Seismic Analysis of Equipment System with Non-Linearities Such as Gap and Friction Using Equivalent Linearization Method

H. Murakami, T. Hirai, M. Nakata
Electric Power Development Co. Ltd., Tokyo, Japan
T. Kobori, K. Mizukoshi, Y. Takenaka, N. Miyagawa
Kobori Research Complex Inc., Tokyo, Japan

1. Introduction

Many of the equipment systems of nuclear power plants contain a number of non-linearities, such as gap and friction, due to their mechanical functions. It is desirable to take such non-linearities into account appropriately for the evaluation of the aseismic soundness. However, in usual design works, linear analysis method with rough assumptions is applied from engineering point of view.

An equivalent linearization method is considered to be one of the effective analytical techniques to evaluate non-linear responses ,provided that errors to a certain extent are tolerated, because it has greater simplicity in analysis and economization in computing time than non-linear analysis.

The objective of this paper is to investigate the applicability of the equivalent linearization method to evaluate the maximum earthquake response of the equipment systems such as CANDU Fuelling Machine (F/M) which has multiple non-linearities.

2. Non-linear characteristics of CANDU F/M

As an object of investigation in this paper, the test rig of CANDU F/M which is used in the seismic verification test program [2] (Banwatt,Murakami, 1987) is selected. The complete test rig schematically shown in Fig.1 is composed of the fuelling machine (F/M), F/M support structure, the fuel channel (F/C), and the F/C support frame. This system contains three kinds of non-linear connecting elements, namely, gap, friction, and multi-linear spring elements, in various connections between components. The characteristics of each non-linear element is illustrated in Fig.2.

3. The method of equivalent linearization

3.1 Equivalent linearization method

The purpose of employing the equivalent linearization method is to predict the maximum value of non-linear response by solving linear equation in which an equivalent stiffness Ke and an equivalent damping coefficient Ce are used instead of non-linear coefficients. For evaluating these equivalent coefficient, Ke and Ce, there are two typical methods. One is the least mean square error method developed by Caughey,T.K. ([3],Caughey,1963), and the other is the geometrical method proposed by Jacobsen,L.S. ([5],Jacobsen,1960). Evaluation method of Ke and Ce by using these methods can be explained (See Table 1) as follows:

(a) Caughey's method : Ke and Ce are determined so as to minimize mean-square error between non-linear equation and linear equation.

(b) Jacobsen's method : Ce is determined so as to equate the energy dissipation of non-linear system to that of the linear system per one cycle of vibration at resonance. Ke is determined independently to be the slope of the line joining the ends of the hysteresis loop.

According to these methods, equivalent stiffness Ke for gap element evaluated from Jacobsen's method is always larger than that from Caughey's method. As for multi-linear spring element, Ke from Caughey's method is, on the contrary, larger than that from Jacobsen's method. There are no difference in equivalent damping coefficient Ce between the two methods.

3.2 Coefficient of effective amplitude

The idea of coefficient of effective amplitude is adopted in order to calculate the response more precisely in equivalent linear analysis against random excitation like earthquakes. Equivalent linear coefficients, Ke and Ce, are determined corresponding to the average response displacement value which is obtained by multiplying certain reduction factor, that is the coefficient of effective amplitude, to the maximum value. In this paper, 1.0 and 0.65 are investigated as the coefficient of effective amplitude.

3.3 Iterative method

As the equivalent stiffness Ke and equivalent damping coefficient Ce are obtained as the functions of unknown quantity of response displacement, iterative calculation procedure is necessary, as shown in Fig.3, to find the numerical results until responses converge to a constant value. In this paper, a simple iterative method and Steffensen's iterative method are investigated.

(a) Simple iteration : displacement assumed in each step is determined as the response value obtained in last step.

