A Unified Inelastic Constitutive Equation in Terms of Anisotropic Yield Function

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INTRODUCTION

In order to describe the material behavior under complicated loading conditions, inelastic constitutive equations accounting for the plasticity-creep interaction have been proposed by several researchers (e.g. Chaboche and Rousselier, 1983; Chan et al., 1988; Krempl, 1979; Pugh, 1983; Valanis, 1971). However, these models are developed to predict the hardening and/or softening phenomena during the inelastic deformation processes, and two important features still remain to be considered; material anisotropy induced by the prior deformation history and inelastic flow or, in another word, directionality of the inelastic strain rate.

This paper deals with a unified constitutive model capable of expressing both the deformation-induced anisotropy and the anisotropic flow. In the first part of the paper, an anisotropic yield function which can simulate both the Bauschinger effect and the cross effect is proposed as an extension of the work carried out by Baltov and Sawczuk (1965). Then, the excess stress theory (Perzyna, 1971) is applied to a viscoplastic constitutive relationship so as to describe the plasticity-creep interaction behavior. The experimental verification is carried out for SUS304 stainless steel at 650 °C in a biaxial stress stress state. Moreover, a generalized flow rule of the inelastic strain rate is also developed, by which the description of the ratcheting process can be improved.

ANISOTROPIC YIELD FUNCTION ACCOUNTING FOR MATERIAL ANISOTROPY

The fundamental assumption of the yield function (Hill, 1950) is laid on

$$F = \frac{1}{2} N_{ijkl} (s_{ij} - \alpha'_{ij})(s_{kl} - \alpha'_{kl}) - \frac{1}{3}\bar{\sigma}_y^2 = 0 \quad . \qquad (1)$$

Here N_{ijkl} is a fourth order tensor representing the material anisotropy, s_{ij} and α'_{ij} mean the deviatoric parts of Cauchy stress σ_{ij} and kinematic back stress α_{ij}, respectively. $\bar{\sigma}_y$ stands for yield stress.

The fourth order tensor cannot take an arbitrary form, but must obey a special representation rule, so-called "the representation theorem for isotropic tensors (Spencer, 1971)", since the yield function must be invariant under orthogonal transformation. In addition to the invariance requirement, the properties of symmetry and incompressibility are also assumed for the tensor. The tensor including two variables ξ_{1ij} and ξ_{2ij} may take the form

$$N_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - \frac{1}{3}\delta_{ij}\delta_{kl} + P_1 \xi_{1ij}\xi_{1kl} + P_2 \xi_{2ij}\xi_{2kl} + \frac{Q}{2}(\xi_{1ij}\xi_{2kl} + \xi_{2ij}\xi_{1kl}) \quad , \qquad (2)$$

in which δ_{ij} indicates the Kronecker's delta function, and the scalar coefficients P_1, P_2 and Q are, in general, expressed in polynomial functions of the invariants of ξ_{1ij} and ξ_{2ij}.

Employing the following dimensionless back stress deviator and stress deviator normalized by the yield stress as

$$\xi_{1ij} = \sqrt{3/2}\,\alpha'_{ij}/\bar{\sigma}_y \quad , \qquad (3)_1$$

$$\xi_{2ij} = \sqrt{3/2}\,s_{ij}/\bar{\sigma}_y \quad , \qquad (3)_2$$

and substituting them into eq.(1), we get

$$F = \frac{1}{2}(s_{ij}-\alpha'_{ij})(s_{ij}-\alpha'_{ij}) + \frac{3P_1}{4\bar{\sigma}_y^2}\{(s_{ij}-\alpha'_{ij})\alpha'_{ij}\}^2 + \frac{3P_2}{4\bar{\sigma}_y^2}\{(s_{ij}-\alpha'_{ij})s_{ij}\}^2 + \frac{3Q}{4\bar{\sigma}_y^2}\{(s_{ij}-\alpha'_{ij})\alpha'_{ij}\} \times \{(s_{kl}-\alpha'_{kl})s_{kl}\} - \frac{1}{3}\bar{\sigma}_y^2 = 0 \quad . \qquad (4)$$

The scalar coefficient P_2 must be zero to assure that eq.(4) coincides with the von Mises criterion in the initial state.

