

Determination of J-R Curve from Only One Experimental Test on One Sample

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SUMMARY

Increasing attention has been given to the J concept not only for determination of the onset of crack propagation but also for studying ductile tearing. For these studies the most important material characteristic is the so called J-R curve which gives the J value as a function of the increase Δa of the crack length. Determination of the R curve is rather costly and time consuming because many tests on many samples are needed. Such a situation can be attributed to the measurement of the crack length. The aim of this paper is to describe a procedure avoiding any direct measurement of the crack length (with or without breaking the sample).

Definition of J concept is discussed at first. It is pointed out that J cannot be obtained by straight measurements, but is always computed in using experimental load deflection curve as an input. The current practice is based on the following assumption : if the load deflection curve is known for one value of the crack length, it is possible to know the load deflection curve for any value of the crack length.

This assumption is generalized and correct formula giving J from the experimental load deflection curve are written, taking into account the effect of crack propagation.

Then it is shown that this assumption can be used to extract the value of crack length from the experimental load deflection curve. In addition of this curve, it is only needed to know initial and final values of the crack length and indications on features of the load-deflection curve when propagation does not occur. Therefore the variation of crack length can be obtained from one test on one sample. The method is fully described and the procedure for CT specimen is detailed.

1. Introduction

The use of J concept in Fracture Mechanics [1] has received considerable attention. This concept is not only used as a criterion of onset of crack propagation, but also to study propagation stability. As a consequence there is a need to obtain the curve giving J as a function of crack extension Δa (the so-called J-R curve). This curve, considered as a material characteristic, is extracted from results of loading tests made on cracked samples, and it is interesting that the extraction be as cheap as possible. The aim of this paper is to show that the J-R curve can be extracted from the results of only one test, it is to say that values of J and values of Δa can be deduced from the load-displacement curve of one cracked sample. The proposed method is an generalization and an extension of the hypothesis currently made to deduce J value from experimental tests.

As this paper is a compact one, it is advised to the reader to refer to ref.[2] for more detailed informations.

2. Operational Definition of J Concept

The definition and meaning of J concept can be considered in many different ways : a path integral, the variation of energy with the crack length, the factor characterizing the crack tip singularity. These definitions are more or less linked to an elastic - but non linear - behaviour of the material. But it is the authors opinion that these definitions are not convenient because they are of no use at all for experimental determination of the value of J. As a consequence it is useful to give an operational definition of J.

Such a definition must be founded on a very general principle of mechanics : the principle of virtual work. In continuum mechanics, the conventional form of this principle is :

stress working variation = virtual work of external forces

$$\delta W = X_i \delta u_i$$

where X_i means for external forces and δu_i for virtual displacements.

It must be pointed out that virtual displacements must be admissible, it is to say that equations of compatibility must be satisfied. Unfortunately this is never possible in fracture mechanics because virtual crack propagation is always considered. Therefore the principle of virtual work must be generalized by introduction of material displacements δa_i . Material displacements [3] [4] [5] are displacements of material particles through the material itself, or in other words, the flow of material properties through the body. Some virtual work is created by material displacement δa_i and the principle of virtual work must be written :

$$\delta W = X_i \delta u_i - J_i \delta a_i$$

where J_i are material forces, it is to say dual variables of material displacements.

For practical applications, it is always supposed that the work of conventional forces X_i can be written $X \delta u$ where X and δu are generalized force and displacement. In the same way, the assumption is made that the work of material forces J_i can be written $J B \delta a$ where B δa is the increase of the crack area - or B is the thickness and δa is the virtual crack propagation. Hence the principle of virtual work can be written :

$$\delta W = X \delta u - J B \delta a \quad (1)$$

This is a more operational definition of J concept. But it must be emphasized that the assumption that $J B \delta a$ is a good expression of the virtual work of material forces, is an approximation, if not an arbitrary hypothesis.

Unfortunately, this assumption is not sufficient for practical application and another one must be made. It will be assumed that, during the loading test of the cracked sample, the forces X and J and the stress working are functions of only the displacements u and a (this does not imply that the material behaviour is elastic).

