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Probabilistic calibration of safety coefficients for flawed components in nuclear engineering

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ABSTRACT: The rules that are currently under application to verify the acceptance of flaws in nuclear components rely on deterministic criteria supposed to ensure the safe operating of plants. The interest of having a precise and reliable method to evaluate the safety margins and the integrity of components led Electricité de France to launch an approach to link directly safety coefficients with safety levels. This paper presents a probabilistic methodology to calibrate safety coefficients in relation to reliability target values. The proposed calibration procedure applies to the case of a ferritic flawed pipe using the R6 procedure for assessing the integrity of the structure.

NOMENCLATURE

X, Z, U:	random variables
μ_x, σ_x:	mean and standard deviation of variable X
V_x:	coefficient of variation of variable X, defined by $Vx = \sigma_x / \mu_x$
β:	reliability index
Pf:	failure probability
$G_x(X)$:	limit state function in the basic space
$G_u(U)$:	limit state function in the standardized variables space
design point:	the most probable failure point in the U-space
u*:	vector of the coordinates of the most probable failure point
x^k:	characteristic value of variable X located at k_x standard deviations
γ_x^k or γ_k:	partial multiplication safety coefficient associated with value x_k
θ_x:	partial safety coefficient associated with $\theta_x = \gamma_x$ if X is a load type variable, $\theta_x = 1/\gamma_x$ if X is a resistance type variable.
x^d:	design value of variable X, such that $x^d = \gamma_k . x^k$
Θ or M:	central safety factor (or coefficient) = margin
Θ_k or M_k:	characteristic safety factor (or coefficient) = margin

1 INTRODUCTION

The rules that are currently under application to verify the acceptance of flaws in nuclear components rely on deterministic criteria. Up to now, their application has avoided any serious incident due to equipment in France. But is should not hide the need to make a better evaluation of the relation between the actual risk level and the selected safety coefficients, on which the design is still based. This is the aim of the probabilistic approach launched by Electricité de France.

In this paper we assume that the main reliability concepts described by Madsen (1986) are known. We use a level 3 approach, in which the FORM/SORM methods presented by Tvedt (1989) are involved, and which allows to take better account of probability distribution tails. We also assume that the code calibration vocabularies are known (characteristic values x^k, design values x^d, safety coefficients Θ_k).

We present the calibration of partial safety factors as a function of the reliability target level, for the case of a flawed ferritic pipe.

2 THE CASE OF A FLAWED PIPE

2.1 *Presentation of the case study*

This example is designed to illustrate the probabilistic methodology for a complex case with many non-gaussian input variables and different failure modes. Although it is based on realistic values of the input parameters, this example doesn't provide any values for use in the French code «RSEM». It is a pipe of thickness t and outside diameter ϕe. It has a semi-elliptical defect visible on the internal surface, with height a, and very long. The load stresses the flawed area with a stress σ_∞ composed of a primary stress σ_p and a secondary stress σ_s arising from loads which do not contribute to plastic collapse, like thermal gradients.

The material is ductile steel in the temperature domain considered. Figure 1 shows a flow chart of the analytical model with the four safety coefficients involved in the failure modes (two calculations at a, two calculations after ductile crack growth Δa), that is respectively: Ma, Mi, Mra, Mraa. Other input variables are defined in table 1 below which shows their reference distributions. This model is drawn from the R6-rule presented by Milne (1988), that gives the expression of the plastic correction applied to the elastic stress intensity factor.

Figure 2 presents events involved in the definition of the failure modes considered. Notations used are: E = event, En = complementary event. If the event E occurs, the corresponding margin is inferior to its critical value (e.g. 1). This study only deals with the risk of tearing initiation E_{ini}, containing several initiation modes (tearing and plastic failure), for which following simplified expression is available.

$$E_{ini} = \text{EMra} \cup (\text{EnMra} \cap \text{EMa}) \qquad (3.1)$$

2.2 *Importance factors*

The risk of initiation E_{ini} can be represented by its main event EMa ; the main advantage of simplifying E_{ini} is that EMa is directly related to a global safety coefficient, Ma, which also is a limit state function. The corresponding reliability index is presented in figure 3.

