# Mises Criterion Modification Conformably to Alternating Loading

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## 1. INTRODUCTION

The Von Mises criterion is widely used for the strength evaluation of high-loaded structural components for nuclear power plant. According to this criterion, equivalent quantity (stress/strain) is determined by

$$S_{eq} = \left[C\, I_2(D)\right]^{\frac{1}{2}}$$

Here D is a tensor-deviator of the quantity S, $I_2$ (D) is the second invariant of deviator D within a constant factor equals energy of distortion, C is constant.

Among other merits the criterion enables one to take account of the loading history and to do without the principal stresses/strains calculations, since it has the invariant recording form. At the same time the criterion is unsuitable for the alternating loading [1] and does not take account of cyclicity under the principal direction rotations [2]. The Von Mises criterion modification without these disadvantages is proposed below.

## 2. CRITERION MODIFICATION

Let deviator D has the $S_{ij}$ components in chosen Cartesian coordinates OXYZ. The second deviator invariant can be respectively written as:

$$I_2(D) = \frac{1}{2}\left(S_{xx}^2 + S_{yy}^2 + S_{zz}^2\right) + S_{xy}^2 + S_{yz}^2 + S_{zx}^2$$

Here quadratic form in the right side of equation is positive one and this fact defines the Von Mises criterion non-sensitivity to the component sign changes under the alternating loading. We shall consider the problem on the basis of the simplest special case for the single-component loading.

Let in some point of structure the pure shear $S_{xy}$ time-varying according to periodic law only acts as shown in Fig.1a. In this case the second invariant $I_2$ (D)= $S^2_{xy}$ varies in terms of pulsed acquiring double frequency and range decreased in half, as shown in Fig.1b. This effect is naturally extended to the Von Mises criterion as a whole.

To avoid this error in Case [4] it is ordered to shift the loading cyclogram to the area with positive signs, as shown in Fig.1c, when transferring the minimum load point (point A, Fig. 1c) to abscissa axis. It is noted that such procedure leads to loss of cyclogram visualization and information as well as to complicated calculation. The situation is this that the right side of the Von Mises criterion is interpretable as radius-vector module in deviator space and hence the transfer of this space origin is equivalent to addition over the entire quantities of some vector. Such transfer provides the parallel cyclogram shift only in the case of simple cyclic loading as shown in Fig.1c. In the general case the shift occurs with the scale distortions. In doing so, for instance, two previously equal peaks can obtain the levels. As a result the facts of achieving the yield limit, plastic reverse an so on cannot be directly defined according to the distorted cyclogram. In the particular case considered for pure shear the problem can be solved without resorting to the cyclogram shift. In that case it is sufficiently to use the product of $S_{xy} \, |S_{xy}|$ in place of $S^2_{xy}$. Indeed, the both quantities are always equal in module, but in so doing the sign of second one coincides with $S_{xy}$ component sign. As a result we obtain the desired cyclogram as shown in Fig.1d.

The proposed method allows the generalization for common case through the change of the second invariant $I_2$ (D) by the corresponding quadratic form with the alternating signs

$$B(D)=\frac{1}{2}\left(b_{xx}\,|b_{xx}|+b_{yy}\,|b_{yy}|+b_{zz}\,|b_{zz}|\right)+b_{xy}\,|b_{xy}|+b_{yz}\,|b_{yz}|+b_{zx}\,|b_{zx}|\,,$$

the sign of which can be considered as the stress/strain state one.

It is necessary to determine the corresponding between the components of $S_{ij}$ and $b_{ij}$ on the basis of the modified criterion non-contradiction condition in reference to the classical the Von Mises criterion as well as to conventional sign principle for normal deviator components: tension-plus, compression-minus.

The first condition on criterion non-contradictions leads to the conclusion that $b_{ij} = \pm S_{ij}$. Let consider some time when the classical Von Mises criterion is true, for example, the first quarter of the first loading cycle, and suppose that $B(D)>0$ in this case. Then $B(D)=I_2(D)$ and consequently all $b_{ij} \geq 0$ since quadratic form of $I_2(D)$ always has positive sign. As far as some deviator components is negative we obtain the following relationship:

$$b_{ij}=\begin{cases} S_{ij} & (S_{ij})_1 \geq 0 \\ -S_{ij}, & (S_{ij})_1 < 0 \end{cases}$$

Here $(S_{ij})_1$ are the deviator component values, for instance, during the first quarter of the first loading cycle. They are singly used to establish the sign rule only providing the agreement with the classical Von Mises criterion.

If under the first loading $B(D) < 0$, then all signs in relationship for $b_{ij}$ must be changed by opposite ones.

Let us agree the sign of B(D) with the conventional sign rule for normal components. For this purpose this rule should be generalized for multiaxis stress/strain state. The consideration of this problem is conducted in principal axes.

