STABILITY ANALYSIS OF THIN SHELLS WITH INITIAL IMPERFECTIONS

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SUMMARY

In this paper the problems of stability analysis of thin shells with initial deformations will be outlined.

Often initial imperfections are not known at the time of analysis. A solution method which uses the eigenvalue-stability analysis as an initial step in finding unfavourable imperfections is presented. The succeeding geometrically nonlinear deformation problem is reduced to an iterative application of equivalent linear problems.

The influence of the initial imperfections on the critical load—for simply-curved shells (cylindrical) and multi-curved shells (here spherical)—will be demonstrated by the example of a containment shell. The containment deals with axi-symmetrical shells including non axi-symmetrical deformations which are calculated by means of Fourier analysis in circumferential direction.

As a result of the problem it will be shown that it is not the absolute size of the initial imperfections but their existence which mainly influence the critical load.

1.0 General

It is a wellknown fact that the stability behaviour of thin shells under compression, such as e.g. nuclear reactor steel containments, is considerably influenced by imperfections of the geometrical shape. The imperfections cannot be taken into account by the classical eigenvalue problem, when this is derived for the ideal shape without imperfections. Further, the imperfections are in general not known at the time of analysis. So a realistic and conservative estimate of the size and shape of possible imperfections is necessary.

2.0 Method of Analysis

2.1 Determination of the Most Unfavourable Shape of Initial Deflection

The problem of finding the most unfavourable shape of an infinitesimal small initial deflection is equivalent with searching for a small deviation from the ideal shape which yields the most flexible shape of the shell under a certain load configuration. In physical expressions, this means that we must find a small deviation from the ideal shape which shows a minimum of deformation energy under a certain load configuration. This physical problem may be solved by the stability-eigenvalue problem for the ideal shape of the shell. The eigenvectors represent such small deviations.
By normalization to some allowed mounting tolerance, which has in general a finite size, we get a realistic estimate for the most unfavourable shape and size of initial deflection.

2.2 Solution of the Second Order Theory Initial Deflection Problem

2.2.1 Basic Equations

We start from the system of equations for second order theory:

$$[K_s]\{u\} + \Big[[K_1(P_1)] + [K_2(\lambda P_2)] + [K_3(F(u))]\Big]\{u+u_0\} = \{P_1\} + \lambda\{P_2\} \qquad (1)$$

u	= deformation vector (solution functions)
u_o	= initial deformation vector
$F(u)$	= forces (generalized)
P_1	= constant load vector
P_2	= variable load vector
λ	= load factor for variable load
K_s	= system stiffness matrix of the undeformed system
K_1, K_2, K_3	= geometric stiffness matrices derived from equilibrium conditions of the deformed system

K_1 = matrix of constant load effects
K_2 = matrix of variable load effects
K_3 = Matrix which takes into account changes of forces due to increasing deformation

Equation (1) is valid for small deflections, i.e. the system stiffness matrix K_s of the deformed system may be represented by the system stiffness matrix of the undeformed system. For large deflections (third order theory) K_s becomes a function of deflections.

Multiplying K_1, K_2 and K_3 by $u_o + u$, equation (1) can be written in the following form:

$$[K_s]\{u\} = \{P_1\} + \lambda\{P_2\} + \{P_3(P_1, \lambda P_2, u, u_0)\} \tag{3}$$

$$\{P_3\} = \Big[[K_1(P_1)] + [K_2(\lambda P_2)] + [K_3(f(u))]\Big]\{u + u_0\}$$

In the following, the load vector P_3 shall be called "<u>equilibrium load vector</u>". This equation may be solved by first order theory, provided P_3 is known.

2.2.2 Equilibrium Load Vector

The equilibrium load vector can be derived directly by writing down the six equilibrium conditions for a deformed shell element (see Fig. 1). When neglecting terms which are small of higher order, these are as presented in Fig. 1 and 2 (equations 4).
The left hand side of these equations gives the terms according to first order theory. The right hand sides represent the contributions due to deformation and hence the equilibrium load vector. The forces and deformations refer to constant and variable loads. Small initial deflections (small, however finite deviations from the ideal shape of the shell) are included in the deformations by setting
$u = u_o + u$, $\varepsilon = \varepsilon_o + \varepsilon$, $\theta = \theta_o + \theta$ etc.
These equations are also valid for beam-like stiffeners.

