ABSTRACT

WU , SUI . School Choice: Theory and Empirical Evidence. (Under the direction of Umut Dur and Robert Hammond.)

The problem of assigning students to limited school seats is a mechanism design problem arising from college admissions. Designing the student assignment mechanism also extends to the public school choice policies.

In Chapter 1, we study assignment policies and reassignments of students, where students apply to a magnet school or request a transfer to another school within the public school system. Policymakers and researchers have expressed concerns that these type of reassignment programs could increase racial and socioeconomic stratification and cause an imbalance of resources across schools. We provide evidence from the Wake County Public School System that the reassignment programs reduce stratification in terms of race, socioeconomic status, student need, and student ability. To place our results in context, we conduct several simulations to compare the observed changes in stratification to what changes are possible. The simulation results suggest that the reassignment programs we study do not increase stratification in terms of race, socioeconomic status, or student need/ability, but they also do not reduce stratification to a particularly large degree.

Chapter 2 studies the conditions making the DA mechanism select efficient outcomes. Using the observations from WCPSS elementary magnet schools, we first formally build a theoretical framework to capture the features of the WCPSS student placement process, including students' priorities and preferences. Within this framework, we prove that, in theory, DA can be Pareto efficient under certain assumptions. These assumptions are met in the elementary magnet schools assignment process in WCPSS. We further our study by applying the DA mechanism to the data from the rising-kindergarten elementary magnet applicants in WCPSS in the 2017-2018 school year. To examine the efficiency of the DA assignments, we apply the Top-Trading Cycle (TTC) mechanism to the same dataset. Furthermore, we apply the Stable Improvement Cycle to the DA results to find possible Pareto improvements. We found strong evidence that the DA results are not significantly different from the TTC results, and the SIC algorithm finds no significant Pareto improvement.

Chapter 3 studies the college admission system in China. The Chinese college admission system involves a centralized matching process carried out by the local authority of each province. Our study extends the Chinese college admission mechanism literature by examining how students are matched to the department level instead of the college level. The local authority first matches students to colleges. Then the college matches students to departments. We formulate this two-step process in China into a mechanism. The mechanism first assigns students to colleges by DA mechanism, then assigns students to departments by DA mechanism. We show that this mechanism is neither strategy-proof nor non-wasteful. We further propose a simple solution by allowing students to apply to programs at the department-level and employing the DA mechanism to match students directly to departments.

School Choice: Theory and Empirical Evidence

by
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DEDICATION

To my parents and my husband for their endless support.

BIOGRAPHY

Sui (Joyce) Wu was born in Guangdong, China. She attended Sun Yat-sen University in China and graduated with a bachelor's degree in economics. She holds a master's degree in financial economics from Duke University. During her doctoral study at North Carolina State University, she pursued her research in school choice and mechanism design under the direction of Umut Dur and Robert Hammond.

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CHAPTER

1

REASSIGNMENT POLICIES AND SCHOOL STRATIFICATION

1.1 Introduction

Choice-based assignment of students to schools presents opportunities and challenges throughout the education system in countries like the United States. We study the effect of school assignment policies on racial stratification, socioeconomic stratification, and stratification based on student need or ability. The policies of interest are reassignment programs, where students seek to change their assignment from the school they were assigned to based on the school's attendance boundary. Our research questions are whether reassignment policies change the composition of schools in meaningful ways and if so, whether the changes increase school stratification in terms of race, socioeconomic status, and other student characteristics. The other characteristics are measures of student need (status as having limited English proficiency (LEP) or special education needs (SPED)) or student ability (status as academically or intellectually gifted (AIG)). Many stakeholders have expressed concerns that choice-based assignment systems will worsen school stratification and deepen imbalances of resources across schools. We find no evidence to support these concerns. In contrast, the reassignment programs we study reduce stratification in terms of race, socioeconomic status, and student need/ability.

The types of school choice programs we study involve students moving between public schools in a structured way within constraints established by the school district. As such, our results may not speak directly to all types of school choice programs, such as charter schools to which any student can apply and oversubscribed schools admit students by a lottery. That said, these types of school

choice programs are common. We analyze reassignments where students move to a magnet school or transfer to another traditional public school by requesting reassignment by the district. The transfer programs include general transfers to any other school in the district and calendar transfers that request reassignment to a school of a different calendar (traditional calendar versus year-round calendar). Transfer programs are very common; see Appendix A.2 for a list of many other districts with programs like the ones we study. Our contribution is to study the effects of several types of school choice programs on stratification. Further, we study stratification in several dimensions to provide a comprehensive picture of the compositional changes induced by reassignments.

We measure stratification by comparing the concentration of students of a particular characteristic among the students who attend the school relative to the concentration among those who were originally assigned via the school's attendance boundary. We refer to students in the attendance boundary as assigned students. Students who change their assignment via a reassignment program are called reassigned students. At their originally assigned school, reassigned students are called leavers. Likewise, students who do not change their assignment are called stayers. We decompose the net compositional change into two parts: sending schools and receiving schools. The sending analysis compares the concentration among stayers to the concentration among all originally assigned students (i.e., stayers plus leavers). The receiving analysis compares the concentration among students who attend the school (i.e., stayers plus reassigned students) to the concentration among stayers.

This decomposition provides insights into the mechanisms driving any observed changes in school composition. Specifically, we ask whether a particular characteristic becomes more concentrated at a school because leavers are disproportionately dissimilar (i.e., stayers are "left behind") or whether the characteristic becomes more concentrated because reassigned students are disproportionately similar (i.e., reassigned students reinforce existing concentration). The analysis separately considers schools with high or low levels of a given characteristic before any reassignments, which allows us to measure the extent to which reassignment policies increase or decrease stratification.

Racial stratification has been a long-standing focus in educational policy, as is stratification in terms of socioeconomic status. We are also interested in stratification along dimensions of student need or ability. In particular, we ask if students with LEP or SPED status are concentrated at certain schools. The question is whether resources are being utilized in a balanced way across schools. In this vein, we look for concentrations of students with AIG status. While concentrations of AIG students imply a different type of imbalance than of LEP or SPED students, imbalances across schools are important to measure and analyze irrespective of the form they take.

School assignment policies affect nearly every aspect of the education system. Consider just two aspects of this complex set of tradeoffs. First, districts seek to strike balance by weighing the district's and parents' transportation costs. Second, districts must also weigh students' and parents' preferences over schools for an understanding of the effect of an assignment policy on student achievement. Studying stratification speaks to both of these factors. First, stratification can be the result of assignment policies of neighborhood schools, which prioritize minimizing

transportation costs. Second, stratification has been argued to negatively affect achievement; among proponents of this view, student achievement increases when racial stratification is reduced and when resources across schools are more efficiently balanced. Our analysis provides crucial insights into the complexities of assignment in the context of reassignment programs. Further, our evidence speaks directly to the stated goals of many reassignment programs. For example, the district we study says that their aims are to promote diverse populations and balance the use of school facilities.

Our empirical setting is the Wake County Public School System (WCPSS), which is the 15th largest school district in the United States (WCPSS, 2018) with more than 160,000 students. Our data are from the 2016-2017 academic year. More than 75% of students in WCPSS attend a school based on the school attendance boundary associated with their residential address, which we refer to as the base school. The remaining students attend schools of choice, and there are three key reassignment programs in which students and parents participate: magnet, calendar transfer, and hardship transfer. We do not study charter schools because they are not included in the data. We have detailed information on every student's base school, their participation in any type of reassignment program within the district, the school they attend, and how they were assigned to the school they attend (base, magnet, calendar transfer, or hardship transfer). The data contain student demographic information and measures of student need and ability, but we do not have information on students' free-lunch status.[1] To overcome this data limitation, we collect data at the census-tract level to measure neighborhood-level socioeconomic status of students' residential area.

While school choice has been increasing in recent decades, it is still the case that the majority of students attend their base school. First, most students attend public schools (87.5% of students in 2015-2016 according to data from the National Center for Education Statistics (NCES)).[2] Second, most public school students attend their base school (78.6% of public school students attend their base school and the remaining 21.4% of public school students attend a school of choice). Third, students who attend a public school of choice are relatively dispersed among various types of reassignment programs, including charter schools, magnet schools, and transfer reassignment programs (among public school students, 5.8% of students attend a charter school and 5.3% attend a magnet school).[3] In total, these figures suggest that schools' attendance boundaries are the major source of school assignments, yet reassignment programs such as magnet assignment play important roles as well. As a result, our work has important implications for assignment policies.[4]

[1] WCPSS does not allow student-level data on free-lunch status to be used for research purposes.

[2] These are 2015-2016 figures taken from the Parent Survey and Parent and Family Involvement in Education Survey of the National Household Education Surveys Program (`https://nces.ed.gov/programs/digest/d17/tables/dt17_206.20.asp`). 87.5% of students in 2015-2016 attended some type of public school, while 9.2% attended a private school and 3.3% were homeschooled.

[3] Data on public schools in 2015-2016 are from the Public Elementary/Secondary School Universe Survey (`https://nces.ed.gov/programs/digest/d17/tables/dt17_216.20.asp`). 49,313,134 students attended a public school, with 2,845,322 of those attending a charter school and 2,604,145 attending a magnet school.

[4] Transfer programs are commonly available in school districts across the country. They are arguably understudied, which highlights the importance of our inclusion of transfers along with the more commonly discussed magnet programs. Two papers on transfer programs are Rich and Jennings (2015) (who study Chicago) and Phillips, Larsen, and Hausman (2015) (who do not name the district they study). We are not aware of NCES data on the use of transfer reassignment

Next we lay out a theoretical framework and discuss the related literature. Section 2.5 discusses our data and the Wake County Public School System that we study. Section 1.4 presents our methodology, where the results from our data are in Section 2.4. To place our results in context, Section 1.6 presents simulation evidence for counterfactual reassignments of students. In particular, we analyze the effects on stratification of random movements of students between schools and the largest changes in stratification that are possible given the volume of reassignments we observe. The results we observe in Section 2.4 are relatively similar to what might be expected under random movement of students (i.e., movement between schools that is uncorrelated with the racial or other makeup of schools). Finally, the results are meaningfully different from either of the maximal reductions or maximal increases in stratification counterfactuals, where what was observed in WCPSS appears to be a relatively small change in stratification relative to the bounds of what is possible in terms of changes in stratification.

1.2 Theory and Literature

1.2.1 Theoretical Considerations

Relative to assignment based on school attendance boundaries, school choice programs weaken the link between residential location and school assignment (Bifulco, Ladd, and Ross, 2009). School choice can be thought of in terms of an integration theory of choice, in which school choice can be a tool for students to pursue equity and equality of educational opportunity. In contrast, a market theory of choice emphasizes the role of competition and incentives to improve the public school system. The market theory of choice emphasizes giving families more control over student placement in schools. Concerned about the potential implications of a market theory of choice, many researchers (as detailed below) have raised concerns that choice-based assignment systems may intensify stratification.

There are three main arguments for how choice could adversely affect demographic or socioeconomic balance across schools. First, parents may prefer schools with peers with similar racial or socioeconomic backgrounds as their children (Schneider, Elacqua, and Buckley, 2006; Bifulco, Ladd, and Ross, 2009; Glazerman and Dotter, 2017; Whitehurst, 2017; Hastings, Kane, and Staiger, 2009). Second, the avoidance theory suggests that school choice would unintentionally promote stratification by making it easier for parents to avoid schools with a concentration of minority or disadvantaged students, known as "white flight" (Saporito, 2003; Sikkink and Emerson, 2008; Ladd, Clotfelter, and Holbein, 2017). Third, preferences for schools may differ significantly by parents' race, educational background, and socioeconomic status, which may increase stratification (Williams and Echols, 1992; Weiher and Tedin, 2002; Frankenberg, Siegel-Hawley, and Wang, 2011; Abdulkadiroglu,

programs, but Appendix A.2 provides a list of many other districts with transfer programs like WCPSS's hardship transfer program. Further, Bifulco, Ladd, and Ross (2009) provide information from Durham, North Carolina in 2002-2003. They find that 59.8% of students attend their base school, 13.3% of students attend a magnet school, 8.2% of students are reassigned via a calendar transfer, 13.3% of students are reassigned via a non-calendar transfer, and 5.5% of students attend a charter school.

Pathak, Schellenberg, and Walters, 2017; Hastings, Kane, and Staiger, 2009). Therefore, this paper examines how reassignment policies change the composition of schools.

Racial stratification has been the focus of the related literature. Racial stratification has two main harms, students' long-term development and their academic achievement. In terms of long-term development, Allport's intergroup contact theory argues that exposure to diverse peers is associated with reduced prejudice (Allport, 1954). Other long-term detrimental effects of racial stratification include lower probability of college enrollment (Cook, 2016; Billings, Deming, and Rockoff, 2013; Mickelson and Heath, 1999) and higher crime rates (Billings, Deming, and Rockoff, 2013). In terms of students' academic performance, there is considerable evidence that attending a more stratified school affects achievement (Mickelson and Heath, 1999; Mickelson, Bottia, and Lambert, 2013; Billings, Deming, and Rockoff, 2013; Gamoran and An, 2016). The harmful effects are larger for black students than for white students (Hanushek, Kain, and Rivkin, 2009).

Four main causal channels have been explored in the literature to explain the negative effect of racial stratification on student achievement. First, stratification decreases average peer achievement at some schools, which induces a negative peer effect (Cook, 2016). Second, stratification deteriorates the educational environment and limits resources to help student's learning (Mickelson and Heath, 1999). Third, an imbalanced racial distribution across schools leads to an imbalanced distribution of teacher quality. Stratified schools are more likely to have inexperienced and less effective teachers, thus lowering student achievement (Clotfelter, Ladd, and Vigdor, 2005, 2010, 2011; Jackson, 2009). Finally, stratification leads to more teacher turnover, which causes a less stable learning environment (Clotfelter, Ladd, and Vigdor, 2005).[5]

Beyond race and socioeconomic status, concentrations along other dimensions are also correlated with school resources, thus affecting school quality. However, relative to the large literature on racial stratification, there is much less work on stratification in terms of student needs or abilities. Therefore, we include three additional student characteristics measuring two dimensions of student need (limited English proficiency (LEP) and special education status (SPED)) and one dimension of student ability (academically or intellectually gifted (AIG) identification) to provide a comprehensive picture of the compositional changes induced by reassignments. All three characteristics are correlated with student socioeconomic status. For example, there is evidence of significant disparities in resources, opportunities to learn, and attainment that disadvantage LEP students relative to their non-LEP peers (Sullivan, 2011; de Cohen, Deterding, and Clewell, 2005). Exploring how school choice policies affect concentrations of students by needs or abilities is also important because programs accommodating LEP, SPED, or AIG students may require additional resources (de Cohen, Deterding, and Clewell, 2005; Mickelson, Bottia, and Southworth, 2008).

[5]There is also evidence that exposure to high-aptitude African American students has positive benefits for white students (Carrell, Hoekstra, and West, 2015). This says that stratified schools are harmful because they provide less exposure to a diverse set of peers.

1.2.2 Controlled Choice Programs

The school choice programs we study are controlled choice programs because the school district plays an important role in determining assignments through the choice program, albeit a different role than it plays when deciding school attendance boundaries (Bifulco, Ladd, and Ross, 2009; Bifulco, Cobb, and Bell, 2009). Other types of school choice programs can be thought of as "uncontrolled" choice programs in the sense that the school district does not have the same ability to limit the effect of choice on the composition of schools. These uncontrolled choice programs include charter schools (Weiher and Tedin, 2002; Frankenberg, Siegel-Hawley, and Wang, 2011; Ladd, Clotfelter, and Holbein, 2017), voucher programs (Abdulkadiroglu, Pathak, and Walters, 2015; Egalite, Mills, and Wolf, 2017), and certain types of open enrollment (Özek, 2009; Tegeler and Hilton, 2017).

The difference between controlled and uncontrolled programs can be illustrated in the context of charter schools, where any student can apply and oversubscribed schools admit students by a lottery. This lottery-based assignment policy limits the school district's ability to address compositional changes induced by movements from traditional public schools to charters. Given that our focus is on programs that fall into the category we deem controlled choice programs, our findings do not necessarily speak to all types of choice programs. Despite this caveat, understanding the role of controlled choice programs on stratification is quite important. As detailed earlier, the types of programs we study are very common. We now discuss additional related papers that study the effect on desegregation from various types of school choice programs.

Magnet schools are one of the most common types of controlled choice programs (Goldring and Smrekar, 2000). Previous work evaluating magnet schools as a desegregation tool has used the racial composition of school-aged peers in the attendance zone as the benchmark to which the changes in racial composition in public schools are compared. The results are mixed. Saporito (2003) studies Philadelphia, Pennsylvania in 1990-1992 and finds that the district's magnet school programs increase racial stratification. Bifulco, Cobb, and Bell (2009) argue that the interdistrict magnet program in Connecticut from 2001-2007 reduces racial stratification. Bifulco, Ladd, and Ross (2009) find stratification-intensifying effects in their study of similar school choice programs as the ones examined in this paper (magnet schools, calendar transfers, and hardship transfers) in Durham, NC. Further, Steele and Eaton (1996) examine data on districts who were supported by the Magnet Schools Assistance Program; they find that around half (47%) of districts met their desegregation goals via their magnet program. Finally, Koedel, Betts, Rice, and Zau (2009) find that the magnet program in San Diego, California serves to integrate the district.

Beyond magnet schools, other types of school choice have also been studied in terms of desegregation, also with mixed results. Glazerman and Dotter (2017) study the unified public school lottery system in D.C. and find that the lottery increases racial stratification but reduces income stratification. Egalite, Mills, and Wolf (2017) find that the Louisiana Scholarship Program, a voucher program, reduces racial stratification.[6] Related to our study, there is a set of literature providing

[6]Carlson, Bell, Lenard, Cowen, and McEachin (2019) also examine how assignment policies affect racial stratification in WCPSS; during their study period between 2000 and 2010, students' socioeconomic status played a direct role in

evidence that magnet schools improve students' academic achievement (Gamoran, 1996; Bifulco, Cobb, and Bell, 2009; Gamoran and An, 2016).[7]

1.3 Empirical Setting

1.3.1 Wake County Public School System

The Wake County Public School System (WCPSS) is the largest school district in North Carolina and the 15th largest district in the United States (WCPSS, 2018) with more than 160,000 students. The school district covers both urban areas and rural areas, ranging from cities such as Raleigh (population of 431,746) and Cary (population of 151,088) to small towns such as Apex, Garner, and Zebulon. The student body in WCPSS is diverse: around half of students are non-Hispanic whites and roughly one-third of students receive free-or-reduced price lunch. As of 2016, there were 167 schools in WCPSS. Students are assigned to a base school and can apply to request reassignment to another school. The base assignment uses a non-choice-based optimization algorithm which WCPSS says "balances proximity to school, building utilization, anticipated growth, and impact of future base attendance areas."[8] The base-assignment algorithm "minimize[s] system-wide transportation costs" subject to constraints that include building capacities and demographic balance.[9] Though the district does not publicly explain the exact workings of its base-assignment algorithm, researchers have documented that the vast majority of students attend one of the closest schools; Carlson, Bell, Lenard, Cowen, and McEachin (2019) state that changes to school attendance boundaries to accommodate demographic balance goals typically affect "no more than 5%" of students per year (Carlson, Bell, Lenard, Cowen, and McEachin, 2019, p. 9).

Our focus is on the primary ways that students can apply for reassignment from their base school. As described in Section 1.2.2, the three programs we study are controlled choice programs in which the district partially constrains moves between schools such that the effect of reassignments on school composition is under district control to a certain extent. These three reassignment programs are explained in more detail now. The district has 40 magnet schools as of 2016, which are partially-choice-based assignment schools with specialized offerings that seek to attract applicants. Around half of magnet school seats are for students in the neighborhood of the school, and magnet schools typically locate in neighborhoods that historically had higher proportions of families of

assignments. They find virtually no effect on average racial segregation but a reduction in extremely segregated schools. Our work provides evidence from a more recent period using assignment policies that do not directly consider student-level data on socioeconomic status.

[7] Beyond the school choice programs that are our focus (magnet schools and transfers), many papers study other types of choice, some of which find benefits in terms of academic achievement (Hoxby, 2000; Hastings and Weinstein, 2008; Deming, Hastings, Kane, and Staiger, 2014; Abdulkadiroğlu, Angrist, and Pathak, 2014), and some find little evidence choice increase student performance (Abdulkadiroglu, Pathak, Schellenberg, and Walters, 2017).

[8] See `https://www.wcpss.net/enrollmentproposal`.

[9] See `https://itre.ncsu.edu/focus/school-planning/ored/ipsac/`. Neither the district nor the company who implements its base-assignment algorithm (the Institute for Transportation Research and Education) specify the exact weight on demographic balance in the algorithm. See 1.3.3 for a brief discussion of the history of assignment in WCPSS.

lower socioeconomic status and/or racial minorities. The remaining half of seats are for choice-based assignment of magnet students. The objective of WCPSS's magnet schools is to "reduce high concentrations of poverty, and promote diverse populations, and maximize use of school facilities, and provide innovative and expanded educational opportunities."[10] Magnet seats are assigned using students' submitted lists of preferences over schools and students' priority points.

The construction of priority points in WCPSS is based on the district's *Four Pillars*: student achievement, stability, proximity, and operational efficiency. For elementary schools, priority points depend on whether the student's sibling will attend the school next year (highest priority), whether the student lives in a high-performing neighborhood based on historical test-score data (second highest), and whether the student's base school is overcrowded (third highest). For middle and high schools, priority points depend on siblings (highest priority), feeder-school pathway (second highest), high-performing neighborhood (third highest), and overcrowded base (fourth highest). The magnet program is a controlled choice program because the district sets priority points to prioritize more affluent families in magnet assignment, which diversifies magnet schools because students who are assigned to magnet schools as their base school tend to be less affluent.

There are two transfer programs, the first of which is a calendar transfer, which involves an application for reassignment to a year-round calendar school from a student whose base school has a traditional calendar or vice versa. A traditional calendar has a summer break of roughly three months; a year-round calendar has the same number of instructional days but shorter instructional periods with shorter intervening breaks. The calendar transfer process allows students to choose from a predefined list of one or two schools with the opposite calendar of their base school. Assignment is based on the available capacity at the requested school and students' priority points. Calendar transfer priorities are distinct from magnet priorities, where the former emphasize calendar alignment of the student with his or her siblings and the student's elementary school and secondary schools. The calendar transfer program is a controlled choice program because the district can choose which school is a given neighborhood's calendar option to limit transfers that would worsen stratification.

Finally, a hardship transfer involves an application for reassignment to any other school in the district. Hardship transfers are intended for students who demonstrate difficulty in attending their base school. Students can apply for up to three schools on their transfer application. Assignment is based on the available capacity at the requested school and students' stated reasons for the hardship. Stated reasons vary widely, with examples including transportation reasons (e.g., working long hours at a location close to the requested school) or reasons specific to the base school (e.g., perceived failure of the base school to accommodate the student's needs or abilities in some dimension). See Appendix A.2 for a list of many other districts with transfer programs like WCPSS's hardship transfer program. The hardship transfer program is a controlled choice program because the district can deny transfers that would worsen stratification.

