Stochastic finite element analysis of nozzle corner response

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ABSTRACT

A light water pressure reactor vessel excited by a static internal pressure is presently the object of a wide research at the JRC of Ispra. Attention is focused in this paper on the connection between the vessel and the nozzle where cracks can propagate. The mechanical and geometrical properties of the cladding are the main sources of uncertainty on the data: they are correctly modeled as random fields. Structural analysis is non linear and conducted on a finite element discretization of the system. In addition to the probability distribution of the response variables of interest (stochastic finite element analysis), their correlations are of basic importance for reliability assessment. The paper proposes a procedure for estimating these correlations.

1. INTRODUCTION

Long term objective of this research is the development of probabilistic models for the estimation of the residual lifetime of components undergoing damage-accumulation processes. Validation of the analytical models and development of special diagnostic techniques is pursued by parallel experimentation on scaled models. In (Lucia, 1982) and (EUR Report) the ongoing fatigue tests on 1:5 scale models of Pressurized Water Reactor (PWR) vessels (Figure 1a) are illustrated. Their aim was to investigate the problem of crack formation and propagation by mechanical fatigue. A test-ring now under construction will be used for studying the effect of repeated-termal-shocks-on-service induced defects.

In view of life prediction and structural reliability analysis for the pressure vessel, the propagation of the cracks located on, or very close to, the interface ferritic-austenitic on vessel wall predominates. This propagation is strongly affected by the condition of the cladding, whose high residual stresses (Bertram, 1975; Lidbury, 1984; Jovanovic, Lucia and Lombardini, 1985) were shown:

i) to make faster the propagation of underclad cracks (Dufresne et al., 1983; Lucia and Volta, 1983);

ii) to increase the stress intensity factor K_I (Jovanovic and Lucia, 1986).

The effects of the conditions of the cladding are magnified in the region of highest stress, i.e. in the nozzle inner corner, section meridian to vessel. This region was discretized in finite element meshing for ADINA code as shown in Fig. 1b).
The structural response depends upon cladding residual stresses whose uncertain nature (as well as the one of its thickness and of the hardening ratio of its material) is prone to a random field description, or, after discretization, to a random vector idealization. Therefore,the stochastic finite element procedure developed in (Faravelli, 1986) was introduced in (Carli, Faravelli and Lucia, 1986) in order to find the cumulative distribution functions of response parameters like stresses in the vessel and inelastic strains in the cladding. This result is of basic importance in view of reliability assessment but has to be completed by estimation of the correlation among these response parameters. An appropriate methodology of correlation analysis is formulated in this paper.

2. STOCHASTIC MODEL OF THE PWR VESSEL

The probabilistic definition of the uncertain variables introduced in (Carli, Faravelli and Lucia 1986) is summarized in Table 1 where one assumes as random variables the Young modulus of the basic material E_b, the Young modulus of the cladding E_c and the yielding stress in the cladding σ_y. The vectors E_b, E_c and σ_y in the different finite elements in which the vessel is discretized (Figure 1b) are idealized by sets of strictly correlated equally distributed random variables. The cladding properties (hardening ratio H, thickness t, and residual stress σ_R) are modeled as random vectors: each of them is described by the random variable which defines the spatial average and the random vector of the deviations from this spatial average. Therefore the structural problem is characterized by six random variables x_j and three random vectors describing the spatial variability of three different element properties (Table 1).
The non-linear analysis is conducted for a static internal pressure which is increased from 0 to the final value 18 N/mm^2 used in the laboratory tests. Attention is focused on the connection between the vessel and the nozzle: for the numerical analysis is sufficient to study the quarter whose finite element idealization is shown in Figure 1b. The code ADINA is used and tridimensional iso-parametric elements with 20 nodes and 27 Gaussian points are considered.
A bilinear constitutive law is adopted.
The response variables selected along the line drawn in Figure 2 are the equivalent stress in point $C(y_C)$ and the inelastic strain in point $A(y_A)$.
Introduce an appropriate transforamation Y of each response variable y, and the standardized variable:

$$X_j = (\ln x_j - E\left[\ln x_j\right])/(\mathrm{Var}\left[\ln x_j\right])^{\frac{1}{2}}$$

for any j. $E[\cdot]$ denotes the mean value and $\mathrm{Var}[\cdot]$ the variance.
The model of the dependence of Y on $\underline{X}$ is, in matrix notation:

$$Y = \theta_o + \underline{X}^T \theta_1 + \underline{X}^T \underline{\Theta}\, \underline{X} + \varepsilon = \hat{Y} + \varepsilon \qquad (01)$$

where ε takes into account both the model error and the effects of the

vectors of the deviations from the spatial averages of t, H and σ_R. Eq.(01) is a response surface expression and ε can be treated by one-way (or multi-way) ANOVA as shown in (Faravelli 1986).
The best transformation Y of y selected in (Faravelli, 1986) in order to reach the greater accuracy in the response surface modeling is:
- for the stress in C

