Approximate seismic analysis of piping or equipment mounted on elastoplastic structures

R.Villaverde
University of California, Irvine, USA

1 INTRODUCTION

In recognition of the importance and complexity of the seismic design of the electrical and mechanical equipment in nuclear power plants and other critical facilities, several approximate methods have been proposed to facilitate such a design. For the most part, however, these approximate methods have been restricted to linear systems, without due consideration to the fact that most of the structures to which equipment is attached are designed to yield under the effect of a strong earthquake. As a result, such methods may overestimate the design seismic forces, and thus they may lead to uneconomical and unrealistic designs.

Few investigators have attempted the derivation of simplified methods for the nonlinear analysis of secondary systems. Most of the effort has been directed towards the development of amplification factors by which one can affect linear floor spectra to take into account the nonlinearity of a supporting structure (Kawakatsu et al. 1979, Lin & Mahin 1985, Viti et al. 1981). This approach, however, may not always be practical or accurate. Many factors influence the response of equipment mounted on a nonlinear structure, and thus it would be extremely difficult to generalize for all the possible configurations a structure might take. In addition, the use of floor response spectra may become even more cumbersome than in the linear case when the secondary system is attached to its supporting structure or primary system at more than one point. Furthermore, with such an approach it is difficult to visualize the effect of changes in the structure or the equipment, which is something that may be helpful in their preliminary designs.

The purpose of this paper is to present an alternative approximate method for the seismic analysis of light secondary systems attached to a nonlinear structure. The method, herein summarized and described in detailed elsewhere (Villaverde 1986), is intended to serve as a tool in preliminary designs, and as such its simplicity is emphasized over its accuracy. It is formulated for elastic multidegree of freedom (MDOF) secondary systems connected at one or two arbitrary points of an elastoplastic MDOF primary structure, when the structure is subjected to a specified ground excitation. However, it is restricted to primary and secondary systems that when independently considered have separated natural frequencies and proportional damping, and those cases in which the mass of the secondary system is small when compared with the mass of the primary one. Throughout its derivation, it assumed too that the damping forces in the combined system remain linear at all levels of response.

2 APPROACH

The method is formulated on the basis of: (a) the consideration of a primary and a secondary system as a single combined system; (b) the derivation of simplified formulas to determine the initial natural frequencies, mode shapes, and damping ratios of such a combined system in terms of the corresponding parameters of the independent components; and (c) a formulation to express the maximum response of the combined system, and hence that of the secondary system, in terms of its initial natural frequencies and mode shapes, and the nonlinear response spectrum of a specified ground motion. To account for the fact that a combined primary-secondary system cannot be considered classically damped, the analysis is based on the modified response spectrum method proposed by Villaverde (1980) for systems with nonproportional damping. In like manner, to validate a linear modal analysis in the response calculation of a nonlinear system, use is made of the modal superposition method suggested by Villaverde (1987) for the analysis of nonlinear structures.

3 APPROXIMATE PROCEDURE

Consider a secondary system attached to one or two arbitrary points of a supporting primary structure. Let N_p denote the number of degrees of freedom of the independent structure and N_s that of the secondary system when it is considered fixed at its points of attachment with the structure. In terms of the parameters of such independent primary and secondary systems, the steps for the calculation of the maximum distortions in the secondary system, when the primary one is subjected to a ground motion specified by its nonlinear response spectra, are as follows:

1) Determine the N_p circular natural frequencies ω_{pi}, unit-participation-factor mode shapes $\{\Phi\}_i$, generalized masses M_i^*, and modal damping ratios ξ_{pi} of the independent structure using its initial elastic properties.

2) Determine the N_s circular natural frequencies ω_{sj}, unit-participation-factor mode shapes $\{\phi\}_j$, generalized masses m_j^*, and modal damping ratios ξ_{sj} of the secondary system considering it fixed at its points of connection with the structure.

