3.1}$$

This will be independent of the initial age distribution of the population, though the time taken by a population to reach stability is independent upon it. A stable population has constant birth and death rates, and hence, a constant rate of growth, which is called the 'intrinsic' rate of growth.

As an extension of Lotka's concept of stable population, Sheps and Menken defined another kind of stable population, given below. The proof of existence of such a population was derived in this study.

Definition:

Let (x,m,i,y) female = a female aged x years, with marital status m, parity i and duration y since last birth,

and $N_t(x,m,i,y)$ = number of (x,m,i,y) females at time t.

A population would be considered stable, if

$$\frac{N_t(x,m,i,y)}{\int_X \int_Y \sum_i \sum_m N_t(x,m,i,y)dxdy} = c(x,m,i,y) \tag{3.2}$$

is independent of time t, and if the mortality rates (specific for x,m,i and y) are constant over time.

3.2.2. Proof of Existence of Stable Population

Let marital status, m, be equal to 1 for single persons, 2 for currently married and 3 for widowed or divorced and let

$q(x,m,i,y)dx + o(x)$ = probability of an (x,m,i,y) woman dying in age interval (x,x+dx)

where $$\lim_{x \to o} \frac{o(x)}{x} = 0 .$$

Also, let

$b(x,2,i,y)dx + o(x)$ = probability of a birth to an (x,2,i,y) woman in age interval (x,x+dx)

$\pi_{m_1 m_2}(x,i,y)dx + o(x)$ = probability that an (x,m_1,i,y) woman becomes $(x+dx,m_2,i,y+dx)$ woman in age interval (x,x+dx)

$B(t)$ = no. of female births at time t

$= N_t(0,1,0,0)$

I_{iz} = probability that a woman has i^{th} birth after exact marriage duration z.

Assumptions:

(i) birth, death and marriage rates are independent of time.

(ii) no remarriages of widows or divorcees.

(iii) no premarital pregnancies.

(iv) no marriage before age 15 years.

(v) all vital events are mutually independent.

(vi) multiple births are considered as one birth.

(vii) births can occur to married women only. Births to divorced or widowed women shortly after such events are also excluded.

(viii) survival probabilities are independent of marital status, parity and parity duration.

(ix) birth probabilities depend upon age only, *i.e.*, $b(x,2,i,y) = b(x,y)$

(x) parity duration is counted only up to age 45, for married women, and until dissolution of marriage for divorcees and widows.

Most of the assumptions can be relaxed, but it will result in a more complicated, though similar, proof.

Now,

The number of single women in age interval $(x,x+dx)$ at time $t = N_t(x,1,0,0)dx$

The number of married women aged $(x,x+dx)$ at time $t = \int_0^{30} \sum_i N_t(x,2,i,y)dydx$

and, number of widowed or divorced women of age (x,x+dx) at

$$\text{time } t = \int_0^{x-15} \sum_i N_t(x,3,i,y)dy\ dx \qquad \text{for } x > 45$$

$$= \int_0^{30} \sum_i N_t(x,3,i,y)dy\ dx \qquad \text{for } x \geq 45$$

Now,

$$N_t(x,1,0,0) = B(t-x)\ p(x)\ e^{-\int_0^x \pi_{12}(u)du} \tag{3.3}$$

where $\qquad p(x)$ = probability of survival from birth to age x.

$$N_t(x,2,i,y) = B(t-x)\ p(x) \int_0^{x-15} \left\{ \pi_{12}(x-z)\ I_{i,(z-y)} \times \exp\left[-\int_0^y b(x-v,v)dv - \int_{x-z}^x \pi_{23}(u)du \right] \right\} dz \tag{3.4}$$

where, $\qquad x-z$ = age at marriage

and,

$$N_t(x,3,i,y) = B(t-x)\ p(x) \left\{ \int_0^{x-15} \int_0^{z_1} \pi_{12}(x-z_1)\ \pi_{23}(x-z_1+z_2)\ I_i(z_2-y) \right\} dz_1\ dz_2 \tag{3.5}$$

where, z_1 = duration since marriage

and, z_2 = duration of marriage (till getting widowed or divorced)

Let i_{max} = maximum parity of a woman.

Then,

$B(t)$ = number of female births in year t

$$= \int_0^{\omega} \int_0^{30} \sum_{i=0}^{i_{max}} N_t(x,2,i,y)\, b(x,y)dy\, dx \tag{3.6}$$

$$= \int_0^{\omega} \int_0^{30} \left\{ B(t-x)\, p(x) \sum_{i=0}^{i_{max}} \int_0^{x-15} \pi_{12}(x-z)\, I_{i,z-y} \times e^{-\int_0^{y} b(x-v,v)dv}\; e^{-\int_{x-z}^{x} \pi_{23}(u)du}\, dz \right\} dy\, dx$$

$$= \int_0^{\omega} B(t-x)\, p(x)\, R(x)dx \tag{3.7}$$

where

$$R(x) = \sum_{i=0}^{i_{max}} \int_0^{30} \int_0^{x-15} \pi_{12}(x-z)\, I_i,(z-y) \times e^{-\int_0^{v} b(x-v,v)dv - \int_{x-z}^{x} \pi_{23}(u)du}\, dz\, dy \tag{3.8}$$

$p(x)$ and $R(x)$ are non-negative for all x.

To solve equation (3.7), let the trial solution be

$$B(t) = \sum_{n=1}^{\infty} Q_n e^{r_n t} \tag{3.9}$$

By substituting eq. (3.9) in eq. (3.7), this yields,

$$\sum_{n=1}^{\infty} Q_n e^{r_n t} = \int_0^{\infty} \sum_{n=1}^{\infty} Q_n e^{r_n (t-x)} p(x) R(x) dx$$

$$= \sum_{n=1}^{\infty} Q_n e^{r_n t} \int_0^{\infty} e^{-r_n x} p(x) R(x) dx$$

Therefore,

$$\int_0^{\infty} e^{-r_n x} p(x) R(x) dx = 1 \tag{3.10}$$

The limits of integral are 0 to ∞ instead of 0 to ω, since the integral vanishes between ω and $+ \infty$.

Since $\int_0^{\infty} e^{-r_n x} p(x) R(x) dx$ is $+ \infty$ for $r = - \infty$ and 0 for $r = + \infty$ and is a monotonic function of r, there is only one real root, r. In addition, there are infinitely many complex roots of equation (3.10). The co-efficients Q_n can be determined by initial conditions. Substitution of Q_n and r_n in equation (3.9) yields for

$B(t)$, a curve with damped oscillations. When t is very large, the oscillations become negligible, and $B(t)$ almost wholly relies on the real root r. At that stage, it is possible to write

$$B(t) = B(t-x)\, e^{rx} \tag{3.11}$$

since $$B(t-x) = Q_1\, e^{r(t-x)} = Q_1\, e^{rt}\, e^{-rx} = B(t)\, e^{-rx}$$

where Q_1 is the co-efficient corresponding to real root r.

Now, N_t = total no. of women at time t

$$= \int_0^{\omega} B(t-x)\, p(x)\, dx, \tag{3.12}$$

Therefore,

$$\frac{N_t(x,1,0,0)}{N_t} = \frac{B(t-x)\, p(x)\, e^{-\int_0^x \pi_{12}(u)\, du}}{\int_0^{\omega} B(t-x)\, p(x)\, dx}$$

$$= \frac{B(t)\, e^{-rx}\, p(x)\, e^{-\int_0^x \pi_{12}(u)\, du}}{\int_0^{\omega} e^{-rx}\, p(x)\, dx} \tag{3.13}$$

which is independent of t.

Similarly,

$$\frac{N_t(x,2,i,y)}{N_t} \text{ and } \frac{N_t(x,3,i,y)}{N_t} \text{ are independent of t for}$$

all x, i and y.

Therefore, the population is stable.

3.3. Derivation of Specifically Defined Stable Population

While the following method of obtaining a stable population holds for continuous as well as discrete time, we would assume time to be discrete for explaining this method.

Let X(t) be a vector, whose elements are number of people at each age, marital status, parity and parity duration (and any other characteristics to be stablized) at time t. Suppose the population is stable at time t = T.

$$\text{Let} \quad X(T) = (X_i(T)) \tag{3.14}$$

$$\text{and} \quad N(T) = \sum_i X_i(T) \tag{3.15}$$

Then

$$\frac{X_i(T)}{N(T)} = g_i \text{, constant for class i, and independent of time T.}$$

$$= \frac{X_i(T+1)}{N(T+1)}$$

Therefore, $$N(T+1)\, g_i = X_i(T+1) \tag{3.16}$$

Now, if r is the intrinsic rate of growth of this stable population,

$$N(T+1) = e^r\, N(T)$$

which yields, by substituting in eq. (3.16)

$$e^r\, N(T)\, g_i = X_i(T+1)$$

or, $$X_i(T+1) = e^r\, X_i(T) \tag{3.17}$$

Since eq. (3.17) holds for all i, it holds for the vector of these elements, too. Therefore,

$$X(T+1) = e^r\, X(T) \tag{3.18}$$

Let M be the matrix of hazard rates, i.e.

$$\underset{n\times 1}{X(T+1)} = \underset{n\times n}{M}\ \underset{n\times 1}{X(T)} \tag{3.19}$$

Then, $$M\, X(T) = e^r\, X(T) \tag{3.20}$$

Thus, X(T) is the characteristic vector corresponding to the latent root e^r of M. Since such a stable vector exists, M does have a latent root equal to e^r.

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be n characteristic roots of M. If λ_1 is the principal root, the absolute value of λ_1 will be greater than that of $\lambda_2, \lambda_3, \ldots, \lambda_n$. Then, the effect of λ_i's ($i \neq 1$) will be smaller with increasing time compared to λ_1^t, and when t is sufficiently large, we have

$$M\, X(T) = \lambda_1\, X(T) \quad , \quad \text{for } t = T.$$

If $M^t = (m_{ij}^{(t)})$, and if for all i and j

$$\frac{m_{ij}^{(t+1)}}{m_{ij}^{(t)}} = k, \text{ a constant}$$

then, $\lambda_1 = k$ is the principal root of matrix M. In most cases, t = 64 or t = 128 will be sufficient to achieve the desired stability.

If one desires stability with respect to age, marital status, parity and parity duration, the size of the matrix M would depend upon the no. of sub-divisions in each of these four charactaeristics. Even for a small number of sub-divisions, the size of M becomes prohibitive, to be manipulated easily. For example, 30 age groups, 3 marital status, 6 parities and 15 parity durations give rise to a square matrix M of order 30 × 3 × 6 × 15 (= 8100). Therefore, in order to investigate the distributions of birth intervals in a stable population, it is essential that we take a hypothetical situation with fewer sub-divisions and much smaller fertile age group. One such hypothetical population, chosen arbitrarity, is described below: Assumptions for the hypothetical population:

(i) fertile age group is limited to (15-19) years only.

(ii) earliest marriage age is 15 years.

(iii) maximum no. of births = 1 per year.

(iv) parity duration remains constant after end of fertile period or at widowhood, whichever comes first.

(v) all events occur at the beginning of the year.

(vi) births can occur only to married women.

(vii) probability of a birth to a married woman depends only upon age of marriage and interval since last birth, and is independent of parity.

(viii) no remarriages.

Using the following notation for marital status:

1 - single

2 - married

3 - widowed or divorced, and

$N_t(x,m,i,y)$ = number of women at time aged x, with marital status m, parity i and parity duration y.

Then, the stable vector, X(t), representing the population distribution at time t can be written as

$$X(t) = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \tag{3.21}$$

where,

$$\underset{8 \times 1}{A} = \begin{bmatrix} N_t(0,1,0,0) \\ N_t(1\text{-}13,1,0,0) \\ N_t(14,1,0,0) \\ N_t(15,1,0,0) \\ N_t(16,1,0,0) \\ N_t(17,1,0,0) \\ N_t(18,1,0,0) \\ N_t(19,1,0,0) \end{bmatrix}$$

$$\underset{3 \times 1}{C} = \begin{bmatrix} N_t(15+,3,0\text{-}4,0\text{-}4) \\ N_t(20+,2,0\text{-}4,0\text{-}4) \\ N_t(20+,1,0,0) \end{bmatrix}$$

and,

$$\underset{15 \times 1}{B} = \begin{bmatrix} N_t(15,2,0,0) \\ N_t(16,2,0\text{-}1,0) \\ N_t(17,2,0\text{-}2,0) \\ N_t(18,2,0\text{-}3,0) \\ N_t(19,2,0\text{-}4,0) \\ N_t(16,2,0,1) \\ N_t(17,2,0\text{-}1,1) \\ N_t(18,2,0\text{-}2,1) \\ N_t(19,2,0\text{-}3,1) \\ N_t(17,2,0,2) \\ N_t(18,2,0\text{-}1,2) \\ N_t(19,2,0\text{-}2,2) \\ N_t(18,2,0,3) \\ N_t(19,2,0\text{-}1,3) \\ N_t(19,2,0,4) \end{bmatrix}$$

The vectors A and C cannot give rise to a birth while vector B consists of women who are eligible to bear a child. Matrix M, in this case, would be of order 26 × 26. A similar abridged matrix for a population with fertile age group (15-44) years will be of order 472 × 472, and obviously unmanageable, in addition to the vast amount of data required for estimating the cell probabilities of matrix M. Subjecting a population to constant schedules of risks over a long period of time, therefore, offers a better choice of obtaining this stability. The time required to achieve this stability, of course, will depend upon the distribution of the initial population, and a near stable population distribution at initial time will lead to this stability over a relatively short duration of time.

Instead of using the matrix M, therefore, as the initial population in this study, an age-stable population was subjected to constant schedules of fertility, mortality, marriage and sterility for a long period of time. In view of the small number of people in the populations studied, however, it is difficult to say whether the differences in distributions of populations by age, marital status, etc., is because the population had not reached stability or because of random variations.

CHAPTER IV. POPSIM: DESCRIPTION, MODIFICATION AND ESTIMATION OF PARAMETERS

4.1. Introduction

As described in Chapter II, it was decided to use a simulation approach to study the distributions of birth intervals. The computer program for this purpose was a program named POPSIM (POPulation SIMulation), and this chapter describes it briefly along with the modifications and estimation of parameters for this study. POPSIM was developed jointly by the Research Triangle Institute and the Department of Biostatistics, UNC at Chapel Hill.

POPSIM is a demographic micro-simulation model, and is so designed that principal demographic events occurring to each member of a human population may be simulated on a computer. It is designed for a two-sex cross-sectional population, but its flexibility permits one sex cohort populations. The behavior of populations with a given hypothetical structure can be investigated as can be the behavior of a population possessing distributions and the risks of vital events very close to some existing population. POPSIM is designed as a tool to permit the investigation of responses of population to various risks.

POPSIM generates vital events for each individual in the population. It is stochastic because the events and their time of occurrence are determined by a random sampling of probability distributions. It allows the input probabilities to change with time and can incorporate a feedback mechanism.

The initial population[2] in POPSIM represents a random sample of individuals, without regard to any familial relationship. Therefore, not everyone in a family is a member of the computer population in this model. As an example, while a woman may be a member of the random sample, her husband may be excluded from this sample.

The POPSIM program consists of three main steps:

(i) initialization of population,

(ii) aging of initial population and generation of vital events over time, and

(iii) tabulation of number of events and population distribution over time.

The initial populations of the POPSIM program are generated by separate programs, while the generation of vital events

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[2] The term 'population' here, in fact, refers to a sample of individuals, who age and have the vital events generated to them over time. It seems more convenient to use the term 'computer population' describing a sample of individuals than the term 'computer sample'. These words, therefore, should be read in this context, here.

over time utilizes a common program which also carries out periodic tabulations of the events and of total population by various characteristics.

Complete histories of all vital events generated to each individual can be written on a tape, and a separate program called SURVEY (described later in this chapter) can be used to carry out tabulations from this history tape.

The open model, used at the University of North Carolina at Chapel Hill, here, will be described briefly, along with the program used in this study.

4.2. POPSIM: Open Model

4.2.1. Initial Population

The initial computer population ought to be as near to the desired population as possible. The economic feasibility of generating a very large population, as well as lack of complete data for all individuals place limitations on this objective. The approach used in POPSIM considers the initial population as a random sample from the desired population, with characteristics distributed as in the population under consideration.

Initial populations in the computer using POPSIM are created by using a series of sub-routines to assign age and sex to the members of population. Then, each member is subjected to all possible events which can occur to him/her from birth to initial time, excluding, of course, death, since these members are alive at the time of initialization.

Assignment of ages is accomplished by use of stratified random sampling. Size of the initial population to be generated, the proportion for each sex, and the proportions in specific (though arbitrary) age groups for each sex, are required as input data. The cumulative distribution function and its inverse are calculated for each of these age-sex classes.

The (0,1) interval is next divided into N intervals, where N is the total size of population to be generated, and then, a uniform random number for each of these N intervals is generated to sample the appropriate inverse. The same procedure is used to distribute the persons in an age group by sex.