The terms which contain P_x, P_y, P_z hold only for loads which do not change the direction with deformation. In many shell problems the inplane strain and curvatures and the shear forces q_x, q_y are small quantities so that the terms on the right hand side of equations (4-1) and (4-2) may be neglected.

The summation of terms with M_{xy}, M_{yx}, N_{xy}, N_{yx} in equation (4-6) yields zero due to $\tau_{xy} = \tau_{yx}$.
The contribution of moments M_x and M_y on the right hand side of equation (4-6) may be represented by an inplane couple of loads. In certain shell problems these have no considerable effect.

Finally the remaining equilibrium load vector of equation (4-3) represents a lateral load which may be written in the following form:

$$\{P_3\} \equiv p_z = N_x(\varkappa_x + \varkappa_{x0}) + N_y(\varkappa_y + \varkappa_{y0}) + N_{xy}(\varkappa_{yx} + \varkappa_{yx0}) + N_{yx}(\varkappa_{xy} + \varkappa_{xy0}) - P_x(\theta_x + \theta_{x0}) - P_y(\theta_y + \theta_{y0}) \tag{5}$$

$$\varkappa = \frac{\partial \theta}{\partial s} = \text{curvature}$$

2.2.3 Iteration Procedure

In equations (4) or (5) the forces, deflections, rotations, curvatures etc. are unknown. A first approximation for these unknowns may be reached by using first order theory solution of equation (3) with $P_3 = 0$. With this solution a first approximation of the equilibrium load vector P_3 is obtained. By adding this equilibrium load vector P_3 to the right hand side of equation (3) a second approximation is obtained, etc.

2.2.4 Criterion for Critical Load

The critical load is reached if, with a small increase of λ, a stable state of equilibrium becomes unstable. A close approximation for this case is the value λ, at which at least one component of the equilibrium load vector diverges linearly with increasing iterations (at constant λ).

3.0 Analysis for a Cylindrical Containment with Spherical Top

The containment shown in Fig. 3 was analyzed by the presented method using a computer code for axi-symmetrical shells under nonsymmetrical loads. The analysis was made for a cylinder section and for the hemisphere with adjoining cylindrical section (Fig. 3). The deflections and equilibrium loads of the cylinder section for increasing iterations at a constant load level (first load level) are shown in Fig. 4 and 5.

The influence of magnitude of the initial deflections is shown in Fig. 6 for the cylinder section and the hemisphere. These results show a considerable decrease of the critical load with increasing initial deflections. This decrease is much higher at the doubly curved hemisphere (factor 8) than at the single curved cylinder (factor 2.5). Further, the rapid decrease of the critical load in the range of very small initial imperfections shows that it is mainly the existence and not so much the magnitude which decreases the critical load in comparison to an analysis for the ideal shape.

4.0 Construction Tolerances for Cylindrical Shells under External Pressure

4.1 Codes for Maximum Deviations

Upper limits for shape deviations of welded pressure vessels are presented in a more or less detailed form in the appropriate pressure vessel codes. However, the tolerance limits given in those rules are used first of all for the dimensional checks during construction, and only special parts of them are considered in the criterions for stability. Table 4.1.1 gives a general view on the tolerances specified in three pressure vessel codes often used in the design and construction [1], [2], [3].

4.2 Existing Imperfections

The deviations from the ideal shape of a pressure vessel depend to a large extent on the manufacturing and construction procedures applied. The existing form of the vessel during operation, however, can be influenced additionally by special loading conditions other than pressure, or by attached internal structures.
Nowadays, for extremely thin-walled pressure vessels, as e.g. containments, the welded design based on single steel plates is applied in most cases. The erection procedure usually takes place on site, in absence of normal shop conditions. Due to the erection sequences and welding, the following deviations from the true shape may occur:

- global out-of-roundness
- local flat spots near openings or weld seams, or in the centre parts of the single plates
- discontinuities at weld seams like offsets or bulged edges.