(b) Steffensen iteration : displacement assumed in each step is estimated according to the following equation.

$$\widehat{R_{n+1}} = R_{n+1} + \frac{R_{n+1} - R_n}{r_n^{-1} - 1}, \qquad r_n^{-1} = \frac{R_n - R_{n-1}}{R_{n+1} - R_n}$$

$\widehat{R_{n+1}}$: Assumed displacement at (n+1)th step by Steffensen iteration

R_{n-1} : Assumed displacement at (n-1)th step

R_n : Response displacement at (n-1)th step (= Assumed displacement at n th step)

R_{n+1} : Response displacement at n th step

4. Investigation of applicability

In this investigation, three kinds of analyses, non-linear analysis, equivalent linear analysis, and conventional linear analysis, are applied to F/M which has three types of non-linear connecting elements such as gap, friction, and multi-linear spring element. Their results are compared in terms of agreement of the maximum response values and computing time consumed for the analysis.

In order to investigate the most appropriate conditions for equivalent linear analysis, several cases of analyses are conducted while changing the analytical conditions.

4.1 Analytical model

Fig.4 shows the analytical model adopted for the analysis. It is basically composed of three dimensional beam elements connected to each other by linear springs or various non-linear connecting elements. It is made by reducing the original detailed model developed for the correlation analysis of the F/M seismic verification test [2] (Banwatt,Murakami,1987). The location and characteristics of the non-linear connecting elements are reported in [2] (Banwatt,Murakami,1987). Characteristics of representative non-linear elements, which have influence on the seismic response of F/M, are shown in Fig.5.

4.2 Input waves

Input waves adopted for the analyses are given as the response at the F/M floor to the basic earthquake ground motions S1 and S2. The floor response acceleration spectra (h=1.0%) are shown in Fig.6. The muximum accelerations of input waves are 204.7 inch/s/s (S1) and 346.5 inch/s/s (S2). The analytical model is excited in Z direction.

4.3 Analytical cases

Analytical cases of equivalent linear analyses are shown in Table 2. In these cases, analytical conditions such as equivalent linearization method, iterative method, coefficient of effective amplitude, and maximum acceleration of input waves are changed as parameters. Non-linear analyses are executed as exact solutions, and conventional linear analyses with rough assumptions are also executed for comparison.

4.4 Analytical conditions

These linear and non-linear response analyses are executed by direct time integration method with trapezoidal expansion using the dynamic analysis code developed for the dynamic analysis of CANDU F/M [1] (Banwatt,Murakami,1985). Time interval of analysis is 0.001 second in non-linear analysis and 0.01 second in linear or equivalent linear analysis.

In the linear analysis, the characteristics of non-linear connecting elements are assumed to be linear with initial stiffness in principle based on the conventional method, but as for those elements whose inelastic displacement seem to grow to a large extent, following assumptions are made:

(1) Multi-linear spring element of (13-14) is replaced by a linear spring with the secant modulus to the second breaking point.

(2) Friction elements of (20-22), (21-22), (26-27), and (26-28) are neglected.

(3) Gap element of (20-22) is replaced by a linear spring with the stiffness of collision.

5. Analytical results

5.1 Response to S1 earthquake

As the result of the non-linear analysis, the responses of seven non-linear elements are in inelastic range. They are ; one gap element of (20-22) in Z direction ; four friction element of (20-22), (21-22), (27-26), and (28-26) in Zdirection ; two multi-linear spring elements of (11-12) in Z direction, and (13-14) in θx direction. The maximum response values of acceleration and displacement obtained from equivalent linear analyses (Case 1,2,&3), and linear analysis are compared with that of non-linear analysis in Fig.7 and Fig.8. Maximum acceleration from equivalent linear analysis are smaller, in general, than those from non-linear analysis ,but the agreement in displacement is relatively good, and much improved in comparison with that of linear analysis. It shows that the effects of non-linearities on the response displacement are considered appropriately in the equivalent linear analysis. The reason why the maximum acceleration are underestimated in equivalent linear analysis is considered that high frequency response due to the collision cannot be analized by this method. Comparison as to stress and displacement of non-linear elements are shown in Fig.9.

The degree of agreement with the results of non-linear analysis can be improved by employing a coefficient of effective amplitude.

Table 2 shows computing time consumed in analysis and the number of times of iteration. Computing time consumed for equivalent linear analysis are about 20% to 40% of that of non-linear analysis.