Table 1 - Model input variable distributions

Description	Variable	Unit	Probability distribution	Mean	Standard-deviation or coefficient of variation
Thickness	t	m	-	0.04	-
outside diameter	Øe	m	-	0.812	-
Young's modulus	E	MPa	Lognormal	191000	10000
Poisson's coefficient	ν	-	-	0.3	-
flaw size	a	m	Lognormal	0.005	0.001
flaw growth	Δa	m	-	0.003	-
yield stress	σ_y	MPa	Normal truncated to 2 st.-dev	212	16
ultimate tensile strength	σ_u	MPa	Normal truncated to 2 st.-dev	525	30
gradient	dJ	MPa	Lognormal	65	13
initiation energy	J_{02}	MN/m	Lognormal	0.109	0.033
stress	σ_∞	MPa	Lognormal	Variable from 150 to 600	10%

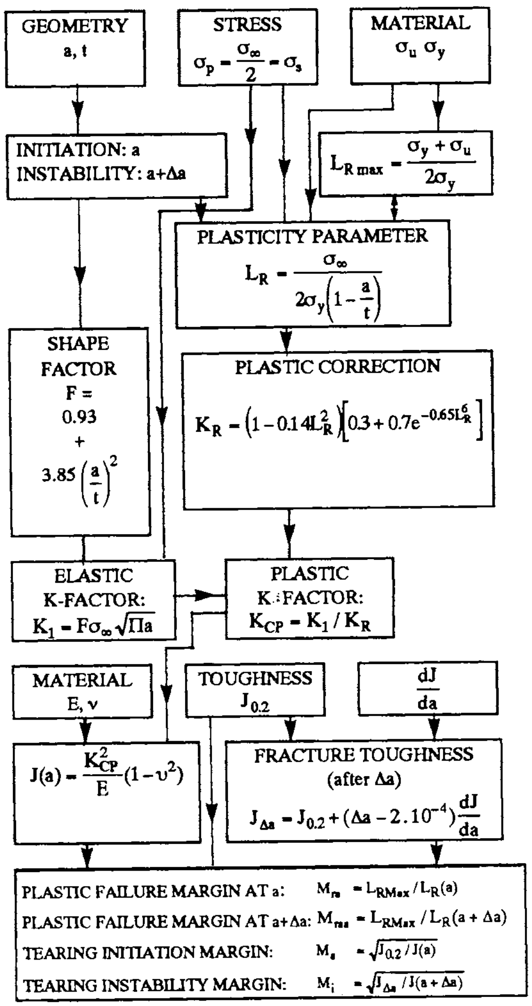


Figure 1 - Flow chart of the analytical model

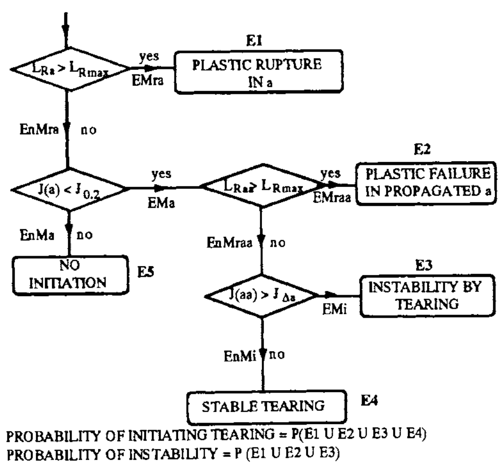
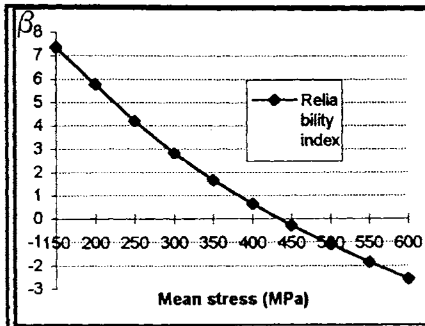


Figure 2 - Schematic representation of failure modes considered

As shown by figure 4, the stress is the main importance factor for the risk of tearing initiation, in other words the variable for which the dispersion has most influence on probability. It is followed by "a" and the yield stress. This explains why the results are presented as a function of the mean stress, which becomes the chief parameter of the model: to a given reliability level corresponds a value of the mean stress.



