Stress Analysis of a Graphite Fuel Element for HTR under Fast Neutron Irradiation

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ABSTRACT

The mechanical behavior of irradiated graphite fuel element is very effective on the safety and reliability of HTR operation . The analytical stress solution for graphite fuel element is given in this paper in the spherical coordinates on basis of the linear viscoelastic theory . The stress calculation of the fuel element under fast neutron irradiation is conducted during it flows through the HTR core .

1 INTRODUCTION

The fuel element of the pebble bed HTR is spherical . About 10^4coated fuel particles are dispersed in a graphite matrix . The coated particles are of triso–type and contain a oxide fuel kernal.The fuel zone is surrounded by a fuel free shell composed of the same graphite material . The overall diameter of the element is 6 cm with a 0.5 cm thick fuel free shell . The strength and reliability of the graphite shell is very effective on operating safety feature of the HTR . The heat generation due to nuclear fuel fission transfers out through the shell , thus the thermal stress is applied in it . Also because of the irradiation–induced dimensional change of the graphite structure , the irradiation induced stress is applied in the shell too . Besides it is necessary to consider the effect of irradiation induced creep which makes the stress relaxation . The above mentioned characteristics are dependent on the temperature and fast neutron accumulative dose , which vary with the distribution of the volumetric heat generation rate and fast neutron flux in a fuel element . For this reason,the detailed sdudy for fuel element in operation has to be conducted in order to ensure the HTR in safe and reliable operation.

The mechanical behavior of the graphite structure under irradiation can be explained with the viscoelastic model . At first the general stress solution in symmetrically spherical coordinates is derived on basis of the viscoelastic theory in hereditary integral form . Then the stress history for the fuel sphere is calculated when it flows through the HTR core .

2 VISCOELASTIC STRESS IN SYMMETRICALLY SPHERICAL COORDINATES

In this paper , the mechanical response of graphite is assumed to be linear viscoelastic and the constitutive relations are formulated in hereditary integral form . Irradiation–induced dimensional changes are treated as equivalent thermal strain . Assumed that the graphite material is isotropical and poissons ratio is independent with D . The dose D is correspondent with time t . Then the stress–strain relations in symmetrically spherical coordinates for little deformation , linear viscoelastic theory are given as follows :

$$\sigma_{rr}(D) = \frac{1}{(1+\mu)(1-2\mu)}\left\{(1-\mu)\int_{0^-}^{D} G(D-D')\frac{\partial \varepsilon_{rr}(D')}{\partial D'}dD' + 2\mu\int_{0^-}^{D} G(D\right.$$

$$- D')\frac{\partial \varepsilon_{\theta\theta}(D')}{\partial D'}dD' - (1+\mu)\int_{0^-}^{D} G(D - D')\frac{\partial[\varepsilon^{0}(D') + \varepsilon^{w}(D')]}{\partial D'}dD'\Bigg\} \quad (1)$$

$$\sigma_{\theta\theta}(D) = \sigma_{\varphi\varphi}(D) = \frac{1}{(1+\mu)(1-2\mu)}\Bigg\{\int_{0^-}^{D} G(D - D')\frac{\partial \varepsilon_{\theta\theta}(D')}{\partial D'}dD' + \mu\int_{0^-}^{D} G(D$$

$$- D')\frac{\partial \varepsilon_{rr}(D')}{\partial D'}dD' - (1+\mu)\int_{0^-}^{D} G(D - D')\frac{\partial[\varepsilon^{0}(D' + \varepsilon^{w}(D')]}{\partial D'}dD'\Bigg\}$$

where σ_{rr} , $\sigma_{\theta\theta}$, $\sigma_{\varphi\varphi}$, ε_{rr} , $\varepsilon_{\theta\theta}$, $\varepsilon_{\varphi\varphi}$ are three stress components and strain components in spherical coordinates.ε^{0},ε^{w} are the thermal and irradiation induced strain . G is the relaxation modulus which is related to the creep compliance J as follows :

$$\int_{0^-}^{D} J(D - D')\frac{\partial G(D')}{\partial D'}dD' = h(D) \quad (2)$$

where h is the unit step function . If G or J is given, the other one can be determined by solving the integral equation (2) .

