Seismic Structural System Reliability of Reinforced Concrete Containment Structures

Jun MIZUNO
Kajima Corp., Tokyo, Japan
A. H.-S. ANG
University of California, Irvine, CA USA

1 INTRODUCTION

The reliability of structural systems has remained as one of the major problems in probabilistic reliability assessment, especially for large and complex redundant structures. In this paper, the seismic structural system reliability of a reinforced concrete(RC) containment structure is evaluated. The system reliability against ultimate failure is evaluated on the basis of the stable configuration approach (SCA) /1/2/ and the three-dimensional finite element method (FEM) of stress analysis.

First, under equivalent static seismic loads, the structural system reliability is evaluated using the dominant stable configurations. Then, the method is extended to directly evaluate the system failure probability through random stress responses obtained from the FEM model.

2 STRUCTURAL MODELING AND INPUT MOTION

A RC containment structure, and associated material properties are shown in Fig. 1 and Table 1,following Shinozuka et al/3/. For simplicity, the initial configuration of the containment structure is assumed to be axisymmetric with orthogonal reinforcements and fixed at the base.

A stable configuration is defined as any damaged state of the structure that can still carry the applied loads. The stable configurations are composed of three-dimensional FEM models with linear material constants, with the failed elements removed, and the stress redistribution taken into account.

The four node shell element, QUAD4 in the NASTRAN code, is used for modeling the RC containment. The FEM model and the shell element are shown in Fig.2. The full FEM model for the stress analysis of the various stable configurations must be used to evaluate the stress states of the asymmetric configurations.

The ground motion is modeled as a zero-mean stationary Gaussian process with a Kanai-Tajimi spectrum. The predominant frequency of the soil, ω_g, and the soil damping factor, ζ_g, are assumed to be 9π and 0.6, respectively. The duration of motion is assumed to be 10 seconds. The dead load is deterministically taken into account in the stress analysis.

3 SYSTEM RELIABILITY BASED ON SCA AND STATIC STRESS ANALYSIS

3.1 Equivalent static seismic force

The RC containment is assumed to be subjected to equivalent static forces from an earthquake with a specified peak ground acceleration, say, the 2SSE that is assumed to be 0.8g (the corresponding rms acceleration is assumed to be $\sigma_a = 0.267g$).

The rms nodal accelerations of the initial FEM model and its maximum response statistics can be evaluated on the basis of linear random response analysis, and the assumption that the maximum responses follow the Poisson occurrence law, in the same manner as the stress evaluation later shown in 4.1. The computed mean maximum accelerations are statically applied to the elements as the body forces.

3.2 Limit state for element failure

It may be reasonably assumed that the ultimate failure of the R/C containment is initiated by the crushing of concrete in the severely stressed area in compression. Thus, the limit state for an element failure is defined as the principal compressive stress, τ^i_{pc}, exceeding the concrete compressive strength, f_c', at the extreme compressive fiber of the element. The performance function for the limit state of element i may then be expressed as $g_i = f_c' - \tau^i_{pc}$, where τ^i_{pc} is expressed in terms of the stress resultant of the shell element shown in Fig. 2.

The statistics of the principal compressive stresses are obtained through the FEM analysis under the mean maximum seismic loads and standard deviations. Assuming that both the maximum compressive principal stresses and the concrete strength are lognormal variates, the element failure probability may be evaluated by the first-order second-moment method. The correlations between element failures can be explicitly taken into account.

3.3 Structural system reliability

The failure of the structural system is formulated as the intersection of the damages to the stable configurations, which in turn involve the unions of element failure events.

For the initial configuration, the element failure probabilities of the most stressed area in compression under the seismic loads are listed in Table 2. The failure probabilities decrease drastically as the stresses decrease. Therefore, the dominant stable configurations are shown in Table 3 with one or two elements that have failed among the elements listed above.

In the SCA, the configurations with few failed elements are more essential than those with large number of failed elements. For the intersection of events corresponding to the initial configuration and all the configuration with only one failed element can be expressed as /2/;

$$P\left(\bigcap_{i=0}^{k} \bar{C}_i\right) = \sum_{i=0}^{k} P(\bar{C}_i) - \sum_{j=1}^{k} P(\bar{C}_0 \cup \bar{C}_j) \qquad \cdots \qquad (1)$$

in which $\bar{C}_0$ is the complement of the initial configuration, and k is the number of configurations considered. The probabilities for unions of events are computed by the second-order bounds, and the geometric average of the bounds is used for calculating the intersection of events.