The relationship between assumed displacement and response value at each iteration step is shown in Fig.10 as for multi-linear spring element between (12-13). It is shown that the convergence can be accelerated by using Steffensen iteration.

5.2 Response to S2 earthquake

In this case, stable response value cannot be obtained because the response values do not converge to a constant value. Therefore additional analyses are performed with using other initial assumed displacement, and iterative methods. The response value at each iteratioin step is shown in Fig.11 for gap and friction elements of (20-22) in Z direction. This figure shows that response values fluctuate repeatedly under any condition within the same range. It is considered that this phenomenon mainly results from an interaction between different non-linear elements.

6. Conclusions

Conclusions are obtained from the results of these investigations as follows :

(1) Equivalent linear analysis is effective to evaluate the maximum response values of CANDU F/M having multiple non-linearities. The reasons are as follows;
1) In spite of disagreement in acceleration, equivalent linear analysis can provide maximum response displacement and reaction forces, which are important in evaluating seismic qualification of F/M.
2) Computing time consumed for equivalent linear analysis are shortened to 20 -40% of that of non-linear analysis.

(2) The findings below are obtained for the applicability of equivalent linearization method. This method can be effectively applied to other similar equipment systems if enough considerations are made regarding the following items ;
1) The degree of agreement with the result of the non-linear analysis can be improved if the equivalent linearization method or coefficient of effective amplitude is appropriately selected.
2) Convergence can be accelerated by utilizing Steffensen iteration.
3) According to the input acceleration levels, response values may be in bi-stable, because of interaction between non-linearities.

Acknowledgement

The authors wish to express their sincere appreciation to Prof.Shibata for his valuable advice and to Atomic Energy of Canada Ltd. for their permission to use data on CANDU F/M.

References

[1] Banwatt,A.S.,Murakami,H., etal. (1985) Developement of Non-Linear Seismic Analysis Code - Full Scale Model Testing for CANDU 600MWe Fuelling Machine. 8th SMIRT Session K. pp.401-407.
[2] Banwatt,A.S.,Murakami,H., etal. (1987) Full scale seismic verification test for 600MWe CANDU fuelling machine and its correlation analysis. 9th SMIRT Session K. pp.1127-1132.
[3] Caughey,T.K. (1963) Equivalent Linearization Technique. The Journal of the Acoustical Society of America, Volume 35 Number 11, pp. 1706-1711.
[4] Conte,s.d.,Cari de Boor, (1972) Elementary Numerical Analysis. McGraw-Hill Book Company.
[5] Jacobsen,L.S., (1960) Damping in Composite Structure. Proc.2nd WCEE, pp 1029-1044.