The remaining characteristics are assigned to each individual as described above. For example, a female, aged 20 years at the time of initialization, has all possible events generated from time = -240 months to time = 0 (initial time is usually set as zero, though special considerations may necessitate a change in it). All the events, except death, are recorded for her. These events may include marriage, birth of one or more children, any spontaneous or induced abortions, use of contraceptives, and widowhood or divorce, etc.

4.2.2. Generation of Vital Events

After the initial population has been created, POPSIM can be used to generate vital events and histories. POPSIM has the flexibility to use any desired time period as simulation period in any specified intervals of time. For example, vital events can be generated for 25 years in steps of 1 year each, 2.5 years each, 5 years each, or 12.5 years each. The steps, however, cannot be longer than 15 years, though the events can be carried over, if so desired, if the steps are shorter than the time to a particular event. A request for a total simulation period of, say 25 years, at 6 years' interval, on the other hand, will be complied with a total simulation period of 30 years, instead of 25. In this study, events were generated for 15 or 30 years in 15-year steps.

Vital events sub-routines can be changed to one's specifications, if so desired.

To generate a date to next event for an individual, an event-sequenced simulation procedure is used. Dates for each competing event, which can occur to an individual, are generated and the event corresponding to the earliest date is deemed the next to occur to that particular individual. This event and its date is carried in the record for this individual. This procedure is correct under assumption that the input parameters are independent probabilities rather than crude rates.

The generation of date of an event is accomplished by the use of inverse of geometric distribution within a time period during which the monthly probabilities of the event remain constant. For example, let us consider a male exactly 21 years old, and let p_6 denote the monthly death probability, constant for male age group (20-24) years. Then, a random number r between (0,1) is generated and

$$t = \frac{\ln(1-r)}{\ln(1-p_6)}$$

gives the date of death of this male, provided $t \leq 48$ months. If $t > 48$, this man would die not earlier than age 25, and a new monthly probability p_7 (say) would be used to generate the date of his death, and if $t \leq 60$, then, date of death = (300+t) months. If $t > 60$, the next age group is used, and the process is repeated, until a date of death within the age group is generated.

4.2.3. Tabulations

POPSIM provides tabulations of the initial population as well as at the end of each "step" of simulated period. The rates (death, birth and marriage, etc.) are printed for the last year of each step at the end of simulation period, as well as after every 20 steps. A vital event history tape is written for each individual in the computer population, and this feature can be used for special tabulations or further analysis of the simulation data.

4.3. Changes Made in POPSIM for Use in this Study

The number of vital events which can occur to an individual was reduced in this study, which cut the data requirement to a minimum, and still served the purpose of analyzing birth intervals.

No males were included in this study. A woman could have only the following events occurring to her:

(i) death

(ii) marriage

(iii) births

(iv) sterility.

The sub-routines to generate date of an event were the same as in POPSIM with the exception of births. All conceptions were assumed to end in a live birth, unless the woman died. In addition, the interval to next birth was assumed to have a lognormal distribution with parameters depending upon parity of a woman. For all other events, date of an event was generated using inverse of the geometric distribution within a time period as in POPSIM used at the University of North Carolina at Chapel Hill.

The events were recorded on tape, and another program, SURVEY, was used to get desired tabulations of birth intervals.

4.4. SURVEY Program

SURVEY provides tabulations of intervals by duration and up to two more cross-variables. The tabulations span all the members of the population, or if desired, a sub-population (e.g., females only, a population aged 15 years and above on survey date, etc.) can be chosen for which any number of tables can be provided. SURVEY can provide tabulations for birth intervals, contraceptive segments, conception intervals and conception delays, and the type of intervals may include open, immediate past closed, straddling, all previous closed or i^{th} closed intervals. Means and standard deviations of intervals are also computed in these tabulations.

4.5. Estimation of Parameters for Generating Events

As described in Section 4.1, generation of events as well as an initial population with the desired demographic characteristics requires use of parameters in POPSIM program. A sample initial population of specified size (approximate size, because of random variations) had the following characteristics assigned to all women:

(i) age,

(ii) marital status,

(iii) age at marriage,

(iv) duration of marriage,

(v) number of children born,

(vi) age at sterility.

These characteristics were assigned by methods explained in Section 4.2.

4.6. Birth

Data for birth intervals by parity of women and age are scarce. Henry [1956] has, however, published some data for ancient Geneva families (16^{th}-20^{th} centuries), reconstructed from their histories. These data include the length of birth intervals for each woman by parity. It was decided to use these data for our purpose of simulating births among women.

A lognormal distribution was fitted to these birth intervals for each parity. Table 4.1 shows the computer chi-square value for the fit for each parity.

Thus, if X_i = length of i^{th} birth interval, then log X_i is approximately normally distributed with mean $(-\log \rho_i)$ and variance α_i. ρ_i and α_i were estimated using maximum likelihood estimation technique. A polynomial in parity was fitted to ρ_i and α_i and the results are shown in Table 4.2.

The fitted polynomials used in this study were:

$$\rho_i = .052567 - .011425\, i + .003269\, i^2$$
$$- .000420\, i^2 + .000019\, i^A \quad ;$$
$$R^2 = .948$$

and,

$$\alpha_i = .49830 - .85058\, i + .72855\, i^2 - .29005\, i^3$$
$$+ .05801\, i^4 - .005622\, i^5 + .0002095\, i^6$$

$$R^2 = .965$$

TABLE 4.1. COMPUTED χ^2 FOR GOODNESS OF FIT OF LOGNORMAL DISTRIBUTION TO BIRTH INTERVALS

PARITY	DEGREES OF FREEDOM	COMPUTED CHI-SQUARE	$\chi^2_{.05}$ (FROM TABLE)	NO. OF INTERVALS
0	6	12.12	12.59	74
1	5	9.51	11.10	67
2	7	3.28	14.10	66
3	5	9.12	11.10	54
4	7	13.22	14.10	48
5	6	6.46	12.59	40
6	6	7.65	12.59	34
7	7	8.00	14.10	25
8	4	5.11	9.49	17
9	1	0.60	3.84	8

TABLE 4.2. COMPARISON OF M.L.E. ESTIMATES OF ρ_i AND α_i WITH ESTIMATES FROM POLYNOMIAL

PARITY	M.L.E. ESTIMATES		POLYNOMIAL ESTIMATES	
	ρ_i	α_i	ρ_i	α_i
0	.527	.4994	.5257	.4983
1	.440	.1307	.4401	.1388
2	.390	.1785	.3974	.1526
3	.386	.1131	.3791	.1576
4	.366	.1839	.3714	.1414
5	.387	.1464	.3651	.1641
6	.337	.2059	.3554	.2110
7	.327	.2069	.3419	.1974
8	.353	.1189	.3291	.1231
9	.318	.3796	.3258	.3789

4.7. Monthly Probabilities of Death by Age

It was decided to use life table functions to estimate the monthly probabilities of death. The Regional Model Life Tables computed by Coale and Demeny [1966] were source of the two life tables used in this study, with $e_o^o = 40$ and $e_o^o = 60$, model West.

Let

ℓ_x = number of survivors at exact age x

and $\quad {}_{60}L_x$ = person years lived ages (x,x+60) months

$$= \int_x^{x+60} \ell_x \, dx \tag{4.1}$$

Then

Probability of survival from age group (x,x+48) months to age group (x+60,x+108)

$$= \frac{{}_{60}L_{x+60}}{{}_{60}L_x} \tag{4.2}$$

Assuming that the event of death has an exponential distribution with parameter δ within an age group, we have, the probability of survival

$$\int_{60}^{\infty} \delta e^{-\delta t} \, dt = e^{-60\delta} = \frac{{}_{60}L_{x+60}}{{}_{60}L_x} \tag{4.3}$$

which yields

$$\delta = -\frac{1}{60} \log \frac{60^{L_{x+60}}}{60^{L_x}} \qquad (4.4)$$

The Weibull distribution did not fit these monthly death probabilities well for ages greater than 15 years. Therefore, the monthly probabilities derived from Regional Model Life Tables were used in POPSIM.

4.8. Age at Sterility

A lognormal distribution for (50-age at sterility) was used for this simulation experiment. If Y is age at sterility, then log(50-Y) was deemed to be distributed normally with mean 1.9 and variance of 0.5.

4.9. Yearly Probabilities of Marriage

Conditional probabilities of first marriage were obtained from the Khanna Study, conducted in Khanna villages of Punjab, India.

Let C_x = cumulative number of marriages to age x.

and N = number of single women at age 15.

Then,

yearly probabilities of marriage in age (x,x+1)

$$= \frac{C_{x+1} - C_x}{N - C_x}$$

Table 4.3 gives the cumulative number of marriages for certain ages.

TABLE 4.3. NUMBER OF MARRIAGES

AGE x	CONDITIONAL PROB. OF FIRST MARRIAGE IN (x,x+4)	NO. OF UNMARRIED WOMEN AT AGE x	NO. OF MARRIAGES IN AGE (x,x+4)	CUMULATIVE TOTAL c_{x+5}
15	.864	10000	8640	8640
20	.904	1360	1230	9870
25	.154	130	20	9890
30	.091	110	10	9900
35	.000	100	0	9900
40	.000	100	0	9900
45	.000	100	0	9900

Source: Khanna Study, (1965)

Polynomials were fitted to the last column c_x in Table 4.3. Cumulative number of marriages at single years x, x+1, x+2, x+3, x+4 were estimated from these polynomials, and yearly probabilities of marriage computed.

One may wonder what the effect of these parameters would be on the ascertainment methods themselves, since these estimates are from different sources. However, for the purpose of our study, we believe that this effect would be minimal, because even though the sources of information, from which these estimates were derived, are different, these are congenial to each other. For example, the initial age distribution was so chosen from Regional Model Life Tables that it reflected the fertility level of Geneva women and the mortality level of $e_o^o = 40$ years. The marriage pattern of Khanna Study women gave rise to early marriages and high fertility levels which were not far from those of Geneva women.

CHAPTER V. SIMULATED POPULATIONS AND ANALYSIS OF BIRTH INTERVALS

5.1. Introduction

As described in Chapter I, a cross-sectional population is a weighted mixture of several cohort populations, the weights determined by the age-sex-marital status distribution of the desired cross-sectional population. To achieve a better understanding of various distributions of birth intervals under different ascertainment plans, it would, therefore, be helpful if these distributions were also determined for some sample cohort populations. It was, therefore, decided to investigate some cohort populations first, and use any experience gained in the analysis for designing the study of the cross-sectional populations.

At the risk of being repetitive, a brief description of the cohort and cross-sectional populations is given next.

All of the computer populations were restricted to female only. While the cross-sectional populations included all the primary members as well as the females born during the simulation period, the cohort populations consisted of only the primary members and the female births were not included.

5.1.1. Cohort Populations

An initial population of 500 women aged 15 and married were the members of first cohort. The sample size of 500 women was based upon the cost considerations of computer time based upon previous experience of POPSIM use. The second population was same as first cohort with the exception that the expectation of life at birth for the second cohort was 60 years as compared with 40 years in the first cohort. The next cohort was same as the first one, except that input birth intervals were increased by 25% after the women reached age 30 years, and that the size of initial population was 1000 women. The last cohort consisted of an initial population of 1000 women aged 15 years and single at the time of initialization. Table 5.1 contains the details of the cohort populations.

Since all members of an age-cohort are the same age, it is simpler to write a computer program to generate the initial population than to generate one at random, since the characteristics of the population such as age, marital status and fertility history of 15 year olds are not subject to random variation. Also, it is faster to read in a population than to generate it with its history. The initial populations in the cohorts, therefore, were read in with appropriate histories of the members of the populations.

5.1.2. Cross-Sectional Populations

For cross-sectional populations, it was decided to use the generate the initial population at random. In POPSIM, the simulated initial computer population is considered a random sample, representative of the parent population. To generate this random sample, age-sex distribution of the parent population is needed, and an appropriate stable population from model stable populations computed by Coale and Demeny was decided upon as the parent population.

TABLE 5.1. DESCRIPTION OF COHORT POPULATIONS

POPN. NO.	INITIAL POPN. SIZE	AGE OF WOMEN AT START OF SIMULATION (in years)	MARITAL STATUS AT START OF SIMULATION	e_o^o (in years)	SIMULATION PERIOD (in years)	INPUT BIRTH INTERVALS	EVENTS GENERATED
1	500	15	Married	40	30	As estimated in Chapter IV	Birth, Death
2	500	15	Married	60	30	"	"
3	1000	15	Married	40	15	"	"
4	872	30 (The initial population in this run is one obtained in run 3, after 15 years of simulation.)	Married	40	15	Increased by 25%	"
5	1000	15	Unmarried	40	30	Same as in Population No. 1	Birth, Death, Marriage

For $e_o^o = 40$, a death rate near 25 per 1000 population seemed reasonable. From the experience gained from generation of fertility histories of women in cohort populations, a rate of growth of about 3.5% per year was considered appropriate. The population selected will be considered in Section 5.4. For this population, we had

Death rate = 24.65 per 1000 population,

Birth rate = 59.65 per 1000 population, and

r = rate of growth = 3.5%

This initial population of 500 women was generated for a period of 30 years, with all female births becoming part of the population. The population thus generated became the initial population for each of the four cross-sectional populations generated for 15 years later. This initial population was called run 1. Run 2 generated vital events for members of population of run 1 for another 15 years without any change in parameters. Run 3 was similar to run 2, except for the level of fertility. The input parameters for this run were changed as to permit an increase in input birth intervals by 50%. The birth intervals obtained in this run were compared to those obtained in run 2 for assessing the effect of decrease in fertility on different ascertainment plans. Run 4 was similar to run 3, with the input birth intervals increased by 50% for only those women who had achieved parity 4 or more. Run 5 had all the parameters of run 2 except for $e_o^o = 60$ instead of $e_o^o = 40$ of run 2 to study the effect of decrease in the level of mortality on mean birth intervals obtained using different ascertainment plans. Table 5.2 gives the details about these cross-sectional populations.

5.2. Birth Intervals in Cohort Populations

The following birth interval ascertainment plans were analyzed for each cohort:

(a) all previous intervals ending by age A.

1. all previous intervals ending before age A.
2. open interval at age A.
3. an i^{th} interval ending at age A.

(b) all intervals ending after age A.

4. an i^{th} interval starting at age A.
5. i^{th} birth occurring before age A and $(i+1)^{th}$ birth occurring after age A--Straddling Interval.
6. all intervals beginning at age A, and ending before age 45--Interior Intervals.

(c) 7. immediately previous closed interval.

TABLE 5.2. COMPUTER CROSS-SECTIONAL POPULATIONS

RUN NO.	INITIAL POPN. SIZE	STARTING SURVEY DATE	SIMULATION PERIOD (in years)	INPUT BIRTH INTERVALS	POPN. SIZE AT END OF SIMULATION	BIRTHS GENERATED IN YRS. 30-45	e_o^o
1	501	0	30	As estimated in Chapter IV	1605	--	40
2	1605	30	15	"	2764	4046	40
3	1605	30	15	As input intervals increased by 50%	2200	2620	40
4	1605	30	15	All input intervals of order 5+ increased by 50%	2387	3244	40
5	1605	30	15	As in Run 1	3243	4118	60

TABLE 5.3. DESCRIPTION OF SIMULATED POPULATIONS , COHORTS

POPN. NO.	POPULATION SIZE			NUMBER OF BIRTHS		
	AT BEGINNING	AFTER 15 YEARS	AFTER 30 YEARS	IN 0-15 YEARS	15-30 YEARS	TOTAL 0-30
1	500	429	355	2767	1757	4524
2	500	474	436	2932	2082	6014
3	1000	872	-	5620	--	--
4	872	-	708	--	2672	--
5	1000	926[a]	712	2836[b]	2394[c]	8711

[a]After 10 years; population size after 20 years = 830.

[b]In (0-10) years; number of births in (10-20) years = 3481

[c]In (20-30) years.

The interval in (c) above is applicable both to (a) and (b). For ascertainment plans 3 and 4 above, a one-year time period was used instead of age A, since the probability of any interval ending or beginning at exact age A is zero, and we would not obtain any birth intervals in these categories, otherwise.

To test whether two mean birth intervals (in the same ascertainment plan) were equal, a t-test was used. Let H_0: $\mu_1 = \mu_2$ be the null hypothesis against the alternative H_A: $\mu_1 \neq \mu_2$. The statistic for testing this hypothesis is

$$t^* = \frac{\bar{X}_1 - \bar{X}_2}{s_p \sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

where s_p^2 is the pooled mean square estimate of σ^2. This statistic was used with (N_1+N_2-2) degrees of freedom. A P-level was obtained for each of the tests, and where applicable, Pearson's P_λ test was used to obtain a combined level of significance. The statistic for P_λ test is

$$P_\lambda^* = -2 \sum_{i=1}^{k} \log_e P_i$$

and is distributed as χ^2 (2k) where k is the number of <u>independent</u> tests. Level of significance was arbitrarily set at $\alpha = .05$.