3) Assume the combined primary-secondary system is a system with N_p+N_s degrees of freedom whose natural frequencies and damping ratios are those of its independent components. Classify a mode of this combined system as a resonant mode if its natural frequency is common to both independent components, and as a nonresonant mode otherwise. In addition, classify each nonresonant mode as one with a frequency of the primary system or one with a frequency of the secondary one.

4) For a given combined system mode, let subscripts I and J respectively identify the parameters of the independent primary and secondary systems in those modes whose frequencies are the closest or coincide with the frequency of the given combined system mode.

5) Calculate the displacements of the secondary system when one of its attachment points is considered free and subjected to a unit force while the other is held fixed. Let $\{\phi\}_c$ be the vector with these displacements, and let f_{cc} be the displacement of the point subjected to the unit force. On the basis of these displacements, construct a vector $\{df\}$ with the distortions of the elements of the secondary system, after they are normalized with respect to f_{cc}. For a secondary system with a single point of attachment, consider $\{df\} = 0$.

6) For each of the component modes, calculate the values of

$$(1) \quad \beta_j = \{\phi\}_j^T [m] \{\phi\}_c / (f_{cc} \{\phi\}_j^T [m] \{\phi\}_0) ;$$

$$(2) \quad \Phi_o(i,j) = \Phi_m(i) + \beta_j [\Phi_n(i) - \Phi_m(i)]$$

where $\{\phi\}_0$ is a vector of size N_s whose elements are unity for the degrees of freedom in the direction of the excitation, and zero elsewhere; $\Phi_m(i)$ and $\Phi_n(i)$ are the amplitudes in the mode shape $\{\Phi\}_i$ of the degrees of freedom of the primary system to which the secondary system is connected. For a single point of attachment, consider $\beta_j = 0$.

7) For each mode of the independent secondary system, construct a vector of mode distortions $\{d\phi\}_j$, whose entries are the distortions in the secondary system when the system, fixed at its points of attachment, vibrates in its jth mode. Similarly, construct a vector of mode distortions $\{d\Phi\}_i$ for each of the modes of the independent structure.

8) Let $SD(\omega, \xi, x_y)$ signify the ordinate corresponding to a natural frequency ω, damping ratio ξ, and yield displacement x_y in the specified nonlinear displacement response spectrum.

9) For any two resonant modes with the same frequency, combined, and any nonresonant mode with a frequency of either the primary or secondary system, compute a vector of maximum modal secondary system distortions, $\{X_s\}_r$, with the formulas given below:

(A) Two resonant modes with equal frequency.- Let $\omega_o = \omega_{pI} = \omega_{sJ}$, $\xi_o = (\xi_{pI}+\xi_{sJ})/2$, $\gamma_{IJ} = m_J^*/M_I^*$, and for any circular natural frequency ω_r and damping ratio ξ_r define an equivalent damping ratio as $\xi'_r = \xi_r + 2/\omega_r s$, where s is an earthquake equivalent duration which may be estimated as proposed by Villaverde (1984). In like manner, let

$$(3) \quad D = [\,|\Phi_o^2(I,J)\gamma_{IJ} - (\xi_{pI} - \xi_{sJ})^2|\,]^{1/2}$$

$$(4) \quad \mu_{sr} = |\Phi_o(I,J)|/2D < |\Phi_o(I,J)|/(\xi_{pI} - \xi_{sJ})^2$$

Then, if $|\xi_{pI} - \xi_{sJ}| > |\Phi_o(I,J)\sqrt{\gamma_{IJ}}|$, calculate $\{X_s\}_r$ as follows:

$$(5) \quad \{X_s\}_r = \mu_{sr} \{d\phi\}_J \sqrt{2(\rho_{mn} - \alpha_{mn})\, SD(\omega_m, \xi_m, x_{ym}) SD(\omega_n, \xi_n, x_{yn})}$$

in which $\omega_m = \omega_n = \omega_o$, $\xi_m = \xi_o - D/2$, $\xi_n = \xi_o + D/2$, and