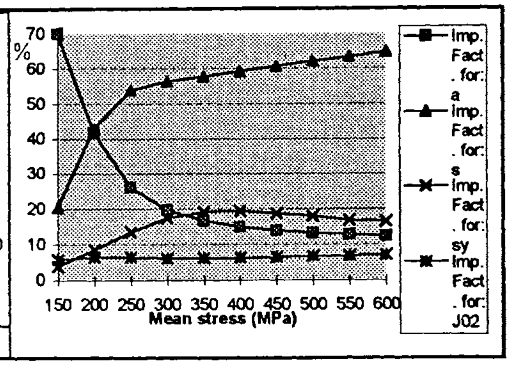


Figure 3 - Reliability index for the risk of initiation of tearing as a function of the mean stress

Figure 4 - Importance factors for the risk of initiation of tearing as a function of the mean stress

2.3. *Calibration of partial coefficients for a given situation*

This is the most important part of this paper. A situation refers to a case in which input distributions of variables are known. For a given failure mode, the code must fix a target reliability value for this situation, and proposes a choice of partial coefficients and characteristic values. The failure mode is here the event EMa. We assume that:

- the reliability objective is the target value β^T (e.g. $\beta^T = 3.09$, for $P_f = 10^{-3}$);
- all input distribution parameters are fixed at their reference values, except the mean stress, which set the link with the reliability target level; therefore we have a given set of known characteristic values, x_i^k (for i = 1 to n-1), plus $x_n^{k^T}$ corresponding to the characteristic value of σ_∞, and determined by β^T;
- there are only four random variables: σ_∞, σ_y, a, $J_{0.2}$; the other variables are fixed at their mean value, as their uncertainties have no importance on the probability;
- the characteristic values are all located at one standard deviation from the mean value, at +1 standard deviation for load type variables (σ_∞ and a) and -1 standard deviation for resistance type variables (σ_y and $J_{0.2}$). This choice will be justified later.

As afore mentioned, if we want to vary the reliability objective β^T, at least one distribution parameter has to be varied. We have selected the mean stress value. Thus, a target reliability of 10^{-3} is equivalent to a mean stress of 290 MPa. To calibrate the partial safety coefficients as a function of β^T, we firstly propose to use as design values the values of the coordinates of the basic variable space point corresponding to the most probable failure point of the U-space (i.e. the design point). This is a well-known method recommended by many practitioners of code calibration (Melchers, 1988). Madsen (1986) even demonstrated that the design point is an optimum when the basic variables are gaussian and the limit state function is linear.

Figure 5 shows the variation of the four coefficients θ_x^k, defined in the nomenclature, as a function of the index β^T. Note that they are all increasing, which agrees with the variation expected for this type of coefficient. The values of the coefficients on "a" and σ_∞ are the highest. The coefficient of "a" varies more quickly starting from $\beta^T = 4$, corresponding to the fast variation in the importance of "a" in the study of importance factors (see figure 4). For a target reliability index of 3.09, corresponding to a probability of 10^{-3}, we have:

$$\theta_a^k = 1.08,\ \theta_{\sigma_\infty}^k = 1.14,\ \theta_{\sigma_y}^k = 1.01,\ \theta_{J_{0.2}}^k = 0.92 \tag{3.2}$$