To test whether the decrease in fertility resulted in a proportional increase in lengths of the different mean birth intervals, the appropriate null hypothesis would be H_0: $\mu_1 \leq 1.25\ \mu_2$ (or 1.50 μ_2, as the case may be) versus H_A: $\mu_1 > 1.25\ \mu_2$ (or 1.50 μ_2), where the increase in input parameters for births was 25% or 50% in different populations.

5.3. Examination of the Ascertainment Plans

5.3.1. All Previous Intervals Ending Before Age A

In an age cohort, the terms "survey date" and "age of woman" are interchangeable, since the difference between those two is constant. For example, since the women were aged 15 years at time zero, a statement referring to the women's age A years automatically refers to the survey date of (A-15).

All previous intervals ending before age A can be determined in two ways; all intervals whether any woman is living or not (prospective ascertainment), and all intervals for living women only (retrospective ascertainment). These intervals were tabulated for parity completed and by age, the different ages being 25 years to 45 years in steps of 5 years.

Table 5.4 gives the mean birth intervals for certain selected group of women at each age. These intervals are computed for all women, whether or not alive at a particular survey date. The women were so selected that each of mean birth intervals in Population No. 1 were most nearly equal to the corresponding input birth intervals. This implied selection of 25 year olds with 4 births, 30 year old women with 6 births, 35 year olds with 8 births, 40 year

TABLE 5.4 . ALL PREVIOUS CLOSED MEAN INTERVALS, PROSPECTIVE ASCERTAINMENT,COHORTS

INPUT MEAN INTERVAL	PARITY	AGE OF MOTHER (IN YEARS)									
		25 4		30 6		35 8		40 9		45 10+	
24.41	0	20.84 21.36	21.08 18.03	21.94 21.71	22.41 18.11	22.00 19.43	21.34 19.74	25.06 21.18	21.50 21.73	23.31 19.96	21.47 22.12
24.35	1	23.73 24.40	24.68 22.55	24.54 24.44	23.59 22.78	24.00 23.25	23.89 23.05	24.75 23.79	23.62 24.12	23.21 23.84	23.56 23.75
27.16	2	28.18 27.59	27.62 24.37	27.17 27.10	26.86 24.60	27.80 25.60	27.33 24.10	30.48 26.15	28.59 26.28	26.66 26.32	25.96 26.33
28.54	3	28.16 28.60	28.32 25.48	28.68 29.01	27.86 26.94	28.43 25.09	26.61 25.94	28.31 27.07	29.27 29.36	29.14 25.39	26.65 27.78
28.89	4	****** ******	****** ******	28.85 28.92	29.43 25.11	26.84 26.90	28.40 25.60	29.40 28.21	30.34 25.81	28.11 27.65	27.97 26.52
29.73	5	****** ******	****** ******	27.28 30.15	30.13 28.38	28.01 26.59	29.15 29.41	31.08 28.71	33.34 29.65	29.46 29.43	29.51 29.26
31.27	6	****** ******	****** ******	****** ******	****** ******	29.25 30.73	30.20 28.87	31.89 36.55	34.61 31.50	29.78 35.51	29.98 30.27
32.27	7	****** ******	****** ******	****** ******	****** ******	31.45 42.19	32.50 27.47	35.15 40.92	34.75 31.09	32.13 38.80	31.53 29.54
32.29	8	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	32.90 39.87	32.86 30.26	31.46 29.52	31.62 30.51
37.05	9	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	34.66 38.29	34.61 31.20
	MEAN	25.23 (179) 25.49 (409)	25.43 (193) 22.61 (234)	26.41 (169) 26.89 (336)	26.71 (180) 24.32 (226)	27.22 (123) 27.47 (229)	27.43 (157) 25.53 (200)	29.89 (116) 30.28 (226)	29.88 (161) 27.76 (226)	28.79 (296) 30.47 (386)	28.29 (361) 27.73 (510)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

olds with 9 births, and 45 year old women who had exactly 10 births. This selection scheme was only to provide an example, and any other group of women would equally be effective in discussing these intervals.

As can be seen from this table, the change in mortality level from $e_o^o = 40$ to $e_o^o = 60$ years has very little effect on mean birth intervals at all ages and parities. The difference was non-significant for all ages of women.

The effect of reduced fertility is not readily apparent on these birth intervals. A closer look, however, indicates the direction in which these intervals would be influenced by change in level of fertility. Let us consider a woman with parity 8 in run 1 and another woman of equal parity in run 4 at survey date 35 years. Both of these women have had equal time for reproduction, but the woman in run 4 had been subjected to lower fertility for 5 years, and her last two intervals total approximately 5 years on the average. The woman in run 1, however, had not been exposed to reduced fertility and her last two intervals were, on the average, shorter than those in run 4. With constant time for reproduction, however, this implies longer intervals up to parity 6 for woman in run 1 compared to the woman in run 4. This phenomenon is observed in Table 5.4, though the increase in the last few intervals is not proportional. The number of intervals which could be affected is determined by the time elapsed since the change in fertility.

TABLE 5.5 . ALL PREVIOUS CLOSED MEAN INTERVALS, RETROSPECTIVE ASCERTAINMENT, COHORTS

INPUT MEAN INTERVAL	PARITY	AGE OF MOTHER (IN YEARS) 25 4		30 6		35 8		40 9		45 10+	
24.41	0	20.82 21.51	21.09 18.05	22.31 21.80	22.48 18.21	22.34 19.45	21.38 19.95	25.51 21.09	21.50 22.18	23.89 20.30	21.40 22.08
24.35	1	23.78 24.52	24.71 22.58	24.54 24.56	23.32 22.85	24.17 23.46	23.94 23.06	25.02 23.96	23.62 24.29	23.34 23.84	23.66 23.82
27.16	2	28.19 27.59	27.56 24.44	27.24 27.13	26.87 24.54	28.04 25.67	27.32 24.24	30.49 25.95	28.59 26.64	26.69 26.46	26.20 26.40
28.54	3	28.54 28.88	28.35 25.46	28.89 29.07	28.03 26.92	28.56 25.28	26.64 26.05	29.14 27.09	29.27 29.62	29.58 25.26	26.72 27.94
28.89	4	****** ******	****** ******	29.00 28.96	29.59 25.22	27.04 26.81	28.44 25.51	29.43 28.37	30.34 25.93	28.13 27.75	28.09 26.76
29.73	5	****** ******	****** ******	27.29 30.40	30.18 28.36	28.08 26.78	29.24 29.32	31.07 28.62	33.34 30.10	29.61 29.87	29.40 29.41
31.27	6	****** ******	****** ******	****** ******	****** ******	29.62 30.34	30.12 29.07	31.97 36.77	34.61 31.88	29.71 35.92	30.19 30.57
32.27	7	****** ******	****** ******	****** ******	****** ******	31.58 42.50	32.54 27.48	35.05 41.30	34.75 31.17	32.38 39.24	31.50 29.85
32.29	8	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	33.39 39.95	32.86 30.07	31.39 29.50	31.60 30.50
37.05	9	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	****** ******	34.79 38.20	34.79 31.61
	MEAN	25.33 (170) 25.62 (394)	25.43 (192) 22.63 (230)	26.54 (159) 26.99 (323)	26.75 (177) 24.35 (220)	27.43 (117) 27.59 (222)	27.45 (155) 25.59 (195)	30.12 (107) 30.34 (214)	29.88 (109) 27.99 (208)	28.95 (273) 30.63 (360)	28.36 (342) 27.90 (478)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

The women in run 5 (unmarried cohort at start) had shorter intervals than those in run 1. This stems from the fact that for equal parities, women in run 1 had a longer time in the marital status --and to reproduce--than those in run 5. For example, for a woman aged 25 years with two births in her fertility history in run 1,

First closed interval + second closed interval + open interval = 10 years

while for a similar woman in run 5,

Time from age 15 to marriage + first closed interval + second closed interval + open interval = 10 years.

This would, therefore, yield shorter closed as well as open intervals for an unmarried cohort (at initial time) than for a married cohort.

Mean birth intervals for living women only are tabulated in Table 5.5. The group of women selected corresponded to those in Table 5.4. Here again, changing mortality level has a non-significant effect on mean intervals at all ages and parities for surviving women.

Although the difference between corresponding intervals in Tables 5.4 and 5.5 is small, the intervals for living women only are slightly longer than those for all women. This may be, because for women who die before survey date, longer intervals may not be included, simply because some women died before giving birth. Because of the way in which POPSIM generates the date of next event, the date of death occurred before date of next birth, and what could have been a long interval would be terminated without being considered as a birth interval. In the case of living women, however, date of death cannot occur until after the survey date, and all intervals, long or short, are included.

The remarks about the effect of reduced fertility for all previous closed intervals for all women, living or not, apply here, too. With constant time for reproduction, longer mean birth intervals are obtained for parities six or less in run 1 compared to the mean intervals for women in run 4, where the fertility was reduced by increasing the input birth intervals by 25%.

As in the case of all previous closed intervals for all women, the cohort with single women as the initial population had shorter mean birth intervals as compared to the cohort with married women as the initial population; because of the shorter marital duration for unmarried cohort.

5.3.2. Open Intervals

An open interval is defined as the time period between the last birth and the survey date and are based upon surviving women at survey date. These intervals were determined by age (25 years to 45 years, at 5-year intervals) and by parity, and are tabulated in Table 5.6.

TABLE 5.6 . OPEN INTERVALS,COHORTS

PARITY	AGE OF MOTHER AT SURVEY DATE (IN YEARS) 25		30		35		40		45	
0	120.00	120.00	180.00	180.00	240.00	240.00	300.00	300.00	360.00	360.00
	120.00	57.15	180.00	136.73	240.00	196.98	300.00	256.98	360.00	316.98
		(27)		(3)		(2)		(2)		(2)
1	39.39	36.79	96.72	127.60	200.31	187.60	260.31	247.60	320.31	307.60
	(8)	(5)	(3)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
	52.78	17.10	138.83	44.96	198.83	197.11	258.83	257.11	316.99	317.11
	(11)	(67)	(6)	(6)	(6)	(1)	(6)	(1)	(5)	(1)
2	28.06	28.30	86.73	100.79	44.71	206.52	229.59	266.52	289.59	326.52
	(26)	(16)	(7)	(3)	(6)	(1)	(5)	(1)	(5)	(1)
	31.74	19.90	110.42	30.72	81.15	20.34	241.15	*******	299.09	*******
	(45)	(189)	(6)	(17)	(5)	(1)	(5)	(0)	(4)	(0)
3	24.80	24.87	28.51	54.70	28.17	103.16	*******	226.62	*******	286.62
	(87)	(81)	(4)	(10)	(1)	(2)	(1)	(1)	(0)	(1)
	24.46	17.13	31.35	24.00	88.30	18.53	217.39	*******	277.39	*******
	(169)	(319)	(15)	(71)	(2)	(3)	(0)	(0)	(1)	(0)
4	18.67	18.29	28.73	31.09	113.53	105.19	159.54	147.00	249.90	207.00
	(170)	(192)	(23)	(18)	(5)	(4)	(5)	(4)	(3)	(4)
	17.50	13.51	33.89	21.37	60.55	30.64	128.82	98.28	266.99	184.93
	(394)	(230)	(45)	(168)	(10)	(22)	(4)	(3)	(2)	(2)
5	10.78	11.07	24.58	25.33	61.17	52.39	153.87	138.38	237.07	198.38
	(131)	(161)	(93)	(83)	(14)	(17)	(7)	(5)	(6)	(5)
	11.22	8.96	24.43	18.13	51.38	25.81	133.49	61.08	211.95	163.29
	(260)	(79)	(179)	(295)	(62)	(74)	(13)	(12)	(11)	(3)
6	6.82	6.53	20.74	19.52	33.73	41.92	113.09	96.95	145.52	187.42
	(25)	(21)	(159)	(177)	(26)	(36)	(4)	(10)	(4)	(5)
	7.88	4.95	18.07	16.07	36.28	24.68	58.97	45.15	136.59	119.36
	(35)	(7)	(323)	(220)	(157)	(184)	(40)	(44)	(14)	(12)
MEAN	18.93	18.05	26.70	26.74	72.82	69.22	178.99	164.36	247.75	246.61
	(452)	(481)	(292)	(297)	(56)	(66)	(24)	(27)	(21)	(22)
	18.87	17.17	25.75	19.72	53.23	27.15	125.14	61.06	229.44	162.16
	(924)	(918)	(581)	(780)	(248)	(287)	(75)	(62)	(42)	(20)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

A decrease in mortality level from $e_o^o = 40$ to 60 years does not significantly change the mean open birth interval. A 25% increase in input mean closed intervals, i.e., a decrease in fertility, however, results in significant difference between corresponding open intervals obtained in populations 1 and 4. The difference, however, is in opposite direction to change in closed birth intervals. Moreover, while the over-all decrease in mean open intervals at each survey date is approximately 25%, the same is not true about these open intervals at each parity.

The resultant decrease in the mean length of open intervals when input closed intervals are increased by 25% may be unexpected at first glance, but can be explained by a simple example below. The example here considers women of parity 2 arbitrarily, and proof for any other parity will be similar. The notation here is for this example only, and should not be confused with earlier ones.

Let

θ_2 = open interval of a woman of parity 2, before fertility change.

w_2 = open interval of woman of parity 2, after increase in closed interval.

Now, w_2 is a weighted average of Θ_{22} and Θ_{32}, where

Θ_{22} = new open interval for those women who would still have two births before survey date,

and Θ_{32} = new open interval for those women who would have two births (instead of three) because of fertility decrease.

There will be very few women with open interval Θ_{12}, i.e., those women who would have two births now (instead of one) because of decrease in fertility, and may be ignored without seriously jeopardizing the argument, here.

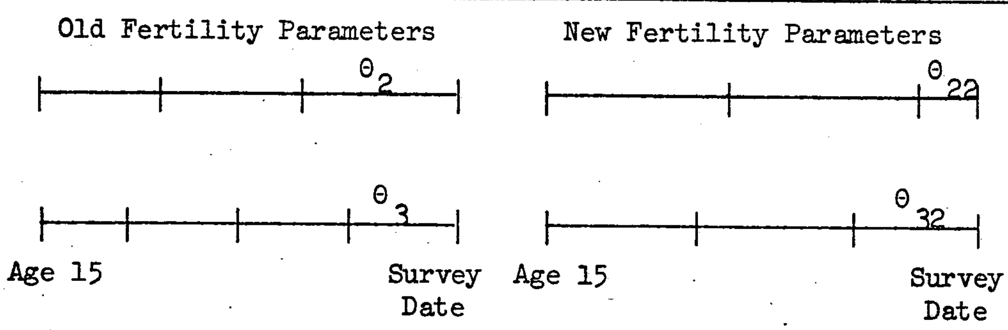

CHART 5.1. OPEN INTERVALS, COHORTS

Now, $\Theta_2 > \Theta_{22}$, $\Theta_3 < \Theta_2$ and $w_2 = c_1\ \Theta_{22} + (1-c_1)\ \Theta_{32}$ where, c_1 is proportion of women with open interval Θ_{22}. Θ_{32} may or may not be less than Θ_3. If $\Theta_{32} \leq \Theta_3$, then $w_2 < c_1\ \Theta_2 + (1-c_1)\ \Theta_2$, i.e., $w_2 < \Theta_2$. Even if, however, $\Theta_{32} > \Theta_3$, one would expect Θ_{32} to be nearly equal to Θ_2, in which case,

$$w_2 = c_1\ \Theta_{22} + (1-c_1)\ \Theta_{32}$$
$$< c_1\ \Theta_2 + (1-c_1)\ \Theta_2 = \Theta_2$$

Even if $\Theta_{32} > \Theta_2$, it would have to be sufficiently larger as to counteract the decrease because of $\Theta_{22} < \Theta_2$. Thus, it seems, that only under very restrictive conditions can w_2 be larger than Θ_2.

The women in run 5 (unmarried cohort at start) had shorter open intervals than those in run 1. The reason for this is explained in Section 5.3.1.

5.3.3. An i^{th} Interval Ending at Age A

Since not many women in a population of 500 or 1000 women have a birth of same order at a fixed age, it is better to use an interval of time as reference point during which a birth interval should end. If, however, this interval of time is too large, any birth intervals beginning and ending in this time interval would be omitted. It was, therefore, decided to use an interval of one year as the reference period in this study.

The change in mortality does not significantly affect mean closed intervals obtained in the aforementioned ascertainment plan, as can be seen from Table 5.7. There are too few women having an interval ending at later ages and even though an increase in input birth intervals apparently resulted in an increase in mean birth intervals, there is no obvious trend in this lengthening. Mean birth intervals for the unmarried cohort, as usual, were shorter than corresponding intervals in the married cohort, because of the short marital duration for women in run 5.