$$(6) \quad \rho_{mn} = \frac{1}{2}\left[\frac{SD(\omega_m, \xi_m, x_{ym})}{SD(\omega_n, \xi_n, x_{yn})} + \frac{SD(\omega_n, \xi_n, x_{yn})}{SD(\omega_m, \xi_m, x_{ym})}\right] ; \quad \alpha_{mn} = 2\sqrt{\xi'_m \xi'_n}/(\xi'_m + \xi'_n)$$

$$(7) \quad x_{yq} = \{\sum_{k=1}^{N_p} [d\psi_k(q)\, F_{yk}]^2\}^{1/2}/\omega_o^2 \hat{M}_q , \quad q = m,n$$

where $d\psi_k(q)$ are the entries of $\{d\psi_p\}_q = \mu_{pq} \{d\Phi\}_I$ and

$$(8) \quad \hat{M}_q = \mu_{pq} M_I^* + \mu_{sr} m_J^*; \; q = m,n$$

$$(9) \quad \mu_{pq} = [1 \pm (\xi_{pI}-\xi_{sJ})/D]/2 < 1/(\xi_{pI}-\xi_{sJ}), \; q = m,n$$

where the negative sign corresponds to μ_{pm} and the positive to μ_{pn}. However, if $|\xi_{pI} - \xi_{sJ}| < |\Phi_o(I,J)\sqrt{\gamma_{IJ}}|$, $\{X_s\}_r$ is given by

$$(10) \quad \{X_s\}_r = \sqrt{2(1-\alpha_{IJ})}\, \mu_{sr} \{d\phi\}_J\, SD(\omega_o, \xi_o, x_{yo})$$

where

$$(11) \quad \alpha_{IJ} = 1/[1 + D^2/4\xi_o'^2] ; \qquad x_{yo} = \{\sum_{k=1}^{N_p} [d\psi_k(r) \; F_{yk}]^2\}^{1/2}/\omega_o^2 \hat{M}_r$$

in which $d\psi_k(r)$ are the entries of $\{d\psi_p\}_r = \mu_{pr}\{d\Phi\}_I$ and

$$(12)\quad \hat{M}_r = \mu_{pr}M_I^* + \mu_{sr}m_J^* \; ; \qquad \mu_{pr} = \left|\Phi_o(I,J)\sqrt{\gamma_{IJ}}\right|/2D \leq 1/(\xi_{pI} - \xi_{sJ})$$

(B) Nonresonant modes with primary system frequency.-

$$(13)\quad \{X_s\}_r = \{d\psi_s\}_r \; SD(\omega_{pI}, \xi_{pI}, x_{yI})$$

where

$$(14)\quad \{d\psi_s\}_r = A_o'(J)[r_c\{df\} + \sum_{j=1}^{N_s} r_j\{d\phi\}_j] \; ; \qquad A_o'(J) = A_o(J)/\sqrt{1+\delta_{IJ}^2}$$

$$(15)\quad r_c = [\Phi_n(I) - \Phi_m(I)]/A_o'(J) \; ; \qquad r_j = \mathrm{sgn}(1-\delta_{Ij})\, A_o'(j)/A_o'(J)$$

$$(16)\quad A_o(j) = \Phi_o(I,j)\omega_{pI}^2/(\omega_{sj}^2 - \omega_{pI}^2); \qquad \delta_{Ij} = (\xi_{pI}\omega_{pI} - \xi_{sj}\omega_{sj})/(\omega_{pI} - \omega_{sj})$$

and sgn is a function which reads as "the sign of." Similarly,

$$(17)\quad x_{yI} = \{\sum_{k=1}^{N_p} [d\psi_k(r)\; F_{yk}]^2\}^{1/2}/\omega_{pI}^2 M_I^*$$

where $d\psi_k(r)$ are the entries of $\{d\psi_p\}_r = \{d\Phi\}_I$.