For context on the reassignment process, consider a rising kindergarten student. For the aca-

[10] See `https://www.wcpss.net/domain/90` for more detail.

demic year that begins in August, the student's parents register starting in January by physically visiting her base school.[11] During January and February, her parents may choose to apply to attend a magnet school by submitting an online application with a rank-ordered list of preferences over magnet schools. If the magnet application is successful, she is assigned to a magnet school, and she "loses" her base assignment in the sense that she has to apply for a transfer to be reassigned to her base school if they change their mind. After magnet results are released in February, the transfer process opens and her parents can apply for a transfer to any other school in the district (hardship transfer) or to a school from a predefined list of one or two schools with the opposite calendar of their base school (calendar transfer). Transfer results are released in March. There is a subsequent appeal process during which students and parents attend an appeal hearing and support their transfer request with additional information regarding their child's fit at the requested school or unsuitability at the base school. We have detailed information on every students' base school, their participation in these reassignment programs, the school they attend, and how they were assigned to the school they attend (base, magnet, calendar transfer, or hardship transfer).[12]

1.3.2 Data Measuring School Stratification

School choice programs may intensify school stratification as measured several dimensions. First, we study stratification in terms of students' race/ethnicity (African Americans and white students). Second, to study stratification in terms of students' socioeconomic status, we use the census-tract level data associated with students' residential address to measure the census-tract level median household income.[13] We identify students as living in a high-income neighborhood if their census-tract level median household income is above the Wake County median household income.[14] Third, we ask whether school choice affects the district's ability to efficiently use school facilities and balance the accommodation of students with particular needs or abilities across schools. The needs/abilities of interest are status as limited English proficiency (LEP), special education (SPED), or academically or intellectually gifted (AIG).

There were 164,959 students registered in WCPSS as of February 2015 across 167 schools, including 108 elementary schools, 35 middle schools, and 24 high schools. There are five early colleges (two middle schools and three high schools) that follow a different admission process.[15] Excluding early colleges from our analysis leaves 162 public schools in total. Throughout, we restrict to

[11] The exact timing of each application process changes year-to-year, and we describe the process as of 2016-2017. For details on the most recent year, see `https://www.wcpss.net/Page/33766`.

[12] If a student was assigned because of a successful appeal, we only observe how she initially requested the school such that, for example, a student whose hardship transfer was rejected but was assigned there on appeal will appear in the data as a hardship transfer.

[13] We also consider the percentage of the population with a bachelor's degree or higher in the tract. See Table A.10.

[14] The drawback of our socioeconomic status analysis is that we do not have a measure of a student's socioeconomic status, only data at the census-tract level. For the other characteristics we study, we look at stratification in terms of the characteristic itself at the student level. But for socioeconomic status, we look at stratification in terms of concentrations of students from high or low income neighborhoods.

[15] We exclude early colleges from our analysis of reassignments because the early college application process involves exam-based and essay-based priorities that are very different from the criteria that determine magnet priorities. See Lauen, Fuller, Barrett, and Janda (2017) for work that focuses on early colleges in North Carolina.

grade-entry students who are rising into kindergarten, sixth, or ninth grade, because the majority of reassignments occur at grade-entry years. Our data are linked at the student level from the 2015-16 school year to the 2016-17 school year, excluding students who exited after 2015-2016 or entered for 2016-2017. The resulting sample consists of 25,482 grade-entry students as shown in Table 1.1. Aggregating across all three reassignment programs, 6,219 students, or 24.4% of students, changed their assignment in these data. This large volume of choice-based assignment provides a large scope for reassignments to affect stratification. Further, as shown in Panel C of Table 1.1, schools are very heterogeneous within the district; for example, the proportion African American at the average school is 18.9%, but splitting schools into those below- and above-average for African Americans divides schools into groups with averages of 9.1% and 35.1%, respectively. Finally, as shown in Panel D of Table 1.1, there is broad use of each reassignment program (magnet (Mag), calendar transfer (CT), and hardship transfer (HT)) by students of each characteristic we study.[16]

1.3.3 Evolution of WCPSS Assignment Policies

The district of interest has been studied numerous times in the related literature. For example, Ayscue, Siegel-Hawley, Kucsera, and Woodward (2018) study WCPSS along with the second largest district in North Carolina, Charlotte-Mecklenburg. They find that both districts saw a "growing share of intensely segregated schools" over the period from 1989-2010. The contribution of our study is to leverage rich data on base assignments and eventual assignments to precisely measure the effect of reassignments on stratification.[17]

We now briefly describe how assignment policies in WCPSS have evolved. This discussion is drawn from several sources that provide more detail on the history of WCPSS (Silberman, 2002; Finder, 2005; Garcia, 2012; Parcel and Taylor, 2015; McDermott, Frankenberg, and Diem, 2015; Ayscue, Siegel-Hawley, Kucsera, and Woodward, 2018; Carlson, Bell, Lenard, Cowen, and McEachin, 2019). The district's assignment policies can be thought of falling into three periods: pre-2000, 2000-2010, and post-2010. The division between periods in 2000 marks the district's shift from race-based assignments to socioeconomic status-based assignments (Finder, 2005). This was a preemptive move to ward off concerns regarding the eventual end of the use of students' race in assignments. The division between periods in 2010 marks the election of a new majority of the Wake County Board of Education (Parcel and Taylor, 2015). The new school board implemented changes that led to a sharp move away from socioeconomic status-based assignments (Ayscue, Siegel-Hawley, Kucsera, and Woodward, 2018). Those changes culminated in a new assignment plan for the 2012-2013 school year that increased school choice options, but this plan was implemented for only one year

[16] Appendix Table A.11 presents a regression analysis of the determinants of the use of each reassignment program as well as being reassigned by any program. To mention one interesting result, relative to non-Hispanic students of races other than African American or white, white students are less likely to use the magnet program to be reassigned and more likely to use the transfer programs.

[17] Monarrez (2018) studies the effect of school attendance boundaries on racial stratification. According to his results, the attendance boundaries in WCPSS have a stratification-reducing effect relative to a counterfactual in which all students attend the nearest school. This is not true for most large US districts or any of the largest districts in North Carolina other than WCPSS.

(Parcel and Taylor, 2015).

Assignment policies in WCPSS since 2013 have neither reinstituted the socioeconomic status-based assignments of 2000-2010 nor deemphasized diversity as of 2012-2013 (Parcel and Taylor, 2015). Since the choice plan assignment of 2012-2013, the district's stated assignment policy continues to list demographic balance as a consideration in its assignments. However, it has been argued that demographic balance now plays less of a role in assignment in WCPSS since 2013 (Parcel and Taylor, 2015).[18] Parcel and Taylor (2015) state that: "By November 2013, it was therefore clear that the board would not return to precisely the same diversity policy" (Parcel and Taylor, 2015, 111).[19]

1.4 Methodology

The analysis considers each reassignment program separately. We look at each school that had at least one student leave or enter the school through the reassignment program. For each reassignment, the base school is called the sending school, and the new school is called the receiving school. For magnet applications, the sending schools could include all schools, but in these years we observe that 149 of the 162 schools "lost" any students through magnet assignment. Likewise, transfers affect most but not all schools, with calendar transfers affecting only 122 sending schools. Receiving schools are those to which students are reassigned via each program.

Students are classified as assigned-stayers (or stayers) if they attend their base school. Otherwise, the student is called an assigned-leaver (or leaver) when studying their originally assigned school and a reassigned student at their new school. The analysis compares school-level characteristics before and after reassignments. We measure stratification by comparing the concentration of students of a particular characteristic among the students who attend the school relative to the concentration among those who were originally assigned there; this is done separately for sending and receiving schools. The sending analysis compares the concentration among stayers to the concentration among all originally assigned students (i.e., stayers plus leavers). The receiving analysis compares the concentration among students who attend the school (i.e., stayers plus reassigned students) to the concentration among stayers.

We use a benchmark for the concentration of students to represent the proportion of the characteristic at each school if students were perfectly evenly distributed across schools. This benchmark composition is the average of the characteristic in the district, which separately considers schools

[18] As stated earlier, as of the time of our data from 2016-2017, the vast majority of students attend one of the closest schools, with Carlson, Bell, Lenard, Cowen, and McEachin (2019) stating that changes to school attendance boundaries to accommodate demographic balance goals typically affect "no more than 5%" of students per year (Carlson, Bell, Lenard, Cowen, and McEachin, 2019, p. 9).

[19] The education reporter for the largest local newspaper stated in 2017 that: "Wake buses fewer students for diversity now" (Hui, 2017).

with high or low levels of a given characteristic before any reassignments.[20,21,22] As explained earlier, we look at stratification in terms of race (African Americans and white students), socioeconomic status, and student need/ability (LEP, SPED, and AIG).[23,24,25] A reassignment program will increase stratification if a school with an above-average proportion of African American students, for example, has a larger proportion of African American students after reassignments. Thus stratification tells us whether reassignments move school composition closer to the average level.

Before presenting the results, we discuss an example to clarify a key component of the analysis, which is how we isolate the effect of each reassignment program separately. Consider a school with 100 students who are assigned there as their base school. Further suppose that 10 students left the school via each reassignment program (30 in total) and 6 students are reassigned to the school via each reassignment program (18 in total). Altogether, the school starts with 100 base students and ends with 88 (=100-30+18) students who attend. The analysis of magnet reassignments for this school excludes the students who used the calendar and hardship transfers programs (and likewise for the other analyses). The sending analysis for this school compares the base students excluding calendar-leavers and hardship-leavers (80 students) to the remaining of them after excluding magnet-leavers (70 students). The receiving analysis compares the students who attend the school excluding all reassigned students (70 students) to the students who attend the school excluding calendar-reassigned and hardship-reassigned (76 students). With this approach, we can isolate the effect of each reassignment program separately.

1.5 Observed Changes in Stratification: Empirical Evidence

The results are in Tables 1.2 to 1.7. In each table, schools are separated according to whether the characteristic of interest is underrepresented (Below Average) or overrepresented (Above Average). The three reassignment programs are magnet (Mag), calendar transfer (CT), and hardship transfer (HT). This is done separately for sending schools (S), receiving schools (R), and the net impact (N). The net effect across all three programs are shown in the final row of each table, where the net effect is calculated by comparing all base students to all students who attend the school, irrespective of the

[20] In Appendix Table A.3, the median is used rather than the mean. The two sets of results are very similar.

[21] Other benchmarks could be used to groups schools, such as the averages for the entire metropolitan area rather than only those students enrolled in the public school system. This might matter if the composition of the public school system is not representative of the area as a whole. Results using a benchmark drawn from the entire metropolitan area are very similar; see Appendix Table A.4.

[22] In Appendix Table A.5, we look at stratification in the extremes of the distribution, rather than top versus bottom half. In particular, we look at the proportion of each characteristic among schools in the top and bottom deciles. The results are very similar to those from the main analysis.

[23] Students classified as having limited English proficiency are identified as speaking a language other than English at home (self-identified) and in need of additional support (self- and staff-identified). In these data, many students listed as LEP are Hispanic students (approximately 70% of Hispanic students are currently or have ever been limited English proficient). This results in a large overlap between the results for Hispanic stratification and stratification by LEP status; see Appendix Table A.9.

[24] For more detail on special education in WCPSS, see `https://www.wcpss.net/special-education`.

[25] Academically or intellectually gifted students are identified via Cognitive Abilities Test scores that exceed 85%; other factors, including classroom performance/growth, can be part of the academically gifted identification process. Students can be formally AIG identified starting in fourth grade.

reassignment program used by reassigned students. The cells in these tables show the average proportion of the characteristic before reassignments (Pre), the change from reassignments (Change), and the statistical test results (t-test). The t-tests ask whether the observed change is statistically significantly different from zero, where "↗" indicates that reassignments increase the concentration of the characteristic on average, "↘" that reassignments decrease the concentration, and "·" that the change is statistically insignificantly different from zero at a significance level of 10%.[26]

We discuss the results by first focusing on the net impact of each reassignment program on racial stratification.[27] The magnet program in WCPSS tends to reduce racial isolation: fewer African American at schools where they were overrepresented (from 35% to 32%) and more white students at schools where they were underrepresented (from 28% to 31%). Changes for African Americans at schools with few African Americans and for white students with many whites also move in the direction of less stratification, but these changes are statistically insignificant. Given that magnet programs historically existed to address racial segregation, our results for magnet schools may seem unsurprising. However, the magnet program in WCPSS has evolved to incorporate additional goals and a more heterogeneous set of schools; this is evident in the characteristics of the receiving schools shown in Tables 1.2 and 1.3, where before reassignments, the average magnet school with few African Americans is only 14% African American and likewise the average magnet school with many white students has 55% whites.

The two transfer programs also reduce racial isolation, and the two sets of results are similar between calendar and hardship transfers. These programs tend to reduce the number of white students at heavily white schools (by two or three percentage points) and increase the number of African Americans at schools with few African Americans (by roughly two percentage points). In total, the net effect of the three reassignment programs (seen in the final rows in Tables 1.2 and 1.3) is a large reduction in the gap between the average school with few of a given race relative to the average school with many. For African Americans, the gap between underrepresented and overrepresented schools goes from 26 percentage points (9% to 35%) to 20 percentage points (11% to 31%). For white students, the gap between underrepresented and overrepresented schools goes from 36 percentage points (33% to 69%) to 29 percentage points (36% to 65%). Overall, there is no evidence that these programs worsen racial stratification; instead the opposite is true.

In terms of socioeconomic stratification, the results are very similar to those from race.[28] In

[26] An alternative analysis is to test whether the proportion of schools where the characteristic becomes more concentrated is statistically significantly larger than the proportion where the characteristic becomes less concentrated. This test of proportions may give different results as illustrated in the following example: among schools with more white students than average, 60% might have fewer white students after reassignments; but among such schools that have more white students after reassignments, the change in proportion white might be very large. In such situations, the average change might move in a different direction than the modal change. Our use of t-tests will reflect the average change, where the alternative of proportions tests would reflect the modal change. Results using proportions tests do indeed reflect some differences, but the qualitative findings we report below are unchanged. These results are shown in Appendix Table A.2.

[27] Appendix Tables A.6, A.7, and A.8 present the results for elementary, middle, and high schools separately. There are fewer statistically significant changes in the high school data, but the general patterns hold across all three levels.

[28] As mentioned in footnote 14, our socioeconomic status analysis is limited by the use of neighborhood-level data, rather than data on each student. Despite this limitation, the analysis produces similar results with student-level data on race, student need, and student ability (all measured at the student level) as with socioeconomic status (measured at the

particular, Tables 1.3 and 1.4 are strikingly similar in qualitative terms. In terms of quantitative magnitudes, the results from socioeconomic status are larger in percentage terms in that there are larger increases in the representation of students from high-income neighborhoods at schools with few such students, relative to white students at schools with few white students. Thus, the results we find for stratification by socioeconomic status provide stronger evidence of reduced stratification, relative to racial stratification. Recall that our socioeconomic status measures are at the neighborhood level, which provides less precise measures of the actual composition of schools compared to our racial measures.

In terms of stratification along dimensions of student need/ability, these programs also reduce stratification.[29] The magnet program reduces the concentration of LEP students at schools where they are overrepresented and has similar effects for both SPED and AIG students. It has a pronounced effect of increasing AIG presence at schools with few AIG students, but this is not statistically significant. Looking at the decomposed effects in Table 1.7, there is evidence that the magnet program has heterogeneous effects for AIG students. The likely explanation is the priority structure of magnet assignment, where AIG students receive higher priority at some schools.[30]

The two transfer programs also appear to reduce stratification in terms of concentration of students with LEP, SPED, or AIG status. Hardship transfers do not appear to meaningfully affect schools that are overrepresented with students of these characteristics or schools that are underrepresented in terms of AIG students. This might reflect the fact that students do not apply for hardship transfers to schools with many LEP or SPED students or to schools with few AIG students. That said, calendar and hardship transfers do increase the concentration of LEP and SPED students are schools with few of them, and calendar transfers reduce the number of LEP and AIG students at schools where they are overrepresented. The overall conclusion is that these reassignment programs reduce stratification in terms of student need.

We now turn to the decomposed analysis that separately analyzes the movement of reassigned students away from their original school (sending analysis) to their new school (receiving analysis). As discussed earlier, this decomposition allows us to interpret the mechanisms through which the observed net effects occur, which is important for policy interpretations. For racial stratification, African Americans become slightly less concentrated at heavily African American schools after sending magnet-leavers out (from 34% to 33%), which might suggest that African Americans are being left behind by the magnet program. However, this conclusion is changed once we observe the small negative change in the proportion of African Americans at schools with few of them (a small change that rounds to 0). Likewise, the proportion of white students goes down after sending magnet-leavers out, irrespective of whether they were initially underrepresented or overrepresented. Taken together, we conclude that the magnet program is reassigning students from a heterogeneous

neighborhood level).

[29] In WCPSS, students can be formally AIG identified starting in fourth grade. Our analysis uses only grade-entry students (kindergarten, sixth, and ninth grade), so our AIG analysis uses only middle and high school students.

[30] There are three middle schools at which AIG identified students receive higher priority for magnet assignment. The magnet priority structure is discussed in detail in Dur, Hammond, and Morrill (2018a).

set of schools in terms of racial composition, rather than magnet-leavers only leaving schools with heavily minority populations or only leaving schools where the leaver is underrepresented. For transfers, the proportion of white students tends to go down after removing leavers, more so for hardship transfers. It appears that these reassignment programs are generating a lot of movement in students and that whites are somewhat more likely to be reassigned.

The receiving analysis for race suggests that the racial gap between underrepresented and over-represented schools narrows because African Americans become less concentration after bringing in magnet-reassigned students at heavily African American schools (from 47% to 40%); this goes hand-in-hand with the result for whites at schools with few whites. The various components of the decomposed analysis send mixed signals in Table 1.3, with some evidence of stratification-increasing effects for whites in schools with few whites for the transfer programs. However, the net effect across all three programs appears to be dominated by the magnet reassignments, and the net effect is a reduction in stratification. For conciseness, we refrain from walking through each additional result from the decomposed analysis. The overall findings are that stratification is generally reduced. The decomposition is mainly useful in showing two points: (1) the effects on sending schools are generally dominated by the effects on receiving schools, and (2) the heterogeneous set of schools to which students are reassigned imply that reassignments are not harmful with respect to stratification.

1.6 Bounds on Changes in Stratification: Simulation Evidence

Our empirical results suggest that the reassignment programs we study reduce stratification in terms of race, socioeconomic status, and student need/ability. But the magnitudes of these reductions are hard to interpret without some auxiliary evidence on what changes in stratification are possible. In this section, we compare the observed changes in stratification to three counterfactuals: (1) random movement of students across schools, (2) maximal reductions in stratification, and (3) maximal increases in stratification. The comparison to random movement is useful as a benchmark for when students move between schools with no consideration for the racial makeup or other characteristics of the sending or receiving schools. The comparison to maximal reductions or increases in stratification is useful for bounds on what is possible.

The simulations consider counterfactual moves of students between schools, separately for each characteristic for each reassignment program. Each student of a given characteristic who moves under a given program is moved to a new school according to the simulation environment in question. For the random environment, leavers are reassigned to a randomly chosen school. For the stratification-decreasing environment, leavers are reassigned to a randomly chosen school where they are underrepresented (i.e., a school that has below-average percent of the characteristic).[31]

[31] For an example of the stratification-decreasing environment, an African American leaver is randomly reassigned to a school that is below average for percent African American, irrespective of the percent African American of his or her original school. For an African American who leaves a school that is below average for percent African American, moving them to another school that is below average for percent African American reduces stratification at the new school. For an African American who leaves a school that is above average for percent African American, moving them to school that is below average for percent African American reduces stratification at both schools. In this example, if the African

For the stratification-increasing environment, leavers are reassigned to a randomly chosen school where they are overrepresented (i.e., a school that has above-average percent of the characteristic). Each environment for each characteristic for each reassignment program uses 1,000 simulation runs.

The results that compare the observed changes to random movements are in Table 1.8 and to maximal changes in stratification are in Table 1.9. The random movements in Table 1.8 are presented as the average change in the percentage across all schools in the group (above average or below average), along with the standard deviation of the changes across schools in a given simulation run. These standard deviations across schools are very large, which reflects the fact that random movements of students can generate wide swings in school composition. The average changes from random movements are small relative to these standard deviations, and the observed changes are relatively similar to the average changes. Overall, we do not see much evidence that the observed changes in stratification are systematically different from what might be seen in students moved between schools without regard to school composition.

The results in Table 1.9 can be thought of as bounds on what changes in stratification are possible. The maximal reductions in stratification are from counterfactuals in which all moves of students between schools are integrative, while the maximal increases in stratification are from counterfactuals in which all moves are disintegrative.[32] Table 1.9 shows that very large changes in stratification can occur, while the observed changes in stratification appear relatively small by comparison. The first implication is that these reassignment programs do not increase stratification, while large increases are possible; for an extreme example, calendar transfers could reduce the percent white at schools with few whites by 8.3% and increase the percent white at schools with many whites by 5.0%. The second implication is that, while these programs reduce stratification, much larger reductions are possible if all moves were integrative. Given that these are controlled choice programs, the absence of larger reductions in stratification may be concerning. Even though these type of controlled choice programs allow the district some control over the effect of reassignments on school composition, the programs we study do not reduce stratification to a large extent.

1.7 Conclusion

School assignment policies matter greatly for the composition of schools in terms of racial and socioeconomic balance as well as the balance of student need/ability across schools. We study the

American student is originally assigned to a school that is below average for percent African American, then he or she is randomly reassigned to a school that is also below average for percent African American, excluding their original school.

[32] One could imagine alternative maximal counterfactuals in which students are moved systematically to the most extreme schools. In the stratification-decreasing environment, for example, white leavers can be systematically sent to the school with the fewest white students, then to the school with the second fewest, etc. In contrast, our approach for stratification-decreasing is to move students randomly to a school where they are underrepresented. Moving students systematically would result in larger changes in stratification, which calls into question our use of the term maximal. However, this alternative counterfactual approach is sensitive to how we specify the maximum number of leavers who can move to a given school, which is important because schools have capacity constraints that may or may not bind ex ante. Our approach avoids this issue, while still generating wide bounds in possible changes in stratification.

effect of reassignment programs in which students apply to change their school assignment. Given the input of students' and parents' preferences into these choice-based assignment systems, there has been concern that such reassignments will worsen school stratification and deepen imbalances of resources across schools. We find no evidence to support these concerns. The reassignment programs we study do not increase stratification in terms of race, socioeconomic status, or student need/ability, but they do not induce large reductions in stratification either.

The analysis divides schools into those that have an assigned population of students with relatively few or many students of a given characteristic, then we study composition changes from reassignments. For race, African American students and white students become less racially isolated after reassignments in most cases. There become fewer African Americans at schools with heavily African American student populations through the magnet program and more African Americans at schools with few African Americans through transfers. Similar stratification-decreasing effects are seen for white students, where again reassignments improve the racial balance across schools. The results for socioeconomic status are quite similar to those for racial, again suggesting less stratification by socioeconomic status. The characteristics of need or ability of interest are whether students have limited English proficiency (LEP), special education needs (SPED), or academically or intellectually gifted (AIG) status. This analysis speaks to the efficient allocation of resources across schools, where imbalance suggests that certain schools have a large share of students with certain needs or abilities. As with race and socioeconomic status, we find that stratification by need is reduced through reassignments.