TABLE 5.7 . I-TH INTERVALS ENDING AT A GIVEN AGE,COHORTS

PARITY	AGE OF MOTHER AT END OF INTERVAL (IN YEARS)							
	20-21		25-26		30-31		35-36	
0	******	******	******	******	******	******	******	******
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	******	40.99	******	22.69	******	******	******	******
	(0)	(31)	(0)	(2)	(0)	(0)	(0)	(0)
1	28.00	39.18	******	******	******	******	******	******
	(7)	(5)	(0)	(0)	(0)	(0)	(0)	(0)
	26.65	26.43	******	13.52	******	******	******	******
	(4)	(60)	(0)	(1)	(0)	(0)	(0)	(0)
2	40.04	41.25	******	14.52	******	******	******	******
	(16)	(17)	(0)	(1)	(0)	(0)	(0)	(0)
	36.82	28.48	35.49	32.22	******	28.39	******	******
	(38)	(112)	(4)	(19)	(0)	(1)	(0)	(0)
3	29.69	32.01	34.61	21.92	******	******	43.44	******
	(58)	(54)	(10)	(2)	(0)	(0)	(1)	(0)
	30.35	26.24	36.80	35.67	72.75	34.85	******	******
	(141)	(112)	(5)	(40)	(1)	(4)	(0)	(0)
4	23.40	26.14	38.28	40.65	19.08	26.94	******	******
	(87)	(95)	(23)	(20)	(1)	(2)	(0)	(0)
	24.67	21.37	34.85	28.41	57.34	27.47	******	26.64
	(155)	(59)	(43)	(100)	(14)	(22)	(0)	(1)
5	20.85	21.63	28.99	32.94	41.25	37.80	******	******
	(19)	(19)	(44)	(60)	(6)	(5)	(0)	(0)
	22.02	24.63	31.63	29.13	55.72	78.79	73.39	37.99
	(27)	(5)	(122)	(90)	(25)	(43)	(4)	(2)
6	21.44	******	28.09	29.09	37.00	37.20	43.18	64.98
	(1)	(0)	(50)	(71)	(27)	(24)	(5)	(4)
	14.72	******	26.44	24.06	55.62	34.45	46.51	42.97
	(2)	(0)	(114)	(42)	(72)	(99)	(18)	(18)
MEAN	27.54	29.00	31.85	32.10	37.22	36.64	43.22	64.98
	(193)	(189)	(128)	(154)	(24)	(31)	(6)	(4)
	28.09	27.36	30.20	29.64	56.04	35.92	51.40	41.81
	(373)	(379)	(288)	(294)	(112)	(169)	(22)	(21)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

5.3.4. An i^{th} Interval Beginning at Age A

As explained in Section 5.3.3, the reference time interval was taken as one year in this case as well, because very few women had an interval starting at same point of time. Table 5.8 tabulates mean birth intervals for selected ages and parities, and discussions in Section 5.3.3 hold for this section also because except for those few women who have completed their parities or die, an i^{th} interval ending in age group (x,x+1) also signifies the beginning of $(i+1)^{th}$ birth interval, starting in age group (x,x+1) years. While one-year age groups selected for this ascertainment are different from those in Section 5.3.3, the results are similar for the two ascertainment plans.

5.3.5. Straddling Intervals

A straddling interval is defined as the closed interval which "straddles" the survey date, i.e., as the interval which begins before survey date and ends after it.

Table 5.9 reveals that as all other ascertainment plans discussed so far, lower mortality has insignificant effect on mean straddling intervals, too. Straddling intervals are also slightly lower for women in run 5 than for those who start as married at age 15 years.

The reduction of fertility, however, has a substantial impact on straddling intervals, which tend to increase with decrease in

TABLE 5.3 . I-TH INTERVALS BEGINNING AT A GIVEN AGE ,COHORTS

	AGE OF MOTHER AT BEGINNING OF INTERVAL (IN YEARS)							
PARITY	24-25		29-30		34-35		39-40	
0	******	******	******	******	******	******	******	******
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	******	24.35	******	******	******	******	******	******
	(0)	(61)	(0)	(0)	(0)	(0)	(0)	(0)
1	23.43	19.55	******	******	27.80	******	******	******
	(11)	(14)	(0)	(0)	(1)	(0)	(0)	(0)
	26.19	23.89	15.26	21.26	******	******	******	******
	(22)	(124)	(1)	(13)	(0)	(0)	(0)	(0)
2	28.66	25.53	23.96	23.07	24.27	******	******	******
	(125)	(129)	(13)	(5)	(0)	(0)	(1)	(0)
	27.52	28.60	49.72	27.42	******	17.80	******	******
	(241)	(84)	(21)	(97)	(0)	(9)	(0)	(1)
3	28.73	29.22	30.03	28.73	******	******	43.44	******
	(26)	(25)	(48)	(37)	(4)	(5)	(1)	(0)
	27.94	27.99	38.08	28.79	******	29.87	******	22.52
	(45)	(5)	(89)	(125)	(0)	(33)	(5)	(2)
4	29.51	29.55	26.67	29.73	30.68	30.25	34.38	******
	(34)	(39)	(4)	(2)	(1)	(0)	(0)	(0)
	28.63	23.26	29.03	28.92	******	25.95	29.65	29.42
	(111)	(149)	(3)	(37)	(0)	(1)	(0)	(0)
5	37.43	******	30.64	31.21	26.24	37.91	******	27.74
	(1)	(0)	(77)	(83)	(4)	(6)	(0)	(1)
	******	******	31.14	30.29	******	29.71	46.81	32.38
	(0)	(0)	(167)	(94)	(0)	(82)	(20)	(12)
6	******	******	32.05	31.59	28.10	25.72	31.62	21.41
	(0)	(0)	(35)	(52)	(38)	(33)	(3)	(5)
	******	******	27.45	33.34	******	33.03	******	30.24
	(0)	(0)	(75)	(19)	(0)	(111)	(0)	(32)
MEAN	28.57	27.91	29.65	30.85	28.08	27.90	34.53	22.46
	(197)	(207)	(177)	(179)	(48)	(44)	(5)	(6)
	27.81	26.42	30.36	29.04	******	30.70	43.37	30.59
	(419)	(423)	(356)	(385)	(0)	(236)	(25)	(47)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

TABLE 5.9 . STRADDLING INTERVALS(IN MONTHS) ,COHORTS

PARITY	AGE OF MOTHER (IN YEARS)									
	25		30		35		40		45	
0	91.63	77.11	******	******	******	******	******	******	******	******
	(29)	(25)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	77.83	40.14	******	71.52	******	******	******	******	******	******
	(53)	(229)	(0)	(24)	(0)	(0)	(0)	(0)	(0)	(0)
1	34.17	29.78	34.66	25.05	17.08	******	******	******	******	******
	(67)	(67)	(6)	(3)	(1)	(0)	(0)	(0)	(0)	(0)
	32.27	28.57	32.85	28.37	******	31.84	******	******	******	******
	(165)	(312)	(5)	(65)	(0)	(4)	(0)	(0)	(0)	(0)
2	31.28	30.47	34.91	37.08	48.09	36.33	26.71	27.44	******	******
	(257)	(267)	(22)	(14)	(2)	(1)	(1)	(1)	(0)	(0)
	31.40	28.39	41.85	33.86	97.52	41.81	******	******	******	******
	(511)	(235)	(40)	(185)	(1)	(17)	(0)	(0)	(0)	(0)
3	30.54	28.49	37.11	38.38	36.96	52.13	39.05	64.41	******	******
	(104)	(118)	(86)	(78)	(4)	(8)	(1)	(1)	(0)	(0)
	29.90	33.00	37.98	34.24	55.78	37.51	62.30	24.82	******	******
	(208)	(34)	(166)	(316)	(14)	(71)	(1)	(3)	(0)	(0)
4	25.66	24.71	33.92	33.03	38.69	42.12	32.27	50.38	20.33	******
	(6)	(2)	(66)	(189)	(19)	(15)	(1)	(1)	(1)	(0)
	26.35	27.25	34.57	29.83	65.93	36.51	59.74	40.22	58.09	67.79
	(7)	(1)	(386)	(226)	(43)	(164)	(8)	(20)	(2)	(1)
5	******	******	29.28	30.97	38.20	40.02	34.71	43.33	26.69	******
	(0)	(0)	(127)	(159)	(86)	(77)	(7)	(10)	(1)	(0)
	******	******	32.08	30.76	60.50	35.96	37.32	40.87	46.67	47.12
	(0)	(0)	(250)	(79)	(164)	(289)	(48)	(70)	(2)	(8)
6	******	******	27.72	25.94	39.46	39.34	47.91	55.34	******	73.65
	(0)	(0)	(23)	(21)	(153)	(173)	(22)	(30)	(0)	(4)
	******	******	31.45	44.22	57.45	37.30	54.22	41.30	55.45	49.70
	(0)	(0)	(35)	(7)	(307)	(215)	(147)	(174)	(26)	(33)
MEAN	35.24	32.29	34.03	33.20	39.51	40.05	43.60	52.58	23.51	73.65
	(463)	(479)	(433)	(465)	(266)	(274)	(32)	(42)	(2)	(4)
	33.79	31.97	34.92	33.40	59.12	36.71	55.20	40.87	55.04	49.64
	(944)	(811)	(884)	(902)	(529)	(760)	(204)	(268)	(30)	(42)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2 fertility. A 25% increase in input closed birth intervals resulted in more than 25% increase in straddling intervals at survey date 15 years (age 30), while at survey date 20 years, resultant increase was 25%. One would be tempted to infer that straddling intervals show the effect of decreased fertility in less than 5 years, but this may lead to wrong conclusions. First, the data is insufficient for assessment at later survey dates, and second, it takes longer than survey period to compute straddling intervals, since by definition, these intervals end after survey date. As will be seen later, however, straddling intervals seem to be a sensitive index, even for cross-sectional populations.

5.3.6. Previous Closed Intervals

These intervals were ascertained by age of mother both at beginning of interval and at end of an interval. Age of mother at survey date can also be used, but in a cohort population, this would be a duplication of efforts, because at survey date, say, T each women in cohort is of age T + C years where C is the age of women at survey date zero. Thus, if we determine birth intervals by age of mother, it also determines the birth intervals by survey date.

The decrease in mortality level does not significantly alter mean birth intervals, classified by either method for mother's age. Also, women in the unmarried cohort had the shorter mean birth intervals than those in the married cohort of run 1.

The decrease in fertility resulted in a corresponding increase in mean intervals in both Tables 5.10 and 5.11.

A summary of the results for the various ascertainment plans appears in Table 5.12.

5.3.7. Interior Intervals

All intervals that begin and end in age group (A,A+y), where y is predetermined, are termed interior intervals, with respect to age. These birth intervals are also interior with respect to survey date, since we are dealing with cohort populations.

These intervals are useful in comparing the original input distribution of birth intervals with empirical distribution, and will be described in greater detail in the section dealing with comparison of parity specific interval distribution, later in this chapter.

5.4. Cross-Sectional Population

In POPSIM, the simulated initial computer population is considered a random sample, representative of the parent population. To generate this random sample, age-sex distribution of the population is needed. It was decided to use a stable population as parent population, from model stable populations computed by Coale and Demeny.

For $e_o^o = 40$, a death rate near 25 seems reasonable. From the experience gained from generation of fertility histories of women

TABLE 5.10. PREVIOUS CLOSED INTERVAL BY AGE OF MOTHER AT
BEGINNING OF INTERVAL,COHORTS

AGE AT BEGINNING OF INTERVAL	SURVEY DATE 5		10		15		20		25		30	
15- 19	24.98	25.93	37.98	40.24	29.27	32.03	25.23	29.15	25.23	29.15	25.23	29.15
	(459)	(466)	(120)	(90)	(36)	(20)	(35)	(19)	(35)	(19)	(35)	(19)
	25.51	21.02	37.06	39.08	26.91	28.84	26.96	25.53	26.96	25.83	26.96	25.83
	(930)	(598)	(217)	(143)	(63)	(33)	(62)	(31)	(62)	(31)	(62)	(31)
20- 24	******	******	24.70	25.55	34.84	38.55	27.40	34.21	27.84	29.50	27.84	29.50
	(0)	(0)	(365)	(400)	(110)	(112)	(30)	(17)	(29)	(16)	(29)	(16)
	******	******	25.80	24.31	36.52	38.12	37.23	28.86	33.03	28.38	31.72	28.38
	(0)	(0)	(764)	(788)	(254)	(186)	(98)	(41)	(76)	(40)	(75)	(40)
25- 29	******	******	******	******	26.29	26.98	39.21	41.37	35.53	45.93	31.67	44.61
	(0)	(0)	(0)	(0)	(341)	(359)	(160)	(160)	(50)	(30)	(47)	(26)
	******	******	******	******	26.46	25.78	50.10	39.61	55.90	33.98	49.83	31.81
	(0)	(0)	(0)	(0)	(666)	(736)	(650)	(267)	(163)	(69)	(121)	(65)
30- 34	******	******	******	******	******	******	26.99	26.01	41.58	39.62	40.54	38.20
	(0)	(0)	(0)	(0)	(0)	(0)	(263)	(295)	(164)	(170)	(79)	(48)
	******	******	******	******	******	******	26.09	26.29	44.24	38.54	51.69	38.46
	(0)	(0)	(0)	(0)	(0)	(0)	(173)	(616)	(365)	(299)	(143)	(127)
35- 39	******	******	******	******	******	******	******	******	25.33	26.44	41.26	43.51
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(210)	(256)	(164)	(220)
	******	******	******	******	******	******	******	******	29.80	25.92	43.90	39.93
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(317)	(516)	(377)	(380)
40- 44	******	******	******	******	******	******	******	******	******	******	26.13	27.84
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(134)	(162)
	******	******	******	******	******	******	******	******	******	******	38.37	25.72
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(205)	(312)
MEAN	24.98	25.93	27.99	28.25	28.44	29.81	30.89	31.42	31.98	32.40	34.12	36.87
	(459)	(466)	(485)	(490)	(487)	(491)	(488)	(491)	(488)	(491)	(488)	(491)
	25.51	21.02	28.29	26.58	29.09	28.29	43.13	30.11	39.56	30.55	40.53	33.60
	(930)	(598)	(981)	(931)	(983)	(955)	(983)	(955)	(983)	(955)	(983)	(955)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

TABLE 5.11. PREVIOUS CLOSED INTERVAL BY AGE OF MOTHER AT END OF INTERVAL, COHORTS

AGE AT END OF INTERVAL	SURVEY DATE 5		10		15		20		25		30	
15- 19	24.98	25.93	24.14	23.85	23.59	24.16	23.59	24.16	23.59	24.16	23.59	24.16
	(466)	(466)	(29)	(13)	(25)	(12)	(25)	(12)	(25)	(12)	(25)	(12)
	25.51	21.02	22.12	19.13	20.89	18.14	20.89	18.14	20.89	18.14	20.89	18.14
	(930)	(598)	(49)	(21)	(39)	(16)	(39)	(16)	(39)	(16)	(39)	(16)
20- 24	******	******	28.23	28.37	23.38	29.85	23.36	30.79	23.36	30.79	23.36	30.79
	(0)	(0)	(456)	(477)	(30)	(18)	(29)	(14)	(29)	(14)	(29)	(14)
	******	******	28.61	26.75	27.71	25.73	28.65	25.78	29.10	25.78	29.10	25.78
	(0)	(0)	(932)	(910)	(69)	(47)	(59)	(37)	(56)	(37)	(56)	(37)
25- 29	******	******	******	******	29.07	29.96	28.13	28.88	28.44	29.26	28.44	28.93
	(0)	(0)	(0)	(0)	(432)	(461)	(42)	(29)	(34)	(18)	(34)	(16)
	******	******	******	******	29.56	28.61	29.45	30.06	29.83	30.33	29.84	30.33
	(0)	(0)	(0)	(0)	(875)	(892)	(178)	(55)	(98)	(46)	(95)	(46)
30- 34	******	******	******	******	******	******	32.21	31.81	30.75	31.83	32.27	34.23
	(0)	(0)	(0)	(0)	(0)	(0)	(392)	(436)	(55)	(51)	(46)	(36)
	******	******	******	******	******	******	49.02	30.53	50.27	31.29	52.73	31.05
	(0)	(0)	(0)	(0)	(0)	(0)	(707)	(847)	(97)	(104)	(62)	(87)
35- 39	******	******	******	******	******	******	******	******	33.85	32.92	34.96	32.06
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(345)	(396)	(95)	(73)
	******	******	******	******	******	******	******	******	41.33	30.96	42.48	32.14
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(693)	(752)	(212)	(163)
40- 44	******	******	******	******	******	******	******	******	******	******	37.10	39.25
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(259)	(340)
	******	******	******	******	******	******	******	******	******	******	42.93	35.49
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(519)	(606)
MEAN	24.98	25.93	27.99	28.25	28.44	29.81	30.89	31.42	31.98	32.40	34.12	36.87
	(459)	(466)	(485)	(493)	(487)	(491)	(488)	(491)	(488)	(491)	(488)	(491)
	25.51	21.02	28.29	26.58	29.09	28.29	43.13	30.11	39.56	30.55	40.53	33.60
	(930)	(598)	(981)	(931)	(983)	(955)	(983)	(955)	(983)	(955)	(983)	(955)

****** IMPLY NO ENTRIES IN THE CELL

THE ORDER OF THE BIRTH INTERVALS IS RUN 1 RUN 2
RUN 4 RUN 5

TABLE 5.12. SUMMARY OF ASCERTAINMENT PLANS FOR COHORTS

KIND OF BIRTH INTERVAL	EFFECT OF DECREASE IN MORTALITY	EFFECT OF DECREASE IN FERTILITY ON BIRTH INTERVALS	RUN 5 COHORT vs. RUN 1 COHORT
1. All previous intervals by age A	No change	Slight Increase	Shorter interval for run 5
2. Open birth intervals	"	Proportional Decrease	"
3. i^{th} intervals ending at age A	"	Haphazard Increase (no obvious trend)	"
4. i^{th} intervals beginning at age A	"	"	"
5. Straddling intervals	"	Proportional Increase	"
6. Immediate past closed interval	"	"	"

in cohort populations, a rate of growth nearing 3.5% per year was considered appropriate. Table 5.13 gives the needed age distribution for females.

