MATERIAL DAMAGE IN STRUCTURAL ANALYSIS

J. HULT, J. JANSON

Chalmers University of Technology, Division of Solid Mechanics, Fack, S-40220 Göteborg, Sweden

Summary

The paper gives a survey of the development of continuous damage mechanics during the 20 years following the originating paper by Kachanov in 1958.

In contrast to the stiffness of a structure its strength may be drastically influenced by local material defects. These defects may be macroscopic, such as finite sharp cracks, or microscopic, such as grain boundary voids, slag inclusions, etc.

The strength of a structure in the presence of one or several finite cracks is studied in Fracture Mechanics (FM). The crack is here assumed to be embedded in a material without microscopic defects. In contrast the strength of a structure in the presence of a great number of microscopic defects, but without macroscopic cracks, may be studied by methods developed in Continuous Damage Mechanics (CDM). Here the material defects are assumed to be continously distributed throughout the material volume. The mechanical state of the material may then be described by a continuous field quantity, denoted damage, in addition to those normally used, viz. stress and strain.

The basic ideas and assumptions in CDM, as well as applications to various problems of structural strength and lifetime analyses, are presented and discussed. Among these are creep buckling under the influence of damage, and quantitative comparisons are made with the case of creep buckling in the absence of damage.

Simultaneous presence of macroscopic cracks and microscopic defects will lead to a decrease in strength, which is larger than that caused by any of these two causes separately. The analytic tools developed in FM and CDM have been combined to study this effect. A Dugdale type crack model is analysed, assuming continuous damage to develop in the thin zones extending ahead of the crack but also in the surrounding medium. The amount of the damage is related to the separation of the two opposite faces in these zones. For constant applied load and ambient temperature, when the material is assumed to be elastic - perfectly plastic, an instability criterion is found, which combines the wellknown Griffith and Dugdale results. For constant load and high temperature, where creep deformation is assumed to occur, the common relation between load an creep rupture lifetime is found. For cyclic load at ambient temperature a relation between crack propagation rate and variation in stress intensity factor is obtained, which closely resembles the Paris law for fatigue crack propagation. Finally, for cyclic load at high temperature strongly non-linear creep - fatigue - interaction is found to result, in close similarity to what is found for most metals.

The various cases give a strong support to further development of CDM as a tool to be used with confidence in design work.

1. Introduction

The term damage was introduced into structural mechanics by Miner [1] in a study of fatigue under cyclic loads with varying amplitude. He restated a formula for the fatigue lifetime previously proposed by Palmgren [2]. If a member is subjected to n_i load cycles at an amplitude σ_i, which would cause fatigue failure after N_i load cycles, then the member is said to have suffered a damage

$$D_i = n_i/N_i \tag{1}$$

If the member is subjected to a sequence of loads with amplitudes σ_1, σ_2, ..., then fatigue failure is assumed to occur, when

$$\Sigma D_i = 1 \tag{2}$$

This linear cumulative damage theory has found wide use in fatigue design, even though experimental results sometimes deviate strongly from (2).

A similar linear damage theory had been proposed by Robinson [3] for creep rupture under varying load. A member subjected to t_i hours of tensile stress σ_i, which would cause creep rupture after T_i hours, is said to have suffered a damage

$$D_i = t_i/T_i \tag{3}$$

If the member is subjected to a sequence of loads σ_1, σ_2, ..., then creep rupture is assumed to occur, when the condition (2) is satisfied. Even though field experience has sometimes shown marked deviations from this linear damage theory, its conceptual simplicity has been much appreciated in design work.

The expressions (1) and (3) are both posterior definitions of damage in the sense that the failure quantities N_i and T_i first have to be determined before D_i may be calculated.

An entirely different damage concept was proposed by Kachanov [4] in a study of brittle creep rupture. It was defined as the relative decrease in load carrying area of a tensile member, due to the formation of microcracks, voids, and cavities occurring during creep. With A denoting the current area, measured externally, and A_n the net load carrying area, Kachanov defined damage as

$$\omega = (A-A_n)/A \tag{4}$$

In the virgin state A_n=A and hence ω=0. In the limiting state of complete inner destruction of the material A_n=0 and hence ω=1. Normally equilibrium will be lost, and hence rupture will occur, before the damage level ω=1 is reached.