As a consequence it is possible to make integration and derivation, and to get an practical expression of J :

$$W = \int_0^u X du \Big|_{a=\text{constant}} \quad J = \frac{\delta W}{B \delta a} \Big|_{u=\text{constant}} \quad (2)$$

3. Extraction of J value from the Load-deflection Curve

3.1 Current Methods

Historically, eqs. (2) were straight used in the first proposed method to obtain J value from experimental tests [6]. It became soon obvious that this method is costly and difficult to perform : many experimental tests were needed and derivative of W had to be get from numerical data. In order to avoid these drawbacks, others methods were proposed [7] [8].

The analysis of these last methods shows that they are based on a new assumption : "If load displacement-curve-X,u- is known for one given value of crack length a, all other load displacement curves (for the same material ant the same shape of sample, but for different values of crack length) are known".

3.2 Fundamental Assumption

This (third) assumption is the generalization of those made in current practice. It can be translated in a scale effect of crack length on applied Force X and on displacement u, leading to the definition of scale functions [9] :

$X = A \varphi$	$u = C \xi$	(3)
X load		
A and C are known functions of only a		
φ is a function of ξ (to be determined)		

$A(a)$ and $C(a)$ are generally given as a result of limit analysis, but it is possible to use more realistic way like experiments or elastic plastic analysis. Nevertheless it is assumed they are known for a given shape of sample.

φ and ξ can be called reduced force and reduced displacement. The relation between φ and ξ is depending of the material and of the geometry of sample (see Figure 1). This curve $\varphi(\xi)$ is not known and must be experimentally determined. In other words, the goal of the experimental test is to obtain the curve $\varphi(\xi)$. It can be noted that geometrical similitude can be applied and that the curves related to samples with same shape made out of the same material, but of different scale give similar curve - φ being proportional to the square of the scale-factor and ξ to the scale-factor itself. Therefore $\varphi(\xi)$ could obtain from smaller or larger samples than the tests done.

3.3 Expression of J

It is easy to apply eqs. (2) in using eq. (3) :

$$W = A C \int_0^{u/C} \varphi \, d \xi$$

$$J = - \frac{1}{B} \frac{d(AC)}{da} \int_0^{u/C} \varphi \, d \xi - \frac{AC}{B} \varphi \frac{d}{da} \left(\frac{u}{C} \right)$$

This result can be written :

$$J = - \frac{1}{B} \frac{d(AC)}{da} \int_0^u \frac{X \, du}{AC} + \frac{1}{B} \frac{dC}{C da} Xu \quad (4)$$

or

$$J = - \frac{1}{B} \left[C \frac{dA}{da} \int_0^u \frac{X \, du}{AC} - A \frac{dC}{da} \int_0^x u \, d \frac{F}{AC} \right] \quad (4')$$

These equations give the correct value of J when there is crack propagation, it is to say when a, A(a) and C(a) are not constant during the test.

When crack does not grow, A and C are constant and eqs. (4) become :

$$J = - \frac{1}{B} \frac{d(AC)/da}{AC} \int_0^u X \, du + \frac{1}{B} \frac{dC/da}{C} Xu \quad (5)$$

$$J = \frac{1}{B} \left[\frac{dC/da}{C} \int_0^x u \, dX - \frac{dA/da}{A} \int_0^x X \, du \right] \quad (5')$$

Eq. (4) can also be simplified if the second scale function C is constant (deflexion not depending of the crack length) :

$$J = - \frac{1}{B} \frac{dA}{da} \int_0^u \frac{X}{A} \, du \quad (6)$$

which can be modified in :

$$J = - \frac{1}{B} \left[\int_0^u \frac{dA}{A da} X \, du + \int_a^a \frac{d^2 A / da^2}{dA/da} J \, da \right] \quad (7)$$

If no crack propagation occurs eq. (6) and eq. (7) become :

$$J = - \frac{1}{B} \frac{dA}{A da} \int_0^u X \, du$$

where it is found the well known area $U = \int_0^x X \, du$ below the load curve.