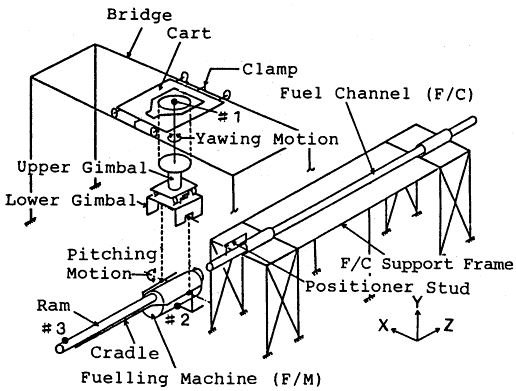


Fig.1 Full scale test rig

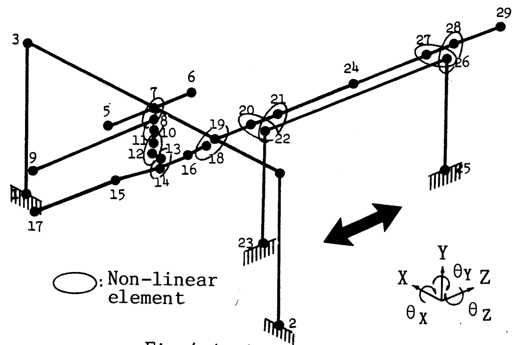


Fig.4 Analytical model

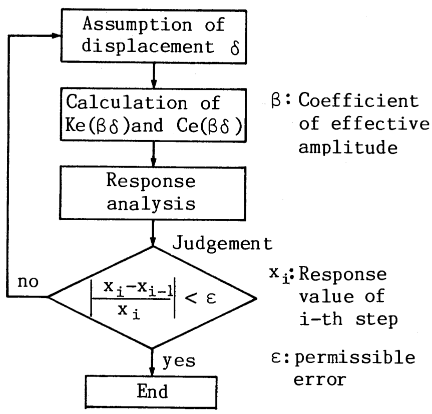


Fig.3 Flow of equivalent linear analysis

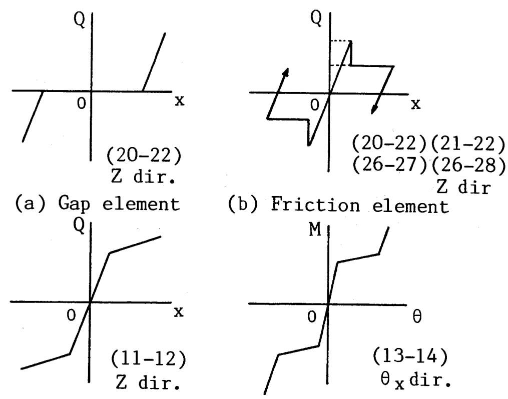


Fig.5 Non-linear elements

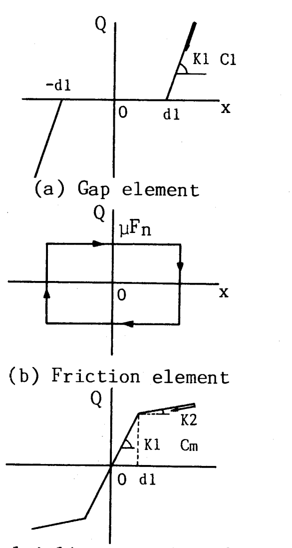


Fig.2 Characteristics of non-linear elements

Table 1 Equivalent linearization method

Non-linear element		Caughey's method	Jacobsen's method
Gap element	Ke	$\frac{K1}{\pi}(\pi-2\theta-\sin2\theta)$	$(\frac{\delta-d1}{\delta})K1$
	Ce	$\frac{C1}{\pi}(\pi-2\theta-\sin2\theta)$	$\frac{C1}{\pi}(\pi-2\theta-\sin2\theta)$
Friction element	Ke	0	$\frac{\mu Fn}{\delta}$
	Ce	$\frac{4\mu Fn}{\pi v}$	$\frac{4\mu Fn}{\pi v}$
Multi-linear spring element	Ke	$\left[\frac{K1-K2}{\pi}\right](2\theta+\sin2\theta)+K2$	$(K1-K2)\frac{d1}{\delta}+K2$
	Ce	Cm	Cm

$\theta=\sin^{-1}\frac{d1}{\delta}$

δ: displacement assumed for Ke and Ce
Ke: equivalent stiffness
Ce: equivalent damping coefficient

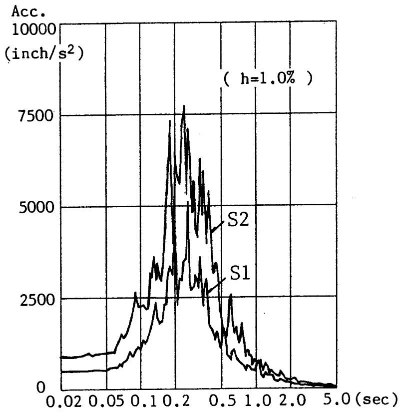
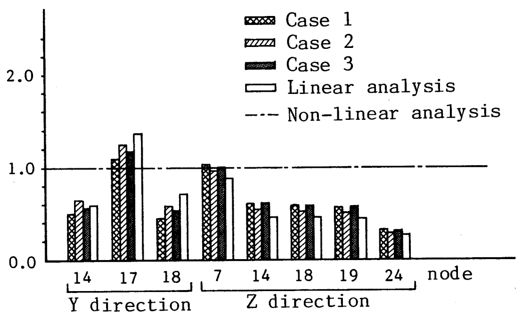