Kachanov postulated the growth law

$$d\omega/dt = Cs^{\nu} \tag{5}$$

where

$$s = \sigma/(1-\omega) \tag{6}$$

denotes the "net stress", i.e. the average stress acting on the net load carrying area A_n. The quantities C and ν are temperature dependent material parameters. The quantity σ in (6) is the "current stress", i.e. the hypothetical stress acting on the current area A.

For a material which shows very little creep deformation, such that $A \simeq A_0$, where A_0 is the virgin area, the current stress σ closely equals the nominal stress σ_0. Eq:s (5) and (6) then yield, after integration and considering the initial condition $\omega(0)=0$

$$\omega(t) = 1 - [1 - (1+\nu)C\sigma_0^{\nu}t]^{1/(1+\nu)} \tag{7}$$

From (5) and (6) or from (7) follows that $d\omega/dt \to \infty$ at time

$$t_R = 1/[(1+\nu)C\sigma_0^{\nu}] \tag{8}$$

which is then a measure of the brittle creep rupture time.

For a sequence of nominal stresses σ_{01}, σ_{02}, ... acting during time intervals t_1, t_2, ... there results the creep rupture condition

$$\Sigma t_i/t_{Ri} = 1 \tag{9}$$

Hence Kachanov's theory is compatible with Robinson's postulate (2). Likewise the simplified form of Kachanov's damage growth law (5), viz.

$$d\omega/dt = \bar{C}\sigma^{\bar{\nu}} \tag{10}$$

due to Taira [5], implies the rupture condition (9). In contrast to the Kachanov theory (5), (6), which describes an accelerated damage growth, the Taira law (10) describes a constant rate of damage growth, which is in opposition to experimental results. Hence the Taira law (5) has not found much use in design work.

Kachanov also considered the influence of creep deformation on the creep rupture process. He then assumed the creep strain rate $d\varepsilon/dt$ to depend on the current stress σ, i.e. to be unaffected by the damage. Rabotnov [6] made the more reasonable assumption that $d\varepsilon/dt$ depens on the net stress s

	Kachanov	Taira	Rabotnov
$d\varepsilon/dt$	$F(\sigma)$	$F(\sigma)$	$F(s)$
$d\omega/dt$	$f(s)$	$f(\sigma)$	$f(s)$

None of these relations describe the effect of instantaneous load changes. In order to take primary creep into account, by modelling it as an instantaneous strain increment, Odqvist [7] supplemented the stationary creep law $d\varepsilon/dt = F(\sigma)$ by a term $G'(\sigma)\cdot d\sigma/dt$. Similarly Broberg [8] supplemented the stationary damage law $d\omega/dt=f(s)$ by a term $g'(s)\cdot ds/dt$, which corresponds to an instantaneous damage increment

	Odqvist	Broberg	
$d\varepsilon/dt$	$F(\sigma) + G'(\sigma)\cdot d\sigma/dt$	$F(s) + G'(s)\cdot ds/dt$	(11)
$d\omega/dt$	-	$f(s) + g'(s)\cdot ds/dt$	(12)

Damage laws for multiaxial stress fields have been developed by Kachanov [4,9], Rabotnov [10], Hayhurst [11], Hayhurst & Leckie [12], Leckie & Hayhurst [13,14,15], Leckie [16], Chaboche [17] and others. These laws are all outgrowths of the original Kachanov theory (4), (5), (6).

2. Tensile and compressive failure of rods

If a tensile load P is applied to a rod in the creep and damage regime, the rod may rupture during load application (instant rupture) or at a later time (creep rupture). Likewise a compressive load P applied to a slightly bent road may cause buckling during load application (instant buckling) or at a later time (creep buckling). All these failure modes may be derived from the creep and damage laws (11), (12) and the geometrical nonlinearities involved in the two cases.