The probability of the intersection of events corresponding to configurations with more than one failed element are evaluated through the second-order bounding method. However, these events are generally much less significant.

The structural system failure probabilities are computed considering both earthquake directions, using one configuration (the initial configuration), three configurations, six configurations, and seven configurations, respectively. The result obtained with seven configurations includes the stable configuration with failures of the most stressed elements, $c_{1,2}$. These results are listed in Table 4. The results show that the initial configuration contributes most significantly to the system failure probability, and the contribution of the stable configuration with two failed elements is negligible. In terms of the failure surface, the significant volume of the safe region is removed by the event, $\bar{c}_0$, and the contribution of the other events is much less significant.

4 SYSTEM RELIABILITY BASED ON RANDOM VIBRATION ANALYSIS

4.1 Random vibration stress responses

The results under the equivalent seismic static loads show that the damage to the initial configuration gives a reasonable upper bound estimate of the system ultimate failure probability. In this light, only the event corresponding to the initial configuration may be used for the system reliability analysis. This makes it possible to directly evaluate the rms stress responses of the FEM model in the frequency domain on the basis of linear random response theory, given that the ground motion is a stationary Gaussian process. The variance of the stresses in the i-th element can be expressed as,

$$\sigma_{\tau\theta}^{i\,2} = \sum_j \sum_k r_j r_k \phi_{\tau j}^i \phi_{\tau k}^i \int_{-\infty}^{+\infty} H_j(\omega)\, H_k^*(\omega)\, S_a(\omega)\, d\omega \qquad \theta = x,y,xy \quad \dots \quad (2)$$

in which ϕ_θ^i is the component of the j-th modal stress vector, $H_j(\omega)$ is the frequency response function of the j-th mode, r_j is the participation coefficient, $S_a(\omega)$ is the power spectral density function of the earthquake.

The principal compressive stresses in the critical section of the containment may be approximated by the vertical stress components, since the stress states in the most stressed section are close to an uniaxial stress state under seismic loads. Assuming further that the maximum responses follow the Poisson occurrence law, the mean maximum stress response and its standard deviation can be expressed as;

$$\bar{\tau}_{y\,m}^{i} = \sigma_{\tau y}^{i} \sqrt{2 \ln (\nu_{\tau y}^{i} t_d)} + 0.577215\, \sigma_{\tau y}^{i} \Big/ \sqrt{2 \ln (\nu_{\tau y}^{i} t_d)} \quad \dots \quad (3)$$

$$\sigma_{\bar{\tau}_{y\,m}^{i}} = \frac{\pi}{\sqrt{6}} \frac{\sigma_{\tau y}^{i}}{\sqrt{2 \ln (\nu_{\tau y}^{i} t_d)}} \quad \dots \quad (4)$$

where $\sigma^i_{\tau y}$ is the rms fiber stress response in the y-direction, evaluated by Eq. 2, using the dominant vibration modes; and $\nu^i_{\tau y}$ is the mean zero crossing rate of the stress. Assuming that the peak stress responses, as well as the concrete strength, are lognormal variates, the element failures may be evaluated by the first-order second-moment method with the response statistics of Eqs.3 and 4.

4.2 Failure probability and fragility function

The annual failure probability P_E may be given by;

$$P_E = \int_{a_0}^{a_{max}} \nu_E\, F_E(a)\, f_{A1}(a)\, da \quad \dots \quad (5)$$

in which $F_E(a)$ is the conditional ultimate failure probability as a function of peak ground acceleration, a; and $f_{A1}(a)$, is the probability density function of the peak ground acceleration, given an earthquake which is obtained from the annual exceedence probability function $F_A(a)$ shown in Fig.

4 /3/; ν_E is the mean occurrence rate of earthquake per year, and assumed to be 0.15; and a_{max} and a_0 are the maximum and minimum peak ground acceleration considered in the integration. For the ultimate structural system failure, $F_E(a)$ is approximated by,

$$F_E(a) = P(\bar{C}_0 \mid a) \qquad \cdots \quad (6)$$

4.3 Computational results

Fig. 5 shows the fragility curve of the structural system. At the peak ground acceleration of 0.8 g, the ultimate failure probability agrees with the result obtained by static analysis, which is 1.09×10^{-4}. Integrating Eq. 5 between 0.05g and 1.0g, the system failure probability is computed as shown in Table 5.