As increasing numbers of students attend schools through choice-based assignment, more attention is now being given to students' and parents' preferences over peers. Concerns over stratification have played an important role in this debate. The reassignment programs that we study are quite different from the types of choice-based assignment systems that have been studied thus far. However, assignment through school attendance boundaries remains the most common method by which students are assigned to schools, and most school districts have reassignment programs (Monarrez, 2018). As a result, our analysis of reassignment programs is timely and relevant for the current policy debates. The reassignment programs we study reduce stratification in terms of race, socioeconomic status, and student need/ability, which suggest that the expansion of these type of controlled choice programs should not be expected to come at the cost of diverse schools with resources that are balanced across the school district.

Given the results we observe in the Wake County Public School System, we conduct a series of simulations to ask what changes in stratification are possible. The first counterfactual considers random movement of students without regard to characteristics (e.g., race) of the sending or receiving schools. The second and third counterfactuals provide bounds on compositional changes. These upper and lower bounds on changes in stratification come from environments in which all students who are reassigned move to a school where they are underrepresented (maximal decrease in stratification) or are overrepresented (maximal increase in stratification). The results suggest that the observed changes in stratification are consistent with random movement of students and are

meaningfully different from either maximal counterfactual. Thus, the reassignment programs we study do not increase stratification in terms of race, socioeconomic status, or student need/ability, but they also do not reduce stratification to a particularly large degree relative to what is possible.

Table 1.1 Number and Characteristics of Schools and Students

Panel A: Number of Schools			
	Sending	Receiving	All
Mag	149	40	156
CT	122	74	126
HT	151	157	162
All	158	162	162
Panel B: Number of Students			
	Leaver	Stayer	All
Mag	2,376	19,081	21,457
CT	2,262	11,840	14,102
HT	1,581	19,263	20,844
All	6,219	19,263	25,482
Panel C: Characteristics of Schools			
	Below-Average	Above-Average	All
AA	9.06%	35.12%	18.87%
White	32.75%	68.53%	51.31%
High Income	14.08%	80.15%	47.52%
LEP	12.78%	35.54%	22.62%
SPED	5.94%	14.78%	10.14%
AIG	16.23%	33.58%	25.53%

Panel D: Fraction of Leavers by Characteristic and Program				
	Mag	CT	HT	All
AA	11.58%	6.42%	6.57%	24.57%
White	8.42%	10.66%	7.06%	26.15%
High Income	7.98%	9.96%	5.42%	23.35%
LEP	7.18%	6.61%	3.98%	17.77%
SPED	5.31%	7.54%	6.44%	19.29%
AIG	16.81%	7.95%	4.50%	29.27%

Notes: These summary statistics are taken from our data covering the Wake County Public School System. The sample includes grade-entry students who are rising into kindergarten, sixth, or ninth grade. Panel A shows the count of schools involved in each reassignment program (magnet (Mag), calendar transfer (CT), or hardship transfer (HT)), which is the number of schools at which at nonzero number of students participated. Sending schools are the originally assigned school of reassigned students, and receiving schools are the new schools to which students are reassigned. Panel B shows the counts of students involved in each reassignment program. At their originally assigned school, reassigned students are called leavers. Students who do not change their assignment are called stayers. Panel C shows average proportion of each characteristic for all schools and separately for below- and above-average groups of schools that are split by the average proportion of the characteristic (African American (AA), white, limited English proficiency (LEP), special education needs (SPED), or academically or intellectually gifted (AIG)). Panel D shows the proportion of students of a given characteristic who are leavers who leave their base school by each reassignment program (Mag, CT, or HT) as well as the proportion who leave by any reassignment program (All).

Table 1.2 African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.09	−0.00	·	0.34	−0.01	↘
Mag-R	0.14	0.02	↗	0.47	−0.07	↘
Mag-N	0.09	0.00	·	0.35	−0.03	↘
CT-S	0.08	0.01	↗	0.31	0.01	·
CT-R	0.06	0.02	↗	0.29	−0.03	↘
CT-N	0.08	0.02	↗	0.30	−0.01	·
HT-S	0.08	0.00	·	0.33	−0.01	·
HT-R	0.08	0.02	↗	0.33	0.00	·
HT-N	0.08	0.02	↗	0.34	−0.00	·
Net	0.09	0.02	↗	0.35	−0.04	↘

Notes: Schools are separated according to whether African American students are underrepresented (Below Average) or overrepresented (Above Average). The three reassignment programs are magnet (Mag), calendar transfer (CT), and hardship transfer (HT). The analysis is done separately for sending schools (S), receiving schools (R), and the net impact (N). The net effect across all three programs are shown in the final row. The cells show the average proportion of African Americans before reassignments (Pre), the change from reassignments (Change), and the results from t-tests shown as "↗" (reassignments statistically significantly increase the concentration), "↘" (reassignments statistically significantly decrease the concentration), and "·" (change is statistically insignificantly different from zero). All tests use a significance level of 10%.

Table 1.3 White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.31	−0.02	↘	0.66	−0.01	·
Mag-R	0.11	0.15	↗	0.55	0.02	·
Mag-N	0.28	0.03	↗	0.65	−0.01	·
CT-S	0.31	−0.03	↘	0.69	−0.04	↘
CT-R	0.29	0.04	↗	0.68	0.00	·
CT-N	0.31	−0.01	·	0.68	−0.03	↘
HT-S	0.29	−0.03	↘	0.66	−0.02	↘
HT-R	0.28	0.01	↗	0.67	−0.00	·
HT-N	0.28	−0.01	·	0.66	−0.02	↘
Net	0.33	0.03	↗	0.69	−0.04	↘

Notes: See the notes to Table 1.2.

Table 1.4 Ratio of Students Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.15	−0.02	↘	0.80	0.01	↗
Mag-R	0.01	0.20	↗	0.44	0.04	·
Mag-N	0.13	0.06	↗	0.80	0.00	·
CT-S	0.17	−0.02	↘	0.83	−0.01	↘
CT-R	0.16	0.05	↗	0.87	−0.04	↘
CT-N	0.17	0.01	·	0.83	−0.04	↘
HT-S	0.16	−0.02	↘	0.81	0.00	·
HT-R	0.13	0.02	↗	0.82	−0.03	↘
HT-N	0.14	0.00	·	0.81	−0.02	↘
Net	0.14	0.08	↗	0.80	−0.05	↘

Notes: See the notes to Table 1.2.

Table 1.5 Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.14	0.01	↗	0.37	0.04	↗
Mag-R	0.22	−0.05	↘	0.59	−0.24	↘
Mag-N	0.15	0.01	↗	0.40	−0.03	↘
CT-S	0.13	0.02	↗	0.38	0.04	↗
CT-R	0.12	0.00	·	0.40	−0.04	↘
CT-N	0.13	0.02	↗	0.38	0.01	·
HT-S	0.13	0.01	↗	0.37	0.04	↗
HT-R	0.14	−0.00	·	0.43	−0.05	↘
HT-N	0.13	0.01	↗	0.39	−0.01	·
Net	0.13	0.03	↗	0.36	−0.05	↘

Notes: See the notes to Table 1.2.

Table 1.6 Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.06	0.01	↗	0.15	0.01	↗
Mag-R	0.04	0.02	↗	0.19	−0.06	↘
Mag-N	0.06	0.01	↗	0.15	−0.01	·
CT-S	0.07	0.01	↗	0.16	0.01	↗
CT-R	0.06	0.01	↗	0.18	−0.03	↘
CT-N	0.07	0.01	↗	0.16	−0.01	↘
HT-S	0.07	0.00	·	0.16	0.00	·
HT-R	0.06	0.01	↗	0.17	−0.01	↘
HT-N	0.06	0.01	↗	0.17	−0.00	·
Net	0.06	0.02	↗	0.15	−0.02	↘

Notes: See the notes to Table 1.2.

Table 1.7 Ratio of AIG Students

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.16	−0.03	↘	0.34	−0.03	↘
Mag-R	0.07	0.23	↗	0.22	0.04	·
Mag-N	0.16	0.04	·	0.34	−0.03	↘
CT-S	0.13	−0.01	·	0.32	−0.02	↘
CT-R	0.12	0.02	·	0.30	0.01	·
CT-N	0.13	0.01	·	0.32	−0.02	↘
HT-S	0.14	−0.01	↘	0.31	0.00	·
HT-R	0.13	0.01	↗	0.30	−0.00	·
HT-N	0.13	−0.00	·	0.30	−0.00	·
Net	0.16	0.06	↗	0.34	−0.04	↘

Notes: See the notes to Table 1.2.

Table 1.8 Comparison of Actual Changes and Random Movement Simulations

		Below Average		Above Average	
		Actual	Random	Actual	Random
Mag	AA	0.38%	−0.00% (3.10%)	−2.81%	−2.25% (7.17%)
	White	2.84%	−1.28% (9.16%)	−1.06%	−0.82% (5.89%)
	High Income	6.26%	0.78% (10.58%)	0.47%	0.15% (5.16%)
	LEP	0.59%	1.05% (3.61%)	−2.86%	2.59% (10.00%)
	SPED	1.12%	0.79% (1.69%)	−0.53%	0.92% (4.02%)
	AIG	4.23%	−1.64% (10.27%)	−2.78%	−2.92% (3.75%)
CT	AA	1.55%	1.22% (4.15%)	−0.66%	−0.76% (7.66%)
	White	−0.89%	1.27% (10.15%)	−3.44%	−4.15% (9.78%)
	High Income	0.63%	4.09% (15.59%)	−3.51%	−3.75% (6.74%)
	LEP	2.03%	1.70% (5.78%)	1.24%	−0.15% (9.95%)
	SPED	1.00%	0.58% (2.46%)	−1.12%	−0.77% (5.12%)
	AIG	1.09%	0.71% (4.13%)	−1.60%	−2.20% (2.85%)
HT	AA	1.64%	0.16% (2.14%)	−0.10%	−0.85% (4.47%)
	White	−0.98%	−1.64% (6.83%)	−1.83%	−1.73% (5.04%)
	High Income	0.33%	−0.48% (5.67%)	−2.33%	−0.06% (4.59%)
	LEP	0.83%	0.96% (3.62%)	−0.64%	2.82% (6.48%)
	SPED	0.55%	0.05% (1.55%)	−0.25%	−0.18% (3.35%)
	AIG	−0.13%	0.40% (3.38%)	−0.05%	−0.22% (1.04%)

Notes: The results from random movements of students between schools are shown as the average change in the percentage of the characteristic, along with the standard deviation of the changes across schools in a given simulation run. In these simulations, leavers are reassigned to a randomly chosen school. The simulations use 1,000 runs.

Table 1.9 Comparison of Actual Changes and Bounds from Maximal Changes in Stratification

		Below Average			Above Average		
		Actual	Reducing	Increasing	Actual	Reducing	Increasing
Mag	AA	0.38%	5.23%	−1.16%	−2.81%	−9.06%	6.13%
	White	2.84%	12.13%	−5.36%	−1.06%	−6.25%	3.51%
	High Income	6.26%	15.72%	−1.56%	0.47%	−2.55%	1.98%
	LEP	0.59%	5.29%	−0.99%	−2.86%	−8.14%	8.93%
	SPED	1.12%	4.01%	0.03%	−0.53%	−1.51%	3.57%
	AIG	4.23%	11.16%	−4.67%	−2.78%	−4.84%	0.68%
CT	AA	1.55%	6.60%	−1.76%	−0.66%	−7.43%	7.15%
	White	−0.89%	12.32%	−8.31%	−3.44%	−13.71%	4.98%
	High Income	0.63%	19.43%	−5.06%	−3.51%	−12.70%	2.20%
	LEP	2.03%	9.23%	−2.23%	1.24%	−7.29%	8.69%
	SPED	1.00%	4.54%	−1.19%	−1.12%	−4.39%	5.47%
	AIG	1.09%	8.94%	−3.11%	−1.60%	−6.72%	4.65%
HT	AA	1.64%	6.20%	−1.50%	−0.10%	−9.36%	5.79%
	White	−0.98%	14.01%	−6.42%	−1.83%	−8.63%	4.22%
	High Income	0.33%	14.63%	−3.19%	−2.33%	−10.15%	2.62%
	LEP	0.83%	4.76%	−1.47%	−0.64%	−9.91%	7.01%
	SPED	0.55%	4.15%	−1.25%	−0.25%	−4.54%	4.70%
	AIG	−0.13%	4.74%	−2.25%	−0.05%	−2.33%	2.35%

Notes: The results from maximal changes in stratification are shown as the maximal change in the percentage of the characteristic in the simulation run. For the stratification-decreasing environment, leavers are reassigned to a randomly chosen school where they are underrepresented (i.e., a school that has below-average percent of the characteristic). For the stratification-increasing environment, leavers are reassigned to a randomly chosen school where they are overrepresented (i.e., a school that has above-average percent of the characteristic). The simulations use 1,000 runs.

CHAPTER

2

EFFICIENCY IN STUDENT-PROPOSING DEFERRED ACCEPTANCE MECHANISM: THE CASE OF WAKE COUNTY PUBLIC SCHOOL SYSTEM

2.1 Introduction and Related Literature

School choice has been one of the central themes of education research in recent decades. One of the core problems involves student placements among schools based on students' preferences and priorities. This problem falls into a broader category of the mechanism design problems allocating indivisible objects based on agents' preferences over objects and their priorities for objects when there are insufficient supplies, and the market cannot employ monetary transfers. Specifically, it is impossible to assign all students to their top choice schools.

In designing a mechanism to implement student assignments, researchers have probed into the properties the mechanism can ideally achieve. Strategy-proofness, fairness, and efficiency are considered to be the most desired properties of student assignment mechanism. The strategy-proofness requires that the mechanism encourages students to report their preferences truthfully. The fairness property eliminates justified envy among students. The efficiency requirement maximizes the students' welfare so that there are no feasible Pareto improvements upon the resulting assignments (Abdulkadiroğlu and Sönmez, 2003; Pathak, 2016). With these goals in mind, many studies have

made endeavors to find the best student assignment mechanism.

In their seminal paper, Abdulkadiroğlu and Sönmez (2003) focuses on the deficiencies of a widely used mechanism called Boston mechanism (BM).[1] They propose the replacement of BM with one of the two different mechanisms, namely the Deferred Acceptance (DA) mechanism and the Top Trading Cycle (TTC) mechanism.

The DA mechanism is first introduced by Gale and Shapley (1962) for the college admission problem, which is a close analogy to the school choice problem. In this paper, the authors formulate the college admission problem as a two-sided matching problem where the applicants have preferences over colleges and colleges have preferences over applicants. When both colleges and applicants have strict preferences, the matching produced by the student proposing DA algorithm is the only stable matching that Pareto dominates other stable matchings in terms of student welfare. One major difference between school choice and college admission is that in school choice problem seats are public resources. Hence schools do not have preferences over students, making student assignments a one-sided matching problem focusing exclusively on students' preferences. Balinski and Sönmez (1999) adapt DA to the one-sided matching problem by introducing the definitions of fairness and strategy-proofness.

Another candidate in matching theory is the TTC mechanism. Since the school choice problem also assembles the housing market allocation, researchers also advocate the TTC algorithm involving the housing market for the school choice problem as it has the attractive properties of being strategy-proof and Pareto efficient (Shapley and Scarf, 1974).

Studies comparing the DA and TTC mechanisms are abundant in terms of the aforementioned desired features of student assignment procedure because there is no mechanism being both Pareto efficient and fair (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003; Kesten, 2004). The DA algorithm is strategy-proof and fair. However, DA is not Pareto efficient. Contrarily, the TTC algorithm is strategy-proof and Pareto efficient but lacks fairness. The choice of mechanism boils down to the trade-off between Pareto efficiency and fairness.

After Abdulkadiroğlu and Sönmez (2003) forms school choice as a mechanism design problem, many school districts favor the DA mechanism over the TTC mechanism, including the Wake County School District (Dur, Hammond, and Morrill, 2018a). Favoring the DA mechanism makes one wonder if that means people care about efficiency less than strategy-proofness. There are a few possible explanations. First, participants are more likely to reveal their true preferences under the DA mechanism than under the TTC mechanism (Chen and Sönmez, 2006). Second, compared to the TTC mechanism, the requirement is weaker for the DA mechanism to yield consistent assignment results with changing participant numbers and resource levels (Ergin, 2002; Heo, 2019). Last but not least, Troyan (2019) shows that TTC is rarely applied because it is hard for the applicants to understand. Despite its appealing properties mentioned above, the DA mechanism still lacks Pareto efficiency. A

[1] Many school districts within the US implement BM and its variants, such as Boston, Cambridge, Denver, Minneapolis, Seattle, and Wake County. This mechanism is also known as the Immediate Acceptance (IA) algorithm since it assigns as many students to their top-ranked schools as possible. For those who rank the school in second place would only have access to it if there is any seat remaining.

rich literature strives to find a way to improve the efficiency in the DA mechanism (Kesten, 2006; Dur, Gitmez, and Yılmaz, 2019). One partial remedy adopted in practice is to use a single tie-breaking method by which assigns each student a single random number for tie-breaking at every school to ensure the minimal amount of efficiency loss (Erdil and Ergin, 2008; Abdulkadiroğlu, Pathak, and Roth, 2009). Wake County Public School System (WCPSS) adopts this single tie-breaking method. Besides the tie-breaking, scholars also find that when the priority is acyclic, the DA becomes Pareto efficient (Ergin, 2002; Kesten, 2006; Ehlers and Erdil, 2010). However, the application of this acyclic priority is very restrictive, as the acyclic priority rarely exists in the real world. The probe into preference profiles is not promising either, as Heo (2019) finds that the preferences making DA Pareto efficient tend to be very hard to identify.

Our work extends the inquiry of finding conditions under which the DA mechanism selects efficient outcomes. Since WCPSS uses the DA mechanism in magnet schools assignment, we are curious how much trade-offs there are between efficiency and fairness. WCPSS assignment data shows that DA is efficient in the elementary magnet school assignment. Given the fact that the TTC mechanism is not fair, the DA mechanism is a better choice. Motivated by this observation, we first formalize a theoretical framework to capture the features of the WCPSS student placement process, including students' priority structures and preference profiles. Within this framework, we prove that DA results in Pareto efficient assignments when some assumptions hold. These theoretical assumptions are on both priority structures and preference profiles. It is worth mentioning that the requirements for the priority structure established in this paper are weaker than the acyclic priority conditions. Besides, the students' preference profiles making DA efficient can be found among the elementary magnet school applicants in WCPSS. Finding weaker and more feasible conditions that make DA efficient is our major theoretical result.

Furthermore, we empirically test these theoretical results and assumptions using data of magnet applicants in WCPSS in the 2017-2018 school year. We restrict our analysis to the rising-kindergartners so that our assumptions hold. Given students' submitted preferences and priority scores, we apply DA-STB to assign students to schools. To examine the efficiency of the DA-STB result, we also apply the TTC to the same assignment problem. Since theories extol the TTC of its virtue of Pareto efficiency, the assignments resulted from TTC serve as the efficiency benchmark to evaluate the efficiency of DA-STB. Following the method used in Abdulkadiroğlu, Pathak, and Roth (2009) and Abdulkadiroglu, Che, Pathak, Roth, and Tercieux (2017), we find no statistically significant evidence that the simulated DA-STB assignments are different from the TTC results. In addition, following Ergin (2002) and Pathak (2016), we apply Stable Improvement Cycles (SIC) to the DA-STB assignments. The results again do not support that the SIC can provide any significant Pareto improvements to the DA-STB assignments. Therefore, we are confident that the DA-STB mechanism is efficient in the setting of the WCPSS elementary magnet applications.

The rest of the paper proceeds as follows. Section 2.2 introduces the background of magnet school programs in the Wake County Public School System (WCPSS). Section 2.3 formulate our theoretical framework. Section 2.4 lays out assumptions making the DA mechanism efficient and proves our

main theoretical results. Section 2.5 describes the dataset we employed from WCPSS. Section 2.6 shows our observations from WCPSS data and discuss the efficiency features of DA-STB. Section 2.7 concludes.

2.2 Background

2.2.1 Wake County Public School System

We choose the Wake County Public School System(WCPSS) as the case study of the efficiency of the DA mechanism because it is a representative sample among the school districts across the country. As of 2019, WCPSS is the largest school district in North Carolina and the 15th largest in the country, with an average daily enrollment of over 160,000 students (WCPSS, 2019b). The student population has almost tripled since 1980, making WCPSS one of the fastest-growing school districts.

WCPSS is famous for its efforts to achieve a more integrated school environment (Clotfelter, Ladd, and Vigdor, 2005). Being formed amid desegregation in education in the 1970s, WCPSS is formed by a voluntary merger between the previous historically white Wake County school system and the former historically black Raleigh City schools (Flinspach and Banks, 2005). As a result, WCPSS covers both urban areas and rural areas. There is much diversity within the student population. Around forty-five percent of the students are non-Hispanic white, and the rest are minority ethnics. About one-third of the students receive free or reduced-price lunch in the 2018-19 school year (WCPSS, 2019a).

As of the 2018-19 school year, there are 191 schools in WCPSS, 116 of which are elementary schools. The total numbers of middle schools and high schools are 37 and 29, respectively.

2.2.2 The Magnet School Assignment

While every enrolled student is assigned to a base school, WCPSS allows parents a variety of public school choices. The drawing of school attendance areas determines students' base school. Although WCPSS does not disclose the algorithm defining the attendance areas, according to the WCPSS Enrollment Plan, the assignment areas will be developed by balancing several goals and factors, including student achievement, stability, proximity, and operational efficiency.[2] Each student can look up her base school using the residential address on the WCPSS website.[3]

Among the options that the students have other than the base school in WCPSS, the magnet program has been the cornerstone of the school choice programs since 1982.[4] As of 2018, there are forty-five magnet schools, among which there are twenty-seven elementary schools, twelve middle schools, and six high schools in WCPSS. As an effort of desegregation, the magnet schools usually locate in higher-minority or more-socioeconomic-disadvantage neighborhoods. By offering incentives to

[2] see https://www.wcpss.net/cms/lib/NC01911451/Centricity/Domain/6073/BOE%20WORK%20SESSION%2011_20_18_%20FINAL%20PLAN_150688h32u30mbcydqpherdh23ijtd.pdf

[3] see http://wwwgis2.wcpss.net/addressLookup/

[4] Information from News and Observer "How NC became a national leader for magnet schools to promote diversity and choice "(https://www.newsobserver.com/news/local/education/article191746189.html)

students through special curricular, magnet schools attract students with higher socio-economic status to attend schools with more diverse peers.

The magnet application typically starts in October and ends in January. During this period, students submit ranked lists of magnet schools they prefer to attend. Each student can rank up to five schools. The assignment process is based on the submitted student preferences and the students' priority levels. Since the magnet program's objectives are to "reduce high concentrations of poverty, promote school integration, maximize the use of school facilities, and provide innovative or expanded educational opportunities," the socio-economic status (SES) of the student's residential area and the projected socio-economic status of the student's next school are two critical determinants of the student's priority.[5] Besides the students' SES status, the priority structure also depends on the district's *Four Pillars*: student achievement, stability, proximity, and operational efficiency. Since the priority structure is relatively simple for the rising-kindergarten applicants, our model and analysis mainly focus on the assignment process for the rising-kindergartners. It is worth mentioning that the highest priority goes to students who have older siblings attending the applied magnet school for elementary entry-graders. Since there are many students with the same priority at the same school, ties are broken by assigning a random number to each student. The same lottery number is used for tie-breaking in all schools. While students can obtain information on their sibling priority status and the SES statuses, they do not observe the lottery number.