The strain–displacement relationship , the equilibrium equation and boundary condition can be written :

$$\varepsilon_{rr}(D) = \frac{du_{r}(D)}{dr} \quad , \quad \varepsilon_{\theta\theta}(D) = \frac{u_{r}(D)}{r} \quad (3)$$

$$\frac{d\sigma_{rr}(D)}{dr} + \frac{2}{r}(\sigma_{rr}(D) + \sigma_{\theta\theta}(D)) = 0 \quad (4)$$

and

$$\sigma_{rr}(D) = 0 \qquad \text{when } r = a,b \text{ for the shell} \quad (5)$$

or

$$\sigma_{rr}(D) = 0 \qquad \text{when } r = b \text{ for the solid sphere}$$

$$u_{r}(D) = 0 \qquad \text{when } r = 0 \text{ for the solid sphere.} \quad (6)$$

The Laplace transformation are applied to eq.(1)、(3)、(4)、(5)、(6), the following expressions can be obtained:

$$\bar{\sigma}_{rr}(\xi) = \frac{1}{(1+\mu)(1-2\mu)}\Big\{(1-\mu)\bar{G}(\xi)\xi\bar{\varepsilon}_{rr}(\xi)$$

$$+ 2\mu\bar{G}(\xi)\xi\bar{\varepsilon}_{\theta\theta}(\xi) - (1+\mu)\bar{G}(\xi)\xi[\bar{\varepsilon}^{0}(\xi) + \bar{\varepsilon}^{w}(\xi)]\Big\}$$

$$\bar{\sigma}_{\theta\theta}(\xi) = \bar{\sigma}_{\varphi\varphi}(\xi) = \frac{1}{(1+\mu)(1-2\mu)}\Big\{\bar{G}(\xi)\xi\bar{\varepsilon}_{\theta\theta}(\xi) \quad (7)$$

$$+ \mu\bar{G}(\xi)\xi\bar{\varepsilon}_{rr}(\xi) - (1+\mu)\bar{G}(\xi)\xi[\bar{\varepsilon}^{0}(\xi) + \bar{\varepsilon}^{w}(\xi)]\Big\}$$

$$\bar{\varepsilon}_{rr}(\xi) = \frac{d\bar{u}_{r}(\xi)}{dr} \quad , \quad \bar{\varepsilon}_{\theta\theta}(\xi) = \frac{\bar{u}_{r}(\xi)}{r} \quad (8)$$

$$\frac{d\bar{\sigma}_{rr}(\xi)}{dr} + \frac{2}{r}(\bar{\sigma}_{rr}(\xi) - \bar{\sigma}_{\theta\theta}(\xi)) = 0 \quad (9)$$

and

$$\bar{\sigma}_{rr}(\xi) = 0, \quad \text{when } r = a,b \text{ for the shell,} \quad (10)$$

or

$$\bar{\sigma}_{rr}(\xi) = 0, \quad \text{when } r = b \text{ for the solid sphere,}$$

$$\bar{u}_{r}(\xi) = 0, \quad \text{when } r = 0 \text{ for the solid sphere,} \quad (11)$$

where ξ is transformation parameter. In substituting eq.(7),(8) into eq.(9) and putting it in order, the following equation is written:

$$\frac{d}{dr}\left[\frac{1}{r^2}\frac{d(\bar{u}_r(\xi))}{dr}\right]=\frac{1+\mu}{1-\mu}\frac{d[\bar{\varepsilon}^{\theta}(\xi)+\bar{\varepsilon}^{w}(\xi)]}{dr} \tag{12}$$

The solution of the eq.(12),ie.$\bar{u}_r(\xi)$ expression can be given.

$$\bar{u}_r(\xi)=\frac{1+\mu}{1-\mu}\frac{1}{r^2}\int_a^r[\bar{\varepsilon}^{\theta}(\xi)+\bar{\varepsilon}^{w}(\xi)]r^2dr+\bar{C}_1(\xi)r+\frac{\bar{C}_2(\xi)}{r^2} \tag{13}$$

In substituting eq.(13) into (7) and (8) following expresstions can be obtained

$$\bar{\sigma}_{rr}(\xi)=-\frac{2}{1-\mu}\frac{1}{r^3}\bar{G}(\xi)\xi\int_a^r[\bar{\varepsilon}^{\theta}(\xi)+\bar{\varepsilon}^{w}(\xi)]r^2dr+\frac{\bar{G}(\xi)\xi\bar{C}_1(\xi)}{1-2\mu}$$

$$-\frac{2}{1+u}\bar{G}(\xi)\xi\bar{C}_2(\xi)\frac{1}{r^3}$$

$$\bar{\sigma}_{\theta\theta}(\xi)=\bar{\sigma}_{\varphi\varphi}(\xi)=\frac{2}{1-\mu}\frac{1}{r^3}\bar{G}(\xi)\xi\int_a^r[\bar{\varepsilon}^{\theta}(\xi)+\bar{\varepsilon}^{w}(\xi)]r^2dr+\frac{1}{1-2\mu}\bar{G}(\xi)\xi\bar{C}_1(\xi)$$

$$+\frac{1}{1+\mu}\frac{1}{r^3}\bar{G}(\xi)\xi\bar{C}_2(\xi)-\frac{1}{1-\mu}\bar{G}(\xi)\xi[\bar{\varepsilon}^{\theta}(\varepsilon)+\bar{\varepsilon}^{w}(\varepsilon)] \tag{14}$$

where $\bar{C}_1$ and $\bar{C}_2$ are integral constants, which can be derived from above mentioned boundary condition.