A simulated example of subsequent yield surface is shown in Fig. 1 where the experimental data are taken from a monotonic tension test at the strain rate of 0.5 %/s. The proposed model can accurately predict both the distortion and the translation of the yield surface. Especially, it predicts the cross effect, which is observed in the hardened region perpendicular to the loading direction, in proportion to the development of the back stress.

A UNIFIED INELASTIC CONSTITUTIVE MODEL

The excess stress theory (Perzyna, 1971) assumes that the excess stress beyond the static yield stress r produces the inelastic strain. When the steady creep rate is expressed in a power formula (Norton's law) and the normality rule is adopted, the evolution of the inelastic strain now follows

$$\dot{\varepsilon}^i_{ij} = \left\langle \frac{F-r}{D} \right\rangle^n \frac{\partial F}{\partial \sigma_{ij}} \quad , \qquad (5)$$

in which D means the drag stress with viscosity, and F indicates the anisotropic yield function represented by

$$F = \{\frac{3}{2}N_{ijkl}(s_{ij}-\alpha'_{ij})(s_{kl}-\alpha'_{kl})\}^{1/2} \quad . \qquad (6)$$

Referring to the Bailey-Orowan's form, the evolution of the back stress may be expressed as

$$\dot{\alpha}'_{ij} = C_1\frac{2}{3}\dot{\varepsilon}^i_{ij} - C_2(s_{kl}-\alpha'_{kl})\dot{\varepsilon}^i_{kl}\alpha'_{ij} - C_3\alpha'_{ij} \quad . \qquad (7)$$

Here, the coefficients C_1, C_2 and C_3 are, in general, non-negative scalar parameters related to all the arguments. The isotropic hardening such as cyclic hardening behavior is assumed to be described by the variation of the static yield stress. And this static yield stress may be divided into the temperature dependent term r_T and stress induced component r_a ($r = r_T + r_a$). When the evolution of the latter term is related to the variation of the inelastic strain, a special expression for the rate of r_a is employed as

$$\dot{r}_a = R_1\{R_2(\bar{\alpha}) - r_a\}(s_{ij}-\alpha'_{ij})\dot{\varepsilon}^i_{ij} \quad . \qquad (8)$$

This formulation satisfies that the saturated value of the isotropic hardening is related to the current value of the back stress.

EXPERIMENTAL VERIFICATION OF THE THEORY

The material employed in the present investigation is SUS304 stainless steel, and thin walled tubular specimens are used to impose the combined stress state of tension and torsion. The experiments are performed at high temperature level of 650 °C. An isotropic constitutive model based on the von Mises yield criterion is also used for the simulation as a comparison.

Cyclic Response for Circular Strain Path

Sinusoidal out-of-phase strain cycles, which is equivalent to a circular shape path in strain space, are imposed to the material with increasing strain ranges (diameter of the strain circle) from 0.8% to 1.2%. The relation between the stress amplitude and the number of cycles is shown in Fig. 2, where the open marks mean the stress amplitude in the out-of-phase cycles, and the solid circles corresponding to the uniaxial strain cycles with the same strain ranges are also plotted for comparison. As is seen from the figure, the non-proportional loading in the out-of-phase path induces much more hardening than the uniaxial cycling. Such a behavior can be predicted by both models, and nevertheless the anisotropic model provides quantitatively better result than the isotropic model.

Transient Softening under Reversal Creep

Another kind of examination is carried out under the condition that the axial stress varies from 165 MPa to -165 MPa while the shear stress component is kept constant at 60 MPa. The main feature which should be observed in the response is the transient softening behavior due to the change in the axial stress just as the Bauschinger effect appears in plastic deformation. The creep curve in this test is shown in Fig. 3, where Fig. 3(a) indicates the axial strain response and Fig. 3(b) means the response of shear strain. The anisotropic model provides the better description in the first stage of creep process while the isotropic model predicts the better result in the final stage. On the other hand, the experiment shows the gradual decrease in axial strain just after the axial stress changes from 165 MPa to -165 MPa while such behavior cannot be observed in the calculated results. This discrepancy may arise from the improper estimation of the direction of the inelastic strain rate.

DEVIATION FROM NORMALITY RULE AND ITS EFFECT ON MECHANICAL RATCHETING

As is mentioned above, the proposed model improves the description of complicated strain history under the biaxial stress state, but it has still some difficulty in predicting the inelastic behavior under stress controlled condition such as biaxial creep. In this section, we provide a constitutive model accounting for the deviation from the normality rule.