TABLE 5.13. AGE DISTRIBUTION FOR FEMALES IN THE INITIAL POPULATION

AGE (in years)	PERCENTAGE OF FEMALES
0-1	5.18
1-4	16.29
5-9	16.36
10-14	13.32
15-19	10.84
20-24	8.74
25-29	6.99
30-34	5.56
35-39	4.39
40-44	3.45
45-49	2.69
50-54	2.07
55-59	1.54
60+	2.58
TOTAL	100.00

Source: Coale, Ansley J. and Demeny, Paul, Regional Model Life Tables and Stable Populations, Princeton University Press, Princeton, New Jersey, 1966, p. 42.

In this population, we had

Death rate = 24.65 per 1000 population,

Birth rate = 59.65 per 1000 population, and

r = rate of growth = 3.5%

5.5. Cross-Sectional Computer Populations

An initial sample of size 500 was generated using the female age distribution in Table 5.12. A history of marriage, births and sterility date was generated for each member of the initial population until time of initialization. Events were generated for each of these women and recorded on the history tape. All female births became members of the computer population. At end of 30 years of simulation, a total female population of 1605 was obtained. This population was the initial population for each of four subsequent runs used in this analysis. These cross-sectional populations are described earlier in this chapter in Table 5.2.

5.6. Birth Intervals in Cross-Sectional Populations

Following are the ascertainment plans used to compute birth intervals in cross-sectional populations.

(i) open birth intervals, by parity, by age of mother at survey time.

(ii) all open intervals, by parity.

(iii) straddling intervals, by parity.

(iv) all previous closed intervals, by parity and by total number of intervals at survey date.

(v) all previous closed intervals, by parity.

(vi) previous closed interval by age of mother

(a) age of mother at beginning of interval

(b) age of mother at end of interval

(c) age of mother at survey date.

5.6.1. <u>Open Birth Intervals, by Age of Mother at Survey</u>

Since the length of intervals increase with parity (and age) of a woman, we would expect longer intervals, both closed and open, as the age of mother increases. This trend is apparent in the case of open intervals in Table 5.14, at each survey date. Change in mortality level does not significantly affect the lengths of open intervals while the decrease in fertility is reflected in a proportional increase in open intervals. We have called an index robust if there is no change in it over time if the factors which do not directly affect fertility are the only ones which undergo changes over time. Open birth intervals by age of mother are robust.

5.6.2. <u>All Open Birth Intervals</u>

Table 5.15 gives all open intervals by parity of mother at different survey date. The decrease in mortality significantly increases open intervals at each survey date, and these intervals decrease over time for each of four computer populations investigated. Therefore, these intervals are not robust even though they are sensitive to changes in fertility. For example, decreased

TABLE 5.14. ALL OPEN INTERVALS, BY PARITY AND AGE OF MOTHER
AT SURVEY DATE, CROSS-SECTIONAL POPULATIONS

	SURVEY DATE 360 MONTHS					
	AGE OF MOTHER AT SURVEY					
PARITY	15-19	20-24	25-29	30-34	35-39	40-44
0	13.20 (48)	31.81 (17)	******* (0)	209.37 (1)	******* (0)	******* (0)
1	10.65 (24)	19.21 (28)	25.32 (5)	******* (0)	******* (0)	******* (0)
2	9.98 (18)	15.84 (38)	15.36 (10)	******* (0)	******* (0)	******* (0)
3	1.11 (1)	10.78 (34)	17.51 (21)	32.00 (5)	******* (0)	******* (0)
4	******* (0)	10.55 (9)	17.10 (21)	17.58 (6)	******* (0)	******* (0)
5	******* (0)	12.47 (2)	13.45 (23)	23.83 (19)	29.07 (3)	******* (0)
6	******* (0)	******* (0)	13.06 (12)	20.16 (32)	32.23 (9)	14.70 (1)
7	******* (0)	******* (0)	9.74 (3)	16.05 (14)	22.05 (17)	68.89 (4)
8	******* (0)	******* (0)	******* (0)	14.47 (12)	20.98 (22)	41.10 (7)
9	******* (0)	******* (0)	******* (0)	2.11 (2)	20.85 (18)	23.83 (38)
MEAN	11.76 (91)	16.93 (128)	15.79 (101)	21.71 (91)	23.03 (69)	29.67 (50)

TABLE 5.14. (CONTINUED)

SURVEY DATE 420

AGE OF MOTHER

PARITY	15-19		20-24		25-29		30-34		35-39		40-44	
0	13.29	19.60	33.75	30.77	31.51	84.63	*******	*******	269.37	269.37	*******	*******
	(64)	(67)	(29)	(40)	(1)	(1)	(0)	(0)	(1)	(1)	(0)	(0)
	12.19	12.38	15.74	16.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
1	9.97	15.74	13.43	23.50	23.18	30.23	*******	*******	*******	*******	*******	*******
	(30)	(30)	(28)	(61)	(4)	(8)	(0)	(0)	(0)	(0)	(0)	(0)
	12.19	12.38	15.74	16.17	16.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
2	7.95	4.92	16.46	16.84	20.21	29.83	46.17	31.43	27.68	*******	*******	*******
	(15)	(2)	(47)	(40)	(18)	(33)	(2)	(5)	(1)	(0)	(0)	(0)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
3	4.19	*******	13.13	18.35	21.03	23.99	11.69	16.11	*******	93.83	*******	*******
	(2)	(0)	(36)	(20)	(37)	(34)	(3)	(10)	(0)	(1)	(0)	(0)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
4	*******	*******	9.79	13.68	16.76	23.30	14.49	29.84	30.67	36.15	*******	*******
	(0)	(0)	(18)	(4)	(30)	(33)	(17)	(21)	(1)	(3)	(0)	(0)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
5	*******	*******	*******	*******	8.52	17.17	18.25	35.09	25.22	36.02	92.55	92.55
	(0)	(0)	(0)	(0)	(22)	(11)	(20)	(25)	(3)	(10)	(1)	(1)
	12.19	12.38	15.74	16.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
6	*******	*******	5.55	*******	13.96	22.58	13.29	31.77	36.67	51.49	57.19	51.12
	(0)	(0)	(1)	(0)	(8)	(3)	(29)	(19)	(16)	(25)	(2)	(3)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
7	*******	*******	*******	*******	3.77	*******	13.65	21.60	23.37	24.90	45.37	45.96
	(0)	(0)	(0)	(0)	(4)	(1)	(22)	(15)	(22)	(22)	(8)	(10)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
8	*******	*******	*******	*******	*******	*******	9.35	*******	16.67	15.49	32.80	39.33
	(0)	(0)	(0)	(0)	(0)	(0)	(5)	(0)	(25)	(13)	(11)	(16)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
9	*******	*******	*******	*******	*******	*******	3.76	*******	20.66	17.14	20.67	30.55
	(0)	(0)	(0)	(0)	(0)	(0)	(3)	(0)	(16)	(13)	(44)	(34)
	12.19	12.38	15.74	15.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)
MEAN	11.5	18.1	17.5	22.9	17.0	25.7	16.1	29.0	26.5	35.1	27.9	37.1
	(111)	(99)	(159)	(165)	(124)	(124)	(101)	(95)	(85)	(88)	(66)	(64)
	12.19	12.38	15.74	16.17	15.92	20.04	20.11	23.94	22.81	32.75	31.91	39.74
	(117)	(108)	(159)	(160)	(128)	(123)	(99)	(97)	(89)	(85)	(68)	(66)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3
RUN 5 RUN 4

TABLE 5.14. (CONTINUED)

SURVEY DATE 480

AGE OF MOTHER

PARITY	15-19		20-24		25-29		30-34		35-39		40-44	
0	12.58	15.06	37.22	33.04	79.18	77.01	*******	*******	*******	*******	329.37	329.37
	(53)	(84)	(27)	(55)	(3)	(14)	(0)	(0)	(0)	(0)	(1)	(1)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
1	11.44	13.18	16.63	21.07	23.60	19.50	*******	2.42	*******	*******	*******	*******
	(43)	(32)	(37)	(57)	(3)	(15)	(0)	(1)	(0)	(0)	(0)	(0)
	12.09	12.90	16.57	18.44	16.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
2	8.70	7.52	13.39	18.69	18.12	31.90	19.08	51.59	*******	*******	*******	*******
	(15)	(6)	(63)	(67)	(20)	(36)	(1)	(4)	(0)	(0)	(0)	(0)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
3	8.17	2.15	14.38	12.75	19.51	20.19	32.02	32.79	*******	*******	*******	*******
	(2)	(1)	(50)	(18)	(33)	(53)	(8)	(26)	(0)	(0)	(0)	(0)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
4	*******	*******	11.42	0.10	15.58	17.91	19.20	23.07	29.68	22.43	32.16	51.30
	(0)	(0)	(15)	(1)	(41)	(32)	(17)	(27)	(4)	(12)	(1)	(1)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
5	*******	*******	8.78	*******	15.22	25.07	21.19	27.72	26.97	35.32	*******	50.72
	(0)	(0)	(6)	(0)	(39)	(7)	(24)	(35)	(6)	(16)	(0)	(5)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
6	*******	*******	*******	*******	8.75	15.09	16.69	18.28	35.40	34.55	47.42	53.96
	(0)	(0)	(0)	(0)	(14)	(1)	(34)	(20)	(16)	(25)	(3)	(8)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
7	*******	*******	*******	*******	10.99	*******	15.24	20.61	23.24	29.34	21.65	28.87
	(0)	(0)	(0)	(0)	(5)	(0)	(22)	(7)	(19)	(19)	(3)	(14)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
8	*******	*******	*******	*******	30.03	*******	15.99	*******	15.63	18.99	26.05	25.29
	(0)	(0)	(0)	(0)	(1)	(0)	(6)	(0)	(26)	(14)	(24)	(24)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
9	*******	*******	*******	*******	*******	*******	12.59	*******	15.56	7.84	24.31	28.02
	(0)	(0)	(0)	(0)	(0)	(0)	(3)	(0)	(26)	(4)	(50)	(28)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)
MEAN	11.5	14.2	17.4	22.7	17.6	27.5	18.7	26.4	21.8	28.4	29.4	35.3
	(113)	(122)	(198)	(198)	(164)	(158)	(115)	(120)	(97)	(90)	(82)	(81)
	12.09	12.90	16.57	18.44	15.95	21.75	18.19	25.71	20.58	30.76	26.32	39.99
	(122)	(111)	(206)	(196)	(157)	(159)	(126)	(117)	(96)	(92)	(84)	(81)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3
RUN 5 RUN 4

TABLE 5.14. (CONTINUED)

PARITY	SURVEY DATE 540 — AGE OF MOTHER 15-19		20-24		25-29		30-34		35-39		40-44	
0	15.55	18.94	33.91	42.51	94.93	80.01	173.43	140.82	*******	*******	*******	*******
	(105)	(111)	(35)	(46)	(5)	(11)	(1)	(4)	(0)	(0)	(0)	(0)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
1	10.35	11.38	16.01	20.15	12.64	23.48	*******	37.16	*******	*******	*******	*******
	(40)	(27)	(43)	(61)	(6)	(23)	(0)	(2)	(0)	(0)	(0)	(0)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
2	7.93	5.08	13.47	13.47	21.85	23.51	*******	39.63	*******	*******	*******	*******
	(12)	(4)	(51)	(80)	(9)	(51)	(0)	(18)	(0)	(0)	(0)	(0)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
3	3.56	*******	14.20	9.31	17.49	20.25	24.65	36.53	*******	56.42	*******	*******
	(1)	(0)	(54)	(18)	(37)	(65)	(8)	(28)	(0)	(2)	(0)	(0)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
4	*******	*******	8.23	0.91	15.53	14.03	14.22	25.90	53.42	33.86	*******	*******
	(0)	(0)	(25)	(1)	(64)	(35)	(19)	(47)	(2)	(18)	(0)	(0)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
5	*******	*******	10.50	7.67	14.40	14.72	17.41	21.26	32.20	33.64	95.31	31.25
	(0)	(0)	(3)	(1)	(43)	(8)	(32)	(35)	(7)	(29)	(2)	(7)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
6	*******	*******	*******	*******	14.12	*******	18.05	16.60	27.03	31.07	65.28	44.39
	(0)	(0)	(0)	(0)	(25)	(0)	(35)	(16)	(13)	(23)	(5)	(17)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
7	*******	*******	*******	*******	7.18	*******	16.00	10.44	18.85	23.54	33.70	38.89
	(0)	(0)	(0)	(0)	(5)	(0)	(35)	(3)	(30)	(25)	(10)	(15)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
8	*******	*******	*******	*******	9.07	*******	11.63	*******	15.62	13.23	37.15	31.01
	(0)	(0)	(0)	(0)	(2)	(0)	(18)	(0)	(29)	(12)	(10)	(25)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
9	*******	*******	*******	*******	*******	*******	10.94	*******	15.20	12.23	24.10	23.66
	(0)	(0)	(0)	(0)	(0)	(0)	(3)	(0)	(27)	(2)	(63)	(16)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)
MEAN	14.4	17.1	16.9	23.4	17.4	25.7	17.4	30.3	19.8	30.1	30.9	33.9
	(158)	(142)	(211)	(207)	(197)	(193)	(152)	(153)	(108)	(112)	(91)	(80)
	14.26	14.23	17.02	20.58	20.98	22.31	19.40	25.17	21.69	26.98	31.25	36.04
	(166)	(151)	(233)	(207)	(210)	(190)	(166)	(148)	(124)	(111)	(92)	(85)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3
RUN 5 RUN 4

TABLE ?.15. ALL OPEN INTERVALS,BY PARITY,CROSS-SECTIONAL POPULATION

PARITY	SURVEY DATE 30	35		40		45	
0	20.97	22.43	26.59	26.68	29.16	26.95	34.05
	(66)	(95)	(109)	(84)	(154)	(147)	(173)
		21.90	19.69	23.81	24.90	28.79	29.92
		(97)	(76)	(103)	(100)	(114)	(160)
1	16.30	12.77	21.69	14.73	18.26	13.24	20.25
	(58)	(62)	(99)	(88)	(105)	(89)	(113)
		13.96	14.74	13.53	15.76	14.99	12.23
		(67)	(82)	(80)	(81)	(90)	(79)
2	21.54	16.23	29.56	14.01	23.47	13.93	24.31
	(67)	(83)	(82)	(99)	(113)	(72)	(153)
		21.36	16.00	14.58	17.16	15.63	15.24
		(87)	(82)	(104)	(81)	(112)	(82)
3	14.65	16.59	22.12	17.62	22.19	16.20	23.18
	(61)	(78)	(65)	(93)	(97)	(100)	(113)
		15.04	16.20	15.43	15.90	15.49	17.59
		(75)	(70)	(90)	(89)	(111)	(93)
4	49.03	17.30	28.74	19.63	24.08	17.29	24.03
	(38)	(67)	(62)	(79)	(74)	(111)	(101)
		32.84	35.05	31.14	25.29	19.16	24.59
		(76)	(99)	(87)	(116)	(103)	(124)
5	18.11	15.66	32.32	19.33	33.08	21.30	28.30
	(57)	(46)	(47)	(76)	(64)	(89)	(82)
		17.09	22.56	17.94	26.10	19.23	28.38
		(53)	(52)	(76)	(102)	(109)	(126)
6	40.00	46.70	58.90	38.87	50.27	29.07	44.13
	([illegible])	(60)	(53)	(71)	(57)	(83)	(62)
		45.36	48.18	29.41	37.01	33.74	33.99
		(49)	(57)	(78)	(73)	(80)	(103)
7	53.89	30.95	47.24	35.60	49.02	33.71	54.28
	([illegible])	(61)	(52)	(55)	(49)	(88)	(62)
		44.36	56.06	53.28	53.08	44.17	46.01
		(63)	(51)	(64)	(63)	(105)	(90)
8	81.51	48.70	67.27	50.62	47.66	60.25	53.21
	(52)	(51)	(39)	(74)	(49)	(77)	(62)
		63.05	66.41	44.54	68.32	48.74	61.25
		(53)	(49)	(69)	(55)	(87)	(64)
9+	184.57	134.05	153.83	111.29	147.03	97.91	130.00
	(199)	(199)	(188)	(229)	(188)	(284)	(178)
		150.08	154.25	126.75	125.38	110.55	117.76
		(215)	(180)	(265)	(174)	(321)	(780)
MEAN	76.51	51.53	61.97	45.92	53.01	43.34	47.12
	(693)	(802)	(796)	(948)	(950)	(1140)	(1099)
		59.97	58.17	52.05	46.88	47.65	43.67
		(835)	(798)	(1016)	(934)	(1262)	(1161)

fertility is reflected by increased birth intervals in runs 3 and 4. This increase, however, is less than the change in parameters used for generating these computer populations.