Fig.6 Response spectra of input waves

Table 2 Analytical case of equivalent linear analyses

Case	E.L.A	C.E.A	Iterative method	Number of times	Computing time (sec)
1	Jacobsen	1.0	Simple	20	121 (0.58)
2	Caughey	1.0	Simple	6	35 (0.17)
3	Jacobsen	0.65	Steffensen	8	46 (0.22)
4	Jacobsen	1.0	Steffensen	14	80 (0.38)

E.L.A:Equivalent linearization method
C.E.A:Coefficient of effective amplitude
Computing time of non-linear analysis of S1 =208 sec.
Computer used:HITACHI M-680D



Eig.7 Comparison of acceleration

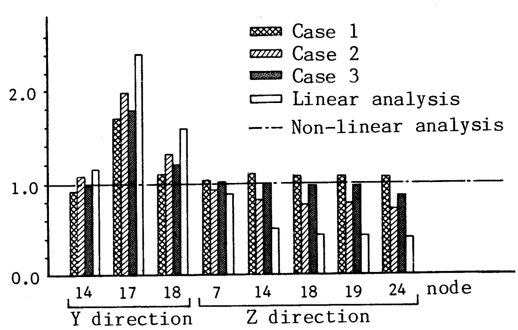
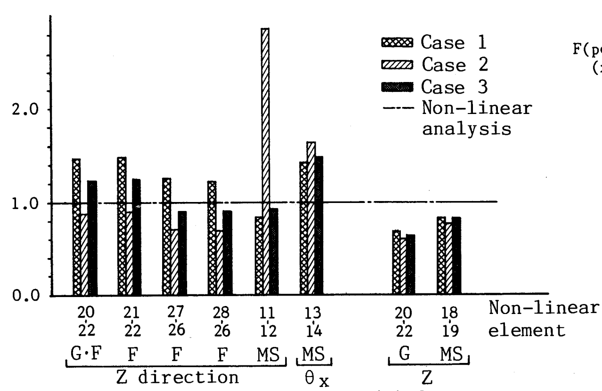
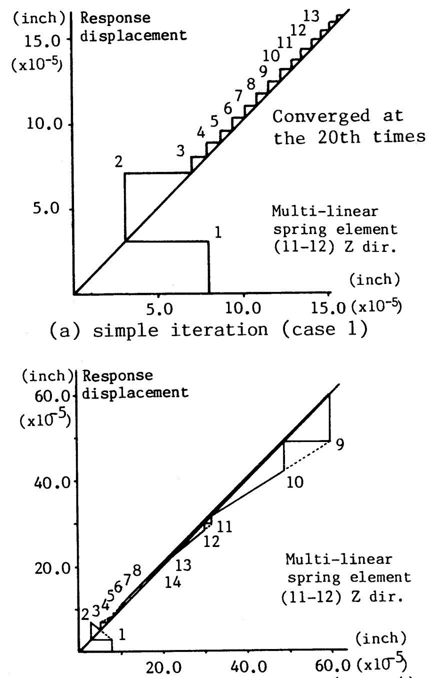


Fig.8 Comparison of displacement



(a) Relative displacement (b) Stress

Fig.9 Comparison of relative displacement and stress



(a) simple iteration (case 1)

(b) Steffensen iteration (case 4)

Fig.10 Response displacement in each calculation step

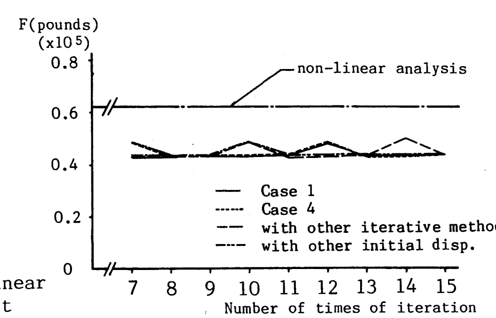


Fig.11 Equivalent linear responses in S2 earthquake

(gap element (20-22) Z dir)