$$Y_C = \ln[(\bar{y} - y)]^{k}.$$

where $\bar{y}$ is 490 N/mm^2 and k = 1
- for the strain in A

$$Y_A = y_A^{k}$$

with k equal to .05. In the following, however, one assume k = 1 for both the response variables.
The coefficinets θ_0, $\underline{\theta}_1$ and $\underline{\Theta}$ (whose total number is 28) are found by regression analysis on the basis of experimental results. The experiments have to be appropriately planned. In (Faravelli, 1986) one considered a "composite central design" with six central points for the estimation of pure error:n=82 is the total number of experiments to be conducted. For the response variable Y_C one finds that the pure error sum of square divided by the number of degrees of freedom (= 5) is s_ε^2 = .053, while the lack of fit sum of squares divided by the appropriate number of degrees of freedom (81-5-27=49) is s_1^2 = .065.
The ratio $\lambda = s_1/s_\varepsilon$ is therefore 1.1.
The same quantities for Y_A are: $s_1^2 = 1.55\ 10^{-6}$; $s_\varepsilon^2 = 1.67\ 10^{-6}$; λ = .96. The ratio λ is regarded in (Faravelli, 1986) as a measure of goodness of fit of the model, λ = 1 being the optimal solution. Once the regression coefficients are known and the variance s_ε^2 of the zero-mean variable ε is calculated, a level-2 reliability method can be applied to Eq.(01) in order to derive the cumulative distribution function (CDF) of Y and hence its mean and variance. Generally, the single variable Y will not be normally distributed even if the X_j' s are Gaussian.

3. CORRELATION ANALYSIS

When the stochastic finite element is solved, both the CDF of Y_A and Y_C are known. One is then interested in their joint distribution or, at least, in their correlation.
For this purpose update Eq.(01) for Y_A in:

$$Y_A = \theta'_{0A} + \underline{X}^T \underline{\theta}'_{1A} + \underline{X}^T \underline{\Theta}_A \underline{X} + c\ Y_C + \varepsilon'_A = \hat{Y}_A + \varepsilon'_A \qquad (02)$$

The solution of this new regression problem provides:
- the pure error, measured by $s_{\varepsilon'}^2 = 1.43\ 10^{-7}$; the large reduction on this term, which mainly accounts for the deviations from the spatial averages of the random vectors H, t and σ_R, shows how this effect is already included in Y_C;
- the lack of fit, measured by $s_{1'}^2 = 3.95\ 10^{-7}$; the previous statement is here confirmed, even if the presence of actual lack of fit makes less significant the reduction on its measure;
- the regression sum of sq. $\Sigma\hat{Y}_A^2 - (\Sigma Y_A)^2/n = 9.16\ 10^{-5}$ (to be compared with $2.71\ 10^{-5}$ obtained from Eq.(01)) against a total sum sq. $\Sigma Y_A^2 - (\Sigma Y_A)^2/n$ of

$11.14 \; 10^{-5}$. Therefore the squared multiple correlation coefficient which measures the strenght of the regression is

$$R^2 = 9.16 \; 10^{-5}/11.14 \; 10^{-5} = .822$$

for Eq.(02) while it was .2432 for Eq.(01) due to the large weight of the random vector deviations. R = .907 can be viewed as being the simple correlation coefficient between Y_A and $\hat{Y}_A$, $R_{Y_A \hat{Y}_A}$.

- The squared partial correlation coefficient

$$r^2_{Y_A Y_C | \underline{X}} = (s_1^2 - s_{1'}^2)/s_1^2 = .745 \qquad (03)$$

which is an estimate of the proportional reduction in the conditional variance of Y_A given $\underline{X}$ due to conditioning on both $\underline{X}$ and Y_C. According to (Kleinbaum and Kupper, 1978) the term $r_{Y_A Y_C | \underline{X}}$ in Eq.(03) can be regarded as the correlation between Y_C and the dependent variable Y_A when the effects of the other independent variables have been removed from both Y_C and Y_A, i.e. the correlation of the residuals of the regressions of Y_A on $\underline{X}$ and Y_C on $\underline{X}$. Therefore, the residuals ε_A and ε_C in Eq.(01), can be assumed jointly mormal with:

$$E[\varepsilon_A] = E[\varepsilon_C] = 0$$

$$Var[\varepsilon_A] = .053 \qquad Var[\varepsilon_C] = 1.55 \; 10^{-6}$$

$$\rho_{\varepsilon_A \varepsilon_C} = r_{Y_A Y_C | \underline{X}} = .863 \; |c \; |/c$$

This means that the conditional distribution of ε_A given ε_C is Gaussian with:

$$E[\varepsilon_A | \varepsilon_C] = (Var[\varepsilon_A]/Var[\varepsilon_C])^{\frac{1}{2}} \; \varepsilon_C \; \rho_{\varepsilon_A \varepsilon_C}$$

$$Var[\varepsilon_A | \varepsilon_C] = Var[\varepsilon_A](1 - \rho^2_{\varepsilon_A \varepsilon_C})$$

Of course $r^2_{Y_A Y_C | \underline{X}}$ can also be estimated by writing Eq.(02) in the form:

$$Y_C = f(\underline{X}) + a \; Y_A + \varepsilon'_C = \hat{Y}_C + \varepsilon'_C$$

In this case one finds: $s^2_\varepsilon = 2.39 \; 10^{-3}$; $s^2_{1'} = 1.648 \; 10^{-3}$; $R^2 = 4.395/5.1977 = .846$ (against $R^2 = .3399$) and $r^2_{Y_A Y_C | \underline{X}} = .745$.