5.6.3. Straddling Intervals

All intervals which include the survey date A straddle A. These intervals, however, cannot be measured at survey date, since these intervals, obviously, have not been completed by that time. Follow up of each woman until her completion of that interval, or until the end of her fertile period, or until her death, whichever occurs first, would yield these straddling intervals. A truncated distribution of these intervals can be obtained at any time after the survey date, and in high fertility situations, where birth intervals are shorter, this truncation effect will be negligible, especially if these intervals are measured at least 5 years after survey date, since only a few (though long) intervals will be missed.

The effect of decreasing mortality on straddling intervals is insignificant, as can be observed from Table 5.16. The effect of a decrease in fertility, however, is reflected by a corresponding increase in the straddling intervals. The change in these intervals over time is insignificant in all computer populations.

5.6.4. All Previous Closed Intervals, by Total Number of Intervals at Survey Date

As in the case of cohort populations, these intervals can be measured in computer populations prospectively and retrospectively,

TABLE 5.16. STRADDLING INTERVALS,CROSS-SECTIONAL POPULATIONS

PARITY	SURVEY DATE											
	30				35				40			
0	41.85 (64)	39.28 (63)	50.72 (64)	42.74 (62)	37.26 (93)	35.92 (90)	53.72 (103)	33.93 (72)	38.26 (70)	33.23 (88)	44.80 (123)	34.48 (79)
1	38.93 (57)	40.72 (58)	50.26 (56)	39.57 (57)	25.80 (62)	27.84 (67)	38.68 (96)	27.90 (80)	27.85 (88)	27.58 (80)	41.05 (103)	29.54 (81)
2	40.17 (63)	41.08 (66)	55.52 (66)	39.78 (65)	32.43 (82)	31.10 (82)	51.39 (77)	31.63 (81)	29.46 (97)	30.69 (102)	46.41 (104)	34.53 (81)
3	42.53 (60)	46.95 (61)	53.95 (59)	41.60 (61)	34.02 (78)	34.19 (74)	48.59 (63)	30.46 (67)	32.31 (91)	31.71 (89)	47.07 (88)	32.24 (84)
4	40.75 (36)	42.89 (36)	56.35 (35)	60.11 (33)	33.39 (55)	31.71 (74)	49.02 (61)	50.09 (94)	31.11 (76)	35.30 (84)	46.23 (55)	46.59 (110)
5	46.67 (50)	46.14 (49)	63.19 (47)	62.03 (50)	31.32 (43)	32.52 (52)	63.26 (46)	50.22 (50)	32.03 (71)	32.76 (73)	53.54 (59)	47.36 (83)
6	52.56 (50)	50.97 (52)	71.69 (51)	67.63 (50)	43.96 (53)	39.66 (44)	64.70 (46)	57.86 (48)	35.06 (59)	35.15 (71)	52.46 (45)	49.50 (55)
7	51.45 (30)	49.82 (33)	67.37 (28)	62.18 (32)	39.85 (54)	39.74 (52)	54.47 (43)	57.69 (39)	35.98 (45)	38.23 (46)	52.12 (33)	50.75 (43)
8	45.47 (36)	48.05 (36)	64.67 (34)	69.28 (33)	34.07 (37)	34.41 (38)	53.98 (26)	53.68 (33)	34.78 (51)	35.22 (48)	54.38 (29)	56.77 (33)
9	55.38 (47)	55.38 (48)	68.67 (47)	72.55 (43)	44.17 (61)	51.01 (69)	71.38 (53)	68.00 (43)	42.11 (84)	47.83 (80)	62.85 (35)	67.52 (35)
MEAN	45.12 (493)	45.37 (502)	59.18 (487)	53.27 (486)	35.55 (523)	35.60 (642)	53.21 (614)	43.94 (612)	33.56 (732)	34.45 (761)	47.85 (684)	42.19 (691)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4 RUN 5

TABLE 5.17. ALL PREVIOUS CLOSED MEAN INTERVALS, PROSPECTIVE AND RETROSPECTIVE ASCERTAINMENT, CROSS-SECTIONAL POPULATIONS

	SURVEY DATE												
PARITY	360			420				480				540	
0	23.63	24.00	23.97	24.84	24.02	23.93	23.98	26.27	23.75	24.10	23.88	27.99	23.83
	23.99	24.20	24.27	25.22	24.12	24.13	24.15	26.60	23.71	24.33	24.02	28.30	23.84
1	24.40	24.95	25.40	26.38	25.25	24.71	25.09	27.83	24.96	24.68	24.68	29.12	24.95
	24.45	25.13	25.45	26.47	25.42	24.84	25.12	28.06	25.06	24.78	24.66	29.42	25.04
2	27.95	28.31	28.57	29.59	28.45	28.22	28.24	31.32	28.26	27.78	27.92	32.92	28.18
	28.41	28.61	28.82	29.96	28.87	28.47	28.39	31.62	28.56	27.93	28.00	33.45	28.39
3	28.17	29.10	29.41	30.18	28.85	29.15	29.32	31.61	28.46	28.96	28.95	33.27	28.40
	27.72	28.73	29.24	29.79	28.39	28.87	29.20	31.35	28.10	28.72	28.79	33.25	28.08
4	28.95	29.15	29.47	29.89	30.31	29.11	29.10	31.80	32.52	28.82	29.28	33.13	34.43
	29.03	29.21	29.55	29.82	30.50	29.13	29.12	31.82	32.72	28.85	29.30	33.31	34.80
5	28.94	30.09	30.45	30.83	31.45	29.48	30.04	32.78	32.71	29.06	29.63	34.59	34.47
	29.02	30.26	30.75	30.87	31.73	29.65	30.22	32.73	33.15	29.16	29.94	34.71	34.96
6	30.53	32.02	32.14	33.06	33.08	32.39	31.76	35.58	35.26	31.84	31.40	36.58	36.48
	30.64	32.01	32.29	33.23	33.41	32.37	31.91	35.84	35.64	31.81	31.49	36.99	36.88
7	32.16	32.89	32.85	33.73	33.66	32.85	32.81	35.61	35.42	32.25	32.57	36.58	35.54
	32.44	33.06	33.17	34.23	34.03	32.91	33.12	36.00	35.82	32.27	32.80	36.85	37.04
8	30.44	32.07	32.14	33.69	33.47	31.91	31.76	35.47	35.93	31.82	31.75	37.01	37.76
	30.47	32.04	32.06	33.62	33.60	31.98	31.69	35.49	36.16	31.93	31.72	37.06	38.04
9	35.37	35.97	35.52	36.26	36.46	34.93	35.29	38.46	37.72	34.65	35.60	39.59	39.26
	35.41	36.04	35.57	36.30	36.39	35.07	35.34	38.55	37.69	34.79	35.63	39.80	39.27
MEAN	28.31	28.95	29.11	29.91	29.34	28.76	28.87	31.56	29.86	28.58	28.72	32.89	30.49
	(4939)	(5910)	(5883)	(5525)	(5729)	(7315)	(7339)	(6466)	(6873)	(8985)	(9057)	(7559)	(8183)
	28.61	29.23	29.34	30.21	29.66	28.98	29.03	31.87	30.17	28.76	28.82	33.27	30.82
	(4275)	(5138)	(5263)	(4772)	(4939)	(6432)	(6685)	(5643)	(5999)	(8059)	(8394)	(5708)	(7272)

THE UPPER LINE REFERS TO PROSPECTIVE ASCERTAINMENT, WHILE THE LOWER LINE REFERS TO THE RETROSPECTIVE ASCERTAINMENT

THE ORDER OF THE INTERVALS IN EACH LINE IS
RUN 2 RUN 3 RUN 5 RUN 4

i.e., for all women, whether living or not, and for living women only; and as in cohorts, the mean birth intervals are slightly longer for surviving women than for all women.

Change in mortality does not significantly affect these intervals, nor do these intervals change significantly at different survey times. Decrease in fertility, however, is reflected by an increase in last few closed intervals only, the number of intervals affected depending upon the time elapsed since change in fertility level.

5.6.5. All Previous Closed Intervals

These intervals are weighted averages of the intervals obtained in Section 5.6.4, the weights being the number of women in each parity. The intervals for all women are slightly shorter than those for living women only. Mortality change does not significantly affect the mean birth intervals, but more importantly, fertility changes do not affect the mean intervals significantly until about 15 years after the decrease in fertility. These indices, therefore, cannot be referred to as sensitive indices.

5.6.6. Previous Closed Interval by Age of Mother

Age of mother was grouped in five-year age groups and ascertained in three ways:

(i) at beginning of birth interval,

(ii) at end of birth interval,

(iii) at survey date.

The discussion in Section 5.6.4 indicated that the last closed interval may be a sensitive fertility index worth looking into, and therefore, it was decided to investigate this particular birth interval in more detail by mother's age at different times. As in other ascertainment plans, the effect of decreased mortality is insignificant. The effect of decreased fertility, however, is mixed, as evident from Table 5.18. For the earlier age groups, the last birth intervals increased proportionately, while for later ages of mothers, the increase in length of previous closed intervals is smaller. This could be due to a truncation effect, as sterility and end of fertile period play their roles. This truncation effect is especially evident in the birth intervals where the age of mother is determined at the beginning of interval. A woman who has a closed birth interval starting at age, 42 years, does not have a very long period before the end of fertile period sets in, and, therefore, that particular interval would be, on the average, of shorter length than if it had started when age of the woman was, say, 30 years.

Previous closed intervals do not significantly change over a period of time, other competing risks being constant, and, therefore, can be regarded as robust.

Table 5.21 summarizes the discussion about various methods of ascertainments above.

TABLE 5.18. PREVIOUS CLOSED INTERVAL BY AGE OF MOTHER AT BEGINNING OF INTERVAL, CROSS-SECTIONAL POPULATIONS

AGE AT BEGINNING OF INTERVAL	SURVEY DATE 30	35		40		45	
15- 19	26.82	25.85	34.81	24.92	36.18	25.32	37.93
	(141)	(157)	(188)	(202)	(207)	(202)	(239)
		28.13	27.58	24.11	24.85	23.52	25.28
		(160)	(173)	(197)	(184)	(219)	(184)
20- 24	27.72	30.17	41.36	26.80	38.37	26.98	39.23
	(125)	(163)	(165)	(210)	(198)	(235)	(234)
		30.85	32.38	27.67	29.79	27.51	30.66
		(180)	(194)	(204)	(253)	(258)	(279)
25- 29	29.36	33.29	42.06	32.48	42.03	28.50	42.73
	(115)	(146)	(129)	(169)	(163)	(223)	(201)
		32.82	41.94	29.36	38.91	28.19	36.12
		(122)	(148)	(173)	(174)	(201)	(223)
30- 34	34.14	37.41	45.24	32.18	43.49	32.62	43.22
	(119)	(142)	(127)	(158)	(143)	(194)	(174)
		37.49	43.41	33.11	44.72	33.01	44.93
		(139)	(125)	(154)	(154)	(206)	(192)
35- 39	41.99	42.41	48.18	40.79	50.16	39.89	50.83
	(109)	(133)	(124)	(181)	(174)	(214)	(211)
		42.38	47.83	40.05	48.34	41.25	50.12
		(139)	(131)	(179)	(165)	(213)	(204)
40- 44	40.30	41.24	41.65	39.32	43.92	39.25	45.93
	(129)	(156)	(143)	(199)	(174)	(247)	(212)
		40.30	41.80	39.56	42.55	41.29	45.02
		(156)	(139)	(203)	(174)	(245)	(209)
MEAN	33.14	34.81	41.63	32.57	42.05	32.22	43.13
	(738)	(897)	(876)	(1119)	(1059)	(1315)	(1271)
		35.10	38.20	32.23	37.27	32.50	38.36
		(896)	(910)	(1110)	(1104)	(1342)	(1291)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3
RUN 5 RUN 4

TABLE 5.19. PREVIOUS CLOSED INTERVAL BY AGE OF MOTHER AT END OF INTERVAL, CROSS-SECTIONAL POPULATIONS

AGE AT END OF INTERVAL	SURVEY DATE 30	35		40		45	
15- 19	18.91	18.58	21.76	18.44	22.51	18.19	24.15
	(67)	(84)	(81)	(114)	(86)	(107)	(94)
		18.67	17.55	18.66	18.86	19.20	18.78
		(73)	(93)	(108)	(97)	(116)	(92)
20- 24	27.21	27.76	37.84	27.05	35.99	26.73	35.80
	(143)	(163)	(163)	(201)	(198)	(218)	(224)
		28.84	29.33	27.06	26.31	24.58	26.40
		(174)	(178)	(204)	(219)	(245)	(233)
25- 29	29.97	31.05	41.49	29.41	40.18	28.20	43.71
	(119)	(147)	(147)	(190)	(171)	(236)	(227)
		33.73	35.51	27.74	32.82	28.91	32.86
		(145)	(146)	(177)	(189)	(230)	(236)
30- 34	31.18	34.24	41.80	31.38	42.32	29.69	42.39
	(112)	(127)	(119)	(156)	(154)	(193)	(182)
		32.67	43.74	30.01	39.69	29.97	40.13
		(126)	(140)	(158)	(152)	(193)	(190)
35- 39	33.62	38.07	46.50	32.14	43.98	32.33	42.29
	(99)	(128)	(129)	(150)	(147)	(178)	(171)
		37.59	45.10	32.91	43.39	33.73	42.42
		(132)	(118)	(150)	(146)	(176)	(181)
40- 44	37.67	39.88	43.11	38.50	45.71	36.73	45.52
	(115)	(152)	(143)	(185)	(176)	(231)	(208)
		37.92	42.39	36.48	44.57	36.03	44.30
		(149)	(142)	(191)	(182)	(223)	(205)
MEAN	30.35	32.38	39.86	30.09	39.66	29.58	40.35
	(655)	(801)	(782)	(996)	(932)	(1163)	(1106)
		32.54	36.11	29.44	64.80	29.30	35.20
		(799)	(817)	(988)	(985)	(1183)	(1137)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3
RUN 5 RUN 4

TABLE 5.20. PREVIOUS CLOSED INTERVAL BY AGE OF MOTHER AT SURVEY DATE, CROSS-SECTIONAL POPULATIONS

AGE AT SURVEY	SURVEY DATE 30	35		40		45	
15- 19	17.24	17.58	17.06	16.61	20.07	17.10	23.07
	(43)	(47)	(32)	(60)	(38)	(53)	(31)
		18.81	15.38	16.68	18.36	18.50	17.39
		(45)	(51)	(55)	(47)	(64)	(47)
20- 24	25.64	24.96	36.13	24.83	32.93	25.64	33.01
	(111)	(130)	(125)	(171)	(143)	(176)	(161)
		25.74	26.83	24.66	23.42	22.70	24.28
		(135)	(142)	(179)	(164)	(201)	(159)
25- 29	30.38	30.56	42.26	29.84	41.71	27.86	42.92
	(101)	(123)	(123)	(161)	(144)	(192)	(182)
		33.54	33.69	28.59	30.21	28.87	30.48
		(128)	(123)	(159)	(156)	(203)	(184)
30- 34	29.39	35.11	45.87	29.19	43.08	28.30	42.84
	(90)	(101)	(95)	(115)	(120)	(151)	(149)
		33.31	44.50	27.94	37.86	28.11	37.06
		(99)	(97)	(126)	(117)	(164)	(147)
35- 39	33.85	35.98	49.43	30.62	45.17	28.25	44.16
	(69)	(84)	(87)	(97)	(90)	(108)	(112)
		38.34	50.24	32.53	45.08	31.79	41.76
		(88)	(84)	(96)	(92)	(124)	(111)
40- 44	35.73	42.26	52.83	34.48	52.17	32.73	43.84
	(50)	(66)	(64)	(81)	(80)	(91)	(80)
		37.79	52.52	33.30	50.11	32.44	48.55
		(68)	(66)	(83)	(90)	(92)	(85)
MEAN	28.93	31.19	42.40	27.98	40.47	27.33	40.11
	(464)	(551)	(526)	(685)	(615)	(771)	(715)
		31.72	36.84	27.63	33.54	27.29	33.42
		(563)	(563)	(698)	(656)	(848)	(733)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3
RUN 5 RUN 4

TABLE 5.21. SUMMARY OF ASCERTAINMENT PLANS FOR CROSS-SECTIONAL POPULATIONS

KIND OF BIRTH INTERVAL	EFFECT OF DECREASE IN MORTALITY	EFFECT OF DECREASE IN FERTILITY ON BIRTH INTERVAL	ROBUSTNESS
1. Open birth intervals, by age of mother at survey	No change	Proportional Increase	Yes
2. All open intervals	"	Slight Increase	No
3. Straddling intervals	"	Proportional Increase	Yes
4. All previous closed intervals, by total number of intervals at survey date	"	Proportional Increase in last few intervals	Yes
5. All previous closed intervals	"	No change	Yes
6. Previous closed interval by age of mother measured at			
(i) beginning of interval (ii) end of interval (iii) survey date	"	Proportional Increase in earlier age groups, smaller increase as age of woman increases	Yes

5.7. Investigation of Sensitivity of Selected Indices

The discussion so far has centered upon whether or not a particular ascertainment plan gives rise to birth intervals which would indicate changes in fertility, if there have been any, and whether these intervals are robust. These analyses, however, were based on survey dates five years apart, and any changes could be reflected no earlier than five years. For an index to be termed sensitive, it is desirable for change to be reflected as soon as possible. This section is devoted to further study of those methods of ascertainment which showed promise as sensitive indices. Previous closed interval was a promising index in the analysis of cross-sectional populations, and it seemed reasonable to investigate this index in two more ways, namely, by parity (for all birth intervals), and by parity (for all intervals ending after survey date 360 months).