2.1 Tensile failure

When the rod in Fig. 1 is loaded, it will elongate and contract uniformly. For isochoric deformation the current stress σ is related to the nominal stress σ_0 as

$$\sigma = \sigma_0(1+\varepsilon) \qquad (13)$$

and hence from (6) the net stress is

$$s = \sigma_0(1+\varepsilon)/(1-\omega) \qquad (14)$$

This combines with (11), (12) to yield a differential equation for $s(t)$ in terms of $\sigma_0(t)$. The use of logarithmic instead of linear measures of strain and damage, cf. Hult [18], would change (14) into

$$s = \sigma_0 \exp(\varepsilon+\omega) \qquad (15)$$

giving the differential equation

$$[1/s - G'(s) - g'(s)]\cdot ds/dt - [F(s) + f(s)] = (1/\sigma_0)\cdot d\sigma_0/dt \qquad (16)$$

For instantaneous load changes (16) degenerates to

$$[1/s - G'(s) - g'(s)]\cdot ds/d\sigma_0 = 1/\sigma_0 \qquad (17)$$

Hence $ds/d\sigma_0 \to \infty$ or $ds/dt \to \infty$, implying instantaneous or delayed rupture, when $s \to s_*$ where

$$1/s_* - G'(s_*) - g'(s_*) = 0 \qquad (18)$$

Hence a rupture locus

$$s_* = \sigma_0 \exp(\varepsilon+\omega) \qquad (19)$$

exists in the ε-ω-σ_0-space (Fig. 3). Tensile rupture occurs when the state point $(\varepsilon,\omega,\sigma_0)$ reaches this surface. For any prescribed loading history $\sigma_0(t)$ the rupture time t_* may be determined from (16) and condition $s(t_*) = s_*$, cf. Westlund [19].

2.2 Compressive failure

When the slightly bent rod in Fig. 2a is loaded in compression, further bending deformation will be concentrated to the midspan region, if the creep and damage laws are strongly nonlinear. A hinge model, shown in Fig. 2b, may then be used to describe the mechanics of the rod, cf. Boström [20]. For small deflections the current bending moment at the hinge is

$$\mu = \mu_0(1+\varepsilon) \tag{20}$$

where $\mu_0 = P\cdot(L/2)\cdot(\alpha_0/2)$ is the nominal bending moment. The close similarity between (13) and (20) should be noted. With damage ω developing in the hinge, a net bending moment may be defined analogously to the net stress s in (6)

$$m = \mu/(1-\omega) \tag{21}$$

and hence

$$m = \mu_0(1+\varepsilon)/(1-\omega) \tag{22}$$

This combines with the creep and damage laws for the hinge

$$\begin{cases} d\varepsilon/dt = F(m) + G'(m)\cdot dm/dt & (23) \\ d\omega/dt = f(m) + g'(m)\cdot dm/dt & (24) \end{cases}$$

to yield a differential equation for $m(t)$ in terms of $\mu_0(t)$. The use of logarithmic instead of linear measures of strain and damage would change (22) into

$$m = \mu_0 \exp(\varepsilon+\omega) \tag{25}$$

Hence all the conclusions drawn for tensile loading have a direct counterpart for compressive loading. In particular an instability locus exists in the ε-ω-μ_0-space (Fig. 4). For any prescribed loading history $\mu_0(t)$ the buckling time t_* may be determined from the condition $m(t_*) = m_*$, where

$$1/m_* - G'(m_*) - g'(m_*) = 0 \tag{26}$$

Numerical evaluations by Bialkiewicz [21] show the relative importance of geometrical and physical (creep and damage) nonlinearities. As expected the overall picture is quite similar to that for tensile failure.

3. <u>Finite crack in damaging material</u>

Fracture mechanics (FM) has been developed to describe the strength of structures containing one or several cracks of finite size embedded in an otherwise defect free continuuum. Continuous damage mechanics (CDM), on the other hand, describes the strength of crack free structures in terms of a continuous defect field. Hence FM and CDM may be considered as dealing with two extremes: FM deals with complete absence of damage and CDM with absence of finite cracks. It is reasonable to expect a closer description of real structural strength by combining FM and CDM.

The specific surface energy, often denoted γ, is a basic parameter in linear elastic fracture mechanics (LEFM). It may be used also to determine the ideal fracture strength (cohesive strength) σ_F' of a linearly elastic material. A simple analysis yields

$$\sigma_F' = \sqrt{\gamma E/b} \tag{27}$$

where E denotes Young's modulus and b the interatomic distance.