Facilitated with the students' submitted preferences and the priority structure, magnet seats are assigned to students via the DA mechanism with some modification due to administrative adjustments. Applicants get the assignment results in February. If one does not get any assignment, the student will attend her base school. Once a student is assigned to a magnet school, she will lose her seat in the base school. Since the magnet program offered by WCPSS is well-established, its influence and its assignment mechanism have been studied by numerous researchers(Garcia, 2012; Parcel and Taylor, 2015; Ayscue, Siegel-Hawley, Kucsera, and Woodward, 2018; Dur, Hammond, and Morrill, 2018b; Carlson, Bell, Lenard, Cowen, and McEachin, 2019; Dur, Morrill, and Phan, 2018). However, the efficiency of the DA mechanism employed by the magnet program has not been examined theoretically or gauged empirically by the literature yet. This niche motivates our key research question that how efficient can the DA mechanism be in the context of WCPSS magnet school assignments.

2.3 The Model

In this section, we formalize the magnet school assignment problem using a theoretical framework. The model representing the student assignment problem consists of the following components:

1. A finite set of students $N \equiv \{1, 2, \ldots, n\}$.
2. A finite set of magnet schools $S \equiv \{s_1, s_2, \ldots, s_m\}$.

[5] see `https://www.wcpss.net/domain/90`

3. A vector listing the capacity of each magnet school $q \equiv (q_{s_1}, q_{s_2}, \ldots, q_{s_m})$, where q_{s_k} is the number of available seats at school s_k.

4. A preference profile $R \equiv (R_1, R_2, \ldots, R_n)$ is a list of students' preferences over schools, where R_i is student i's preference over $S \bigcup \{\varnothing\}$. $\varnothing$ is the option of being unassigned. Any school ranked lower than $\varnothing$ is unacceptable. Assume that any student has the option of being unassigned, $q_\varnothing = |N|$. We assume R_i is complete and transitive. For any two schools $s_k, s_l \in S$, if student i weakly prefers school s_k to school s_l, then we have $s_k R_i s_l$. We further use P_i to denote the strict preferences of R_i. Let $P \equiv (P_1, P_2, \ldots, P_n)$.

5. A strict priority structure $\succ \equiv (\succ_{s_1}, \succ_{s_2}, \ldots, \succ_{s_m})$ is a list of students' priority orders at each school. For each $s_k \in S$, $\succ_{s_k}$ is the priority orders of school s_k over all applicants N. This order is complete and transitive. For any two students $i, j \in N$, if $i \succ_{s_k} j$, student i has a higher priority at school s_k than student j.

With the aforementioned definitions, a school assignment problem can be described by a 3-tuple of the students' preference profile, the priority structure, and the capacity vector of all schools $(P, \succ, q)$, given that N and S are fixed.

A **matching** allocates available magnet school seats to students so that each student is admitted by at most one magnet school. Formally, we can write the matching as a function $\mu : N \to S \bigcup \{\varnothing\}$ such that for each $s_k \in S$, $|\mu^{-1}(s_k)| \leq q_{s_k}$. If student i does not gain a placement in any magnet school, she remains unassigned, that is, $\mu(i) = \varnothing$

A matching μ is **individually rational** if all students weakly prefer their assignment than remaining unassigned, that is, for any $i \in N$, $\mu(i)\ R_i\ \varnothing$.

A matching μ is **fair** if there exists no student-school pair (i, s) such that (i) student i strictly prefers school $s \in S$ to her assignment $\mu(i)$, that is, $s\ P_i\ \mu(i)$, (ii) there exists a student $j \in N$ who received a seat at school s under μ, that is, $\mu(j) = s$, and (iii) student j has a strictly lower priority at school s than student i, $i \succ_s j$.

A matching μ is **non-wasteful** if there is no student-school pair (i, s) such that (i)$s\ P_i\ \mu(i)$, and (ii) school s has a vacant seat under μ, that is, $|\mu^{-1}(s)| < q_s$.

A matching μ is **stable** if it is individually rational, fair, and non-wasteful.

A matching μ **Pareto dominates** μ' if $\mu(i)\ R_i\ \mu'(i)$ for all $i \in N$, and $\mu(j)\ P_j\ \mu'(j)$ for some $j \in N$. A matching μ is **Pareto efficient** if there is no other matching Pareto dominates it.

A **mechanism** φ selects a matching for each school assignment problem. Let $\varphi(P, \succ, q)$ denote the matching selected by φ for the school assignment problem $(P, \succ, q)$.

A mechanism φ is **strategy-proof** if it is a weakly-dominant strategy for every student to report her preference truthfully in the game induced by φ. No student can make herself better-off by misrepresenting her preferences.

We next introduce the **acyclicity condition**, due to Ergin (2002) and Ehlers and Erdil (2010).

Given a priority structure $\succ$, a **cycle** consists of students $i, j, k \in N$ and schools $s_x, s_y \in S$ such that (i) $i \succ_{s_x} j \succ_{s_x} k \succ_{s_y} i$, and (ii) there exist disjoint sets $N_{s_x}, N_{s_y} \subseteq N \setminus \{i, j, k\}$ (possibly $N_{s_x} = \emptyset$ or $N_{s_y} = \emptyset$)

such that $N_{s_x} \subseteq \{l \in N : l \succ_{s_x} j\}$, $N_{s_y} \subseteq \{l \in N : l \succ_{s_y} i\}$, $|N_{s_x}| = q_{s_x} - 1$ and $|N_{s_y}| = q_{s_y} - 1$.
A priority structure is **acyclic** if no cycles exist.

Corollary 1. *(Theorem 1 of Ergin (2002)) When the priority structure is acyclic, the student-proposing DA mechanism is Pareto efficient, group strategy-proof, and consistent.*

Now we introduce the preference profile making the DA mechanism efficient, due to Heo (2019).

Corollary 2. *(Heo (2019)) When the preference profile is constructed as follows, the DA mechanism selects efficient results.*

- *All students have the same preferences over all schools except for school s_x and s_y. Let the capacity of school s_x and s_y be q_{s_x} and q_{s_y}, respectively.*
- *For student $i \in \{1, \ldots, q_{s_x}\}$, s_x is the most preferred school. s_y is the least preferred school.*
- *For student $j \in \{q_{s_x} + 1, \ldots, n-1\}$, s_y is the most preferred school. s_x is the least preferred school.*
- *For student n, s_y is the most preferred school. s_x is the second preferred school.*

In Section 2.4, we will show that our assumptions making the DA mechanism selects the efficient outcomes are weaker than the acyclic priority condition and the preference requirement defined by Heo (2019).

2.3.1 The Student-Proposing Deferred Acceptance Mechanism

In this section, we describe the deferred acceptance (DA) mechanism. The DA mechanism is first introduced by Gale and Shapley (1962) for the college admission problem. As illustrated by the previous studies (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003), one aspect distinguishing the school choice problem from the college admission problem is that available school seats are objects to be allocated instead of being agents with preferences. Therefore, we only care about students welfare. We focus on the student-proposing DA mechanism, which is fair, strategy-proof, and student-optimal compared to any other fair mechanisms (Balinski and Sönmez, 1999). In WCPSS magnet school assignment, student priorities at schools are coarse instead of being strict. During the matching process, WCPSS assigns each student a single number drawn from a random lottery and uses this number for priority tie-breaking at every school [6]. Now we formally describe the student-optimal DA mechanism as follows:
Step 1: Each applicant applies to her top-ranked choice, possibly $\varnothing$. Each school $s \in S \bigcup \{\varnothing\}$ tentatively accepts the top q_s applicants according to $\succ_s$ and rejects the rest.
Step k, $k \geq 2$: Each student who is rejected in step $k-1$ applies to her next choice, possibly $\varnothing$. Each school $s \in S \bigcup \{\varnothing\}$ considers the students tentatively admitted at step $k-1$ and the new applicants together and tentatively admitted the top q_s students according to $\succ_s$ and rejects the rest.

[6] According to Abdulkadiroğlu, Pathak, and Roth (2009), this single tie-breaking method is more efficient compared to the multiple tie-breaking method.

The above process stops when all students are tentatively admitted or are rejected by all the listed schools. At termination, each student is placed in the schools that tentatively admit her or stays assigned.
It is well documented that the DA mechanism comes with some nice features for the student assigning problems such as strategy-proofness and fairness (Abdulkadiroğlu and Sönmez, 2003; Pathak, 2016). However, the major shortcoming of DA is its lack of Pareto efficiency. It has been shown that Pareto efficiency and fairness cannot be achieved at the same time(Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003). In the following section, we will show our main results that under the assumptions fitting the situation of elementary magnet school matching in WCPSS, the DA mechanism becomes Pareto efficient.

2.4 Theoretical Results

In this section, we provide our theoretical results. These results hold under some restrictions on preferences and priorities.
We start with presenting the assumptions that we make to derive our theoretical results and defining some notions that we use in this section.
Let $b : N \to S \bigcup \{\emptyset\}$ be the student's sibling school function. That is, for any student $i \in N$, $b(i)$ is student i 's sibling school. If a student has no sibling priority school, $b(i) = \emptyset$.
The first assumption is on the priority structure.

Assumption 1. *(Assumption on the priority structures) There are two priority classes for each school: sibling priority and others. If $b(i) = s$ for some i, then student i has sibling priority at school s. Sibling priority is the highest priority at any school $s \in S$. Each student may have at most one school where she has sibling priority, $|b(i)| \leq 1$.*

The second assumption is on the student preferences.

Assumption 2. *(Assumption on the preference profiles with a popular school) Each student with a sibling school ranks her sibling school as either first or second choice. For the ones who are not ranking their sibling schools as top choice and the ones who do not have sibling schools, they all rank the same school, which we call popular school, as top choice.*
Denote the popular school as s^*. Then Assumption 2 says that:

- *If $b(i) \neq \emptyset$, it is either $b(i)\ P_i\ s$ for any $s \neq b(i)$, or $s^*\ P_i\ s$ for any $s \notin \{s^*, b(i)\}$*
- *If $b(i) = \emptyset$, then $s^*\ P_i\ s$, for any $s \neq s^*$.*

As an alternative restriction on the preference, we present our last assumption.

Assumption 3. *(Assumption on the preference profiles without any popular school) Each student with a sibling school ranks her sibling school as the first choice. For the ones do not have sibling schools, they randomly rank any one school as their top choice.*

Facilitated with the above assumptions, we now demonstrate that the DA mechanism selects Pareto efficient outcomes when the above assumptions hold. We start with a case in which there exists a popular school.

Proposition 1. *(With a popular school) Under Assumption 1 and 2 DA selects Pareto efficient outcomes.*

Proof. According to Assumption 2, any student i with $b(i) = s^*$, i.e. has sibling priority at the popular school, ranks that school as top choice. We consider two cases:

Case 1 $|\{i \in N : b(i) = s^*\}| \geq q_{s^*}$.

In this case, the number of students having sibling priorities at the popular school is greater or equal to the capacity of the popular school. Hence, independent of the preferences of all other students, only students with sibling priorities at s^* can be assigned to s^*. If student i with $b(i) = s^*$ is rejected by s^*, then it means that there are at least q_{s^*} students with sibling priorities at s^* and they all have better tie-breaker. Moreover, any student rejected by s^* cannot cause the rejection of the student temporarily held by school s^*. That is, any student applying to s^* in the following steps cannot cause any student tentatively assigned to s^* to be rejected. Hence, the tentative assignment to s^* in *Step 1* is in fact permanent. Thus it is enough to consider the reduced problem such that all students assigned to the popular school $\{i \in N : \mu(i) = s^*\}$ and the popular school s^* is removed.

Then, by Assumption 2, if student i is remaining and $b(i) = s_k \neq s^*$, she ranks s_k as top choice. By using the same arguments above and the fact that the sibling priority is the highest priority, all students having sibling priorities tentatively accepted by that school s_k cannot be replaced in the further steps. Therefore, for any $s_k \in S$, we can remove all accepted students with sibling priorities at s_k and the filled school seats of s_k (i.e. $|i \in N : b(i) = s_k \& \mu(i) = s_k|$) from the problem.

In this reduced problem, we have a problem in which all remaining students have exactly the same priorities at all remaining schools. As a result, the DA mechanism turns into the Serial Dictatorship mechanism for the remaining students and the outcome is Pareto efficient (Gibbard et al., 1977).

Case 2 $|\{i \in N : b(i) = s^*\}| < q_{s^*}$

As explained in Case 1, students with sibling priorities at s^* who are tentatively admitted cannot be replaced in the further step. Consequently, the tentative admission is in fact final. According to Assumption 1, since the sibling priority is the highest priority and $|\{i \in N : b(i) = s^*\}| < q_{s^*}$, all students with $b(i) = s^*$ is admitted by s^*. Again, we can remove all students with sibling priorities at the popular school $\{i \in N : b(i) = s^*\}$ from N, and remove the filled school seats $|\{i \in N : b(i) = s^* \& \mu(i) = s^*\}|$ from q_{s^*} to form the reduced problem. Moreover, if i with $b(i) = s^*$ is tentatively accepted in *Step 1* and j with $b(j) \neq s^*$ is rejected, then i has a

higher priority than j does. And any student applying to s^* in the further steps has a lower priority than i does. Hence any tentatively accepted student by s^* in *Step 1* cannot be replaced in further steps. For the remaining part of the proof we refer to the second paragraph of Case 1, following which DA mechanism turns into the Serial Dictatorship mechanism for the remaining students and the outcome is Pareto efficient.

□

Next, we consider the case when there is no popular school.

Proposition 2. *(Without any popular school) Under Assumption 1 and 3, DA selects Pareto efficient outcomes.*

Proof. Similar to the proof we show for Proposition 1, according to Assumption 3, student i with $b(i) = s_k \in S$ top ranks s_k. Under Assumption 1, if she is temporarily placed in s_k in *Step 1*, this placement is in fact permanent. We can remove all students with $\{i \in N : b(i) = s_k \& \mu(i) = s_k\}$ and the school seats $|\{i \in N : b(i) = s_k \& \mu(i) = s_k\}|$.
The reduced problem becomes a problem in which all remaining students have exactly the same priorities at all remaining schools. As a result, the DA mechanism turns into the Serial Dictatorship mechanism for the remaining students and the outcome is Pareto efficient. □

Now we show that our assumptions making the DA mechanism efficient are weaker than the restrictions proposed by previous studies. We start by showing that our assumption on priority allows violation of the acyclicity condition.
Consider three students $i, j, k \in N$ and two schools $s_x, s_y \in N$. Assume that $b(i) = s_x$ while $b(j) \neq s_x$ and $b(k) \neq s_y$. The tie between j and k is broken by a random lottery resulting in $j \succ_{s_x} k$. Further assume that $b(k) = s_y$ and $b(i) \neq s_y$. It is easy to see that the above priority satisfies Assumption 1. It violates the acyclicity condition as $i \succ_{s_x} j \succ_{s_x} k \succ_{s_y} i$. Therefore, our assumption on priority is weaker than the acyclicity condition.
Next we consider the restriction on student preferences. Under Assumption 2, each student with a sibling school ranks her sibling school as either first or second choice. For the ones who are not ranking their sibling schools as top choice and the ones who do not have sibling schools, they all rank s^* as top choice. This is clearly weaker than the preference profile constructed in Heo (2019) as our assumption does not pose any restriction on the least preferred schools.
Therefore, our assumptions on priority and preference making the DA mechanism efficient are weaker than the restrictions proposed by previous studies.

2.5 Data

Our goal is to evaluate the performance of the DA mechanism with single tie-breaking in the context of WCPSS rising-kindergartner magnet school assignments. We use the student-level registration and application data from WCPSS in the 2017-2018 school year. The dataset contains each student's

submitted rankings of up to five magnet schools, the student's priority point at each magnet school, the schools where the student has sibling priorities, and the assignment results for each student. This rich student-level dataset captures the priority structure and the preference profiles among WCPSS magnet applicants. At the school level, we observe the number of available seats in each magnet school.

To ensure that our theoretical assumptions hold, we restrict our analysis to the elementary entry-graders who apply for magnet schools. In the 2017-2018 school year, there are 1,340 entry-grade elementary magnet applicants, 27 elementary magnet schools, and a total of 907 magnet seats available.

2.6 Empirical Results

In this section, we use empirical data to evaluate the performance of our theoretical results. Our theoretical framework yields two major testable implications. The first testable implication is that our assumptions hold for WCPSS magnet applicants. The second one is that the DA-STB mechanism selects Pareto efficient assignments. We start our empirical analysis by inspecting whether our assumptions hold in the context of WCPSS elementary magnet school matching. Depending on the assumptions proved to hold, we check which propositions can be used for further empirical examination. In the section subsection, we empirically test whether applying DA-STB yields efficient outcomes in WCPSS data.

2.6.1 Tests on Assumptions

In this subsection, we test whether our assumptions hold in the data. We start by testing for the assumption of student priority structures. Our assumption on priority is that every student has at most two priority levels and at most one sibling schools, where the sibling priority is the highest priority level. The analysis falls into three parts: whether the students have at most two priority levels; whether the students have at most one sibling schools, and whether the sibling priority is the highest priority.

Table 2.1 shows the results addressing the first test on priority. The second column shows the number of priority classes each student has across all WCPSS schools. About 66% elementary magnet applicants have at most two priority levels, and about 95% applicants have at most three priority levels. Therefore, in a broader view, the first part of the assumption on priority is met by 66% of students. Compared to this broader view, it may be more relevant if we narrow our scope to include only schools listed by students. Hence we ask how many priority levels are there within the schools listed by students. The results are reported in the fourth column of Table 2.1. About 96% of the students have at most two priority levels. In this sense, we can say that most of the students have no more than two priority levels.

The second test on priority examines whether students have at most one sibling priority schools.

Table 2.2 shows the number of each student's sibling priority schools observed in the data. The result provides strong evidence supporting our assumption on the number of sibling schools, as about 99% of students have at most one sibling school.

The last check on our assumption on priority structure is straight-forward: by the construction of the students' priorities in WCPSS, sibling priority is the highest priority by definition. In conclusion, most of the students have no more than two priority levels, at most one sibling priority school, and the sibling priority is the highest priority level. Therefore, Assumption 1 holds in our data.

While Assumption 1 is on students' priority structures, Assumption 2 and 3 are assumptions on students' preference profiles. These assumptions can be translated into two cases: the first case is that students top rank either their high priority schools or the popular school, given there exists one popular school among students. The second case is that when there is not a single popular school among students, students always top rank their high priority schools.

Compared to the first case, the second case under the assumption that there exists no popular school is easier to verify in data. Because we only need to check if students top rank their high priority schools. In WCPSS, students' high priority schools can either be their sibling priority schools or other high priority schools.

As introduced in Section 2.2, the students have information on the schools where they have sibling priorities, and their priority level at each school that they are interested in applying based on SES status. Although the tie-breaker and the exact number of priority points is not disclosed to students, each student has an estimate of her priority levels. Based on this fact, we check whether students would highly rank schools where they have high priority levels or sibling priorities.

We first look at the effect of sibling priorities on students school preferences by showing the ranked order of the students' sibling schools. Table 2.3 shows that among the students with sibling priorities, about 92% of students top rank the schools where they have sibling priorities. Therefore, we can conclude that most students would highly rank the schools where they have sibling priorities.

To examine how the priority levels impact students school rankings, we start our analysis by excluding students with only one priority level. Besides, since many students tend to have a high priority because of sibling priority, we need to isolate the students' preferences for high priority schools from the students' tendency to attend the same school as their siblings. To separate the high-priority effect from the sibling effect, we exclude sibling schools from both students' submitted rankings and student's priorities. With sibling schools excluded, we categorize the remaining schools on a student's preference list into two groups: the schools where the student has the highest priority and the other lower priority schools. With this classification, we analyze the propensity of students top ranking one of the highest priority schools. To set a benchmark, we assume that students top rank a school randomly without regard to their priority and calculate the probability of each school get top ranked. Then we calculate the actual chances that a school gets top ranked by each student in the data. By running a T-test comparing the benchmark probability and the observed probability in the data, we can show if a higher priority school gets a higher chance of getting on top of the list. To be more specific, we illustrate it with the following example. Suppose the students has the

same priority at 10 schools and this is their highest number of priority points (so 17 schools where they have less priority). Without regard to priority, they would have a 10/27 chance to top rank a highest priority school by chance. So if such students are top ranking a highest priority school at a rate higher than 10/27, it suggests that these students are more likely to top rank a highest priority school.

Table 2.4 shows the number of students having the same priority points at different number of schools. To make our T-test results statistically valid, we choose to run the T-test for students having the same highest priority level at 3, 9, and 10 schools. We also run the same analysis with all student pooled together. Table 2.5 reports the T-test results. It shows that for students having the same highest priorities at 9 and 10 schools, as well as the pooled data, the T-test results are significant at a significance level of 1%. These results support our argument that students have a higher propensity to highly rank the schools where they have high priority levels.

Sibling priority and priority scores are two importance factors affecting students preferences over schools. The speculation why sibling priority and priority points can play a role in students preferences is that one would have a higher chance to gaining access to a magnet school seat by high ranking schools where they have sibling priorities or high priority points. To see if this is true, we compare the assignment results of students who top-rank their sibling schools, and the assignment results of students who top-rank schools where they have high priority levels, to the average assignment results with all students pooled together. The comparison can be found in Table 2.6. This table shows that a student has a probability of 98% getting the first choice if she top-rank her sibling school, indicating that top-ranking sibling schools almost guarantee students access to magnet seats where they have sibling priorities. It also gives strong evidence that top-ranking schools where students have high priorities yields a 63% chance of gaining a magnet seat. Compared to the fact that the probability an average applicant getting an assignment is about 50%, top-ranking high priority school increases this probability by a meaningful margin.

Since we find evidence supporting that students are likely to top rank either sibling schools or high priority schools, it is reasonable to believe that there exists no popular school ranked by students other than their high priority school. Table 2.7 shows the schools got ranked by students that are neither their sibling schools nor their high priority schools. There is not a single school got listed by a dominant number of students. Thus in WCPSS, there is no popular school, and students are likely to highly rank high priority schools. Assumption 3 on students' preference profiles holds in our data. In summary, our Assumption 1 and Assumption 3 hold in WCPSS. These assumptions lead to Proposition 2, which means that DA selects Pareto efficient outcomes in WCPSS.

2.6.2 Efficiency Comparison

In the above subsection, we have shown that in the context of the entry-grade elementary magnet school application within WCPSS, Assumption 1 and 3 are mostly met. In this subsection, we can empirically examine whether Proposition 2 is true using the actual application data.