3.4 Example : Deep Crack in Bending

The current assumption is that the load is proportional to the square of the remained

ligament $b = W - a$

$$A = (W-a)^2 = b^2$$

$$\frac{dA}{da} = -2(W-a) = -2b$$

Eq. (6) gives :

$$J = \frac{2b}{B} \int_0^u \frac{X}{b^2} du$$

if no propagation occurs ($b = \text{cte}$) $J = \frac{2}{Bb} \int_0^u X du$

3.5 Example : CT Specimen

A current practice assumes that C is constant and that A is given by $A(a) = \alpha b \sigma_y$ where $b = W - a$

and α is solution of $\alpha^2 + 2 \left[\frac{2a}{b} + 1 \right] \alpha - 1 = 0$

$$\frac{dA}{da} = b \sigma_y \frac{d\alpha}{da} - \alpha \sigma_y = - \frac{2(1+\alpha)}{1+\alpha^2} \alpha \sigma_y$$

and eq. (6) gives :

$$J = \frac{2\alpha(1+\alpha)}{(1+\alpha^2)B} \int_0^u \frac{X}{ab} du$$

if no propagation occurs, $J = \frac{2}{Bb} \frac{1+\alpha}{1+\alpha^2} \int_0^u X du$

3.6 Example : Center Cracked Plate

The current practice is the use of the following assumption :

$$\frac{u}{b} = \text{function of } \frac{X}{b}$$

$$b = W - a \quad [2W = \text{width}, 2a = \text{crack length}]$$

$$\text{it is to say } A = b = W - a \quad C = b = W - a$$

$$J = \frac{2b}{B} \int_0^u \frac{X du}{b^2} - \frac{1}{Bb} Xu$$

or

$$J = \frac{b}{B} \int_0^u \frac{X du}{b^2} - \frac{b}{B} \int_0^x \frac{u dX}{b^2}$$

4. Extraction of Δa value from Load-deflection Curve

4.1 State of the art

As shown before, J value can be extracted from the load deflection curve, but if crack propagation occurs, crack length measurement is needed in order to apply eqs. (4).

In any way, knowledge of crack length is required to obtain J - R curve.

There is a lot of different crack length measurement techniques [10]. The most reliable ones are optical and fracture surface observations, but breakdown of the samples is required

and only one point of the curve can be deduced from one sample. Other methods are non destructive ones like compliance methods, potential difference, acoustic emission, ultrasonics, etc., but the validity of these methods is still questionable.

Recently it has been noted that the hypothesis (see article 3.2) used for the determination of J value can be also used for determination of the value of crack length [9] [11] [12] [4]. The method considered in this paper is based on the utilisation of the fundamental assumption made to compute J value.

4.2 Principle of the Proposed Method

In eq. (3), φ is a function of ξ which characterizes the material and the geometry of the sample. If this function is known before the test is performed, it is possible to draw a family of curves giving the load X as a function of the deflection u for given constant values of the crack length. The intersections of the experimental curve with each curve of the family will provide with the values of crack length during the test (see Figure 2).

Therefore the only problem is how to get the good curve φ as a function of ξ . Two different ways have been proposed. One is to store the curve corresponding to the material and the shape (geometrical similitude) of the tested sample ; this curve is obtained from preceeding tests. Such a method does not take into account the heat to heat variations of the mechanical properties of material. The other way is to adopt a theoretical shape of the φ ξ curve and to adjust it with test results. Such a method could be considered as too restrictive about the real curves.

The method proposed here uses both the knowledge of general features of the φ - ξ curve, deduced from preceeding tests, on same material and some sample shape, and this of measurements giving initial crack length a_0 and final crack length a_f .

It is assumed that the following items are known :

- 1-initial value of crack length a_0
- 2-final value of crack length a_f
- 3-general features of φ as a function of ξ

1 and 2 are obtained of surface examination of broken specimen, 3 is deduced of available preceeding tests, or from one test on an sample with blunted crack (in order to avoid propagation) which can be a small specimen.