A basic, linear CDM model material is defined by the strain and damage relations

$$\varepsilon = s/E\ , \qquad \omega = s/D \tag{28}$$

corresponding to F=0, G'=1/E, f=0, g'=1/D in (11), (12). From (18) then follows $s_* = ED/(E+D)$ and the fracture damage

$$\omega_F = E/(E+D) \tag{29}$$

Combination of the LEFM and CDM approaches yields the lowered fracture stress (Janson & Hult [22])

$$\sigma_F = \sigma_F^{\prime}\sqrt{1-\omega_F} = \sigma_F^{\prime}\sqrt{D/(E+D)} \tag{30}$$

Similarly the interaction between the singular stress field around a sharp, finite crack and a surrounding damaging medium may be considered. The analysis in general becomes highly complex, and calls for a numerical approach at an early stage. The great simplification offered by the Dugdale model in studying elastic-plastic media may be utilized also here, cf. Jansson [23,24, 25,26]. The state of stress in the thin zones extending ahead of the crack may be considered to be uniaxial. The amount of damage in these zones has been related to the separation of the two opposite faces, and in addition a simple expression has been adopted for the smaller damage accumulation occurring in the surrounding medium. Four kinds of loading have been analyzed, and the following results have been obtained:

3.1 Constant load, ambient temperature

A relation for the critical crack length is found (Fig. 5), which is smaller than that obtained by LEFM (Griffith's result). For small crack lengths Dugdale's result is approached.

3.2 Constant load, elevated temperature

Creep rupture lifetimes are obtained, which increase with decreasing initial crack length (Fig. 6). The standard creep rupture curve is approached for zero crack length.

3.3 Cyclic load, ambient temperature

After a certain incubation period the crack begins to extend in a stepwise manner. The rate of growth is related to the variation in stress intensity factor (Fig. 7). This relationship is found to depend very weakly on various assumptions made in the analysis and to be resemble closely the one given by Paris [27].

3.4 Cyclic load, elevated temperature

Strongly concave creep-fatigue interaction curves are obtained (Fig. 8) for a wide range of parameter variations. This general non-linear type of interaction is characteristic of most metals tested so far in creep-fatigue loading.

4. Conclusions

This survey of the fundamentals of continuous damage mechanics (CDM) and its use in the analysis of rupture phenomena, separately or in combination with fracture mechanics (FM) concepts, indicates that CDM may find increasing use in design work. Even though an unambiguous physical interpretation of the damage quantity itself, as defined by Kachanov and others, is still lacking, its conceptual simplicity makes it a natural addition to the classical parameters used in analyses of structural strength.

References

[1] Miner, M.A., J.Appl.Mech. 12 (1945), A-159.

[2] Palmgren, A., Zeitschrift VDI 68 (1924), 339.

[3] Robinson, E.L., Trans. ASME 60 (1938), 253.

[4] Kachanov, L.M., Izv. Akad. Nauk SSSR, No 8 (1858), 26.

[5] Taira, S., Proc. IUTAM Colloquium on Creep in Structures, Stanford 1960 (ed. N.J. Hoff), Springer, Berlin 1962, 96.

[6] Rabotnov, Yu.N., Proc, XII IUTAM Congr., Stanford 1968 (ed. M.Hetényi & W.G. Vincenti), Springer, Berlin 1969, 342.

[7] Odqvist, F.K.G., VII IUTAM Congr., Istanbul 1952. Roy. Inst. Tech. Stockholm Trans. No. 66, 1953.

[8] Broberg, H., J. Appl.Mech. 41 (1974), 809.

[9] Kachanov, L.M., Foundations of mechanical rupture (in Russian). Nauka, Moscow 1974.

[10] Rabotnov, Yu.N., Creep problems in structural members, North-Holland, Amsterdam 1969.

[11] Hayhurst, D.R., J.Mech.Phys.Sol. 20 (1972), 381.

[12] Hayhurst, D.R. & Leckie, F.A., J.Mech.Phys.Sol. 21 (1973), 431.

[13] Leckie, F.A. & Hayhurst, D.R., Proc. Roy. Soc. London A 340 (1974), 323.

[14] Leckie, F.A. & Hayhurst, D.R., Mech Res. Comm. 2 (1975), 23.

[15] Leckie, F.A. & Hayhurst, D.R., Acta Met. 25 (1977), 1059.

[16] Leckie, F.A., Phil. Trans. Roy. Soc. London A 228 (1978), 27.

[17] Chaboche, J.-L., Diss. Univ. de Paris VI (1978).

[18] Hult, J., Trans. 3rd SMiRT Conference, London 1975, paper L4/8.