In substituting the derived $\bar{C}_1$ and $\bar{C}_2$ into eq.(14) and then applying Laplace inverse transformation to it, the following stress formulas are obtained:

$$\sigma_{rr}(D)=\frac{2}{1-\mu}\left\{\beta_1 f_2(r,D,D')-\frac{1}{r^3}f_1(r,D,D')\right\}$$

$$\sigma_{\theta\theta}(D)=\sigma_{\varphi\varphi}(D)=\frac{2}{1-\mu}\left\{\beta_2 f_2(r,D,D')+\frac{1}{2r^3}f_1(r,D,D')-\frac{1}{2}f_3(r,D,D')\right\} \tag{15}$$

where,f_1, f_2, f_3, a series of integral expressions. They are:

$$f_1(r,D,D')=\int_{0^-}^{D}G(D-D')\frac{\partial\left[\int_a^r(\varepsilon^{\theta}(D')+\varepsilon^{w}(D'))r^2dr\right]}{\partial D'}dD'$$

$$f_2(r,D,D')=\int_{0^-}^{D}G(D-D')\frac{\partial\left[\int_a^b(\varepsilon^{\theta}(D')+\varepsilon^{w}(D'))r^2dr\right]}{\partial D'}dD' \tag{16}$$

$$f_3(r,D,D')=\int_{0}^{D}G(D-D')\frac{\partial[(\varepsilon^{\theta}(D')+\varepsilon^{w}(D')]}{aD'}dD'$$

where a is internal radius of the shell or $a=0$ for the solid sphere.

The konstant coefficients in eq.(15) can be expressed as follows:

$$\beta_1=\frac{r^3-a^3}{r^3(b^3-a^3)}\quad,\quad\beta_2=\frac{2r^3+a^3}{2r^3(b^3-a^3)}\quad\text{for the shell,} \tag{17}$$

$$\beta_1=\frac{1}{b^3}\quad,\quad\beta_2=\frac{1}{b^3}\quad\text{for the solid sphere} \tag{18}$$

In general,the thermal and irradiation induced strains can be given by following forms:

$\varepsilon^{\theta}(D)=\alpha(T,D)\circ T(r,D)$

$\varepsilon^{w}(D)=A(T)D+B(T)D^2$

where T−temperature distribution,α thermal expansion coefficient, A and B−constant coefficients. $D=\int\varphi dt$, in which φ is fast neutron flux. If α,T,A,B,φ are independent with t and φ is well-distributed,then stress formulas in eq.(15) can be simplified as follows:

$$\sigma_{rr}(D) = G_1(D)\sigma^{\theta}_{rr} + G_2(D)\sigma^{w}_{rrA} + G_3(D)\sigma^{w}_{rrB}$$

$$\sigma_{\theta\theta}(D) = \sigma_{\varphi\varphi}(D) = G_1(D)\sigma^{\theta}_{\theta\theta} + G_2(D)\sigma^{w}_{\theta\theta A} + G_3(D)\sigma^{w}_{\theta\theta B} \quad (19)$$

where

$$G_1(D) = \frac{1}{E}G(D)$$

$$G_2(D) = \frac{1}{E}\int_{o-}^{D} G(D-D')dD' \quad (20)$$

$$G_3(D) = \frac{1}{E}\int_{o-}^{D} 2G(D-D')D'dD'$$

E is the elastic modulus of unirradiated qraphite.