However, there is one caveat worth mentioning before we look into the actual assignment result

observed in our data. The assignment mechanism employed by WCPSS deviates from DA-STB due to some additional requirements and adjustments made by WCPSS. A typical requirement is that all siblings should be assigned to the same school. More detailed discussion can be found in Dur, Morrill, and Phan (2018). Therefore, we cannot examine that efficiency of DA-STB by simply look at the actual student assignment.

To obtain the assignment results that would be generated by DA-STB, we run simulations with the students' application data, keeping the priority structure and the submitted preferences unchanged. The DA-STB assignments are then compared to the actual assignments. The comparison results are reported in the second through the fifth column of Table 2.8. We measure the efficiency by calculating the numbers and percentages of students who get their first, second, third, fourth, and fifth choice, as well as who remain unassigned. The more students get magnet seats at higher ranked schools, the more better-off students are. We observe efficiency gain by changing from the actual assignments to the DA-STB assignments, as the numbers of students who get their first, second, third, and fourth choices all increase while the number of assigned students decreases.

Now we need to show that DA-STB is not only more efficient than the actual assignment, but also Pareto efficient. One concern when applying our theoretical results to the observed data is that the number of schools one can list is limited to five. If this constraint is binding, the stability and efficiency of the DA-STB assignments would be affected (Calsamiglia, Haeringer, and Klijn, 2010). Hence we check whether the limit on submitted preferences are binding in our data. In fact, 91.3% of the students rank less than five schools, indicating that most students preference list are not affected by the limiting number of submitted rankings. It is reasonable to believe that choice limits would not incur any strategic response or affect efficiency.

To evaluate the efficiency of DA-STB assignments, we use the TTC assignments as our benchmark for efficient outcomes. The simulated results generated by TTC are reported in column eight and column nine of Table 2.8. The DA-STB results are very close to the TTC results. We further compare TTC and DA-STB with a statistical sign test. The sign test asks if the rank order of the DA-STB assignment tends to be the same as the rank order of the TTC assignments. If DA-STB selects efficient outcomes, then we should not be able to reject the null hypothesis in the following test:

$$H_0 : Rank_{DA-STB} = Rank_{TTC}$$
$$H_A : Rank_{DA-STB} \neq Rank_{TTC}$$

All two-sided sign tests yield insignificant results among all 1,000 simulation runs at the 10% significance level. We also calculate the average rank for a student across all simulation runs for DA matched at the student-level to a data set that has the average rank across all runs for TTC. The sign test for the average rank is insignificant as well. Therefore, we conclude that the DA-STB assignments are not significantly different from the TTC assignments.

To verify that the DA-STB assignments are indeed efficient, we apply the Stable Improvement Cycle (SIC) introduced by Erdil and Ergin (2008) to the assignments generated by each DA-STB simulation

run and see how many students can obtain Pareto improvement by exchange their assignments with each other. The SIC results are reported in the sixth and seventh column of Table 2.8. Again, these results are very similar to the DA-STB results. Table 2.9 reports the number and percentage of students who obtain efficiency gain. Again, the number of students who are made better-off by SIC is very small. These changes are all insignificant when we consider the standard deviations from the simulation runs.

In conclusion, the DA-STB selects efficient assignments in the WCPSS data. This efficiency feature makes DA-STB especially favorable in the elementary magnet school assignment process. Because although TTC is efficient, it does not eliminate justified envy. Panel B in Table 2.8 reports the numbers and percentages of students who have justified envy under each mechanism. Under TTC, there are 157 or 11.72% of students who have justified envy. Switching from TTC to DA-STB eliminates justified envy while preserving Pareto efficiency.

2.7 Conclusion

This paper provides a theoretical framework to prove that DA selects efficient outcomes under certain assumptions on priority structure and preference profiles. Our main result is that we can find a feasible combination of priority structures and preference profiles in real life data that make DA efficient. The requirement for the priority structure is that there are at most two priority levels, with the sibling priority being the highest priority. This priority assumption is weaker than the acyclic priority conditions established in previous literatures.

In addition, the students' preferences profiles making DA efficient requires that students either top rank their sibling/high priority schools, or the single popular school when there is one. This kind of preference can be found among the elementary magnet school applicants in WCPSS. The weaker and more feasible conditions making DA efficient is our major theoretical result.

Moreover, we empirically examine our theoretical results using data of magnet applicants in WCPSS in the 2017-2018 school year. Restricting our analysis to the rising-kindergartners, we empirically confirm that our assumptions on priorities and preferences hold. Given students' submitted preferences and priority scores, we apply DA-STB to assign students to schools. To examine the efficiency of the DA-STB result, we also apply the TTC to the same assignment problem. The assignments generated by DA-STB is not statistically different from the TTC results. Since the TTC results are Pareto efficient, so are the DA-STB results.

Furthermore, we apply Stable Improvement Cycles (SIC) to the DA-STB assignments. The results again show no evidence that the SIC can provide any significant Pareto improvements to the DA-STB assignments. Therefore, we are confident that the DA-STB mechanism is efficient in the setting of the WCPSS elementary magnet applications and our theory holds.

Table 2.1 Number of Priority Levels (2018)

Number of Priority Levels	Percent of Students (All Schools)	Percent of Students (Sib Schools Excluded)	Percent of Students (Only School Listed by Students)
1	33.18%	16.65%	82.55%
2	32.36%	41.90%	13.94%
3	29.01%	41.23%	3.43%
4	5.44%	0.22%	0.07%
Total	100%	100%	100%

Notes: This table summarizes the number of each student's priority levels. For example, the row of one priority level shows the percentage of students who have the same priority across all schools. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year

Table 2.2 Number of Students' Sibling Schools (2018)

The Number of Student's Sib Schools	Percent of Students
0	76.81%
1	23.04%
2	0.15%
Total	100%

Notes: This table summarizes the number of each student's sibling schools. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year.

Table 2.3 The Ranked Order of Students' Sibling Schools(2018)

The Ranked Order of Sib School	Percent of Students
1	91.61%
2	0.65%
3	0%
4	0%
5	0%
Not Listed	7.74%
Total	100%

Notes: This table summarizes how students rank their sibling schools in their submitted preferences. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year.

Table 2.4 The Number of Highest Priority Schools, with Sib Schools Excluded

Number Highest Priority School	Number	Percent
1	3	0.40%
2	1	0.13%
3	368	49.33%
9	41	5.50%
10	333	44.64%
Total	746	100.00%

Notes: This table summarizes the number of each student's highest priority schools with all sibling schools excluded. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year. There are 27 elementary magnet schools in total. To get the numbers reported in this table, we first drop each student's sibling schools. If a student has one sibling school, she faces a choice pool consisting of 26 non-sibling magnet schools. Further assume that this student has the same priority level at 10 schools and this is her highest priority level besides the sibling priority (so there are 16 schools where she has lower priority). Without regard to priority, she would have a 10/26 chance to top rank a highest priority school by chance. The row with 10 highest priority schools in this table shows that there are 333 students have the same priority at 10 schools and this is their highest priority level besides the sibling priority. We can compare the observed probability of these 333 students top-ranking one of the 10 highest priority schools to the probability of them top-ranking schools randomly. A T-test can show whether such students are top ranking a highest priority school at a rate significantly higher than the random case. If the T-test result is significant, it suggests that these students are more likely to top rank a highest priority school.

Table 2.5 Propensity of Top-ranking High Prio School, with Sib Schools Excluded (2018)

Number of Highest Prio Schools	Prob(Top Rank Highest) (If Randomly Selected)	Prob(Top Rank Highest) (Actual)	T-test H_A: $P(Actual) > P(Random)$
3	11.13%	7.74%	·
9	34.62%	100.00%	***
10	37.09%	59.66%	***
Average	23.71%	33.33%	***

Notes: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. See the notes to Table 2.4.

Table 2.6 Assigned Choice Number (2018)

Panel A: Assigned Choice Number, All Students (2018)		
	Number	Percent
Assigned 1st Choice	546	40.75%
Assigned 2nd Choice	70	5.22%
Assigned 3rd Choice	30	2.24%
Assigned 4th Choice	11	0.82%
Assigned 5th Choice	11	0.82%
Unassigned	672	50.15%
Total	1,340	100.00%
Panel B: Assigned Choice Number if Top-rank Sibling School (2018)		
	Number	Percent
Assigned 1st Choice	278	97.89%
Assigned 2nd Choice	0	0.00%
Assigned 3rd Choice	0	0.00%
Assigned 4th Choice	0	0.00%
Assigned 5th Choice	0	0.00%
Unassigned	6	2.11%
Total	284	100.00%
Panel C:Assigned Choice Number if Top-rank Highest Priority School (2018)		
Assigned 1st Choice	288	60.25%
Assigned 2nd Choice	3	0.63%
Assigned 3rd Choice	5	1.05%
Assigned 4th Choice	2	0.42%
Assigned 5th Choice	1	0.21%
Unassigned	179	37.45%
Total	478	100.00%

Notes: This table summarizes the assignment results. For example, the row beginning with "Assigned 1st Choice" shows the number and percentage of students who got their top choices. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year.

Table 2.7 Top-Ranked Non-Sib Non-High-Priority Schools

School ID	Number of Students	Percent
920336	46	5.25%
920344	66	7.53%
920352	18	2.05%
920376	92	10.50%
920380	16	1.83%
920396	24	2.74%
920414	55	6.28%
920415	13	1.48%
920416	43	4.91%
920440	19	2.17%
920446	9	1.03%
920448	95	10.84%
920452	61	6.96%
920456	18	2.05%
920460	36	4.11%
920476	16	1.83%
920496	12	1.37%
920525	16	1.83%
920532	13	1.48%
920536	35	4.00%
920560	6	0.68%
920564	9	1.03%
920572	36	4.11%
920596	78	8.90%
920600	24	2.74%
920620	12	1.37%
920632	8	0.91%
Total	876	100.00%

Notes: This table summarizes the number of students top-ranking each school besides their sibling schools. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year.

Table 2.8 The Comparison of the Efficiency of Assignment Results

	Actual Result		DA-STB		SIC		TTC	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Panel A: Statistics on Assignment Results								
Assigned 1st Choice	546	40.75%	604.68	45.13%	605.19	45.16%	608	45.37%
Assigned 2nd Choice	70	5.22%	110.42	8.24%	110.33	8.23%	104	7.76%
Assigned 3rd Choice	30	2.24%	38.91	2.90%	39.08	2.92%	38	2.84%
Assigned 4th Choice	11	0.82%	14.47	1.08%	14.44	1.08%	15	1.12%
Assigned 5th Choice	11	0.82%	9.70	0.72%	9.68	0.72%	10	0.75%
Unassigned	672	50.15%	561.82	41.93%	561.29	41.89%	565	42.16%
Total	1340	100%	1340	100%	1340	100%	1340	100%
Panel B: Statistics on Justified Envy								
Students with Justified Envy	249	18.58%	0	0%	0	0%	157	11.72%

Notes: Notes: This table compares the simulated assignment results generated by three mechanisms as well as the actual assignment results. The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year.The DA-STB and TTC results are generated by running the DA-STB or TTC mechanism to the actual application data with student priorities and submitted preferences. The SIC results are generated by running SIC to the DA-STB assignment results after each simulation run. The results of DA-STB, SIC, and TTC use simulations of 1,000 runs. The results of TTC is consistent and being Pareto optimal.

Table 2.9 Improvement by Stable Improvement Cycles

Improvement by SIC	Number of Students	Percent
+1	0.25 (1.56)	0.02% (6.37)
+2	0.09 (0.58)	0.01% (6.71)
+3	0.05 (0.36)	0.00% (7.26)
+4	0.10 (0.68)	0.01% (6.66)
+5	0.46 (2.80)	0.03% (6.08)

Notes:The numbers are from our data covering the grade-entry elementary students in the Wake County Public School System in the 2017-2018 school year. The improvement is defined as the improvement of student assignments by running SIC to the DA-STB results. The DA-STB results are generated by running the DA-STB mechanism to the actual application data with student priorities and submitted preferences. The SIC results are generated by running SIC to the DA-STB assignment results after each simulation run. The results use simulations of 1,000 runs. The standard deviations are reported in parentheses.

CHAPTER

3

DEFICIENCIES IN CHINESE COLLEGE ADMISSION MECHANISM

3.1 Introduction and Related Literature

The problem of assigning students to limited school seats is one of the most widely studied topics in education policies. This problem arises from college admissions. The study of the college admission mechanism can be traced to the canonical paper by Gale and Shapley (1962), which first forms the college admission as a mechanism design problem. In this paper, the authors introduce the deferred-acceptance (DA) mechanism and prove that it selects stable admission results, and Pareto dominates other stable matchings. Other related studies find that the DA mechanism is strategy-proof for students (Dubins and Freedman, 1981; Roth, 1982). Since Abdulkadiroğlu and Sönmez (2003) have formulated the public school choice problem as a mechanism design problem, the research on the DA mechanism extends to the student assignment problem among public K-12 schools.

The strategy-proofness of the DA mechanism makes it desirable in designing student assignment mechanisms. Preference manipulation is unfavorable as it benefits sophisticated students at the cost of sincere students (Pathak and Sönmez, 2008; Dur, Hammond, and Morrill, 2018a).

Like the school choice in the US, the Chinese college admission mechanism has been under extensive research and debates. Assigning applicants to college seats in China forms one of the largest matching markets in the world. In 2019, there are about 10 million high school seniors competing

for 8 million seats (including both undergraduate and postsecondary education).[1]

The Chinese college admission system involves a centralized matching process carried out by the local authority of each province. The matching of students to college seats across China is based on each student's submitted preference list and a single score that determines the student's priority at all colleges. This priority score is the test score a student gets in the Chinese National College Entrance Exam, also called the "Gaokao" exam score. A student's submitted preference consists of a ranking of colleges, as well as a ranking of departments under each listed college.

Before 2001, the immediate acceptance (IA) mechanism is the only mechanism implemented in college admissions in China.[2] However, the IA mechanism has a major limitation that it is not strategy-proof and encourages preference manipulation (Abdulkadiroğlu and Sönmez, 2003; Nie, 2007). In order to alleviate this problem, the parallel mechanism was introduced into the Hunan province college admission in 2001. Other provinces soon adopt the parallel mechanism as well. In 2019, most provinces in China had implemented the parallel mechanism to assign students to colleges.[3]

In previous researches, the Chinese Parallel mechanism is deemed equivalent to a serial dictatorship (SD) mechanism based on one single priority score (Nie, 2007; Feng, 2011; Wu and Zhong, 2014). However, more recent literature formulates the parallel mechanism in the scope of a parametric family of modified DA mechanisms.[4] This family of mechanisms puts the parallel mechanism in between the IA mechanism and the DA mechanism in terms of the stability and the efficiency (Chen and Kesten, 2015; Song, Tomoeda, and Xia, 2019; Chen and Kesten, 2019).

In the matching literature, the student-proposing DA mechanism is considered to be optimal for students in the context of the college admission problem, as it is fair, strategy-proof, and Pareto dominates any other fair mechanisms (Balinski and Sönmez, 1999). Since the parallel mechanism deviates from the DA mechanism, many scholars show concerns over the Chinese college admission mechanism's performance.

One concern is that the number of schools that can be listed is usually limited, so students cannot list all schools they would like to apply. When this constraint on listing preferences of colleges is binding, students may take strategic responses by including a "safe school" in their application (Calsamiglia, Haeringer, and Klijn, 2010).

Second, students still have the incentive to misreport their preferences under the parallel mechanism. The fewer parallel options there are, the stronger this incentive is (Chen and Kesten, 2017; Song, Tomoeda, and Xia, 2019).

Third, local education authorities usually submit applications exceeding a college's planned admission quotas to the college for review.[5] As a result of this over submission, once a student's listed departments have been filled with higher score students, her application will be returned (Feng,

[1] Data from `https://gaokao.eol.cn/gkbm/`.

[2] We describe the history of China's Gaokao in more detail in Appendix.

[3] In 2019, Inner Mongolia is the only province that had not yet adopted the parallel mechanism.

[4] When all schools rank students in the same order, the DA mechanism and the SD mechanism are equivalent.

[5] The local authority administering the centralized college admission would submit 105% of a college's planned admission quotas for the college to review. More detailed discussions can be found in the Appendix.

2011).

Given the concerns raised in the literature, the local education authorities in China have made efforts to improve the parallel mechanism. In 2019, some provinces switched to the DA mechanism to assign students to colleges. [6] One may think that the local authority implementing the DA mechanism would make the Chinese college admission strategy-proof. However, all of the existing literature on Chinese college admission formulate the admission process as matching students to colleges. In practice, there is one more step taken by the colleges to assign students to departments. Therefore, studying only the college-level admission mechanism may miss some crucial features in the Chinese college admission process. This niche motivates our research. In this paper, we try to capture some missing pieces in the previous studies by examining both the college-level and the department-level assignments.

Given the fact that students need to list a ranking of colleges, then list a ranking of departments under each listed college, we show that some students have no way to report their preferences truthfully. As students are first assigned to colleges then to departments, we show that such a procedure cannot be strategy-proof.

Furthermore, the Chinese college admission mechanism may be wasteful by assigning students to colleges then to departments.

To show that matching students to colleges by the DA mechanism does not eliminate the deficiencies in Chinese college admission, we formulate the Chinese college admission mechanism as follows: the mechanism first assigns students to colleges by DA mechanism, then assigns students to departments by DA mechanism. We use an illustrative example to show that students have the incentive to manipulate their preferences under this system. Besides, this matching mechanism may generate an assignment that is wasteful. Finally, we propose a simple solution to make the admission process strategy-proof and non-wasteful. The solution is to break down the two-step admission feature and make it a complete student-proposing DA mechanism.

The rest of the paper proceeds as follows. In Section 3.2, we introduce the background of the Chinese Gaokao exam and the Chinese college admission process. Section 3.3 formulates the Chinese college admission mechanism as a matching mechanism satisfying our two-step assumption. Section 3.4 presents our main result that the hypothetical mechanism formulated in Section 3.3 is not strategy-proof. Section 3.5 proposes a simple solution to the preference manipulation problem. Section 3.6 concludes.

3.2 Background: Chinese College Admission

The Chinese college admission system, or the "Gaokao" system, assigns college applicants to available college seats based on students' preferences and students' priorities. In this section, we will introduce these three key components: the students' priorities, the students' preferences, and the mechanism matching students to college seats.

[6] In 2019, the provinces using the DA mechanism to assign students to colleges are Beijing, Shanghai, Jiangsu, Guangdong, Shandong, Laioning, and Tianjing.

The college admission process in China is administered separately at the provincial level by the local Education and Examination Authority (EEA). Within each province, a single priority score determines each student's priority at all colleges. This priority score is the total score that a student gets in the Chinese National College Entrance Examination. Both the Chinese National College Entrance Examination and the Chinese college admission system are commonly known as "Gaokao."

The Gaokao exam is an academic examination held annually on June 7th and June 8th. The Gaokao exam consists of standardized tests on three national compulsory subjects and one elective subject. The majority of Gaokao scores are out of 750. The elective subject divides students into the science track and the art track. Students of different tracks are ranked separately based on their Gaokao scores.

The Gaokao grades are announced to students online, including the overall grade and the grade for each subject. Right after announcing the grades, the provincial EEA also reports the number of students at each grade level, with the science track students and the art track students counted separately.

Colleges announce the admission quotas before the Gaokao exam. Each college sets the planned admission number of each department in a province. At the department level, the quotas allocated to the art track students and the science track students are specified separately. The sum of these numbers is the total quota allocated to that province by the college. The college informs the provincial EEA of this total quota but not the quotas of its departments.

With the knowledge of their scores and the college quotas, students need to submit their preferences online to enter the admission process. This preference includes a ranking of colleges, as well as a ranking of departments under each listed colleges.

With the students' priorities and preferences, the local EEA collaborates with colleges in the admission phase. The process of admitting students to the departments of different colleges proceeds in two steps. The first step involves the local EEA let the college to review a certain number of applications based on the admission quotas set by the college. In the second step, the college assigns the reviewed students to each department based on their submitted rankings. The dominant mechanism employed in the first step is the Chinese parallel mechanism. The mechanism used to assign students to departments varies by colleges. But the serial dictatorship mechanism ($f\bar{e}n\ sh\grave{u}\ y\bar{o}u\ xi\bar{a}n$) is the most widely employed.

The parallel mechanism works as follows. In China, colleges are divided into two or three tiers. Each college belongs to a tier based on its reputation and overall ranking. Colleges in the first tier, the second tier, and the third tier are in descending prestige. Students submit their college preferences in two or three choice bands according to the number of college tiers. Within each choice band, students select several "parallel" choices from the corresponding tier college. For example, a student's first choice band may contain three tier-one colleges, A, B, and C, while her second choice band may contain three other tier-two colleges, D, E, and F.[7]

[7] Please refer to the Appendix for a more detailed discussion on college tiers. In this paper, we do not model college tiers or choice bands.

Based on the Gaokao exam score, the local EEA sorts students in descending order. Ties are broken based on a pre-announced rule.[8] The first parallel choice band contains e colleges. Starting from the student with the highest score, each student applies for her first choice. If the first choice college still has an unfilled quota, the local EEA submits the student's application to the college for further review. Otherwise, the student continues to apply for her second choice. When the ith student has either been submitted for review or applied for all e colleges in the first choice band, the $(i+1)$th student applies to her first choice. This process continues until all students have applied for the first band colleges. Students who remain unadmitted enter the applications for the second choice band, in which a similar process repeats.

The local EEA usually submits 105% of a college's planned admission quotas for the college to review. Since the local EEA only observes the college-level quota but not the department-level quota, the submission of applications to colleges does not guarantee admission. Instead, the student's application enters the second phase of the admission process, where the college admission office matches the students to the departments. For simplicity, in this paper, we only discuss the serial dictatorship mechanism($f\bar{e}n\ sh\grave{u}\ y\bar{o}u\ xi\bar{a}n$).

The serial dictatorship mechanism used in colleges works as follows. Once all applications are submitted to a college, the college sorts the students from the highest score to the lowest score in a strict order. Tie-breaking rules are usually the same as the one used by the local EEA. Starting with the highest score, each student applies for parallel departments in the order of her listed ranking. The student is assigned to the first department with an unfilled quota. Since the reviewed number is usually more than the quota, some students remain unassigned by the end of the mechanism. If the student is willing to accept any department assignment, she will be admitted by departments with unfilled quotas or departments with enough resources to increase enrollment. Otherwise, the application is returned.[9] Since all departments rank students based on the same Gaokao score, the department-level serial dictatorship mechanism is also equivalent to the department-level DA mechanism.

In the next section, we formulate the aforementioned matching mechanism in a theoretical framework.

[8] For example, when there are ties in the overall score among the art track students, the first tie-breaking rule is to rank the Chinese subject scores in descending order. The integrated arts exam score serves as the second rule when there is still a tie, the math is the third rule, and the Foreign Language is the fourth. For the science track students, when there are ties in the overall score, the first tie-breaking rule is to rank the math subject scores in descending order. The integrated science exam score serves as the second rule when there is still a tie, the Chinese score is the third rule, and the Foreign Language is the fourth.

[9] Please refer to the Appendix for more details about the department-level assignment mechanism. Every year before Gaokao starts, each college announces the mechanism used for department-level assignment. We refer the audience to `http://www.eol.cn/e_html/gk/zszc/index.shtml`.