4.3 General Description of the Method

4.3.1 Experimental Curve in the Field φ - ξ

From the experimental load deflection curve X, u, deduced the reduced experimental curve :

$$\varphi = \frac{X}{A(a_0)} \quad \xi = \frac{u}{C(a_0)} \quad \text{See Figure 3}$$

4.3.2 Reference Curve in the Field φ - ξ

If N is the point representing the final state of the test (end of the experimental curve), this point is reported to its reference place N^* such that

$$\varphi(N^*) = \varphi(N) \frac{A(a_0)}{A(a_f)} \quad \xi(N^*) = \xi(N) \frac{C(a_0)}{C(a_f)}$$

Now, with the help of the known general features of reference curve, draw the reference curve as follows :

- for low values of φ and ξ this curve is identical to the experimental curve,
- it finishes at the point N^* ,
- it is as similar as possible to the aspect of already known curves obtained with the same shape and same material (if necessary, make the correction needed for change in geometrical scale - See article 3.2).

4.3.3 Crack Length at the Point M

At the point M, the real crack length can be obtained as follows :

- Draw the connecting curve

$$\varphi = \frac{A(a)}{A(a_o)} \varphi(M) \qquad \xi = \frac{C(a)}{C(a_o)} \xi(M)$$

with values of a decreasing from a_o to a_f .

- The intersection of the reference curve and of this connecting curve is the point M^* corresponding to the point M.

- Crack length at M is the root of the following equations :

$$A(a_M) = \frac{\varphi(M)}{\varphi(M^*)} A(a_o) \quad \text{or} \quad C(a_M) = \frac{\xi(M)}{\xi(M^*)} C(a_o)$$

5. Example : CT Specimen

5.1 Introduction

It is easier to proceed with CT specimen because the scaling function C is constant.

There is only a scaling function $A = \alpha b \sigma_y$ applied to the load (see 3.5), α being the positive root of $\alpha^2 + 2 \left[\frac{2a}{b} + 1 \right] \alpha - 1 = 0$. Therefore introduction of ξ is not needed and it is possible to use the diagram reduced load ξ -deflection u . Obviously connecting curves are vertical straight lines parallel to ξ axis.

5.2 Reference Curve

Experimental load deflection curve is drawn on the diagram φ - u (see Figure 4), φ being given by $\varphi = X/\alpha_o b_o$ where $b_o = W - a_o$ and $\alpha_o = \alpha(a_o)$.

The final point N of this curve is corrected in order to obtain the point N^* :

$$u(N^*) = u(N) \qquad \varphi(N^*) = \frac{\alpha_o b_o}{\alpha_f b_f} \varphi(N)$$

α_f and b_f corresponding to the final value a_f of the crack length.

Then the Reference curve is drawn, being identical to the experimental curve in the vicinity of the origin and joining the point N^* . This curve must be as similar as possible to the known curve for CT specimen made out of the same material. Obviously similitude (dimensional, it is to say u proportional to the geometrical dimension and X to the square of it - Not to forget to take into account the effect of crack length) can be used. If no results are available, it is recommended to make a test on one CT sample with blunted crack.

5.3 Values of J and a

At a point M of the curve, a and J values are obtained as follows :

- M^* being the point of the reference curve with the same value of the deflection u as the point M, the crack length is the root of :

$$\frac{\alpha_b}{\alpha_o b_o} = \frac{\varphi(M)}{\varphi(M^*)}$$

- J value is expressed by :

$$J = \frac{2\alpha(1+\alpha)}{(1+\alpha^2)B} \int_0^u \varphi^* du$$

the integral being computed on the reference curve (and not on the experimental curve).

6. Conclusion

The general assumption made to obtain J can be used to write correct expressions of J when crack propagation occurs.

The same assumption can be also used to extract the value of the crack length from the experimental load-deflection curve. The only needed knowledges are initial and final values of the crack length, and indications on the general features of load deflection curve for the same type of sample made of the same material when crack propagation does not occur.

Such a method has been successfully applied and validated. It is a very efficient, quick and inexpensive method giving the J-R curve from only one test on one sample.