Direction of Inelastic Strain Rate in Mechanical Ratcheting

When cyclic shear straining is imposed under some axial stress, the axial strain is accumulated with the shear cycles, and this phenomenon is known as "mechanical ratcheting." Fig. 4 shows a subsequent yield surface during the mechanical ratcheting process. The open circles indicate 0.02% proof stresses while the solid marks mean 0.05% proof stresses, and the arrows in the figure represent the direction of inelastic strain rate (increment) on the yield points. Here, we point out that the axial strain rate shows negative value at the reversal point, and that the deviation from so-called normality rule can be also seen.

Evolution of Inelastic Strain

Taking into account of the deviation of the inelastic strain rate from the normality law, we propose the following form

$$\dot{\varepsilon}^{i}_{ij} = \left\langle \frac{F - r}{D} \right\rangle^{n} \frac{\partial F}{\partial \sigma_{ij}} + \kappa \frac{3}{2} \dot{s}_{ij} \tag{9}$$

for the evolution equation of the inelastic strain. Here, the second term of the right hand side is the additional term representing such deviation. The parameter κ should be small enough not to give any disturbance on the uniaxial stress-strain relation, and moreover, it must affect certain influence on the inelastic behavior in the multiaxial stress state.

Fig. 5 indicates the accumulated axial strain under the axial stress level of 67 MPa during the shear cycling with the strain range of 1.5% at $\gamma/\sqrt{3}$=0.002 %/s. It has been reported by the authors (Inoue et al., 1985) that most constitutive models reveal to overestimate the strain accumulation. The simulated results obtained in the present investigation still produce too large ratcheting strain while the anisotropic model with the deviation term improves the prediction of the accumulation behavior. The distorted shape of the yield surface as well as the deviation from the normality rule result in the different direction of inelastic strain rate, so that the anisotropic model seems to have the capability to describe the decrease of the axial strain increment per cycle.

CONCLUDING REMARKS

An anisotropic yield function is proposed by use of a fourth order tensor, and applied to a unified constitutive equation capable of describing the inelastic behavior of materials under complex loading path. Based on the representation theorem for isotropic tensors, a rational form of the yield function accounting for the deformation-induced anisotropy is obtained, by which both the distortion and the translation of the yield surface can be accurately predicted.

In the second part, a unified inelastic constitutive model is formulated, and the validity of the theory as well as the applicability to predict the complicated material behavior is evaluated for SUS304 stainless steel at 650 C. Moreover, the deviation from the normality rule is also taken into account of the proposed model, by which the simulation of the biaxial mechanical ratcheting is well improved.

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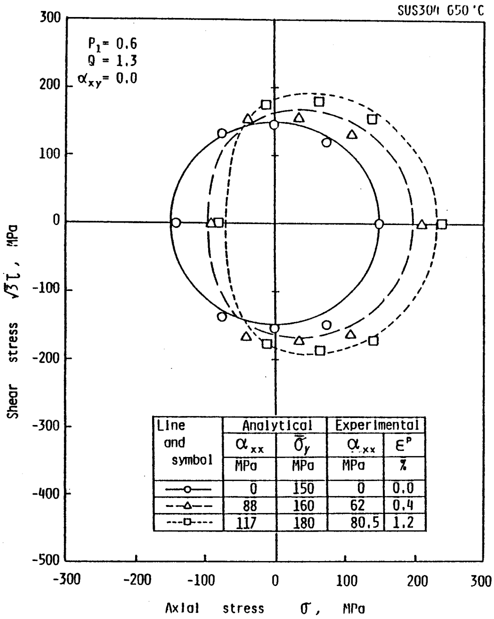

Fig. 1 Calculated result of the yield surface under monotonic tension.

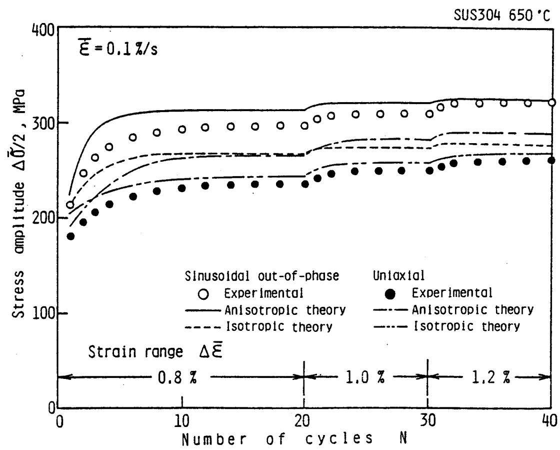

Fig. 2 Relation between the stress amplitude and the number of cycles in the out-of-phase cycles.