4. CONCLUSIONS

Reliability assessment requires often the knowledge of the joint distribution of the response variables. Stochastic finite element techniques express the single response variable as a function of the input random variables and a residual (error) term with zero mean and calculated variance. This paper shows how the correlation between residuals can be computed in order to have a satisfactory probabilistic joint description of the response variables.

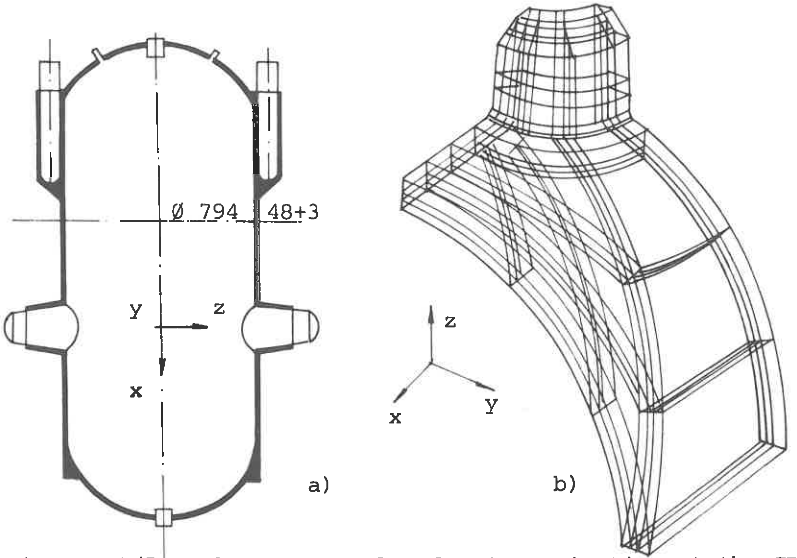

Figure 1 - a) 1/5 scale LWR vessel under investigation at the JRC;
b) meshing for stochastic finite element analysis.

Table 1 - Stochastic model of the LWR pressurized vessel. Probabilistic definition of the uncertain variables.

Physical Quantity	Symbol	Correlation among the vector elements,	Nominal value	Central value (scalar)			Deviation (vector)		
				Type of distribution	Median	Log. standard deviation	Type of distribution	Median	Standard deviation
				Components of vector $\underline{X}_A$			Spatial variabilities		
Young's modulus (ASME SA 508 & SA 533)	$E_b=E_b^*E_{bo}$	1.00	E_b^* = 209000 N/mm²	E_{bo} Lognormal	1.0	.02	....		
Young's modulus (AISI 347)	$E_c=E_c^*E_{co}$	1.00	E_c^* = 168000 N/mm²	E_{co} Lognormal	1.0	.04	....		
Yielding stress (AISI 347)	$\sigma_y=\sigma_y^*\sigma_{yo}$	1.00	σ_y^* = 366.1 N/mm²	σ_{yo} Lognormal	1.0	.05	....		
Post-yielding stiffness (AISI 347)	$\underline{H}=H^*H_o\underline{H}'$	0.75	H^* = 2187 N/mm²	H_o Lognormal	1.0	.0866	H_i' Lognormal	1.0	.05
Thickness	$\underline{t}=t^*t_o\underline{t}'$	0.75	t^* = 3 mm	t_o Lognormal	1.0	.1719	t_i' Lognormal	1.0	.10
Residual stress σ_R	$\underline{\sigma}_R=\max((410\, r^*r^\circ r'/\gamma, \underline{\sigma}_y)$	0.50	r^* = 160 N/mm²	r_o Lognormal	1.0	.14074	r_i' Lognormal	1.0	.14075

* γ=1, 1.2 or 1.5 in vessel, nozzle and connection respectively

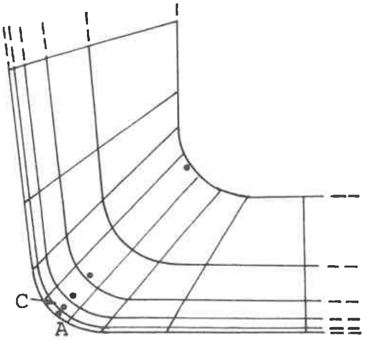

Figure 2 - Points A and C where the structural response is investigated.

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