All these influences can be controlled and held within the specified tolerances under application of good erection sequences and welding techniques. However, for extremely thin-walled vessels the minimum local and global deviations are mostly in ranges exceeding the actual wall thickness. In addition, attention should be paid to the fact that, due to the use of steel plates with equal dimensions, the deviations mentioned above may occur periodically not only in circumferential but also in meridional direction. Periodical deviations may result in a considerable loss of stability if the pattern of the deviations is similar to the buckling mode of the component. During operation, the influences of local temperature distributions or structural discontinuities will result also in deviations from the true shape. These deviations will act together with the erection tolerances and are to be expected mainly in the region of stiffening rings, external constraints, and near internal structures with special operating conditions.

4.3 Measurement of Deviations

To establish the load capacity of thin-walled pressure vessels under external loading, a stability analysis can be performed as described in paragraphs 1 to 3 by using the as-built shape of the structure. Special techniques must be applied for the measurements of the exact shape of large thin-walled vessels to guarantee sufficient accuracy of the results. In particular, attention must be paid that the structure will be in a preloaded condition (dead weight, temperature, residual stresses) different from the operating conditions. Measurements during operation are often impossible due to obstructed accessibility. For this reason, the measurements are normally taken just after completion of the vessel, e.g. following the pressure and leakage tests.

5.0 Safety Considerations

Due to the great discrepancies between experimental results and analytical derivations using ideal shell geometry, large safety factors have to be considered according to the codes. Unfortunately, these safety factors are overall safety factors given in either implicit or explicit form.
The factors cannot be splitted for different influences, such as e.g.

1) Loads
2) Method of analysis
3) Model for analysis
4) Manufacture and construction procedure
5) Material and welding defects
6) Residual stresses
7) Risk

The presented method of analysis allows to reduce considerably the contributions of items 1-3 to the safety factor.

REFERENCES

[1] ASME Boiler and Pressure Vessel Code Section III, 1971 edition, with Addenda to and including summer 1972 Addenda

[2] AD-Merkblatt H1, Ausgabe April 1968

[3] ISO/TC11 Pressure Vessels Appendix 1 to Section IV, Section V

[4] ZUDANS, Z., "New Formulation and Evaluation of Elastic Shell Theory", Dissertation, University of Pennsylvania, 1966

[5] TIMOSHENKO, S., "Theory of Elastic Stability", McGraw Hill, 1961

[6] GIRKMANN, K., "Flächentragwerke", Springer Verlag, Wien 1963, 6. Aufl.

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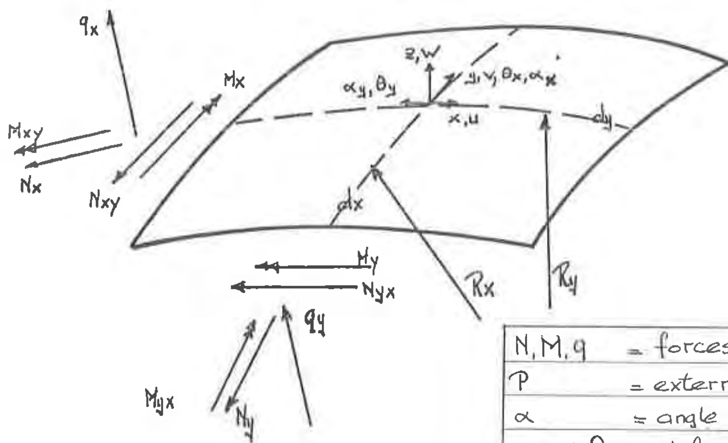
TABLE NO. 4.1.1 TOLERANCES FOR CYLINDRICAL SHELLS