The different birth intervals investigated for short-term sensitivity changes are as follows:

(i) all open intervals, by parity.

(ii) straddling intervals, by parity.

(iii) previous closed intervals, by age of mother

  (a) at survey date

  (b) at beginning of interval

  (c) at end of interval.

(iv) previous closed interval, by parity.

(v) previous closed interval, by parity; only those intervals to be considered which end after survey date

360 months. (The parameters for fertility decrease were changed after survey date 360 months in runs 3 and 4).

Runs 2, 3 and 4 correspond to the numbered runs in the cross-sectional computer populations in Table 5.2, earlier.

Instead of tabulating the above intervals at survey dates 5 years apart, we tabulated them at survey dates 384 months, 396 months and 408 months. Since the effect of mortality had been determined as non-significant in all methods of ascertainments, this effect was not studied in these tabulations.

The intent was to find the shortest period of time for each of the above ascertainment plans after the change in fertility had been put in effect so that the increase in birth intervals was proportional to the change in input parameters.

5.7.1. All Open Intervals

These intervals were tabulated by parity of the mother at survey date 32 years, 33 years and 34 years, *i.e.*, two, three and four years after the fertility decrease had been effected. As can be seen from Table 5.22, these intervals show increase in two years, but the increase is not as much as 50%, the amount of the increase in input parameters. Further, with the advent of time, these intervals tend to decrease within each run, even though no parameters have been changed. This index, therefore, while sensitive, is not robust.

TABLE 5.22. ALL OPEN INTERVALS,CROSS-SECTIONAL POPULATIONS

PARITY	SURVEY DATE								
	32			33			34		
0	23.13	26.85	23.05	20.18	23.05	22.77	19.88	24.11	23.99
	(73)	(104)	(76)	(75)	(98)	(80)	(87)	(96)	(73)
1	17.41	24.74	18.20	13.56	19.59	14.85	13.84	18.14	13.71
	(77)	(76)	(76)	(75)	(101)	(81)	(70)	(106)	(81)
2	27.10	40.46	28.83	21.65	38.93	24.71	14.67	31.21	14.33
	(73)	(71)	(73)	(72)	(66)	(75)	(81)	(77)	(80)
3	23.97	33.21	24.21	16.25	28.82	17.71	17.80	25.59	15.16
	(64)	(62)	(61)	(73)	(64)	(62)	(77)	(63)	(71)
4	23.79	68.63	44.52	16.42	52.47	36.90	18.08	25.80	35.71
	(45)	(38)	(63)	(52)	(51)	(84)	(62)	(55)	(89)
5	28.46	37.67	40.97	21.42	39.68	43.34	23.43	34.85	30.76
	(47)	(51)	(49)	(45)	(42)	(43)	(41)	(46)	(45)
6	49.55	54.40	52.74	46.35	51.43	55.05	45.79	54.15	48.68
	(60)	(57)	(56)	(62)	(64)	(56)	(59)	(61)	(57)
7	41.91	71.18	78.31	36.87	61.02	70.99	32.65	58.50	65.33
	(48)	(43)	(38)	(55)	(44)	(46)	(59)	(49)	(53)
8	81.24	85.57	79.13	70.93	69.31	69.76	64.69	73.99	71.34
	(43)	(49)	(54)	(37)	(47)	(47)	(45)	(42)	(47)
9+	157.92	188.26	185.90	151.01	181.90	168.72	140.78	168.67	162.10
	(192)	(185)	(181)	(203)	(187)	(179)	(196)	(197)	(181)
MEAN	66.13	80.59	76.17	60.44	73.62	67.42	54.66	66.62	62.52
	(722)	(736)	(727)	(749)	(764)	(753)	(777)	(782)	(777)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

TABLE 5.23. STRADDLING INTERVALS,CROSS-SECTIONAL POPULATIONS

PARITY	SURVEY DATE 32			33			34		
0	41.17 (71)	47.34 (101)	37.26 (73)	38.08 (71)	48.90 (94)	36.59 (75)	38.43 (85)	50.38 (91)	36.77 (69)
1	29.89 (76)	47.10 (73)	33.15 (76)	26.11 (74)	42.73 (98)	28.85 (79)	26.42 (69)	39.23 (103)	28.61 (80)
2	35.77 (72)	57.02 (68)	36.34 (72)	30.98 (70)	55.14 (64)	32.49 (74)	31.90 (80)	49.76 (73)	32.00 (78)
3	40.73 (64)	53.76 (62)	39.67 (61)	34.55 (73)	53.89 (61)	35.63 (60)	34.19 (77)	50.90 (58)	30.34 (68)
4	36.45 (43)	59.00 (35)	54.88 (57)	32.51 (51)	52.20 (49)	50.78 (79)	32.08 (61)	48.93 (54)	50.37 (83)
5	43.32 (46)	64.46 (47)	63.48 (48)	35.84 (44)	70.22 (39)	63.85 (42)	35.42 (39)	67.63 (44)	54.75 (44)
6	48.90 (54)	71.41 (52)	66.59 (49)	45.58 (55)	68.03 (58)	66.20 (49)	43.84 (52)	66.52 (54)	62.14 (46)
7	48.75 (39)	67.12 (31)	65.24 (29)	42.38 (46)	61.51 (34)	61.73 (34)	43.21 (51)	61.22 (38)	59.20 (40)
8	48.68 (30)	64.92 (35)	68.79 (37)	40.69 (23)	65.73 (34)	70.60 (32)	35.65 (31)	63.67 (29)	68.00 (31)
9	50.56 (58)	72.68 (47)	76.26 (43)	44.58 (68)	75.44 (43)	75.54 (43)	43.47 (60)	76.50 (47)	70.55 (46)
MEAN	41.36 (553)	58.09 (551)	50.33 (545)	36.50 (575)	56.47 (574)	47.81 (567)	36.05 (605)	56.46 (591)	45.39 (585)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

5.7.2. Straddling Intervals

Table 5.23 contains the straddling intervals by parity which straddle different survey dates. These intervals show proportional increase three years after the reduced fertility has been in effect. HOwever, it would take longer than three years to obtain all straddling intervals, because all of them would end after survey date 33 years.

These intervals are robust, since these do not change if any parameters directly affecting fertility remain unchanged.

5.7.3. Previous Closed Intervals

These intervals were classified by age of mother, and by parity. The age of mother was determined at survey date, at the beginning of the birth interval, and at the end of the interval. Tables 5.24 through 5.28 give the mean previous closed birth intervals cross-tabulated by the above classifications and at survey dates 32 years, 33 years, and 34 years.

All the classifications show these intervals to be robust, but the minimum time required for showing change in fertility in each case was at least four years which was for intervals ending after the fertility change had been effected.

5.8. Comparison of Distribution of Birth Intervals--Input and Output

In the present study, known distributions of birth intervals by parity were used as input in generating births in the computer populations. It would be of interest to know how the distributions of

TABLE 5.24. PREVIOUS CLOSED INTERVAL,AGE OF MOTHER AT
BEGINNING OF INTERVAL,CROSS-SECTIONAL POPULATIONS

AGE AT BEGINNING OF INTERVAL	SURVEY DATE								
	32			33			34		
15- 19	29.77 (165)	27.14 (161)	27.34 (156)	29.47 (158)	32.45 (179)	28.23 (167)	26.92 (160)	33.95 (185)	27.55 (168)
20- 24	28.34 (137)	28.57 (130)	30.19 (161)	33.25 (156)	33.41 (145)	32.04 (174)	32.96 (169)	39.16 (159)	32.58 (191)
25- 29	32.08 (121)	29.80 (120)	30.35 (117)	36.71 (131)	33.78 (127)	35.24 (119)	34.58 (133)	39.09 (133)	42.30 (134)
30- 34	34.11 (121)	34.32 (118)	34.52 (115)	36.42 (117)	36.66 (122)	37.03 (122)	36.97 (129)	40.30 (125)	39.11 (122)
35- 39	41.47 (121)	41.69 (113)	41.32 (114)	43.21 (133)	42.30 (109)	43.66 (116)	44.08 (129)	44.28 (112)	45.15 (121)
40- 44	41.27 (146)	41.18 (143)	41.11 (144)	41.61 (152)	41.77 (151)	41.13 (148)	42.08 (162)	42.44 (156)	42.14 (152)
MEAN	34.34 (811)	33.51 (785)	33.80 (807)	36.58 (847)	36.42 (833)	35.64 (846)	36.00 (882)	39.45 (868)	37.34 (888)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

TABLE 5.25. PREVIOUS CLOSED INTERVAL, AGE OF MOTHER AT END OF INTERVAL, CROSS-SECTIONAL POPULATIONS

AGE AT END OF INTERVAL	SURVEY DATE 32			33			34		
15- 19	20.61 (75)	18.97 (79)	20.08 (82)	18.91 (77)	20.50 (82)	19.53 (86)	17.05 (83)	21.85 (87)	18.42 (91)
20- 24	30.33 (166)	28.02 (155)	29.54 (153)	31.57 (156)	33.45 (173)	30.70 (165)	30.29 (160)	36.80 (166)	28.93 (167)
25- 29	30.57 (122)	30.87 (119)	30.84 (140)	36.02 (136)	36.39 (123)	34.42 (142)	33.90 (148)	41.47 (137)	37.36 (147)
30- 34	32.85 (120)	31.68 (117)	31.23 (113)	36.59 (129)	34.62 (124)	35.21 (123)	35.57 (119)	37.73 (124)	40.83 (136)
35- 39	35.12 (102)	33.74 (101)	34.46 (102)	37.94 (109)	37.01 (111)	37.31 (105)	38.81 (121)	41.48 (122)	41.73 (113)
40- 44	44.30 (226)	45.23 (214)	44.94 (217)	45.15 (241)	45.42 (220)	45.64 (225)	45.98 (251)	46.61 (232)	46.54 (234)
MEAN	34.34 (811)	33.51 (785)	33.80 (807)	36.58 (847)	36.42 (833)	35.64 (846)	36.00 (882)	39.45 (868)	37.34 (888)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

TABLE 5.26. PREVIOUS CLOSED INTERVAL, AGE OF MOTHER AT SURVEY DATE, CROSS-SECTIONAL POPULATIONS

AGE AT SURVEY	SURVEY DATE 32			33			34		
15- 19	18.85 (41)	18.97 (29)	19.72 (37)	17.62 (46)	20.98 (40)	18.49 (54)	16.42 (56)	20.51 (45)	17.32 (60)
20- 24	29.49 (124)	24.59 (112)	27.20 (128)	31.05 (126)	33.96 (118)	29.37 (124)	28.48 (119)	36.85 (116)	27.22 (126)
25- 29	32.39 (109)	30.83 (109)	30.67 (108)	37.08 (117)	36.35 (117)	34.80 (113)	35.36 (126)	43.09 (129)	36.12 (126)
30- 34	30.70 (87)	30.14 (91)	29.73 (88)	36.36 (87)	34.82 (90)	35.70 (90)	34.84 (91)	40.28 (88)	44.06 (89)
35- 39	33.38 (74)	30.59 (70)	32.74 (72)	38.25 (78)	34.40 (74)	37.35 (73)	38.95 (82)	41.55 (84)	45.60 (81)
40- 44	39.70 (187)	39.89 (185)	40.08 (184)	41.48 (194)	40.10 (198)	41.05 (191)	43.13 (195)	42.02 (201)	42.90 (198)
MEAN	32.99 (622)	31.76 (596)	32.21 (617)	35.89 (648)	35.66 (637)	34.66 (645)	35.16 (669)	39.57 (663)	36.95 (680)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

TABLE 5.27. PREVIOUS CLOSED INTERVAL, BY PARITY, CROSS-SECTIONAL POPULATIONS

	SURVEY DATE								
OF INTERVAL	32			33			34		
0	28.86 (83)	27.22 (83)	26.79 (82)	28.03 (82)	31.47 (110)	24.95 (87)	24.26 (78)	31.29 (115)	23.20 (88)
1	27.66 (82)	26.32 (77)	28.33 (89)	29.43 (81)	33.03 (73)	32.04 (82)	27.22 (92)	38.12 (84)	30.46 (88)
2	30.47 (77)	26.73 (76)	29.13 (73)	32.81 (86)	32.39 (78)	32.46 (74)	31.34 (90)	39.42 (78)	31.79 (84)
3	31.21 (54)	29.79 (46)	29.10 (73)	36.90 (63)	34.73 (60)	31.91 (96)	35.18 (73)	40.84 (65)	33.62 (102)
4	29.19 (54)	31.02 (58)	31.41 (56)	33.40 (52)	33.24 (50)	34.94 (56)	31.43 (49)	38.15 (55)	39.50 (52)
5	33.57 (75)	32.64 (72)	32.58 (71)	36.32 (77)	36.11 (79)	35.78 (72)	35.49 (74)	37.94 (76)	41.62 (74)
6	32.84 (59)	31.15 (52)	32.31 (46)	36.40 (66)	34.06 (55)	35.40 (55)	39.10 (73)	40.18 (61)	42.17 (62)
7	36.81 (64)	35.95 (71)	35.73 (76)	39.95 (59)	37.38 (71)	36.84 (72)	38.41 (67)	38.72 (67)	38.92 (72)
8	33.84 (58)	33.57 (49)	33.03 (46)	38.11 (68)	36.03 (51)	36.79 (52)	39.57 (64)	41.85 (54)	40.66 (57)
9	43.01 (205)	43.02 (201)	42.99 (204)	43.53 (213)	43.38 (206)	43.94 (206)	44.31 (222)	44.26 (213)	45.30 (209)
MEAN	34.34 (811)	33.51 (785)	33.80 (807)	36.58 (847)	36.42 (833)	35.64 (846)	36.00 (882)	39.25 (868)	37.34 (883)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

TABLE 5.28. PREVIOUS CLOSED INTERVAL ENDING AFTER SURVEY DATE 30 YEARS
CROSS-SECTIONAL POPULATIONS

	SURVEY DATE								
OF INTERVAL	32			33			34		
0	29.64 (77)	27.92 (76)	26.62 (76)	29.76 (76)	32.27 (103)	24.65 (81)	24.71 (72)	32.05 (108)	22.78 (82)
1	27.73 (72)	25.73 (70)	27.58 (72)	29.75 (71)	33.12 (66)	31.71 (74)	27.23 (82)	38.65 (77)	29.99 (80)
2	30.69 (64)	26.46 (62)	29.39 (61)	33.61 (73)	33.97 (64)	33.36 (62)	31.84 (77)	41.94 (64)	32.46 (72)
3	29.93 (44)	28.08 (35)	27.76 (60)	36.90 (53)	34.45 (49)	31.38 (83)	41.83 (63)	33.37 (54)	****** (89)
4	28.04 (46)	30.70 (47)	31.59 (48)	32.96 (44)	33.48 (39)	35.82 (42)	30.57 (41)	39.58 (44)	41.17 (44)
5	32.65 (54)	31.90 (52)	31.69 (50)	36.73 (56)	36.63 (59)	35.22 (51)	35.60 (53)	39.14 (56)	44.36 (53)
6	34.03 (40)	32.06 (31)	34.11 (29)	38.85 (47)	36.69 (34)	36.15 (38)	42.19 (54)	35.63 (40)	47.05 (45)
7	32.25 (30)	32.18 (35)	31.97 (39)	38.74 (25)	35.07 (35)	33.83 (35)	35.91 (33)	37.68 (31)	38.12 (35)
8	32.57 (30)	30.82 (21)	29.46 (17)	40.14 (40)	36.49 (23)	38.88 (23)	42.96 (36)	48.53 (26)	46.39 (28)
9	29.04 (31)	27.03 (27)	29.53 (27)	34.76 (39)	31.85 (32)	37.22 (29)	40.01 (48)	38.76 (39)	46.73 (32)
MEAN	30.39 (488)	28.83 (456)	29.44 (479)	34.29 (524)	34.07 (504)	32.77 (518)	33.51 (559)	39.11 (539)	35.68 (560)

THE ORDER OF THE BIRTH INTERVALS IS RUN 2 RUN 3 RUN 4

TABLE 5.29. SELECTED BIRTH INTERVALS: CROSS-SECTIONAL POPULATION

KIND OF BIRTH INTERVAL	TIME REQUIRED FOR INTERVAL TO INCREASE PROPORTIONATELY	ROBUSTNESS
1. All open intervals	Less than proportionate increase in two years	No
2. Straddling intervals	Three years*	Yes
3. Previous closed interval, by age of mother at		
(i) survey date	Between four and five years	Yes
(ii) beginning of interval	More than five years	Yes
(iii) end of interval	More than five years	Yes
4. Previous closed interval	Five years	Yes
5. Previous closed interval; for intervals ending after survey date 360 months	Four years	Yes

*It will take more than 3 years to obtain <u>all</u> the straddling intervals at survey date 396 months.

birth intervals are affected by changes in fertility, and whether we can make any predictions about the actual (input) distributions basing our judgment on the output distributions, *i.e.*, whether the distributions of output birth intervals reflect any changes in fertility. This section, therefore, is devoted to comparisons of these input distributions with those obtained as output from the use of POPSIM and SURVEY programs.