The stress components in eq.(19) can be expressed by a series of equations:

$$\sigma^{\theta}_{rr} = \frac{2E}{1-\mu}\left\{\beta_1\int_a^b \alpha T(r)r^2dr - \frac{1}{r^3}\int_a^r \alpha T(r)r^2dr\right\}$$

$$\sigma^{\theta}_{\theta\theta} = \sigma^{\theta}_{\varphi\varphi} = \frac{2E}{1-\mu}\left\{\beta_2\int_a^b \alpha T(r)r^2dr - \frac{1}{2r^3}\int_a^r \alpha T(r)r^2dr - \frac{1}{2}\alpha T(r)\right\}$$

$$\sigma^{w}_{rrA} = \frac{2E}{1-\mu}\left\{\beta_1\int_a^b AT(r)r^2dr - \frac{1}{R^3}\int_a^r AT(r)r^2dr\right\}$$

$$\sigma^{w}_{\theta\theta A} = \sigma^{w}_{\varphi\varphi A} = \frac{2E}{1-\mu}\left\{\beta_2\int_a^b A(T)r^2dr - \frac{1}{2r^3}\int_a^r A(T)r^2dr - \frac{1}{2}A(T)\right\} \quad (21)$$

$$\sigma^{w}_{rrB} = \frac{2E}{1-\mu}\left\{\beta_1\int_a^b B(T)r^2dr - \frac{1}{r^3}\int_a^r B(T)r^2dr\right\}$$

$$\sigma^{w}_{\theta\theta B} = \sigma^{w}_{\varphi\varphi B} = \frac{2E}{1-\mu}\left\{\beta_2\int_a^b B(T)r^2dr - \frac{1}{2r^3}\int_a^r B(T)r^2dr - \frac{1}{2}B(T)\right\}$$

The meaning of a, β_1, β_2 in eq.(21) is the same as previous described.

3 NUMERICAL CALCULATION OF A FUEL SPHERE.

For extrusion moulding graphite fuel element, it is treated as a solid sphere in stress analysis.

At first the temperatare field in the fuel element has to be given in order to calculate it's irradiation—induced stresses.The temperature distribution can be easily derived from heat conductivety equation in symmetrically spherical coordinates.

The relaxation modulus of the graphite creep is given in the appendix 1.The constant coeffieient A(T),B(T),K(T) are defined in the reference[1] behind.

3.1 Stress calculation in a sphere with constant temperature and fast neutron distribution.

The eq.(19),(20),(21) as analytical solution can be directly used to solve this problem and to check the numerical computation behind. The fast neutron flux φ is well–distributed and equal to $10^{14}cm^{-2}$(EDN). The heat generation rate q, the ambient temperature Tg around the sphere, heat conductivity λ and the convective heat transfer coefficient H are given as follows:

$q = 4.0 w/cm^3$,	$Tg = 650℃$
$\lambda = 0.350 w/cm.℃$,	$H = 0.150 w/cm^2℃$
$\alpha = 6\times10^{-6} 1/℃$,	$\mu = 0.15$,
$E = 9.8\times10^5 N/cm^2$.	

The temperature and thermal distribution are described in figure 1, while the irradiation induced stress history in the sphere with fast neutron accumulative dose increase is given in figure 2. It shows clearly that in the beginning the stress decrease with dose increase. But after the irradiation deformation reaches the turning point, the stress will increase.

3.2 The fuel element numerical computation.

When a fuel element flows through the core, it is affected from the variable temperature and fast neutron flux. For this problem, the general formulas(15),(16) have to be used, where the linear integral equations can be treated by numerical integral–differential procedures. In using the parameter data of the example problem in section 3.1, the numerical calculation is carried out and it's results are given in figure 2. The both results agree fairly with each other. It is shown that the numerical method is credible. The error is less than 1.0% in the case of the dose interval of $2.5 \times 10^{20} cm^{-2}$(EDN).

To simulate the actural operation of the fuel element , the data similar to those in 200MW HTR are chosen in the calculation. The fast neutron flux distribution, helium temperature distribution and heat generation rate in the fuel element are given in figure 5 and 6.

In the figure 3 and 4 expressed are the temperature variation and irradiation induced stress history at the inner and outer wall of the shell with the dose increase. It can be seen from the figures that the total stress level are not high, in which the thermal stress is prominent and the irradiation induced stresses are lower.

It must be pointed out that both cases should be considered. In one hand, when the fuel element bears transient load,e.g.cool water or steam into the core or much quicker power increase, the considerably higher stresses can exist in the shell. In other hand, with repeated feed model, the irradiation induced dimensonal change can be intensive. This is the problem that should be further studied.

4 REFERENCE

[1] S.J.Chang,C.E.Pugh,S.E.Moore,(1970) Viscoelastic analysis of graphite under neutron irradiation and temperature distribution, ORNL–2407.

5 APPENDIX 1

The creep relaxation modulus of graphite material under fast neutron irradiation can be obtained by solving the eq.(2),when the creep compliance is given as follows

$$J(D) = \frac{1}{E} + K(T)D + \frac{1}{2E}(1 - e^{-A_0 D})$$

Then

$$G(D) = (S - A_0)Ee^{-SD} - (R - A_0)Ee^{-RD}$$

where

$$S = 1.5A_0 + K(T)E - \sqrt{\frac{1}{4}(1.5A_0 + K(T)E)^2 + EK(T)A_0}$$

$$R = 1.5A_0 + K(T)E + \sqrt{\frac{1}{4}(1.5A_0 + K(T)E)^2 + EK(T)A_0}$$

A_0 is transient creep constant and equal to $2.0 \times 10^{-20} cm^{-2}$(EDN).