3.3 The Chinese College Admission Problem and the Mechanism

3.3.1 The Chinese College Admission Problem

In this section, we formally present the Chinese college admission as a mechanism design problem, which consists of the following components:

1. A finite set of students who take the Gaokao exam $N \equiv \{1, 2, \ldots, n\}$.

2. A finite set of colleges $C \equiv \{c_1, c_2, \ldots, c_m\}$.

3. A finite set of departments D_c in each college $c \in C$, $D_c \equiv \{d_c^1, d_c^2, \ldots, d_c^{h_c}\}$, where h_c is the total number of departments in collge c. Therefore, the set of department-college combination is defined as $\mathbf{D} \equiv \left\{\{d_{c_1}^1, d_{c_1}^2, \ldots, d_{c_1}^{h_{c_1}}\}, \{d_{c_2}^1, d_{c_2}^2, \ldots, d_{c_2}^{h_{c_2}}\}, \ldots, \{d_{c_m}^1, d_{c_m}^2, \ldots, d_{c_m}^{h_{c_m}}\}\right\}$.

4. A vector listing the admission quotas of all colleges $q \equiv (q_{c_1}, q_{c_2}, \ldots, q_{c_m})$, where q_c is the admission quota set by college $c \in C$.

5. A vector $\vec{q_c}$ listing the admission quotas set by all departments in college $c \in C$, $\vec{q_c} \equiv (q_c^1, q_c^2, \ldots, q_c^{h_c})$, where h_c is the total number of departments in college c. Therefore, we have $q_c = \sum_{d=1}^{h_c} q_c^d$. Furthermore, we define the department-level quota set by all colleges as $\mathbf{Q} \equiv \left((q_{c_1}^1, q_{c_1}^2, \ldots, q_{c_1}^{h_{c_1}}), (q_{c_2}^1, q_{c_2}^2, \ldots, q_{c_2}^{h_{c_2}}), \ldots, (q_{c_m}^1, q_{c_m}^2, \ldots, q_{c_m}^{h_{c_m}})\right)$

6. A preference profile $R \equiv (R_1, R_2, \ldots, R_n)$ is a list of students' preferences over all department-college combinations, where R_i is student i's preference over $\mathbf{D} \bigcup \{\varnothing\}$, where $\varnothing$ is the option of not attending any department in college. Any department ranked lower than $\varnothing$ is unacceptable. Assume that any applicant has the option of not attending any department in college, $q_\varnothing = |N|$. We assume R_i is complete and transitive. For any two departments $d_{c_k}^x, d_{c_l}^y \in \mathbf{D}$, if student i weakly prefers $d_{c_k}^x$ to $d_{c_l}^y$, then we have $d_{c_k}^x R_i d_{c_l}^y$. We further use P_i to denote the strict preferences of R_i. Let $P \equiv (P_1, P_2, \ldots, P_n)$.

7. A vector of students' Gaokao exam scores $f \equiv (f_1, f_2, \ldots, f_n)$, where f_i is the Gaokao exam score of student i. Since tie-breaking rules are defined in detail in reality, we assume that there exists no tie between students. That is, for any $i, j \in N$, we have $f_i \neq f_j$. All colleges and departments use the same f to rank students.

With the aforementioned definitions, the Chinese college admission problem can be described by $(P, f, \mathbf{Q})$, given that N and $\mathbf{D}$ are fixed.

A **matching** allocates available department-level seats to students so that each student is admitted by at most one department in one college. Formally, we can write the matching as a function $\mu : N \to \mathbf{D} \bigcup \{\varnothing\}$ such that for each $d_c^x \in \mathbf{D}$, $|\mu^{-1}(d_c^x)| \leq q_c^x$. If student i is not admitted by any college, then we have $\mu(i) = \varnothing$

A matching μ is **individually rational** if all students weakly prefer their admission results than remaining unadmitted, that is, for any $i \in N$, $\mu(i)\ R_i\ \varnothing$.

A matching μ is **fair** if there exists no student-department pair (i, d_c^x) such that (i) student i strictly prefers department $d_c^x \in \mathbf{D}$ to her assignment $\mu(i)$, that is, $d_c^x \ P_i \ \mu(i)$, (ii) there exists a student $j \in N$ who was admitted to d_c^x under μ, that is, $\mu(j) = d_c^x$, and (iii) student j has a strictly lower Gaokao score, $f_j < f_i$.
A matching μ is **non-wasteful** if there is no student-department pair (i, d_c^x) such that (i)$d_c^x \ P_i \ \mu(i)$, and (ii) department d_c^x has a vacant seat under μ, that is, $|\mu^{-1}(d_c^x)| < q_c^x$.
A matching μ is **stable** if it is individually rational, fair, and non-wasteful.
A matching μ **Pareto dominates** μ' if $\mu(i) \ R_i \ \mu'(i)$ for all $i \in N$, and $\mu(j) \ P_j \ \mu'(j)$ for some $j \in N$. A matching μ is **Pareto efficient** if there is no other matching Pareto dominates it.
A **mechanism** φ selects a matching for each college admission problem. Let $\varphi(P, f, \mathbf{Q})$ denote the matching selected by φ for the college admission problem $(P, f, \mathbf{Q})$.
A mechanism φ is **strategy-proof** if it is a weakly-dominant strategy for every student to report her preference truthfully in the game induced by φ. No student can make herself better-off by misrepresenting her preferences. There are many desirable features of a college admission matching mechanism. In this paper, we focus on two properties: non-wastefulness and strategy-proofness.

3.3.2 The Chinese College Admission Mechanism

Now we offer a formal definition of the Chinese college admission process. As mentioned in Section 3.2, the Chinese college admission process proceeds in two steps: the college-level assignment and the department-level assignment. For simplicity purposes, we assume that the local authority matches the number of submitted applications to the college's admission quotas. That is, there is no over-submission problem.
We would like to show that the Chinese college admission still has some deficiencies, even if the DA mechanism is implemented in the college-level assignment. In order to focus on the problem caused by the two-step feature of the Chinese college admission, we simplified the Chinese college admission mechanism as follows.
First, we describe the information observed by the provincial authority and the college. The local authority observes students' submitted preferences over colleges, students' Gaokao scores f , and the college-level admission quotas $q = (q_{c_1}, q_{c_2}, \ldots, q_{c_m})$, where q_c is the total admission quota set by college c. Each college $c \in C$ observes students' submitted preferences over its departments, students' Gaokao score f, and its department-level admission quotas $\vec{q_c} = (q_c^1, q_c^2, \ldots, q_c^{h_c})$.
With the above information, we make the following assumption to simplify the Chinese college admission mechanism:

Assumption 4. *In the Chinese college admission process, the local authority first matches students to colleges by the DA mechanism. When a college received the student applications assigned to it, the college matches students to its departments by the DA mechanism.*

Next, we formally describe the Chinese college admission mechanism.

3.3.2.1 The College-Level Assignment Mechanism

In the college-level assignment phase, the provincial admission authority matches students to colleges and submits their applications to the college admission offices. The authority observes the college-level admission quota $q = (q_{c_1}, q_{c_2}, \ldots, q_{c_m})$, where where q_c is the total admission quota set by college c. The authority also knows the student i's reported college-level preferences P_i over $\{c_1, c_2, \ldots, c_m\} \bigcup \{\varnothing\}$. It is worth mentioning that the local authority does not observe any department-level information in the college-level assignment process.

The mechanism employed in college-level admission is the DA mechanism. The mechanism proceeds as follows:

Round 1:

Step 1: Each applicant applies to her top-ranked choice, possibly $\varnothing$. Each college $c \in C \bigcup \{\varnothing\}$ tentatively accepts the top q_c applicants according to f and rejects the rest.

Step k, $k \geq 2$: Each student who is rejected in step $k-1$ applies to her next choice, possibly $\varnothing$. Each college $c \in C \bigcup \{\varnothing\}$ considers the students tentatively admitted at step $k-1$ and the new applicants together and tentatively admitted the top q_c students according to f and rejects the rest.

This round stops when all students are tentatively admitted or are rejected by all the listed schools. At termination of Round 1, each student's application is submitted to the college that tentatively admit her for the department-level admission or stays unadmitted.

It is possible that a college c still has unfilled quota after Round 1. Denote the number of students admitted by college c in Round $t-1$ ($t \geq 2$) as $q_c(t-1)$. Then college c's unfilled quota in Round t ($t \geq 2$) is $q_c^{Remaining}(t) = q_c - \sum_{z=1}^{t-1} q_c(z)$.

Round t, $t \geq 2$:

Step 0: The students whose application were submitted to the college in round $t-1$ for the department-level admission but not accepted by any department go back to the college-level application pool.

Step 1: Each returned student applies to her next choice, possibly $\varnothing$. Each college $c \in C \bigcup \{\varnothing\}$ that still has unfilled quotas considers the new applicants and tentatively admitted the top $q_c^{Remaining}(t)$ students according to f and rejects the rest.

Step k, $k \geq 2$: Each student who is rejected in step $k-1$ applies to her next choice, possibly $\varnothing$. Each college $c \in C \bigcup \{\varnothing\}$ that still has unfilled quotas considers the applicants tentatively admitted at step $k-1$ and the new applicants together and tentatively admitted the top $q_c^{Remaining}(t)$ students according to f and rejects the rest. This round stops when all students are tentatively admitted or are rejected by all the listed schools. At termination of Round t, each student's application is submitted to the college that tentatively admit her or stays unadmitted.

There are no more rounds once all students are either admitted by a department or rejected by all colleges and departments listed on the submitted preference.

3.3.2.2 The Department-Level Assignment

When the college admission office receives the students' applications from the provincial authority, it matches these students to departments based on their reported department preferences and their Gaokao scores. The college $c \in C$ observes the department-level admission quota $\vec{q}_c = (q_c^1, q_c^2, \ldots, q_c^{h_c})$, the student i's preference over department P_i over $\left\{d_c^1, d_c^2, \ldots, d_c^{h_c}\right\} \bigcup \{\varnothing\}$.
As introduced in Section 3.2, although the mechanism used for department-level assignment varies by colleges, the most common one is the serial dictatorship mechanism(*fēn shù yōu xiān*). This mechanism works the same as the DA mechanism where all departments in college c rank students by the same set of Gaokao scores.
The department-level DA mechanism works as follows:
Step 1: Each applicant applies to her top-ranked department. Each department $d_c^x \in D_c$ tentatively accepts the top q_c^x applicants according to f and rejects the rest.
Step k, $k \geq 2$: Each student who is rejected in step $k-1$ applies to her next choice, possibly $\varnothing$. Each department $d_c^x \in D_c \bigcup \{\varnothing\}$ considers the students tentatively admitted at step $k-1$ and the new applicants together and tentatively admitted the top q_c^x students according to f and rejects the rest.
The above process stops when all students reviewed by the college admission office are either tentatively admitted by a department or rejected by all their listed departments. At termination, each student is placed in the department that tentatively admits her. If a student is not admitted by any department, her application is returned to the college-level admission.

3.4 Results

Previous researches have shown that the student-proposing DA mechanism is stable and strategy-proof (Dubins and Freedman, 1981; Roth, 1982; Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003). However, when we take the department-level assignment into consideration, the DA mechanism in the college-level assignment process does not make the Chinese college admission stable and strategy-proof.
Under Assumption 4, the mechanism described in Section 3.3 still has some major shortcomings, one of which is the lack of strategy-proofness. An intuitive explanation is that the local authority takes the applicants' at the college level instead of the department level. Since the department-level assignment happens after students are assigned to a college, students are required to rank the departments of a college consecutively. That is, suppose a student has the following preference:

$$\text{College A (Accounting)} \succ \text{College B (Biology)} \succ \text{College A (Physics)} \succ \text{College C (Chemistry)}$$

She can only report her preference as:

$$\text{College A (Accounting)} \succ \text{College B (Biology)} \succ \text{College C (Chemistry)}$$

or

College A (Accounting) $\succ$ College A (Physics) $\succ$ College B (Biology) $\succ$ College C (Chemistry)

Neither of them are the student's true preference.
Formally, we have the following proposition:

Proposition 3. *The mechanism satisfying Assumption 4 is neither strategy-proof nor non-wasteful.*

Proof. Proposition 3 can be proof by illustrating a case where the mechanism satisfying Assumption 4 is neither strategy-proof nor non-wasteful. Consider the following example.
Assume there are four colleges, College A, B, C, and D. The admission quotas are shown in Table 3.1. There are five applicants, Abby, Ben, Charlie, David, and Emma. Their true preferences and their scores are shown in Table 3.2.
Now assume that all applicants submit the ranking according to their true preferences. The submitted forms look like the ones shown in Table 3.3.
The local education authority observes that the college-level admission quotas of College A, B, C, and D are 2, 1, 1, and 1, respectively. The mechanism described in Section 3.3 works as follows. In the first step, the DA mechanism yields the following assignment results:

Abby	Ben	Charlie	David	Emma
College A	College B	College A	College C	College D

While all assignments are still tentative, the applications are submitted to corresponding colleges for further review. In the second step, College A reviews the applications by Abby and Charlie. Since both of them list the Department of Accounting as the only choice, Abby is admitted to Accounting while Charlie's application is returned to the college-level application pool.
When the local authority sees Charlie's application returned, the college-level DA algorithm continues. Charlie applies to the next college listed, which is College B. However, since College B's quota has been filled as Ben is admitted, Charlie is rejected. Then Charlie applies to College C. Again, College C no long has any available seat after admitting David, thus Charlie is rejected. After Charlie has applied to all listed colleges, the admission process is terminated and Charlie remains unassigned. The college-level assignment looks like:

Abby	Ben	Charlie	David	Emma
College A	College B	Unassigned	College C	College D

Now if we look at the department-level assignment again, It is easy to see that the final department-level admission result is:

Abby	Ben	Charlie	David	Emma
College A	College B	Unassigned	College C	College D
Accounting	Biology	Unassigned	Chemistry	Math

The above assignment result is not non-wasteful. Because David prefers the Physics Department of College A to his assignment, while the available seat at the Physics Department of College A remains unfilled.
Next, we show that the assumed mechanism is not strategy-proof by showing that Charlie can improve his admission result by unilaterally manipulate his preferences. Suppose that Charlie misreports his preferences by ranking the Chemistry Department of College C as his top choice:

$$\text{College C (Chemistry)} \succ \text{College A (Accounting)} \succ \text{College B (Biology)}$$

The submitted student preferences become the ones shown in Table 3.4. It is easy to see that the final assignment results are:

Abby	Ben	Charlie	David	Emma
College A	College B	College C	College A	College D
Accounting	Biology	Chemistry	Physics	Math

Charlie gets a more favorable assignment result by ranking a "safe school" as his top choice. In fact, this is a common strategy observed in the actual Chinese college admission process.
With the above example, we show that the mechanism satisfying Assumption 4 is neither non-wasteful nor strategy-proof. □

3.5 Policy Implication

The student-proposing DA mechanism is advocated in the literature as being strategy-proof, fair, and student-optimal. However, even if we adopt the student-proposing DA mechanism in the Chinese college admission mechanism, it is still neither strategy-proof nor non-wasteful if we preserve the two-step feature of the admission process. In this section, we propose a simple way to solve this problem. The solution is the remove the problematic two-step feature in the Chinese college

admission process.

If the local education authority managing the centralized college application process matches students directly to the department level, the admission mechanism becomes the DA mechanism. In this case, The college admission offices would provide the department-level quotas to the local central admission authority. The students no longer apply to colleges. Instead, students submit rankings of departments or programs offered by colleges in their application. Application at the program-level allows students to list programs offered by the same college inconsecutively. Based on students' preferences and Gaokao scores, the local central admission authority can use the student-proposing DA to match students to programs.

We think the implementation of the solution proposed above is feasible. The computerization of the Chinese college admission process started in 1998. The Ministry of Education offers the college admission software to all provinces. As more and more provinces embrace the DA mechanism at the college-level admission, the software is updated continuously. The provincial admission authorities can also afford the computational capacity required to carry out the student-to-department matching. Moreover, the DA mechanism is used in the centralized college admission process in other countries, such as Turkey, Chile, Spain, and Ireland (Mora and Romero-Medina, 2001; Chen, 2012; Hastings, Neilson, and Zimmerman, 2013; Saygin, 2016).

3.6 Conclusion

The Chinese college admission system involves a centralized matching process carried out by the local authority of each province. The matching of students to college seats across China is based on each student's submitted preference list and a single Gaokao exam score that determines the student's priority at all colleges.

The existing literature on the Chinese college admission formulates the admission process as matching students to colleges. However, the actual Chinese college admission involves assigning students to departments. The local authority first matches students to colleges. The matching is not final. The college reviews the student applications assigned to it and matches students to departments.

The mechanism employed by the local authority to assign students to colleges is the Chinese parallel mechanism, which has been well examined by previous studies (Chen and Kesten, 2017, 2019; Song, Tomoeda, and Xia, 2019). Compared to the DA mechanism, the Chinese parallel mechanism is less stable, less efficient, and not strategy-proof. In this paper, we show that changing the college-level assignment mechanism to the DA mechanism does not solve the problem once we take the department-level assignment into account.

We formulate the Chinese college admission process into a hypothetical two-step mechanism. The mechanism first assigns students to colleges by DA mechanism, then assigns students to departments by DA mechanism. We show that this mechanism is not strategy-proof nor non-wasteful. We further propose a simple solution by allowing students to apply for programs at the

department-level and employing the DA mechanism to match students directly to departments.

Table 3.1 Example: Admission Quotas

		Deparment Quotas	College Quotas
College A	Accounting	1	2
	Physics	1	
College B	Biology	1	1
College C	Chemistry	1	1
College D	Math	1	1

Table 3.2 Example: Scores and True Preferences

Student	Score	True Preference			
Abby	630	College A Accounting	College B Biology	College C Chemistry	College D Math
Ben	629	College B Biology	College C Chemistry	College D Math	
Charlie	620	College A Accounting	College B Biology	College C Chemistry	
David	617	College A Physics	College B Biology	College C Chemistry	College D Math
Emma	613	College B Biology	College C Chemistry	College D Math	

Table 3.3 Example: Submitted Preferences

Student	Score	Submitted Preference			
Abby	630	College A	College B	College C	College D
		Accounting	Biology	Chemistry	Math
Ben	629	College B	College C	College D	
		Biology	Chemistry	Math	
Charlie	620	College A	College B	College C	
		Accounting	Biology	Chemistry	
David	617	College A	College B	College C	College D
		Physics	Biology	Chemistry	Math
Emma	613	College B	College C	College D	
		Biology	Chemistry	Math	

Table 3.4 Example: Manipulated Preferences

Student	Score	Submitted Preference			
Abby	630	College A	College B	College C	College D
		Accounting	Biology	Chemistry	Math
Ben	629	College B	College C	College D	
		Biology	Chemistry	Math	
Charlie	620	College C	College A	College B	
		Chemistry	Accounting	Biology	
David	617	College A	College B	College C	College D
		Physics	Biology	Chemistry	Math
Emma	613	College B	College C	College D	
		Biology	Chemistry	Math	

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APPENDICES

APPENDIX

A

REASSIGNMENT POLICIES AND SCHOOL STRATIFICATION: SUPPLEMENTAL APPENDIX

A.1 Robustness Checks

This appendix provides additional results that demonstrate the robustness of the results included in the main text. Appendix Table A.1 reproduces the main results, summarizing the outcomes on the five characteristics in terms of the net effects from each reassignment program and the overall net effect.

Table A.1 Main Results for Comparison

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.09	0.00	·	0.35	−0.03	↘
CT-N	0.08	0.02	↗	0.30	−0.01	·
HT-N	0.08	0.02	↗	0.34	−0.00	·
Net	0.09	0.02	↗	0.35	−0.04	↘

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.28	0.03	↗	0.65	−0.01	·
CT-N	0.31	−0.01	·	0.68	−0.03	↘
HT-N	0.28	−0.01	·	0.66	−0.02	↘
Net	0.33	0.03	↗	0.69	−0.04	↘

Panel C: Ratio of Students Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.13	0.06	↗	0.80	0.00	·
CT-N	0.17	0.01	·	0.83	−0.04	↘
HT-N	0.14	0.00	·	0.81	−0.02	↘
Net	0.14	0.08	↗	0.80	−0.05	↘

Panel D: Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.15	0.01	↗	0.40	−0.03	↘
CT-N	0.13	0.02	↗	0.38	0.01	·
HT-N	0.13	0.01	↗	0.39	−0.01	·
Net	0.13	0.03	↗	0.36	−0.05	↘

Panel E: Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.06	0.01	↗	0.15	−0.01	·
CT-N	0.07	0.01	↗	0.16	−0.01	↘
HT-N	0.06	0.01	↗	0.17	−0.00	·
Net	0.06	0.02	↗	0.15	−0.02	↘

Panel F: Ratio of AIG Students

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.16	0.04	·	0.34	−0.03	↘
CT-N	0.13	0.01	·	0.32	−0.02	↘
HT-N	0.13	−0.00	·	0.30	−0.00	·
Net	0.16	0.06	↗	0.34	−0.04	↘

Notes: This table summarizes the results from the main text. See the notes to Table 1.2.

Table A.2 Robustness to Use of Proportions Test

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	PR-Test	Pre	Change	PR-Test
Mag-N	0.09	0.00	↗	0.35	−0.03	·
CT-N	0.08	0.02	↗	0.30	−0.01	·
HT-N	0.08	0.02	↗	0.34	−0.00	·
Net	0.09	0.02	↗	0.35	−0.04	·

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	PR-Test	Pre	Change	PR-Test
Mag-N	0.28	0.03	·	0.65	−0.01	↗
CT-N	0.31	−0.01	·	0.68	−0.03	↘
HT-N	0.28	−0.01	↘	0.66	−0.02	↘
Net	0.33	0.03	·	0.69	−0.04	↘

Panel C: Ratio of Students Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	PR-Test	Pre	Change	PR-Test
Mag-N	0.13	0.06	↗	0.80	0.00	·
CT-N	0.17	0.01	·	0.83	−0.04	↘
HT-N	0.14	0.00	·	0.81	−0.02	↘
Net	0.14	0.08	↗	0.80	−0.05	↘

Panel D: Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	PR-Test	Pre	Change	PR-Test
Mag-N	0.15	0.01	↗	0.40	−0.03	·
CT-N	0.13	0.02	↗	0.38	0.01	↗
HT-N	0.13	0.01	·	0.39	−0.01	·
Net	0.13	0.03	↗	0.36	−0.05	·

Panel E: Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	PR-Test	Pre	Change	PR-Test
Mag-N	0.06	0.01	↗	0.15	−0.01	↗
CT-N	0.07	0.01	↗	0.16	−0.01	·
HT-N	0.06	0.01	·	0.17	−0.00	·
Net	0.06	0.02	↗	0.15	−0.02	·

Panel F: Ratio of AIG Students

	Below Average			Above Average		
Type	Pre	Change	PR-Test	Pre	Change	PR-Test
Mag-N	0.16	0.04	↘	0.34	−0.03	↘
CT-N	0.13	0.01	·	0.32	−0.02	·
HT-N	0.13	−0.00	·	0.30	−0.00	·
Net	0.16	0.06	·	0.34	−0.04	↘

Notes: This table presents the same results as in Appendix Table A.1, but the statistical tests are from proportions tests. This test asks whether the proportion of schools where the characteristic becomes more concentrated is statistically significantly different than the proportion where the characteristic becomes less concentrated.