	ASME CODE CLASS MC	AD-MERKBLATT H1	ISO/TC11 SECT. IV/V
Mean Diameter	—	$\|D_{NOM}-D_{MEAN}\| \leq 0.015\ D_{NOM}$	—
Out-of-roundness	$(D_{MAX}-D_{MIN}) \leq 0.01\ D_{NOM}$	$\frac{2(D_{MAX}-D_{MIN})}{D_{MAX}+D_{MIN}} \leq 0.015 D_{NOM}$	$(D_{MAX}-D_{MIN}) \leq 0.01\ D_{NOM}$
Local deviations Depth e	e depending on: L/D_{NOM}; D_{NOM}/T; T e = 0,3 T. min.	$e \leq 0.01\ L_T$	e depending on: L/D_{NOM}; D_{NOM}/T; T
Local deviations Length of template	L_T depending on: L/D_{NOM}; D_{NOM}/T; D_{NOM} $L_T = 0.25\ D_{NOM}$ max.	L_T (not specified)	e depending on: L/D_{NOM}; D_{NOM}/T; D_{NOM}
Offset of welds to ($12 < T < 50$ mm)	$t_o \leq 1/8$ inch	longitudinal weld seams to $\leq 0.05 \cdot T$ (max. 2 mm) circumferential weld seams: to $\leq 0.10 \cdot T$ (max. 3 mm)	longitudinal weld seams: to $\leq T/10$ (max. 3 mm) circumferential weld seams to $\leq T/10 \cdot 1$ (max. 4 mm)
Wall thickness	$S \geq S_{NOM}$	$S \geq S_{NOM}$	$S \geq S_{NOM}$

Fig. 1



N, M, q	= forces
P	= external Load
α	= angle of shell
u, v, w, θ	= deformation

equilibrium condition in X-direction:

$$\frac{\partial N_x}{\partial x}\,dx\,dy + dy\,\frac{\partial N_{xy}}{\partial y}\,dx - q_x\,\frac{dx}{R_x}\,dy + P_x\,dx\,dy - q_y\,\frac{\partial \alpha_x}{\partial y}\,dx\,dy$$

$$= -N_x\,\frac{\partial \varepsilon_y}{\partial x}\,dx\,dy - N_{yx}\,\frac{\partial \varepsilon_x}{\partial y}\,dy\,dx - N_y\,\frac{\partial^2 u}{\partial y^2}\,dy\,dx \qquad (4\text{-}1)$$

$$- N_{xy}\,\frac{\partial^2 u}{\partial y\,\partial x}\,dx\,dy - P_z\,\theta_x\,dx\,dy + q_x\,\frac{\partial \theta_x}{\partial x}\,dx\,dy + q_y\,\frac{\partial \theta_x}{\partial y}\,dx\,dy$$

equilibrium condition in Y-direction:

$$\frac{\partial N_y}{\partial y}\,dy\,dx + \frac{\partial N_{yx}}{\partial x}\,dx\,dy - q_y\,\frac{dy}{R_y}\,dx + P_y\,dy\,dx - q_x\,\frac{\partial \alpha_y}{\partial x}\,dx\,dy$$

$$= -N_y\,\frac{\partial \varepsilon_x}{\partial y}\,dy\,dx - N_{xy}\,\frac{\partial \varepsilon_y}{\partial x}\,dx\,dy - N_x\,\frac{\partial^2 v}{\partial x^2}\,dx\,dy \qquad (4\text{-}2)$$

$$- N_{yx}\,\frac{\partial^2 v}{\partial x\,\partial y}\,dy\,dx - P_z\,\theta_y\,dy\,dx + q_y\,\frac{\partial \theta_y}{\partial y}\,dx\,dy + q_x\,\frac{\partial \theta_y}{\partial x}\,dy\,dx$$

Fig 2

equilibrium condition in z - direction:

$$-N_x \frac{dx}{R_x} dy - N_y \frac{dy}{R_y} dx - \frac{\partial q_x}{\partial x} dx\, dy - \frac{\partial q_y}{\partial y} dy\, dx +$$

$$-P_z\, dx\, dy - N_{xy} \frac{\partial \alpha_y}{\partial x} dx\, dy - N_{yx} \frac{\partial \alpha_x}{\partial y} dx\, dy \qquad (4\text{-}3)$$

$$= + N_x \frac{\partial \theta_x}{\partial x} dx\, dy + N_y \frac{\partial \theta_y}{\partial y} dx\, dy + N_{xy} \frac{\partial \theta_y}{\partial x} dx\, dy + N_{yx} \frac{\partial \theta_x}{\partial y} dx\, dy$$

$$- P_x\, \theta_x\, dx\, dy - P_y\, \theta_y\, dx\, dy$$

moment equilibrium in x - direction:

$$\frac{\partial M_y}{\partial y} dy\, dx + \frac{\partial M_{xy}}{\partial x} dx\, dy - q_y\, dx\, dy = 0 \qquad (4\text{-}4)$$

moment equilibrium in y - direction:

$$\frac{\partial M_x}{\partial x} dx\, dy + \frac{\partial M_{yx}}{\partial y} dy\, dx - q_x\, dy\, dx = 0 \qquad (4\text{-}5)$$

moment equilibrium in z - direction:

$$-M_{xy} \frac{dx}{R_x} dy + M_{yx} \frac{dy}{R_y} dx - N_{yx}\, dy\, dx - N_{xy}\, dx\, dy + M_x \frac{\partial \alpha_y}{\partial x} dx\, dy - M_y \frac{\partial \alpha_x}{\partial y} dy\, dx \qquad (4\text{-}6)$$

$$= + M_{xy} \frac{\partial \theta_x}{\partial x} dx\, dy - M_{yx} \frac{\partial \theta_y}{\partial y} dy\, dx - M_x \frac{\partial \theta_y}{\partial x} dx\, dy + M_y \frac{\partial \theta_x}{\partial y} dy\, dx$$

Fig. 3

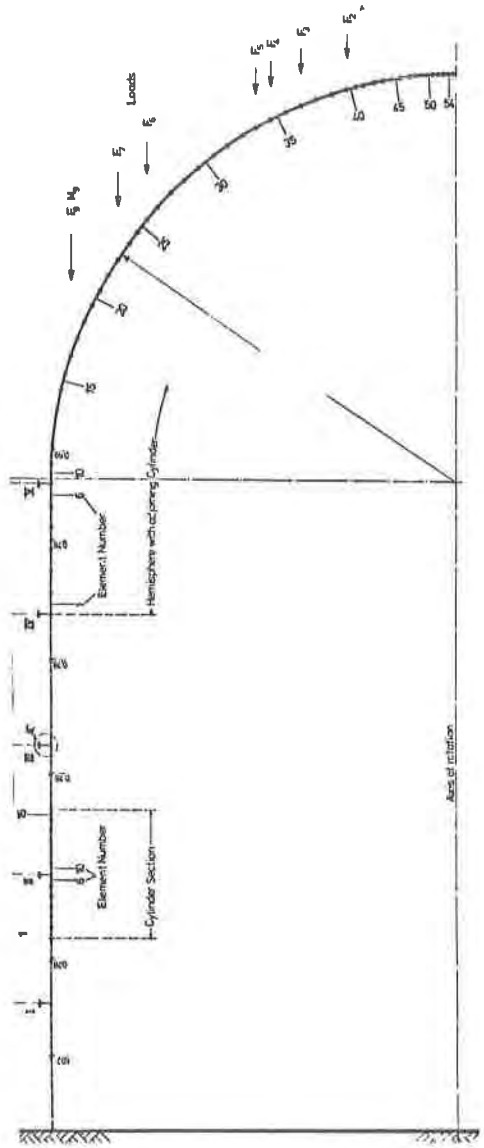

Loads
Element Number
Hemisphere with adjoining Cylinder
Element Number
Cylinder Section
Axis of rotation