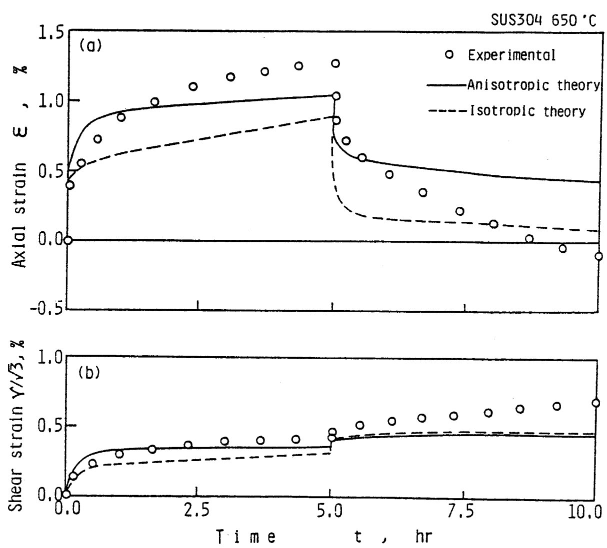

Fig. 3 Creep curves under the reversal creep loading condition.

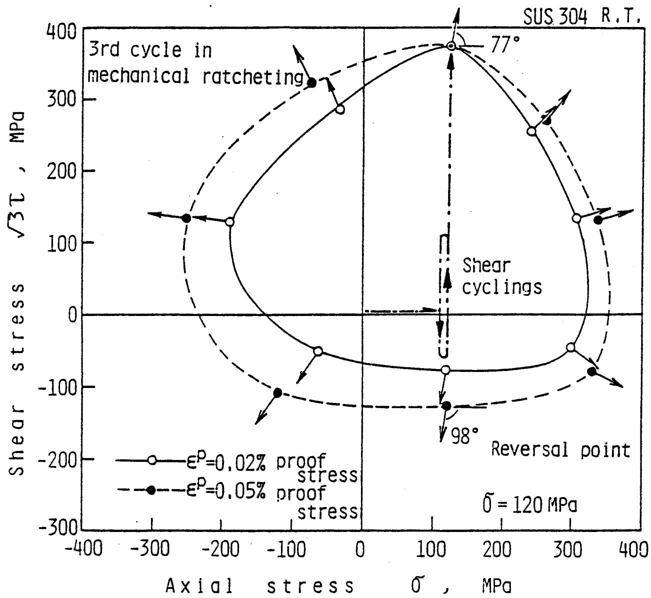

Fig. 4 Variation of yield surface in mechanical ratcheting.

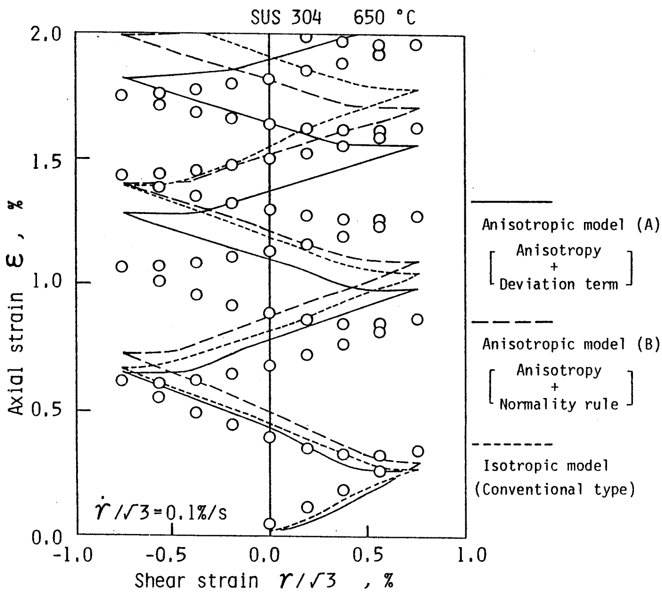

Fig. 5 Accumulation of the axial strain during the mechanical ratcheting process.