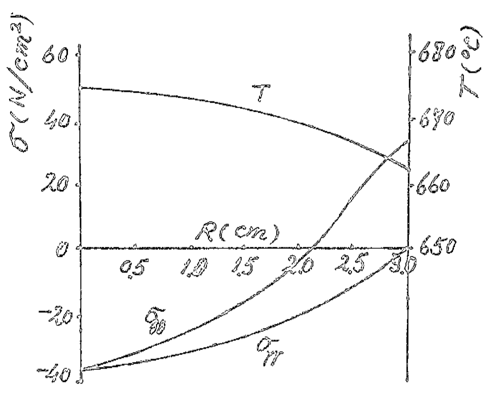

Fig.1 Temperature and thermal stress distribution in the sphere .

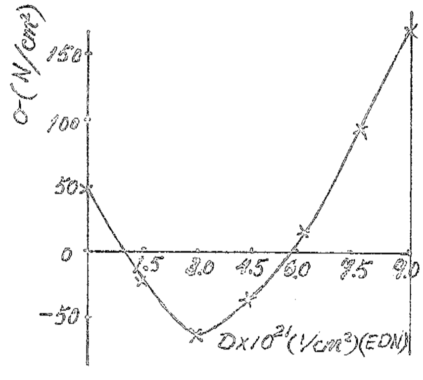

Fig.2 Stress history versus fluence in outer surface of the sphere .

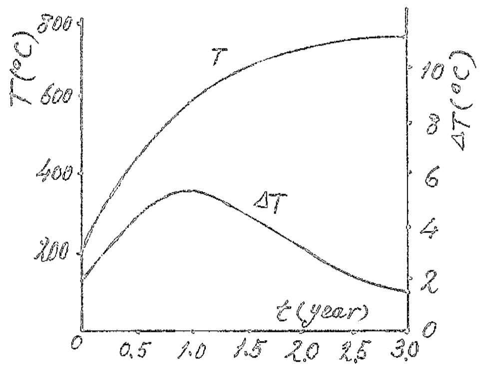

Fig.3 Outer wall temp. and inner-outer wall temp. difference .

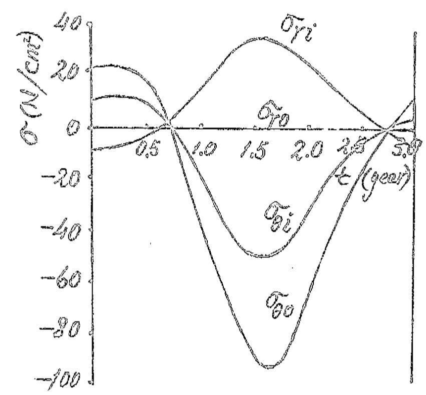

Fig.4 Stress history in outer wall of the sphere .

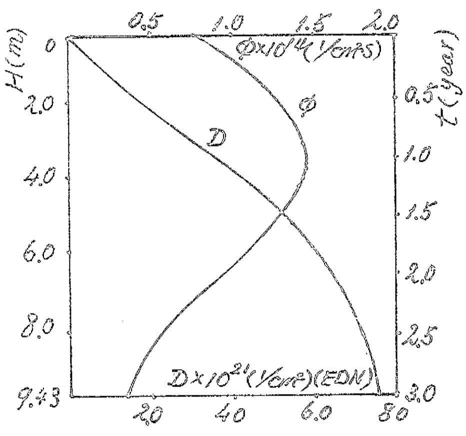

Fig.5 Fast neutron flux and fluence along the core hight .

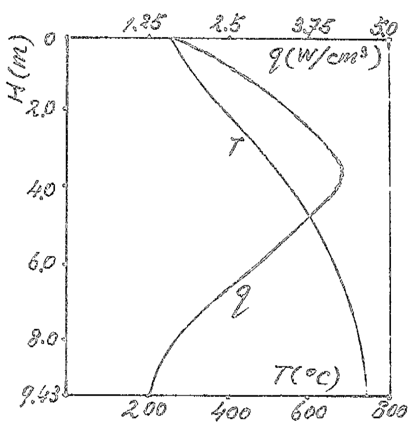

Fig.6 Helium temperature and heat generation along the core height .