Fig 4 DEFLECTIONS

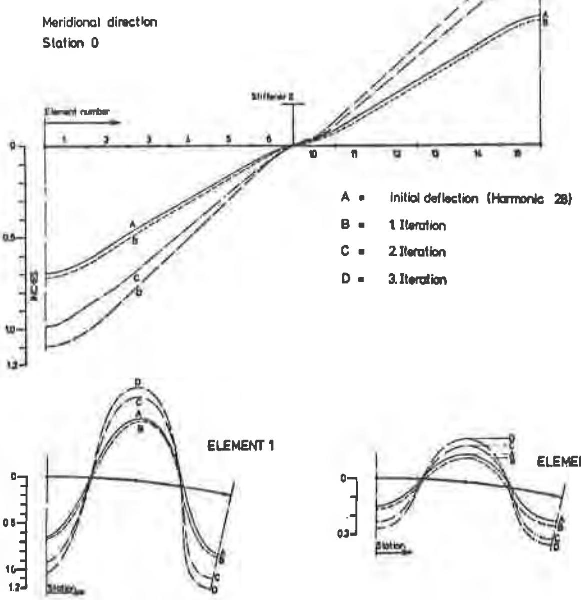
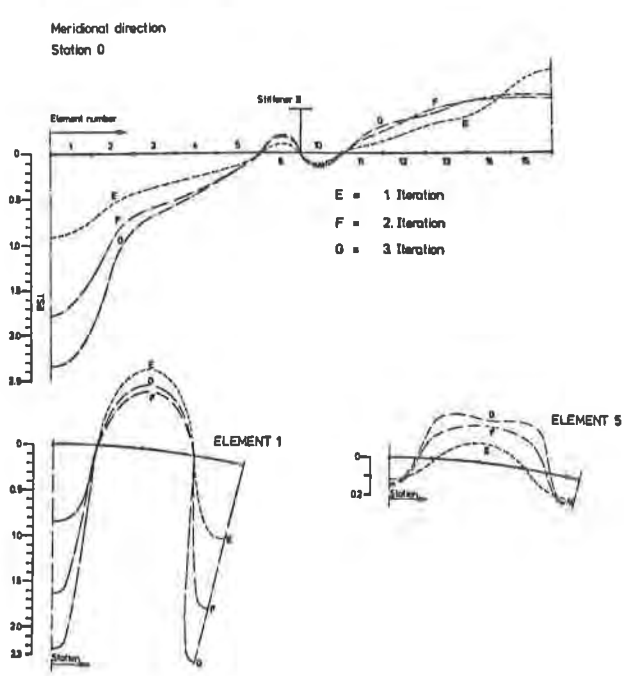


Fig. 5 EQUILIBRIUM LOADS



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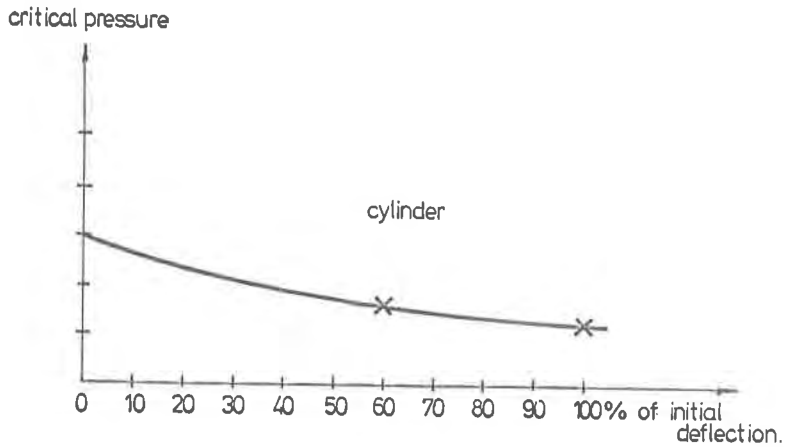

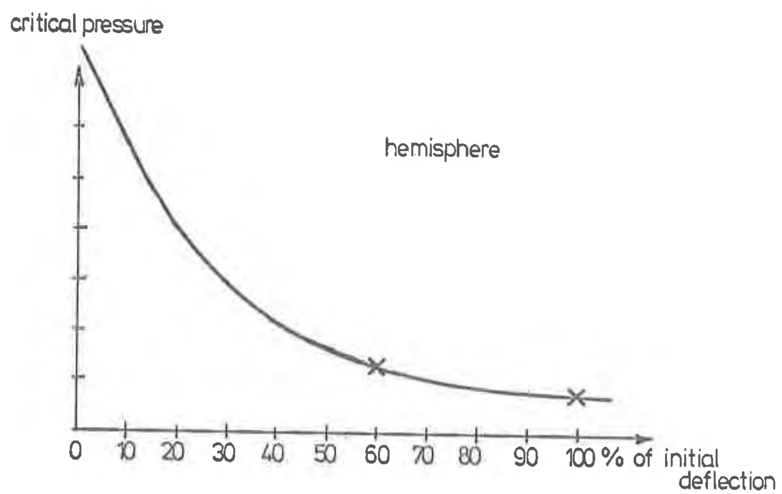

Fig. 6 Influence of magnitude of initial deflections on critical pressure.