The probability of initial yielding is also evaluated; the event corresponding to the initial yielding in the structure may be expressed as, $E_Y = \cup B_{Yi}$, where B_{Yi} is the yielding of the reinforcement in the i-th element. Disregarding the concrete tensile strength, the corresponding performance function for the yielding of the reinforcement may be defined as, $g_i = p_t f_y - \tau^i{}_y$, where p_t is the reinforcement ratio of the section, f_y is the yield strength of the reinforcement, and $\tau^i{}_y$ is the tensile normal stress in the critical direction, i.e., y-direction. The probability of yielding in an element is obtained in the same manner as the probability of crushing.

The result in /3/ is also shown, which is the probability of yielding of the most critical element in tension. The difference is not very significant.

The difference between the probabilities of the initial yielding and ultimate system failure is rather significant. The difference may be interpreted as the reserved safety margin between initial yielding and ultimate system failure.

5 CONCLUDING REMARKS

The ultimate structural system reliability of RC containment structures is evaluated on the basis of the SCA concept and FEM. The results show that the annual probability of ultimate containment failure under seismic hazard is of the order of 10^{-9}, whereas the corresponding probability of first yielding of the reinforcement is 10^{-5}. The difference between the probabilities of initial yielding and ultimate system failure may be interpreted as the reserved safety margin of the structure.

REFFERECES

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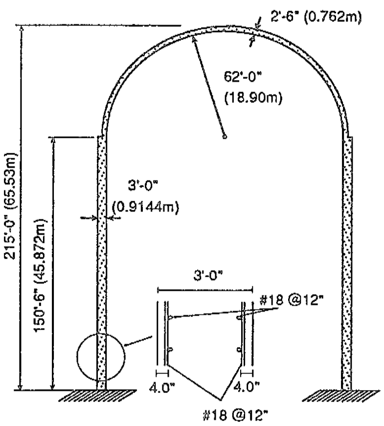

Fig. 1 RC Containment

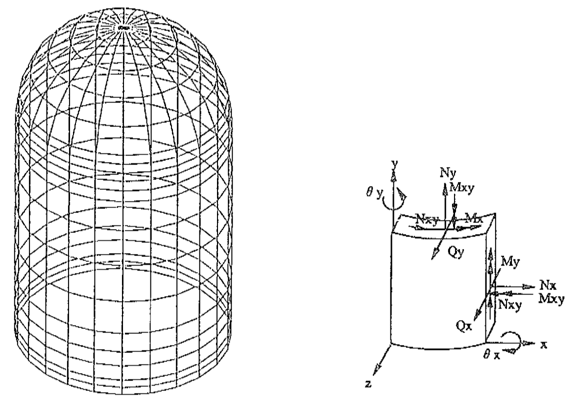

Fig. 2 Finite Element Model and Shell Element

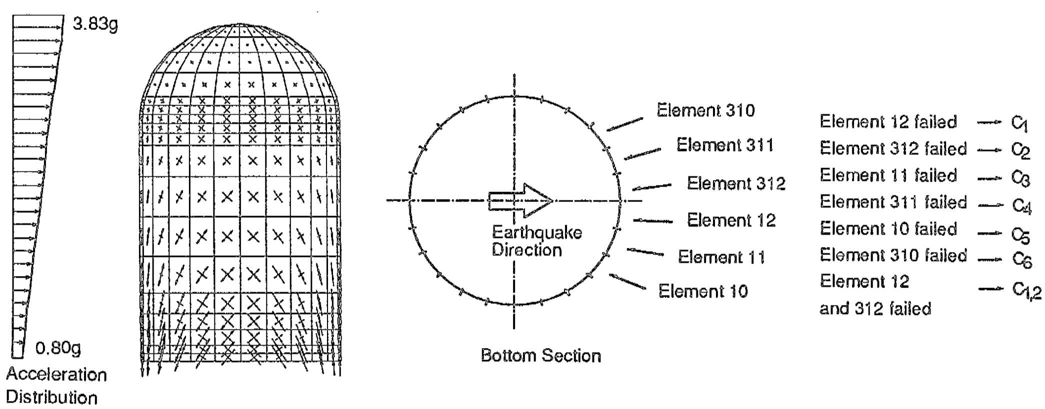

Fig. 3 Principal Stresses on Original Structure and Damage States under Equivalent static Seismic Load