The functions chosen for comparison are distribution functions. The Kolmogorov-Smirnov test will be used to compare the time and the empirical distribution functions.

Let H be any completely specified distribution, and let F be an empirical distribution. We want to test H_0: $F = H$ against the general alternative H_A: $F \neq H$. Then, the Kolmogorov-Smirnov test uses the statistic

$$D = \sup_x \left| F_X(x) - H(x) \right|$$

as test criterion. Tabulations of this criterion are available (Miller [1956]). This test is conservative for grouped data.

5.8.1. Cohort Populations

The distributions considered here were distributions of *interior* birth intervals interior to age groups (15,30), (15,35), (15,40) and (15,45) years, *i.e.*, all the intervals which started and ended within a particular age group. Both the lower and upper age limits are arbitrary and chosen only as means of providing an example. Any other age groups would also serve the purpose adequately, though any upper limit less than 30 years would not include any intervals affected by changes in parameters for fertility.

Table 5.30 gives the empirical distribution function for the married cohort (run 1) compared to the theoretical distribution function. In this table,

$$H_i(x) = \int_{-\infty}^{x} h_i(x)dx, \text{ theoretical distribution function for birth order } i$$

and $$F_i(x) = \sum f_i(x)dx, \text{ empirical distribution function for birth order } i$$

We find that as long as the fertility level does not change, $H_i(x)$ are approximated nicely by $F_i(x)$ with $P > .20$ in lower parities and $.05 < P < .10$ in higher parities. We cannot make the same claim, however, when the parameters affecting fertility are changed, as can be seen in Table 5.31. In this table, in lower parities, there has not been any changes in fertility level, because most of the lower parity birth intervals were completed before the new parameters were put into use for simulation. In these lower parities, there is no evidence which would enable us to reject H_0: $F_i = H_i$, but as soon as the parameters change (in 6^{th} and higher parities), H_0: $F_i = H_i$ is rejected. These null hypotheses were rejected because at any survey date after changed fertility, the distribution of birth intervals is a mixed distribution comprised of some births occurring before the fertility change and the rest after the change, and does not favorably compare either with H_i before change or with the new

TABLE 5.30. COMPARISON OF EMPIRICAL AND THEORETICAL DISTRIBUTION FUNCTIONS, COHORT POPULATIONS, RUN 1

x (in months)	$H_1(x)$	$F_1(x)$*	$H_4(x)$	$F_4(x)$*	$H_7(x)$	$F_7(x)$ UPPER AGE LIMIT			$H_8(x)$	$F_8(x)$ UPPER AGE LIMIT		
						360	420	480 540		360	420	480 540
12	.2578	.247	.0239	.027	.0314	.024	.028	.027	.0039	.022	.020	.018
24	.6293	.626	.4052	.419	.3632	.331	.359	.358	.2482	.300	.300	.293
36	.8170	.816	.7823	.770	.7033	.652	.674	.677	.6852	.679	.654	.641
48	.9050	.900	.9332	.936	.8770	.838	.864	.862	.9049	.847	.836	.836
60	.9485	.939	.9807	.980	.9505	.956	.954	.953	.9744	.927	.921	.921
72	.9706	.961	.9943	.996	.9793	.987	.982	.980	.9934	.964	.960	.959
84	.9821	.967	.9982	.998	.9913	.987	.985	.983	.9978	.985	.985	.983
Number of Women		488		453		294	393	401		137	350	375
Maximum $\mid H_i(x)-F_i(x) \mid$		.015		.0148		.051	.029	.026		.058	.069	.069
P		> .20		> .20		> .20	> .20	> .20		> .20	.05 < P <.10	.05 < P < .10

*All 1st and 4th order intervals ended by age 360 months.

TABLE 5.31. COMPARISON OF EMPIRICAL AND THEORETICAL DISTRIBUTION FUNCTIONS, COHORT POPULATIONS, RUN 4

x (in months)	$H_{1\dagger}(x)$	$F_{1*}(x)$	$H_{4\dagger}(x)$	$F_{4*}(x)$	$H_{7\dagger}(x)$	$F_7(x)$			$H_{8\dagger}(x)$	$F_8(x)$		
						UPPER AGE LIMIT				UPPER AGE LIMIT		
						360	420	480, 540		360	420	480, 540
12	.2578	.274	.0239	.021	.0314	.026	.025	.025	.0039	.025	.014	.013
24	.6293	.625	.4052	.401	.3632	.283	.268	.264	.2482	.170	.176	.177
36	.8170	.825	.7823	.793	.7033	.512	.517	.514	.6852	.354	.432	.459
48	.9050	.914	.9332	.932	.8770	.654	.672	.675	.9049	.520	.632	.683
60	.9485	.946	.9807	.976	.9505	.791	.810	.822	.9744	.715	.791	.834
72	.9706	.970	.9943	.990	.9793	.875	.891	.904	.9934	.841	.875	.909
84	.9821	.985	.9982	.997	.9913	.929	.941	.954	.9978	.903	.925	.958
Number of Women		983		920		607	762	816		275	574	706
Maximum $\lvert H_i(x)-F_i(x) \rvert$		.016		.011		.223	.205	.202		.385	.273	.226
P		> .20		> .20		< .01	< .01	< .01		< .01	< .01	< .01

*All 1st and 4th intervals ended by age 360 months.

†These functions are same as for run 1, since all 1st and 4th order and most of 7th and 8th order intervals began before age 360 months.

theoretical distribution (after fertility change). The parities 0, 4, 6 and 7 here were chosen solely as examples, and not for any particular reason.

5.8.2. Cross-Sectional Population

Since the empirical distributions in cohort populations compared favorably with the theoretical distributions (in the absence of any changes in parameters affecting fertility sub-routine), it was decided to investigate whether the same thing would be true in case of cross-sectional populations. Table 5.32 reveals that even in the absence of any changes in fertility, H_0: $F_i = H_i$ is rejected with $P < .01$ for all parities investigated. It should be noted that distribution functions at lower values of x are reasonably close, and the maximum $| H_i(x) - F_i(x) |$ is achieved near the farther end. A truncation effect can be advanced as an explanation. Some of the prospective intervals never reach that status because either the woman dies or becomes sterile before the interval is complete. The sterility effect is particularly severe, because the woman remains alive and, in computing the empirical distribution, is used in the denominator, and $F_i(x) \not\to 1$ as $x \to \infty$. It is possible to eliminate all sterile women, and recompute the empirical distributions so that $F_i(x) \longrightarrow 1$ as $x \to \infty$. In this case, however, another kind of truncation effect plays its role. Suppose we are considering a woman of parity (j-1), who would give birth to her j^{th} child. Then, the $(j-1)^{th}$ closed birth interval would have to be completed before she

TABLE 5.32. COMPARISONS OF EMPIRICAL AND THEORETICAL DISTRIBUTION FUNCTIONS, CROSS-SECTIONAL POPULATION, RUN 2

x (in months)	$F_1(x)$			$F_4(x)$			$F_7(x)$			$F_8(x)$		
	UPPER AGE LIMIT			UPPER AGE LIMIT			UPPER AGE LIMIT			UPPER AGE LIMIT		
	360	420	480	360	420	480	360	420	480	360	420	480
12	.2714	.2592	.2627	.0265	.0246	.0231	.0305	.0296	.0326	.0277	.0255	.0248
24	.6002	.5789	.5917	.3709	.3561	.3469	.3122	.2978	.2980	.2936	.2870	.3048
36	.7567	.7398	.7668	.7003	.6726	.6784	.6221	.6095	.6075	.6150	.5787	.6019
48	.8447	.8282	.8527	.8411	.8199	.8282	.7840	.7732	.7720	.7590	.7292	.7562
60	.8765	.8600	.8838	.8858	.8704	.8767	.8357	.8323	.8322	.8310	.7986	.8381
72	.8936	.8788	.9026	.8924	.8854	.8910	.8545	.8619	.8583	.8670	.8310	.8686
84	.9034	.8888	.9149	.8990	.8909	.8954	.8592	.8738	.8713	.8781	.8542	.8895
Number of Women	818	1007	1222	604	733	908	426	507	614	361	432	525
Maximum $\|H_i(x)-F_i(x)\|$ *	.0787	.0933	.0690	.1019	.1089	.1033	.1321	.1175	.1200	.1459	.1757	.1487
P	< .01	< .01	< .01	< .01	< .01	< .01	< .01	< .01	< .01	< .01	< .01	< .01

*$H_i(x)$ are same as in Table 5.

becomes sterile or dies. This is particularly true of women with higher parities, since they have less time to complete an interval than those with lower parities.

In cross-sectional populations with decreased fertility (runs 3 and 4), H_0: $F_i = H_i$ was rejected at $P < .01$.

Some conclusions based on the results of this chapter are presented in Chapter VI.

CHAPTER VI. CONCLUSIONS

6.1. Introduction

This study dealt with computer populations as opposed to real life populations. An attempt was made to produce cross-sectional populations to be as close as possible to real populations in a high fertility situation. The input parameters for mortality, fertility and marriage were chosen to approximate this situation. The manner in which the fertility decrease was effected are not representative of the real life situations, since the mean birth intervals would not usually increase uniformly at each parity in real life. Other limitations which should be borne in mind are:

Females were the only ones considered in the study, and any relationship with males which may result in divorce or widowhood was not considered. Fertility was taken as a function of parity which would also make it a function of age indirectly. A direct relationship of fertility with age would have been preferable, but the lack of data barred this consideration. Changes in fertility were effected uniformly over a group of women, *e.g.*, all women in a population, all women with parity 4 or higher, etc. A more realistic approach might be introduction of various contraceptive methods and different acceptance rates of contraceptives by practicing couples, but this would make the analysis more complicated. The birth intervals were considered continuous time variables and measured accurately. Measurement errors, were not considered.

The artificiality of the studies of this kind should be recognized en passe. The variables can be manipulated according to the desire of the investigator. There are no memory lapses among the members of population about reporting any of the vital events, because each event is recorded at the time of its occurrence. This artificiality, nevertheless, is one of the reasons for making such a study an important one, because one can control the experimental conditions in a way such that effect of any variable or a combination of variables can be studied. This study was restricted to investigation of the relative sensitivity of birth interval indices.

6.2. Conclusions

This study investigated ascertainment plan(s) to produce indices which are sensitive, i.e., would reflect changes in birth intervals, when the fertility level has been altered and which are robust, i.e., would not change if none of the parameters directly affecting fertility undergo any changes.

Mathematical expressions for the distributions of birth intervals under different ascertainment plans involved numerous approximations, and the attempt was abandoned. The conclusions here, therefore, relate to the simulation part of this study.

Open intervals had been earlier advanced as a sensitive index of fertility (Srinivasan [1966a]). This claim was supported by the present study, but it was seen that classification of open birth intervals by age of mother at survey date made this index robust, in addition to being sensitive.

Straddling intervals were shown to be both sensitive and robust. These intervals, however, cannot be determined at the time of survey, and are, therefore, useful mainly as potential tools for follow-up studies.

The immediate previous closed interval is another sensitive and robust birth interval. This interval reflected changes earlier when classified by parity of the mother than when groupings were made by age of the mother, presumably because the changes in fertility level were linked directly to parity. If the changes in input birth intervals had been made by considering the age and parity of the mother, there seems no reason to believe that this would be the case. The immediately previous closed intervals which end after the date of implementation of family planning program, however, show the fertility changes earlier than all previous closed intervals.

Mean birth intervals are not sensitive indices, except when these intervals are studied by completed parity of women at the time of survey. In that case, the last few closed intervals reflect any changes in fertility level. This leads us back to previous closed interval as the index of preference rather than mean closed birth interval by mother's completed parity.

6.3. Practical Implications of the Study

If birth intervals are used for evaluation of a family planning program, the kinds of ascertainment plans, suitable for use, will depend upon the kind of surveys, i.e., whether the surveys are one-time only cross-sectional surveys, or whether the population is followed over a period of time.

Suppose a family planning program is put in effect at time T and the population is observed for a period of b years. Let $0 < a < b$. At time T + b, one

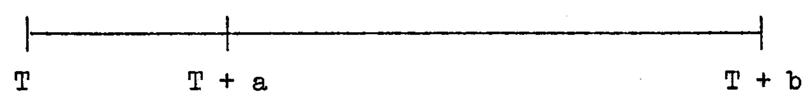

CHART 6.1. TIME SEQUENCE OF SURVEY DATES

could ascertain all types of birth intervals, except those which straddle (T+b). It is possible, however, to determine those intervals which straddle (T+a), if all intervals beginning before (T+a) end before or at (T+b). If b is sufficiently large, i.e., at least 5 years, only a very few intervals, which straddle (T+a) and end after (T+b), will be missed. If, however, a one-time survey is taken at time (T+a), only a few straddling intervals which straddle a point of time occurring after the implementation of the program can be recorded and this ascertainment plan will be of limited interest. It is possible to conduct a one-time survey at time (T+b) and get straddling intervals at (T+a) where a is arbitrarily chosen. The problem of recall of the events may be an important consideration, in that event.

It appears, therefore, that the selection of an index depends upon the type of survey and its purpose. For example, if the purpose of the survey is to estimate fertility level for maternity needs in a hospital, an index describing the frequency of women giving births would be the choice, even if it may not be the most sensitive. Evaluation of a family planning program, however, would require a sensitive and robust index.

6.4. Suggestions for Further Research

The problem of measuring fertility changes over time is faced by anyone who wishes to evaluate a family planning program. Almost every fertility index will show the change, if there has been any, if the time since the implementation of the program had been sufficiently long. In most cases, however, it is desirable to know as soon as possible if there has been a change in the level of fertility of the population under observation, and the need for a sensitive index, therefore, cannot be over-emphasized. On the other hand, the desirable index should not change if the level of fertility has not changed, to avoid any misleading conclusions.

The present study has tried to pinpoint such indices under the specified limiting assumptions and risks of happenings of various vital events. These assumptions may be relaxed and the changes in fertility should be effected so as to more closely resemble the real-life situations. It would be of interest to know whether the conclusions of this study hold for the analyses of latter type in which all or some of the following changes have been incorporated:

(i) inclusion of males in computer population,

(ii) widowhood and divorce of the women,

(iii) births to unmarried as well as to widows and divorcees,

(iv) fertility parameters dependent upon age, parity and other socio-economic variables,

(v) contraception as one of the means of changing fertility level.

The above list is by no means exhaustive, and any changes which would bring the computer populations to be a representative of the real-life populations may be incorporated.

There is further need to compare these indices with conventional indices. Introduction of measurement errors would bring the experiment closer to reality. Finally, the need to test these results with real life data cannot be overlooked.

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