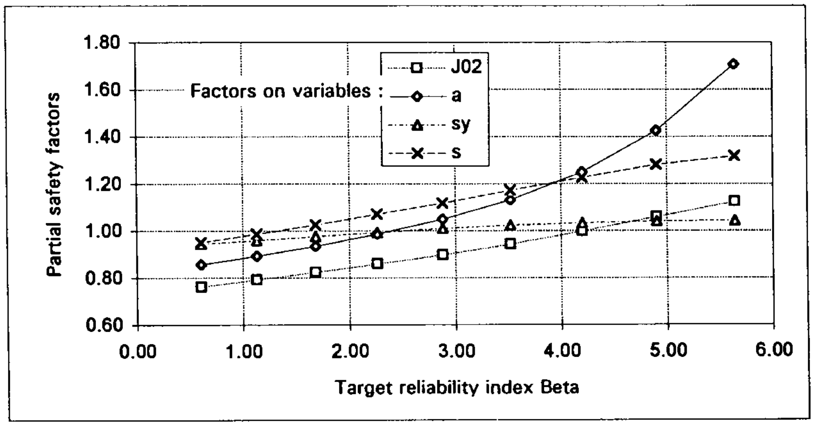


Figure 5 - Variation of partial coefficients as a function of the target reliability index

Secondly, we are interested in having a relation between the partial safety factors depending on the reliability target value. This relation can be obtained by remembering that β^T concerns the event EMa, that is «$Ma(a, \sigma_\infty, \sigma_y, J_{0.2}) < 1$», as the other basic variables are fixed at their mean value. This expression of EMa also gives the design equation that must verify the design values:

$$Ma\left(a^d, \sigma_\infty^d, \sigma_y^d, J_{0.2}^d\right) = 1 \tag{3.3}$$

Conversely, if a set of four values follows this equation, they are on the limit state surface and can be considered as design values shown in table 1. Thus the characteristic values a^k, σ_y^k and $J_{0.2}^k$ are fixed. The distribution of σ_∞ is then determined by the value of β^T, corresponding to the mean stress $\mu_{\sigma_\infty}^T$ and characteristic value $\left(\sigma_\infty^k\right)^T$. Using the notations of the nomenclature, the equation (3.4) becomes:

$$Ma\left(\theta_a^k\, a^k,\ \theta_{\sigma_\infty}^{k^T}\, \sigma_\infty^{k^T},\ \frac{\sigma_y^k}{\theta_{\sigma_y}^k},\ \frac{J_{0.2}^k}{\theta_{J_{0.2}}^k}\right) = 1 \tag{3.4}$$

or

$$Ma_{a^k, \sigma_y^k, J_{0.2}^k, \left(\sigma_\infty^k\right)^T}\left(\theta_a^k, \theta_{\sigma_\infty}^k, \theta_{\sigma_y}^k, \theta_{J_{0.2}}^k\right) = 1 \tag{3.5}$$

The last equation gives an implicit relation between the partial safety factors depending on the target reliability index. There are 3 degrees of freedom in this relation, therefore a great flexibility is introduced. This implicit relation can be linearized using the least square regression around the characteristic point. For $\beta^T = 3.09$, the following simplified relation is obtained:

$$1.83\,\theta_a^k + 4.87\,\theta_{\sigma_\infty}^k + 2.87\,\theta_{\sigma_y}^k + 0.93\,\theta_{J_{0.2}}^k = 11.11 \qquad (3.6)$$

Replacing the coefficients by their value from formula (3.2), the first member has a value of 11.28, close to the expected value ; the design point is on the limit state surface and the coordinates of its image verify (3.3). This result is satisfactory since relation (3.6) is a linear approximation, that should be used carefully as a rough check as follows:

- desired values are given to three of the factors;
- the linear relation is used to check the value of the last factor;
- its real value is calculated using the exact implicit relation.

Note that the $\theta_{\sigma_\infty}^k$ coefficient is the highest, and the $\theta_{J_{0.2}}^k$ coefficient is the lowest. This confirms the predominant role of the load on the failure probability, and the lesser role of toughness.

Note also that this approach is drawn from an analogy with the classical (R, S) case presented by Muzeau (1991).

3 CONCLUSIONS

The probabilistic method is applied to the complex case of a flawed pipe in which four main random variables and two failure modes are considered. The study concerns the risk of tearing initiation and a given situation. It shows that a relation between partial safety factors can be obtained for any reliability target level, which introduces flexibility in their choice.

Then it is possible to calibrate each coefficient using an optimization criteria, such as the choice of the most probable failure point, the so-called design point. Other problems can be treated:

- simultaneous consideration of several failure modes, by the code;
- taking account of several situations;
- other choices of characteristic values.

They will be solved by optimization methods.

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