Table 1 Material Properties

Concrete	
Compressive Strength	Mean 6086 psi (428.1 kgf/cm^2)
	COV 0.106
Young's Modulus	3.6x10^6 psi (253.5 tonf/cm^2)
Weight/unit volume	150 lbs/ft^3 (2.40 tonf/m^3)
Poisson's Ratio	0.2
Reinforcement	
Yield Strength	Mean 71.1 ksi (5000kgf/cm^2)
	COV 0.036
Young's Modulus	29.0x10^6 psi (2040tonf/cm^2)
Poisson's Ratio	0.3

Table 2 Element Failures used for the Initial Configuration

Event		β	$P(B_{i10})$
B_{110}	Failures of Element 12	4.06	2.43×10^{-5}
B_{210}	Failures of Element 312	4.06	2.43×10^{-5}
B_{310}	Failures of Element 11	4.28	9.34×10^{-5}
B_{410}	Failures of Element 311	4.28	9.34×10^{-5}
B_{510}	Failures of Element 10	4.72	1.17×10^{-6}
B_{610}	Failures of Element 310	4.72	1.17×10^{-6}

Under the equivalent static earthquake load corresponding to the 2SSE

Table 3 Element Failure Events in Each Complementary Event

Event	Element Failures
$\overline{C}_0$	$\bigcup_{i=1}^{n} B_{i\|0}$
$\overline{C}_i$	$(\bigcup_{i=1}^{n} B_{j\|0}) \cup (\bigcup_{j\neq i}^{n} B_{j\|i})$
$\overline{C}_{i,j}$	$(\bigcup_{i=1}^{n} B_{k\|0}) \cup (\bigcup_{k\neq i\neq j}^{n} B_{k\|i,j})$

n=6

$B_{k|i,j}$: event corresponding to element k failing given that element i, j have already failed.

Table 4 Probabilities of Intersection of events

Intersection of Events	Probabilities
One Configuration $\overline{C}_0$	1.29×10^{-4}
Three Configurations $\overline{C}_0 \cap \overline{C}_1 \cap \overline{C}_2$	1.14×10^{-4}
Seven Configurations $\overline{C}_0 \cap \overline{C}_1 \cap \overline{C}_2 \cap \overline{C}_3 \cap \overline{C}_4 \cap \overline{C}_5 \cap \overline{C}_6$	1.09×10^{-4}
Eight Configurations $\overline{C}_0 \cap \overline{C}_1 \cap \overline{C}_2 \cap \overline{C}_3 \cap \overline{C}_4 \cap \overline{C}_5 \cap \overline{C}_6 \cap \overline{C}_{1,2}$	1.09×10^{-4}

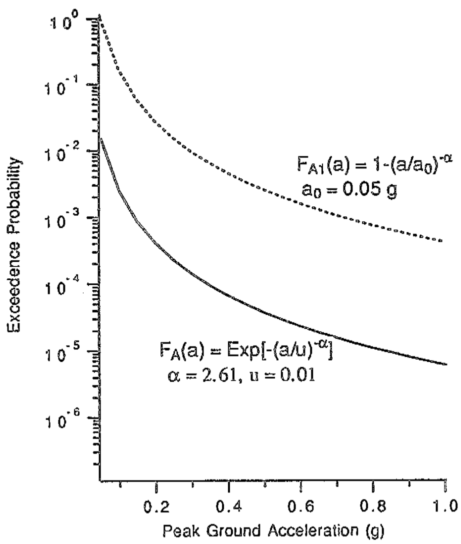

Fig. 4 Seismic Hazard Curve

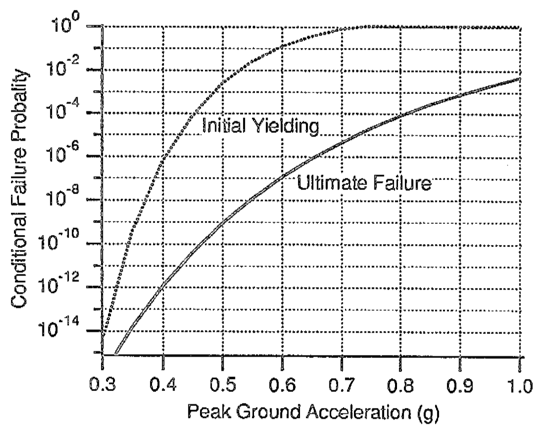

Fig. 5 Fragility Function $F_E(a)$

Table 5 Structural Failure Probabilities

		System Failure Probabilities per year
Ultimate System Failure		6.69×10^{-9}
Initial Yieding	Present Result	1.74×10^{-5}
	Shinozuka (1984)	9.83×10^{-6}