Table A.3 Robustness to Use of Median Benchmark

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.07	0.00	·	0.32	−0.02	↘
CT-N	0.06	0.02	↗	0.28	−0.00	·
HT-N	0.07	0.02	↗	0.32	0.00	·
Net	0.07	0.02	↗	0.31	−0.02	↘

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.30	0.03	↗	0.66	−0.01	↘
CT-N	0.31	−0.01	·	0.69	−0.03	↘
HT-N	0.30	−0.02	↘	0.68	−0.01	↘
Net	0.33	0.03	↗	0.69	−0.04	↘

Panel C: Ratio of Student Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.13	0.06	↗	0.80	0.00	·
CT-N	0.18	0.00	·	0.84	−0.04	↘
HT-N	0.13	0.01	·	0.80	−0.02	↘
Net	0.15	0.08	↗	0.81	−0.05	↘

Panel D: Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.13	0.01	↗	0.37	−0.03	↘
CT-N	0.13	0.02	↗	0.37	0.01	·
HT-N	0.12	0.01	↗	0.38	−0.00	·
Net	0.12	0.03	↗	0.34	−0.03	↘

Panel E: Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.06	0.01	↗	0.15	−0.00	·
CT-N	0.06	0.01	↗	0.16	−0.01	↘
HT-N	0.06	0.01	↗	0.16	−0.00	·
Net	0.06	0.02	↗	0.15	−0.01	↘

Panel F: Ratio of AIG Students

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.17	0.04	·	0.34	−0.03	↘
CT-N	0.13	0.01	·	0.32	−0.02	↘
HT-N	0.14	−0.00	·	0.31	−0.00	·
Net	0.17	0.06	↗	0.34	−0.04	↘

Notes: This table separates schools according to whether the characteristic underrepresented or overrepresented based on a benchmark of the median value of the characteristics in these data, rather than the mean value as is done in Appendix Table A.1.

Table A.4 Robustness to Use of MSA-Wide Benchmark

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.10	0.00	·	0.37	−0.03	↘
CT-N	0.10	0.01	↗	0.36	−0.02	·
HT-N	0.09	0.02	↗	0.37	−0.00	·
Net	0.10	0.02	↗	0.37	−0.04	↘

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.39	0.01	·	0.77	−0.00	·
CT-N	0.38	−0.01	·	0.75	−0.04	↘
HT-N	0.39	−0.02	↘	0.78	−0.01	·
Net	0.41	0.00	·	0.76	−0.04	↘

Notes: This table separates schools according to whether the characteristic underrepresented or overrepresented based on a benchmark of the mean value in the entire metropolitan area (MSA), rather than the mean value in the Wake County Public School System data as is done in Appendix Table A.1. The MSA-wide benchmark is based on the 5-year population estimates from the American Community Survey (2016), using the school-age population age 5 to 19. This is done only for race based on the available data in the American Community Survey.

Table A.5 Robustness to Use of Top and Bottom Decile of Schools

Panel A: African American Student Ratio

	Bottom 10%			Top 10%		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.01	0.02	·	0.55	−0.10	↘
CT-N	0.01	0.01	↗	0.47	−0.03	·
HT-N	0.01	0.04	↗	0.53	−0.00	·
Net	0.01	0.04	↗	0.53	−0.11	↘

Panel B: White Student Ratio

	Bottom 10%			Top 10%		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.07	0.10	↗	0.85	−0.02	·
CT-N	0.10	−0.02	↘	0.84	−0.01	·
HT-N	0.08	−0.02	·	0.87	−0.03	↘
Net	0.11	0.10	↗	0.86	−0.06	↘

Panel C: Ratio of Students Living in High-Income Neighborhood

	Bottom 10%			Top 10%		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.00	0.15	↗	0.99	0.00	·
CT-N	0.00	0.03	↗	0.99	−0.02	↘
HT-N	0.00	0.03	↗	0.99	−0.02	↘
Net	0.00	0.15	↗	0.99	−0.02	↘

Panel D: Ratio of Students with LEP

	Bottom 10%			Top 10%		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.03	0.02	↗	0.60	−0.18	↘
CT-N	0.04	0.05	↗	0.55	−0.01	·
HT-N	0.03	0.02	↗	0.60	−0.05	·
Net	0.03	0.06	↗	0.54	−0.18	↘

Panel E: Ratio of Students with SPED

	Bottom 10%			Top 10%		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.01	0.03	↗	0.21	−0.03	·
CT-N	0.03	0.01	↗	0.23	−0.04	↘
HT-N	0.01	0.02	↗	0.24	−0.03	↘
Net	0.01	0.04	↗	0.21	−0.08	↘

Panel F: Ratio of AIG Students

	Bottom 10%			Top 10%		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.07	0.19	↗	0.46	−0.04	↘
CT-N	0.06	0.03	·	0.43	0.00	·
HT-N	0.06	0.00	·	0.43	−0.00	·
Net	0.08	0.15	·	0.46	−0.05	↘

Notes: This table considers schools in the top and bottom deciles for each characteristic.

Table A.6 Elementary School Students Only

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.06	0.01	·	0.32	−0.02	·
CT-N	0.05	0.02	↗	0.25	−0.01	·
HT-N	0.06	0.02	↗	0.32	−0.01	·
Net	0.06	0.02	↗	0.31	−0.02	·

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.29	0.02	·	0.68	−0.02	↘
CT-N	0.32	−0.02	·	0.71	−0.04	↘
HT-N	0.31	−0.02	↘	0.72	−0.02	↘
Net	0.34	0.01	·	0.73	−0.05	↘

Panel C: Ratio of Students Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.09	0.06	↗	0.80	0.00	·
CT-N	0.16	0.00	·	0.86	−0.04	↘
HT-N	0.12	0.01	·	0.82	−0.03	↘
Net	0.11	0.09	↗	0.81	−0.07	↘

Panel D: Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.13	0.02	↗	0.43	−0.04	·
CT-N	0.13	0.03	↗	0.41	0.02	·
HT-N	0.12	0.01	·	0.42	−0.00	·
Net	0.12	0.04	↗	0.40	−0.06	↘

Panel E: Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.04	0.01	↗	0.13	0.00	·
CT-N	0.06	0.01	↗	0.15	−0.02	↘
HT-N	0.04	0.01	↗	0.14	0.00	·
Net	0.05	0.02	↗	0.13	−0.01	·

Notes: This table reproduces the main analysis including only elementary school students. Students can be formally AIG identified starting in fourth grade; our analysis uses only grade-entry students and there are no kindergarten students who are AIG identified, so this characteristic is excluded here.

Table A.7 Middle School Students Only

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.15	0.00	·	0.42	−0.06	↘
CT-N	0.14	0.02	↗	0.36	−0.00	·
HT-N	0.16	0.00	·	0.41	0.00	·
Net	0.14	0.02	↗	0.40	−0.07	↘

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.24	0.06	↗	0.56	0.00	·
CT-N	0.26	0.01	·	0.59	−0.02	·
HT-N	0.22	0.00	·	0.56	−0.00	·
Net	0.28	0.08	↗	0.59	−0.03	↘

Panel C: Ratio of Students Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.17	0.07	↗	0.79	0.00	·
CT-N	0.17	0.02	·	0.79	−0.04	↘
HT-N	0.16	−0.00	·	0.79	−0.01	↘
Net	0.16	0.12	↗	0.77	−0.05	↘

Panel D: Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.17	0.00	·	0.31	−0.02	·
CT-N	0.15	0.01	↗	0.33	−0.01	·
HT-N	0.16	0.00	·	0.32	−0.00	·
Net	0.14	0.02	↗	0.29	−0.03	↘

Panel E: Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.11	0.01	↗	0.19	−0.02	·
CT-N	0.11	0.01	↗	0.19	−0.00	·
HT-N	0.12	0.00	·	0.21	−0.01	↘
Net	0.11	0.01	↗	0.19	−0.04	↘

Panel F: Ratio of AIG Students

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.16	0.05	·	0.35	−0.03	↘
CT-N	0.13	0.01	·	0.33	−0.02	↘
HT-N	0.11	0.00	·	0.30	−0.00	·
Net	0.17	0.08	↗	0.35	−0.05	↘

Notes: This table reproduces the main analysis including only middle school students.

Table A.8 High School Students Only

Panel A: African American Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.16	0.00	·	0.44	−0.04	·
CT-N	0.19	0.01	·	0.49	−0.01	·
HT-N	0.15	0.00	·	0.38	0.00	·
Net	0.16	0.01	↗	0.43	−0.04	·

Panel B: White Student Ratio

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.21	0.05	·	0.58	0.00	·
CT-N	0.19	0.00	·	0.57	−0.01	·
HT-N	0.26	−0.01	·	0.60	−0.00	·
Net	0.22	0.05	·	0.58	−0.00	·

Panel C: Ratio of Students Living in High-Income Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.25	0.04	·	0.79	−0.00	·
CT-N	0.21	0.00	·	0.67	−0.01	·
HT-N	0.22	−0.01	·	0.78	−0.01	·
Net	0.25	0.03	·	0.79	−0.01	·

Panel D: Ratio of Students with LEP

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.14	0.00	·	0.26	−0.01	·
CT-N	0.13	−0.00	·	0.27	0.01	·
HT-N	0.14	0.00	·	0.27	0.00	·
Net	0.14	0.00	·	0.25	−0.01	·

Panel E: Ratio of Students with SPED

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.11	0.00	↗	0.16	−0.01	·
CT-N	0.11	−0.00	·	0.15	0.00	·
HT-N	0.11	−0.00	·	0.16	0.00	·
Net	0.11	0.00	↗	0.16	−0.00	·

Panel F: Ratio of AIG Students

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-N	0.17	0.03	·	0.33	−0.03	↘
CT-N	0.13	−0.00	·	0.26	−0.00	·
HT-N	0.16	−0.00	·	0.31	−0.00	·
Net	0.17	0.03	·	0.32	−0.03	↘

Notes: This table reproduces the main analysis including only high school students.

Table A.9 Ratio of Hispanic Students

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.07	0.00	↗	0.30	0.05	↗
Mag-R	0.09	−0.01	·	0.47	−0.20	↘
Mag-N	0.07	0.00	↗	0.30	0.00	·
CT-S	0.08	0.01	↗	0.32	0.05	↗
CT-R	0.07	0.00	·	0.34	−0.03	↘
CT-N	0.08	0.01	↗	0.32	0.03	↗
HT-S	0.07	0.00	↗	0.29	0.04	↗
HT-R	0.07	0.00	·	0.36	−0.03	↘
HT-N	0.07	0.00	↗	0.31	0.01	·
Net	0.06	0.01	↗	0.25	0.00	·

Notes: See the notes to Table 1.2.

Table A.10 Ratio of Students Living in High-Education Neighborhood

	Below Average			Above Average		
Type	Pre	Change	T-Test	Pre	Change	T-Test
Mag-S	0.13	−0.01	↘	0.81	0.00	·
Mag-R	0.04	0.16	↗	0.69	0.04	·
Mag-N	0.10	0.05	↗	0.79	0.00	·
CT-S	0.10	0.00	·	0.82	−0.02	↘
CT-R	0.15	0.04	↗	0.86	−0.04	↘
CT-N	0.10	0.03	↗	0.82	−0.04	↘
HT-S	0.11	−0.00	·	0.80	0.00	·
HT-R	0.12	0.01	↗	0.82	−0.03	↘
HT-N	0.11	0.00	·	0.80	−0.03	↘
Net	0.10	0.07	↗	0.80	−0.04	↘

Notes: See the notes to Table 1.2.

Table A.11 Regression Analysis of Reassignments

	(1) Mag Mfx / SE	(2) CT Mfx / SE	(3) HT Mfx / SE	(4) All Mfx / SE
African American	-0.00	-0.02	0.01	-0.01
	(0.01)	(0.01)*	(0.01)	(0.01)
White	-0.05	0.03	0.02	-0.01
	(0.01)***	(0.01)***	(0.01)***	(0.01)
Hispanic	-0.08	-0.00	0.01	-0.06
	(0.01)***	(0.01)	(0.01)	(0.01)***
High-Income	-0.06	-0.01	-0.03	-0.06
	(0.00)***	(0.01)	(0.00)***	(0.01)***
Female	0.01	0.00	-0.00	0.00
	(0.00)	(0.01)	(0.00)	(0.01)
AIG	0.13	0.02	0.00	0.11
	(0.01)***	(0.01)*	(0.01)	(0.01)***
SPED	-0.04	-0.01	0.00	-0.03
	(0.01)***	(0.01)	(0.01)	(0.01)***
LEP	-0.05	-0.05	-0.04	-0.09
	(0.01)***	(0.01)***	(0.01)***	(0.01)***
Grade=6	-0.10	-0.02	-0.07	-0.10
	(0.01)***	(0.01)***	(0.01)***	(0.01)***
Grade=9	-0.16	-0.19	-0.09	-0.28
	(0.01)***	(0.01)***	(0.01)***	(0.01)***
Observations	20465	13434	19883	24315

Notes: The dependent variable in these regressions equals one if the student was reassigned via the magnet program (Column (1)), a calendar transfer (Column (2)), a hardship transfer (Column (3)), or any of the three programs (Column (4)). The regressions use a Probit model, and average marginal effects are shown. Standard errors are shown in parentheses; $*$, $**$, and $***$ denote significance at the 10%, 5%, and 1% level, respectively.

A.2 Other School Districts with Transfer Programs

This appendix provides a list of selected other school districts that have transfer programs that are similar to WCPSS's programs that we study, along with a webpage that discusses the district's transfer program. This list includes many of the largest US districts and arbitrarily chosen other districts.

- Miami-Dade County Public Schools (Florida):
  `https://schoolchoicemiami.com`
- Clark County School District (Nevada):
  `https://ccsd.net/district/policies-regulations/pdf/5112_R.pdf`
- Broward County Public School (Florida):
  `https://www.browardschools.com/Page/41692`
- Hillsborough County Public Schools (Florida):
  `https://www.sdhc.k12.fl.us/doc/569/hillsborough-choice-options/`
- Fairfax County Public Schools (Virginia):
  `https://www.fcps.edu/registration/student-transfer-information`
- Hawaii Department of Education (Hawaii):
  `http://www.hawaiipublicschools.org/ParentsAndStudents/EnrollingInSchool/Pages/Geographic-exceptions.aspx`
- Dallas Independent School District (Texas):
  `https://www.dallasisd.org/Page/56408`
- Montgomery County Public Schools (Maryland):
  `https://www.montgomeryschoolsmd.org/info/transfers/`
- Charlotte-Mecklenburg Schools (North Carolina):
  `https://cmschoice.org/more-info/transfer-information/`
- Prince George's County Public Schools (Maryland):
  `https://www.pgcps.org/student-transfers/`
- Shelby County Schools (Tennessee):
  `http://www.scsk12.org/schoolchoice/files/2018/General-Choice-Faqs-2019-20.pdf`
- Cobb County School District (Georgia):
  `http://www.cobbk12.org/generalinfo/transfers/schoolchoice.aspx`

- Cypress-Fairbanks Independent School District (Texas):
https://www.cfisd.net/en/parents-students/registration/student-transfers/transfer-schools/
- Baltimore County Public Schools (Maryland):
https://www.bcps.org/system/policies_rules/rules/5000Series/RULE5140.PDF
- Pinellas County Schools (Florida):
https://www.pcsb.org/Page/2821
- Northside Independent School District (Texas):
https://nisd.net/pupil-personnel/student-transfers
- Albuquerque Public Schools (New Mexico):
https://www.aps.edu/schools/transfers/aps-student-transfers
- Baltimore City Public Schools (Maryland):
https://www.baltimorecityschools.org/transfer-requests
- Austin Independent School District (Texas):
https://www.austinisd.org/transfer
- Metro Nashville Public Schools (Tennessee):
https://www.mnps.org/transfers
- Anne Arundel County Public Schools (Maryland):
https://www.aacps.org/Page/1084
- Greenville County Schools Online (South Carolina):
https://www.greenville.k12.sc.us/Schools/main.asp?titleid=choice
- Alpine School District (Utah):
https://alpineschools.org/registration-enrollment/
- Fresno Unified School District (California):
https://www.fresnounified.org/dept/stafed/transfers/
- Loudoun County Public Schools (Virginia):
https://www.lcps.org/cms/lib/VA01000195/Centricity/Domain/11494/Special%20Permission%20Overview.pdf
- Guilford County Schools (North Carolina):
https://www.gcsnc.com/Page/11605
- Omaha Public Schools (Nebraska):
https://district.ops.org/DEPARTMENTS/StudentandCommunityServices/

- Columbus City Schools (Ohio):
  https://www.ccsoh.us/Page/2754
- Stockton Unified School District (California):
  https://www.stocktonusd.net/cms/lib/CA01902791/Centricity/Domain/151/
- Boise School District (Idaho):
  https://boise.ss8.sharpschool.com/cms/
- Indianapolis Public Schools (Indiana):
  https://www.myips.org/enrollment-options/enrolling-in-ips/
- Rochester City School District (New York):
  https://www.rcsdk12.org/Page/23033
- Garland Independent School District (Texas):
  https://www.garlandisd.net/content/school-transfers
- Birmingham City Schools (Alabama):
  https://www.bhamcityschools.org/Page/21411

APPENDIX

B

EFFICIENCY IN STUDENT-PROPOSING DEFERRED ACCEPTANCE MECHANISM: SUPPLEMENTAL APPENDIX

B.1 TTC Mechanism in School Assignments

Step 1: Assign counters for each magnet school to track how many seats remain available. Each student points to her top-ranked school, and each school points to the student with the highest priority. Ties are broken using one single lottery assigning one random number to each student. There must exist least one cycle, with student 1 pointing to school 1, school 1 to student 2, ..., student i to school s, and school s pointing to student 1.) Each student is part of at most one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school is reduced by one, and if it reaches zero, the school is removed.
Step k: Each remaining student points to her favorite school among the remaining schools, and each remaining school points to the student with the highest priority among the remaining students. There is at least one cycle. Every student in a cycle is assigned a seat at the school she points to and is removed. The counter of each school in a cycle is reduced by one, and if it reaches zero, the school is removed.
The procedure terminates when each student is assigned a seat (or all submitted choices are considered).

B.2 Pseudo-Codes

B.2.1 The DA-STB Mechanism

INITIALIZE M to empty matching.
WHILE (some student i is unmatched and has not applied to every listed schools)
Let Student i apply for the first school s on Student i's list to which Student i has not yet proposed.
IF (School s still has available seats)
Add $i \rightarrow s$ to matching M. School s's available seat is reduced by 1
ELSEIF (Student i has a higher priority compared to some temporary seat holder Student j at School s)
Replace $j \rightarrow s$ with $i \rightarrow s$ in matching M.
ELSE School s rejects Student i. Mark School s as proposed on i's list.

B.2.2 The TTC Mechanism

WHILE ((cycles connecting student and school) is not empty):
(remaining students) = students that are not assigned in the previous while loop
Loop over each student in (remaining students):
Loop over the student's preference schools:
IF the school capacity is not full:
append the school to the (target schools list) for this the student
append the student to the (target student list) for this school
sort (target student list) by defined priority score
create the student as a source vertex
create the all the schools in the (target schools list) as target vertexes
connect edges from this student to schools in (target schools list)

Loop over all schools:
IF the school capacity is not full
create the school as a source vertex
get the first student in the (target student list), the one with highest priority score
create this student as a target vertex
connect an edge from this school to this student

Call graph_tool function to find (cycles connecting student and school) from created edges

IF (cycles connecting student and school) is not empty

Loop over every cycle in (cycles connecting student and school)
Loop over all the vertexes of the cycle
On each vertex, the student is found
On the next vertex, the school is found
The student on the current vertex is assigned to the school on the next vertex
ELSE:
exit the **WHILE** loop

B.2.3 The SIC Mechanism

While (cycles connecting school_a and school_p) is not empty:
Loop over each student in all students:
Loop over each school_p in the student's preference schools:
IF the student has an assigned school_a:
IF the preferred school_p ranked higher than the assigned school_a in the student's wish list:
append the student to the PI candidate list of school_p
ELSE
append the student to the PI candidate list of school_p

Loop over each school_p in school list:
In this school_p's PI candidate list, divide students into groups with the same priority score
IF stable improvement is turned on:
we choose only the first two groups to do the PI (what you got is kind of random)
ELSE:
we choose all of the groups to do the PI

Loop over chosen groups with different priority scores:
Loop over each student in the current group with the same priority scores:
Under the probability of "using PI", default 30%, run the following assignment:
IF the student has an assigned school_a, and school_p ranked higher than school_a in the student's wish list:
create school_a as a source vertex
create school_p as a target vertex
connect an edge from school_a to school_p

Call graph_tool function to construct (cycles connecting school_a and school_p) from created edges

IF use longest cycle is turned on:

choose the longest cycle

ELSE

use the first cycle (what you got is kind of random)

Loop over all the vertexes of the cycle:
On each vertex, the school_a is found
On the next vertex, the school_p is found
Identify the student who belongs to the current edge
Move the student from school_a to school_p

APPENDIX

C

DEFICIENCIES IN CHINESE COLLEGE ADMISSION MECHANISM: SUPPLEMENTAL APPENDIX

C.1 The Gaokao Exam and Admission Process

C.1.1 Timeline

The Chinese National College Entrance Examination is an academic examination held annually in China. It is also commonly known as "Gaokao." The Gaokao exam consists of a set of standardized tests on several subjects. These standardized tests are the prerequisite for students' admission to most of the tertiary education institutes within the country, especially for the ones at the undergraduate level. Therefore, Gaokao is considered to be the most important exam a high school student will ever take.

The purpose of this article is to document the whole Gaokao process, from the student registration for the Gaokao Exam to the final admission results. Most of the information on Gaokao policies and college admission rules mentioned in this article are from the Ministry of Education (2020), as well as from the official website of the Provincial Education and Examination Authorities such as the Beijing Education Examination Authority (2020), the Shanghai Education Examination Authority (2020), and the Education Examination Authority of Guangdong Province (2020). We start with the timeline of the Gaokao Exam in this section.

Although the Gaokao Exam and the College Admission process is administered separately by the

Education and Examination Authority (EEA) of each province or municipality, the National Ministry of Education takes the primary role in coordinating the process across different regions. Hence the schedules of the Gaokao exams are almost the same across the country.[1]

Although the high school senior students who plan to take the Gaokao exam may start the preparation much earlier, the process of Gaokao starts with the student registration for the exams. Depending on the policy set by local authorities, the registration time is usually between October and November each year if one wants to take the next year's Gaokao exam. The average period of the registration window is one week.

In April, all colleges announce their admission quotas for each province. Colleges also disclose the minimum admission thresholds from the last year, which is the minimum score of those finally admitted to a college or a department of the college.

The Gaokao exams are taken on June 7th and June 8th each year across the country, following roughly the same schedules which last for a total of 9 hours.

After the students taking the exam, teachers who are recruited by the local education authorities will start the grading process. The grading takes about two weeks.

The grades are announced to students on June 24th or June 25th, along with the number of students at each grade. The latter one gives students a general idea about their priority rankings.