References

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FIG. 1

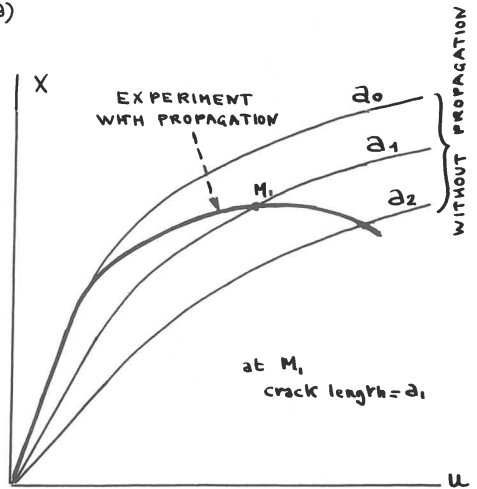


FIG. 2

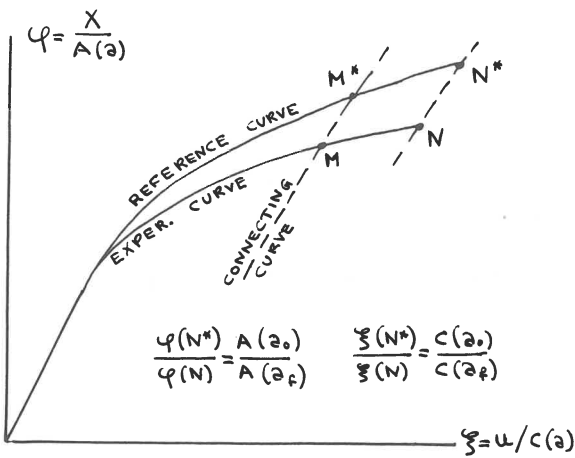


FIG. 3

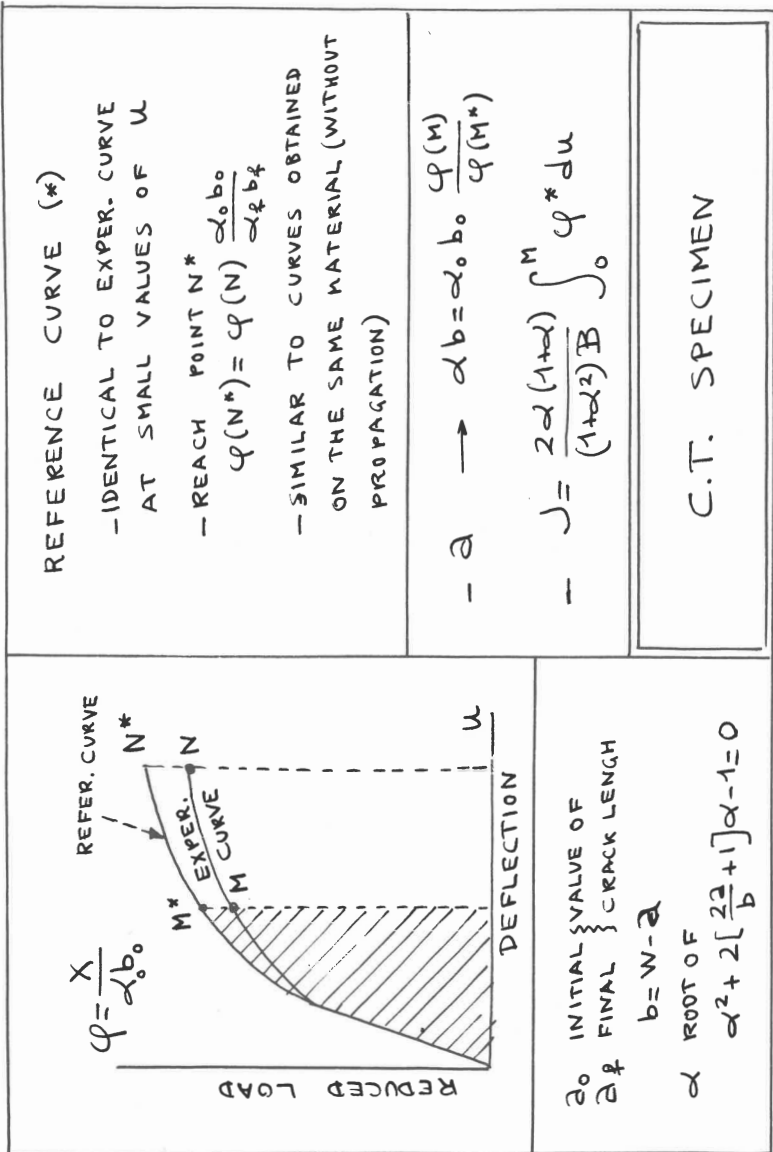


FIG. 4