It should be noted that both the positive and negative principal components contribute to the energy of distortion. Consequently, $I_2(D)$ can be written as

$$I_2(D)=A^{+}+A^{-}$$

where $A^+$ and $A^-$ - the contribution of the positive (tension) and negative (compression) components respectively. Let us introduce the following parameter into the consideration

$$\chi=6\frac{A^{+}-A^{-}}{A}$$

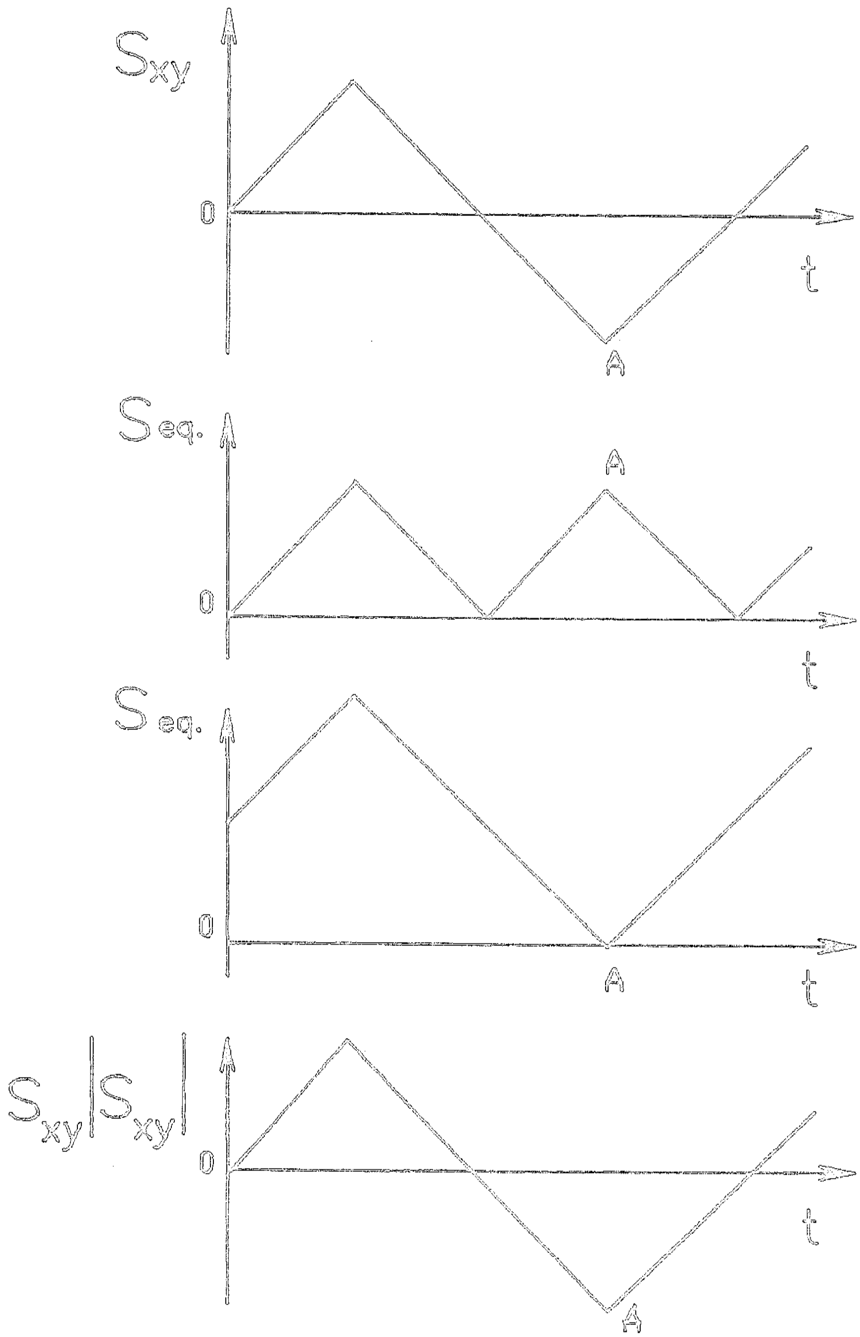
that represents the analogy to the Nadai-Lode parameter since under single-axis tension, pure shear and single-axis compression this parameter assumes the following values +1, 0, -1 respectively. It is easy to show that its sign is opposite to sign of $S_2$ and, due to the inequality $S_1 \geq S_2 \geq S_3$, $S_1 > 0$, $S_3 < 0$, coincides with the third invariant sign:

$$sign\,\chi = sign\left(I_3(D)\right)$$

Thus, the expression for components of $b_{ij}$ assumes the form:

$$b_{ij}=\begin{cases} S_{ij}\cdot sign\left(I_3(D_1)\right), & (S_{ij})_1 \geq 0\,; \\ -S_{ij}\cdot sign\left(I_3(D_1)\right), & (S_{ij})_1 < 0\,. \end{cases}$$

It will be recalled that $I_3(D) = \det D$, $D_1 = \{ (S_{ij})_1 \}$.




a)

b)

c)

d)

As a result we obtain the following expressions for the modified the Von Mises criterion in terms of stresses:

$$\sigma_{eq} = sign\left( B\sigma \right) \left( 3 \mid B\sigma \mid \right)^{\frac{1}{2}}$$

and in terms of strains:

$$\varepsilon_{eq} = sign\left( B\varepsilon \right) \left( \frac{4}{3} \mid B\varepsilon \mid \right)^{\frac{1}{2}}$$

Here the quadratic forms of B($D\sigma$) and B($D\varepsilon$) are denoted by $B\sigma$ and $B\varepsilon$ respectively.

In addition it may be remarked that in the cases of simple loading the criterion can be adopted with the minimum modification:

$$\sigma_{eq} = sign\left( I_3\left( D_6 \right) \right) \left[ 3\, I_2\left( D\sigma \right) \right]^{\frac{1}{2}}$$

It is well known that the symptom of simple loading is the direction tensor constancy, i.e. the performing of the following condition:

$$\frac{D}{I_2\left( D \right)} = const$$

## 3. CONCLUSION

There is no the sign rule for shears in continuum mechanics and the signs of these components depend on choosing the coordinate system. Therefore the analysis of the entire structure loading history should be performed in unified outer coordinate system that is not connect with loaded structure. Note that the analysis of the work [2] on the basis of the proposed modified criterion results in the agreement with experiment.

## REFERENCES

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