At the same time, the local education authority determines the threshold for each tier of colleges. Students with grades higher than the tier thresholds are eligible to apply for colleges in that tier.

From late June to early July, students need to submit their ranked college applications online. The application consisted of a ranked list of colleges and a ranked list of departments for each listed college. The number of colleges and the number of departments one can list depend on the local policies.

From early July to mid-July, after the students submit their applications, the EEA of each province or municipality collaborates with the admission office staff from each college to complete the student admission process.

In late July, all colleges complete the admission process. The final admission results are announced to students.

In the following sections, we will explain the procedures of the Gaokao process mentioned above in more detail, following the chronological order.

C.1.2 Gaokao Registration

C.1.2.1 The Hukou system and Gaokao

In October or November each year, the students in their senior year of the high school register for the Gaokao exam online. However, there is one more restriction tied to the Chinese household registration system, which is commonly known as the Hukou system. The Hukou system is a residency status system connecting the social services provided by the local government, including

[1] We refer the audience to the website `https://www.eol.cn/e_html/gk/gktime/index.shtml` for more detailed information on Gaokao timelines and schedules of different provinces and municipalities.

educational resources, to one's officially registered address. To be eligible to register for the Gaokao in a province, the student must belong to a household with a registered address in the same province. One major reason is that the funding allocated to high schools is linked to the number of students registered locally, rather than based on the actual number of students attending the local high schools. The admission quotas for universities are also linked to the Hukou system, favoring the provinces with major metropolitan cities. We use an example to illustrate this Hukou restriction. If a student's parents have a Hukou registered in a rural province A, but they work in a municipal city B, the student needs to go to province A for Gaokao registration even if he/she attends high school in city B.[2]

Since 2012, many provinces have strived to modify existing restrictions on migrant students' access to tertiary education. However, the threshold for eligibility is relatively high, making it impossible for most of the migrant students to take the Gaokao exam in the city where they live. For example, the Beijing EEA requirements are as follows:[3]

> A migrant student is eligible to apply for local Gaokao registration if the following requirements are met:
>
> 1. The student's parent has a valid residence card or work residence permit.
> 2. The student's parent has a legal and stable residential address.
> 3. The student's parent has maintained a legal working status for at least six years.
> 4. The student's parent has consecutive payments of endowment insurance premiums and medical insurance premiums for at least six years.
> 5. The students have spent three years attending one of the local high schools in Beijing.

C.1.2.2 The Exam Subjects

When register for the Gaokao exam, a student also needs to specify the subjects he/she is going to take. Although the format of the Gaokao may vary by provinces, it is the most common that a student needs to take three compulsory subjects and one elective subject. The three compulsory subjects are Chinese, mathematics, and foreign language. For the foreign language test, a student can choose from English, Japanese, Russian, German, Spanish, or French. For the elective subject, the options are either an integrated science test or an integrated arts test. The integrated science test assesses a student's knowledge of chemistry, biology, and physics, while the integrated arts test evaluates the knowledge in history, geography, and political education. Students at senior high schools generally choose to specialize in either a science or arts route for their final two years. Based

[2] Please see Burkhoff (2015) for more detailed information on the relationship between higher education and the Hukou system.

[3] The original document can be found at `https://gaokao.eol.cn/bei_jing/dongtai/201809/t20180917_1625414.shtml`.

on each student's choice of the elective subject, students from the same region are categorized into two groups: the Art track students and the Science track students.

C.1.3 Gaokao Assessment and Grading

C.1.3.1 Exam Time

Arranged on June 7th and June 8th every year, The Gaokao examinations last nine hours spread over two days. As mentioned in Section C.1.1, since the National Ministry of Education coordinates the process of Gaokao across the country, every province shares a similar schedule of tests. Table **??** shows the most commonly used schedule in China.

Table C.1 The Gaokao Exam Schedule

Time	June 7th		June 8th	
	9:00-11:30	15:00-17:00	9:00-11:30	15:00-16:40
Subjects	Chinese	Math(Arts Track) Math(Science Track)	Integrated Arts Test Integrated Science Test	Foreign Language

C.1.3.2 The Test Contents

Initially, the Gaokao is a national exam administered by the Chinese Ministry of Education since the resumption of Gaokao after the end of the cultural revolution in 1977. In 1985, Shanghai and Guangdong started to offer their version of the Gaokao Exams, followed by other provinces. In 2019, 6 provinces and municipalities offered their own versions of the exam, although they are taken on the same days and to the same timetables.[4]

The Gaokao exam consists of tests on three national compulsory subjects and one elective subject. The three national compulsory subjects are Chinese, Math, and Foreign Language. The options for Foreign Language are English, Japanese, Russian, German, Spanish, or French. All the national compulsory subject scores are out of 150.

The elective subject varies across provinces. The most widely used options are the integrated science test and the integrated arts test. The integrated science test includes components of chemistry, biology, and physics, while the components of the integrated art test are history, geography, and political education. The elective subject scores are out of 300. Therefore, the majority of Gaokao scores are out of 750.[5]

The content of the tests comprises a mix of multiple-choice and short answer questions covering each of the subjects. Besides, the Chinese exam includes an essay section that provides students a

[4] Please see Chen and Kesten (2015) for more details on the history of Gaokao.

[5] There exists some variation of the elective test. For example, in 2019, Shanghai EEA allows students to choose any three subjects from chemistry, biology, physics, history, geography, and political education, regardless of the science or art classification. Each subject scores are out of 70, making the total Gaokao score out of 660.

paragraph describing a scenario or a position. Students are required to write an extended response to the paragraph.

C.1.3.3 Grading

After the students complete the tests, the answer sheets are scanned and managed by the local EEA. Multiple-choice questions are answered on bubble sheets and graded by grading machines. The short answer questions and the essays are sent electronically to two randomly selected teachers for manual grading, with the student's name blinded. If there is a discrepancy larger than five points between the two manual grading results, the answer sheet is sent to a third teacher for the final mark.

The grading procedure takes about two weeks to complete. The scores from the tests on the four subjects are combined to produce an overall Gaokao score. The results are announced to students in late June, between June 22nd to June 25th, depending on the provinces. Students can use their registration number and their password to log in to a website managed by local EEA to check for the results.

C.1.4 Students' Information before Submitting the Ranked Applications to Colleges

C.1.4.1 The Grade and Their Rankings

As described in Section C.1.3.3, after the completion of the Gaokao grading process, the students can see their Gaokao grades online, including both the overall grade and the grade for each subject. The students in the science track and the students in the arts track are then ranked separately based on the overall grade. The ranking of the overall grade is the primary student priority determinant. Right after announcing the grades, the provincial EEA also reports the number of students at each grade level, with the science track students and the art track students counted separately. Table C.2 is an example showing a part of the table that the science track students taking the Beijing 2019 Gaokao can see on the official website.[6] It shows that there are 24 students with an overall score higher than 700. There are seven students with an overall score of 699. Therefore, if a student gets a score of 699, his/her ranking is between 25th to 31st. Some provinces also display the estimated student rankings along with the student's overall score on the same score-inquiring webpage.

C.1.4.2 The College Admission Quotas and Tier Thresholds

Although the colleges announce the planned admission quota for provinces and municipalities in April each year, the actual number of students admitted to each college may change due to the negotiation and adjustment between the provincial EEA and the college admission office during the actual admission process.

Each college sets the planned admission number of each department in a province. The sum of these

[6] Please see `https://www.eol.cn/e_html/gk/gkfsd/index.shtml` for information on rankings of other provinces.

numbers is the total quota allocated to that province by the college. This information is publicly available to students on the official Gaokao website.[7] For example, Table C.3 shows part of the admission quotas of different departments of Tsinghua University that is allocated in Beijing in 2019. The quotas of departments and colleges give students a general idea of how competitive it is to apply for some colleges and departments. Specifically, applying for a department with a quota of 1 is more competitive than applying for a department with a quota of 25.

Moreover, some departments only accept the applications by science track students, while some other departments only accept the applications by arts track students. However, many departments are willing to accept the applications from both art track and science track students. Given the distinction between these two tracks, the quotas allocated to the art track and the science track are specified separately. For example, in Table C.3, it shows that the Department of Economics and Finance of Tsinghua University plans to admit six science track students and two arts track students. By summing up the quotas offered by all departments, the provincial EEA yields the total number of science track and arts track students one college plans to admit. Depending on the provinces, the matching algorithm, which will be discussed in detail in the next section, divides colleges into two or three tiers. In 2019, most provinces still maintained the three-tier division.[8] By summing up the quota offered by all colleges within one tier, the local EEA gets the approximate total number of students one tier can admit. This number determines the tier threshold. The tier threshold makes sure that the students eligible to apply for the tier are no more than 120% of the total quotas. Therefore, satisfying the first tier threshold only enables a student to apply for colleges in the first tier. There is no guarantee that the students will be admitted by the college she applies for. Neither is there any information on the chances of admitting by a specific department.

C.1.4.3 The Admission Mechanisms for Colleges and Departments

Besides the admission quotas and the tier thresholds, students can also get to know the matching mechanism employed by the local EEA assigning students to colleges. The mechanism each college uses to place students in departments is also stated in the admission guide available to download on the college website. We introduce the mechanisms in the next section.

C.1.4.4 The Minimum Admission Thresholds from Last Year

Colleges also make it public the enrollment statistics from the last year, including the application submission threshold, and the lowest ranking of the threshold score. Table C.4 shows the colleges' application submission thresholds and the corresponding rankings in Guangdong Province in 2019. For example, the application submission threshold.[9] for Peking University is 652 in Guangdong for arts track students. The lowest ranking gets into Peking University is 30. However, these statistics

[7] Please see `http://www.eol.cn/html/gkcx/jh2015/11/140.htm`.

[8] In 2019, Beijing, Tianjin, Shanghai, and Guangdong start to divide colleges into two tiers, the undergraduate level colleges and the career colleges

[9] The application submission threshold is the minimum score for a student's application to be submitted to the college for review.

from last year may not be good predictors for the ones this year. For further information, students can also make calls to the college admission office after the grades are announced, asking for the estimated application submission threshold this year. However, the estimates provided by the admission staff may not be accurate either.

C.1.5 Students' Submitted Ranking of Colleges and Departments

From late June to early July, students need to submit their ranked college applications online. The application consisted of a ranked list of colleges and a ranked list of departments for each listed college. The number of colleges and the number of departments one can list depend on the local policies. Figure C.2 shows part of the college application form that the students taking the Beijing Gaokao need to submit. In the form, it is clearly stated the number of parallel choices a student can submit within each choice band, as well as the number of departments or majors a student can list for each college. In this example, a student who took the Beijing 2019 Gaokao can list up to 16 colleges as her parallel choice from the first tier colleges. The first tier colleges include all colleges at the undergraduate level. The second tier colleges include the career colleges. For each college listed, a student can rank up to 6 departments or majors in which she is interested. The student is also asked whether she is flexible about the choices of departments. If the answer is yes, it means that the student is willing to study in any department that still has seats available in the case that she does not get in any listed departments.

C.1.6 The Admission Mechanisms

C.1.6.1 Priority Structure and Tie-breaking

As mentioned in section C.1.3.2, a student's priority solely depends on the single overall Gaokao score, which is obtained by adding up all subject scores. Based on this score, the local EEA sorts students in descending order. The science track students and the arts track students are sorted separately. Ties are broken based on a pre-announced rule, which varies among provinces. The most common tie-breaking rules are as follows.

For the arts track students, when there are ties in the overall score, the first tie-breaking rule is to rank the Chinese subject scores in descending order. The integrated arts exam score serves as the second rule when there is still a tie, the math is the third rule, and the Foreign Language is the fourth.

For the science track students, when there are ties in the overall score, the first tie-breaking rule is to rank the math subject scores in descending order. The integrated science exam score serves as the second rule when there is still a tie, the Chinese score is the third rule, and the Foreign Language is the fourth.

C.1.6.2 The Admission Mechanism Assigning Student to Colleges

As mentioned in Section C.1.1, per illustration purposes, we divide the Gaokao admission into two steps: allocating students to colleges and placing students to departments.

The mechanism employed in the first stage is the parallel mechanism in most of the provinces and municipalities. Although the program codes used to implement the parallel mechanism are not released to the public, explanations about the mechanism are available in the official documents provided by local EEAs.

Now we use Figure C.1 to illustrate how the parallel mechanism works. The parallel mechanism first rank students into strict orders from the highest score to the lowest score; ties are broken with the tie-breaking rule introduced above. The first parallel choice band contains six colleges, listed as College 1, College 2, until College 6. Starting from the student with the highest score, each student first applies for College 1. If College 1 still has an unfilled quota, the student's application is submitted to College 1 by the local EEA for further review. Otherwise, the student continues to apply for College 2. When the ith student has either been submitted for review or applied for all six colleges in the first choice band, the $i+1$th student applies for College 1. This process continues until all students have applied for the first band colleges.

The local EEA usually submits 105% of a college's planned admission quotas for the college to review. However, the submission of applications to colleges does not guarantee admission. A student may not satisfy the requirements set by the departments for which she applies. In this case, if the student is flexible about the department assignment, the college will generally increase its admission quota and admit the student to the department with enough resources. Contrarily, if the student is not willing to attend any other departments, the student's application returns to the application pool, meaning that the student has missed the first round enrollment. This also creates a difference between the lowest score submitted for review and the lowest score admitted. Therefore, the admission threshold is usually higher or equal to the application submission threshold.

If a college returns too many reviewed applications, it may still have quotas unfilled at the end of the first round. Therefore, at the end of the first round, the first-tier colleges with unfilled quotas will announce the number of available quotas and corresponding departments. The students whose applications are returned would have a chance to re-submit a new list of colleges and departments until all first-tier colleges fill their quotas. This stage works as the repechage game for the students who missed the first round admissions. It is worth pointing out that it is considered harmful to a college's reputation if a college cannot fill its quotas in the main round in China. Hence all colleges set the review number to 105% of the planned quotas even if they often need to increase the admission number.

C.1.6.3 The Admission Mechanism Assigning Students to Departments

The second stage of the Gaokao admission process happens after the college receives the students' applications from the local EEA for review.

In this stage, the mechanism assigning students to departments is set by each college. There are three the most common mechanisms, the parallel mechanism, the gradational priority mechanism, and the sequential mechanism. In this section, we will introduce them in detail.

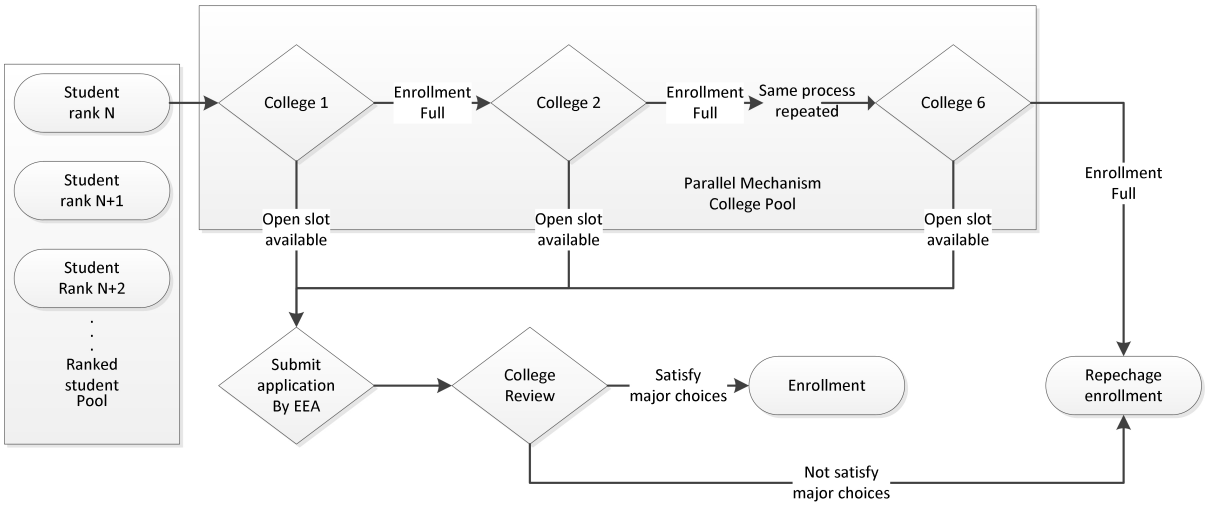

Figure C.1 The College Admission Diagram

C.1.6.3.1 Parallel Mechanism (Deferred Acceptance)

The parallel mechanism for the department assignment resembles the one for the college assignment. Once all applications are submitted to a college, the college sorts the students from the highest score to the lowest score in a strict order. Tie-breaking rules are usually the same as the one used by the local EEA. Starting from the highest score student, each student applies for parallel departments in the order of her listed ranking. The student is assigned to the first department with an unfilled quota. The key difference is that all colleges using the parallel department-assigning mechanism do not divide departments into choice bands. Therefore, the parallel mechanism used within a college is, in fact, the DA mechanism.

Since the reviewed number is 105% of the quota, some students remain unassigned by the end of the mechanism. If the student is willing to accept any department assignment, she will be admitted by departments with unfilled quotas or departments with enough resources to increase enrollment. Otherwise, the application is returned.

C.1.6.3.2 Score Gradation based on Preference Order

The second most common within-college mechanism is the gradational priority mechanism. This mechanism is similar to the parallel mechanism besides a key feature of the priority structure. While the determinant of a student's priority is still the overall Gaokao score, this priority score is reduced by a series of preset changes following the department preference orders. For example, let us assume a college follows a gradation of $\{1,1,0,0\}$ in priority. If a student list Department A as her first choice, her priority score is the same as her Gaokao score when compared with other applicants for Department A. If the student list Department B as her second choice, her priority score becomes the Gaokao score minus one when compared with other applicants for Department B.

To make it more specific, consider the following five applicants reviewed by College U, as shown

in table C.5. Let the department admission quotas are as shown in table C.6. Further, assume that College U follows a gradation of $\{1,1,0,0\}$ in priority.

Since Student 1 has the highest score, she is the first to apply for departments. Student 1 applies to the Department of Physics, which is her first choice. The Department of Physics has one quota. Therefore, Student 1 is admitted by the Department of Physics.

In the next step, Student 2 applies to the Department of Chemistry, which is her first choice. The Department of Chemistry has one quota. Therefore, Student 2 gains a seat in the Department of Chemistry.

The third applicant is Student 3. He first applies to the Department of Physics, which is his first choice. However, the quota of the Department of Physics is filled. So he moves on to the second choice, the Department of Chemistry. At this time, the gradational priority kicks in. Since Student 3 lists the Department of Chemistry of the second choice, his priority score is now $620-1=619$ when compared to other students applying for the Department of Chemistry. Unfortunately, the quota of the Department of Chemistry is also filled.

Now Student 3 applies for the Department of Biology. Because he lists the Department of Biology as the third choice, his priority score becomes $620-1-1=618$ among other Biology applicants. Student 4, who lists the Department of Biology as the first choice, has a priority score of 619. Since $619>618$, Student 4 gets to apply for the Department of Biology first. The Department of Biology admits Student 4.

Student 3 has to move to the fourth choice, the Department of Management. Since the changes in priority between the third choice and the fourth choice are zero, Student 3's priority remains 618. Since the quota of the Department of Management is unfilled, Student 3 gets in.

Student 5 is the last one to apply. However, both departments listed by Student 5 are filled. If the student agrees to take any department assigned to her, she is admitted to the Department of Geology, which still has an available seat.

C.1.6.3.3 Sequential Mechanism (Immediate Acceptance)

The last mechanism used to place students into departments is the sequential mechanism. This mechanism was widely used since the beginning of the Gaokao. In recent years it has lost popularity among colleges. There are still a few colleges using this mechanism for in-school assignments.

The sequential mechanism lets students apply for their first choices. Departments will accept students according to the quotas. Once a student is accepted, the offer is permanent. In fact, the sequential mechanism is the IA mechanism in the matching literature.

Table C.2 Student Counts per Score of the Science Track, 2019 Beijing Gaokao

Score	Number of Students	Accumulated Number of Students
≥700	24	24
699	7	31
698	8	39
697	10	49
696	8	57
695	10	67
694	8	75
693	6	81
692	12	93
691	15	108
690	9	117
689	12	129
688	8	137
687	8	145
686	20	165
685	20	185
684	18	203
683	25	228
682	23	251
681	20	271

Notes: The overall scores are out of 750.

Table C.3 The Enrollment Quotas of Tsinghua University in Beijing (2019)

Major	Quota type	Level	Track	Quota
Social Science Advanced Program	General Admission	Undergrad	Science	1
Social Science Advanced Program	General Admission	Undergrad	Arts	1
Human Science Advanced Program	General Admission	Undergrad	Arts	1
Economics and Finance	General Admission	Undergrad	Science	6
Economics and Finance	General Admission	Undergrad	Arts	2
Law	General Admission	Undergrad	Arts	2
English	General Admission	Undergrad	Arts	1
Journalism	General Admission	Undergrad	Science	1
Journalism	General Admission	Undergrad	Arts	1
Math and Applied Math	General Admission	Undergrad	Science	2
Mathematical Science	General Admission	Undergrad	Science	2
Bioscience	General Admission	Undergrad	Science	1
Mechanical Engineering	General Admission	Undergrad	Science	10
Vehicle Engineering	General Admission	Undergrad	Science	7
Material Science	General Admission	Undergrad	Science	9
Energy and Power Engineering	General Admission	Undergrad	Science	3
Electric Engineering and Automation	General Admission	Undergrad	Science	11
Computer Engineering	General Admission	Undergrad	Science	25
Automation	General Admission	Undergrad	Science	8

Table C.4 Enrollment Statistics of the Arts Track Students in Guangdong(2019)

University Code	University Name	Enrollment Quota	Actual Enrollment	Application Submission Threshold	Ranking of Threshold Score (by lowest)
10001	Peking University	21	21	652	30
10002	Renmin University of China	52	55	637	131
10003	Tsinghua University	2	4	658	10
10004	Beijing Jiaotong University	7	7	599	2234
10007	University of Science and Technology Beijing	4	4	604	1634
10010	Beijing University of Chemical Technology	19	19	573	7882
10012	Beijing Institute of Fashion Technology	8	5	546	20649
10016	Beijing University of Civil Engineering and Architecture	2	2	552	16706
10022	Beijing Forestry University	17	17	585	4546

Table C.5 Example: Student Department Preferences

Student	Gaokao Score	Department Preference			
1	630	Physics			
2	629	Chemistry			
3	620	Physics	Chemistry	Biology	Management
4	619	Biology	Management		
5	617	Management	Biology		

Table C.6 Example: Department Admission Quotas

Department	Admission Quota
Physics	1
Chemistry	1
Biology	1
Management	1
Geology	1

University Enrollment Form of 2019 Beijing Gaokao

(Submit between June 25 8:00 – June 29 20:00)

Student ID: Name:

Tier	University		Item	Code	Full Name	Item	Code	Full Name
Undergraduate General Admission	Parallel Mechanism	A	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		B	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		C	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		D	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		E	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		F	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		G	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		
			Major 3			Major 6		
		H	University			Flexible on major assignment: □Yes □No		
			Major 1			Major 4		
			Major 2			Major 5		

Figure C.2 College Application Form for the Beijing Gaokao(2019)