[19] Westlund, R., J.Mech.Engng.Sci. 18 (1976), 175.

[20] Boström, P.-O., Int.J.Sol.Struct. 11 (1975), 765.

[21] Bialkiewics, J., Paper submitted to 21st Polish Solid Mechanics Conference 1979.

[22] Janson, J. & Hult, J., J.Mech.Appliquée 1 (1977), 69.

[23] Janson, J., Engng. Fracture Mech. 9 (1977), 891.

[24] Janson, J., Engng. Fracture Mech. 10 (1978a), 651.

[25] Janson, J., Engng. Fracture Mech. 10 (1978b), 795.

[26] Janson, J., Engng. Fracture Mech. 11 (1979),

[27] Paris, P.C., Proc. 10th Sagamore Army Materials Res. Conf., Syracuse Univ. Press 1964.

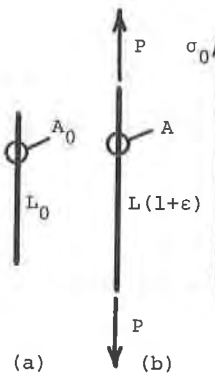


Fig. 1

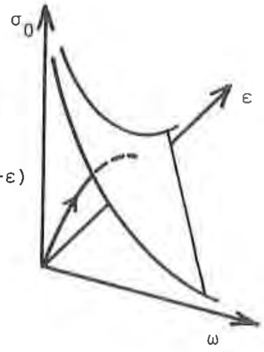


Fig. 3

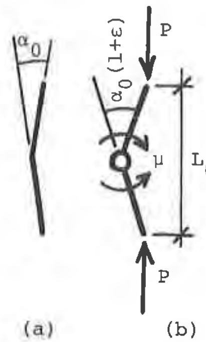
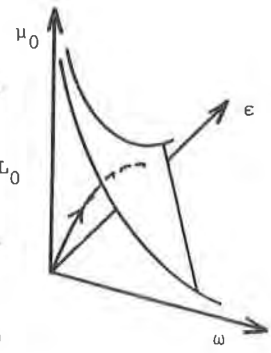


Fig. 2



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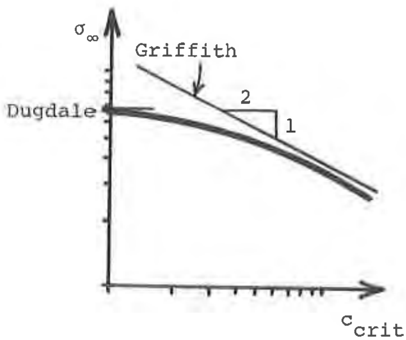

Fig. 5

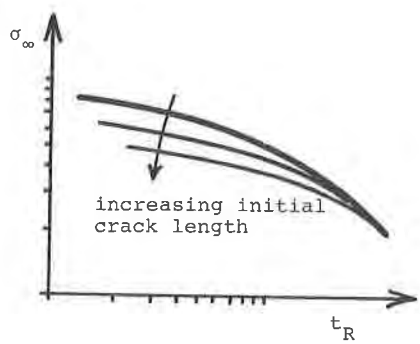

Fig. 6

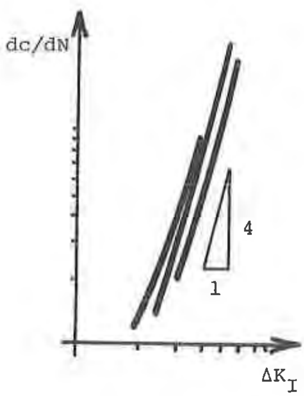

Fig. 7

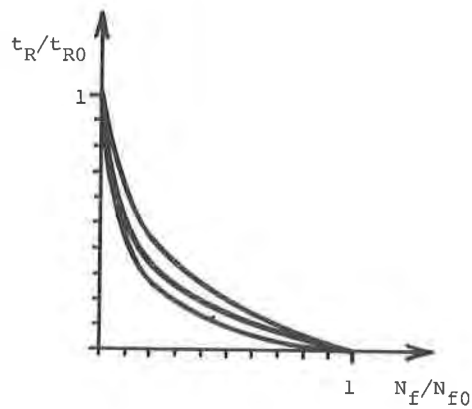

Fig. 8