(C) Nonresonant modes with a secondary system frequency.-

$$(18)\quad \{X_s\}_r = \mu_{sr}\{d\phi\}_J \; SD(\omega_{sJ}, \xi_{sJ}, x_{yJ})$$

in which

$$(19)\quad \mu_{sr} = \{[1 + \sum_{i=1}^{N_p} B_o'(i)]^2 + [\sum_{i=1}^{N_p} B_o'(i)\delta_{iJ}]^2\}^{1/2}$$

$$(20)\quad x_{yJ} = \{\sum_{k=1}^{N_p}[d\psi_k(r)\; F_{yk}]^2\}^{1/2}/\omega_{sJ}^2\hat{M}_r \; ; \qquad B_o'(i) = B_o(i)/(1+\delta_{iJ}^2)$$

$$(21)\quad B_o(i) = \Phi_o(i,J)\omega_{sJ}^2/(\omega_{pi}^2 - \omega_{sJ}^2) \; ; \quad \delta_{iJ} = (\xi_{sJ}\omega_{sJ} - \xi_{pi}\omega_{pi})/(\omega_{sJ} - \omega_{pi})$$

where $d\psi_k(r)$ are the entries of $\{d\psi_p\}_r = \mu_{sr}B_o'(I)\gamma_{IJ}\{d\Phi\}_I$ and $\hat{M}_r = \mu_{sr}m_j^*$.

10) Estimate the secondary system's maximum distortions by combining the above vectors of maximum modal distortions on the basis of the square root of the sum of the squares.

In using the method, a mode of the combined primary-secondary system with natural frequency ω_r and damping ratio ξ_r is in resonance with a close mode with a natural frequency ω_q and damping ratio ξ_q if

$$(22)\quad \left|\omega_r^2 - \omega_q^2\right|/\omega_r^2 \leq \left|\Phi_o(I,J)\sqrt{\gamma_{IJ}}\right|/\sqrt{1+\delta_{IJ}^2}$$

4 NUMERICAL EXAMPLE

Consider the structure-equipment system shown in Figure 1, when the base of the structure is subjected to the first ten seconds of the N-S component of El Centro, May 18, 1940, earthquake ground acceleration. The structure and the equipment are idealized as 3 and 2-DOF shear-beam systems, respectively, whose parameters and initial dynamic properties are listed in Tables 1 and 2. The damping ratios of the structure and the equipment in their respective fundamental modes are considered to be 5 and 0.1 % of critical, respectively, and their individual damping matrices assumed proportional to their respective stiffness matrices. The load-deformation behavior of each of the floors of the structure is considered elastoplastic defined by the initial stiffnesses and yield strengths given in Table 1.

Table 1. Parameters of structure-equipment system

Mass (Mg)	M_1 3.0	M_2 1.5	M_3 1.0		m_1 0.0009	m_2 0.0003
Stiffness ($4\pi^2$ kN/m)	K_1 9.0	K_2 6.0	K_3 3.0	k_1 0.0054	k_2 0.0009	k_3 0.0006
Yield Strength (kN)	F_{y1} 8.33		F_{y2} 4.00		F_{y3} 2.00	

Table 2. Initial dynamic properties of structure

System	Primary			Secondary	
Mode	1	2	3	1	2
Frequency (Hz)	1.0	2.0	3.0	2.0	$2\sqrt{2}$
Gen. Mass (Mg)	4.5	0.9	0.1	0.0009	0.0003
Damping Ratio	0.05	0.10	0.15	0.0010	0.0014
Mode Shape	0.5	0.4	0.1	0.5	0.5
	1.0	0.2	-0.2	1.5	-0.5
	1.5	-0.6	0.1		

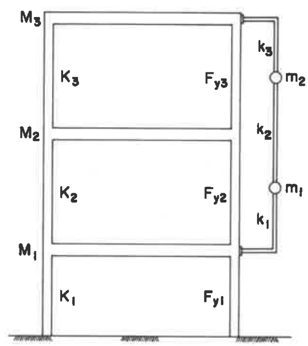

Figure 1. Structure-equipment system in numerical example

According to the established procedure, the combined primary-secondary system represents thus a 5-DOF system whose initial natural frequencies are approximately 1.0, 2.0, 2.0, $2\sqrt{2}$, and 3.0 Hz. As such, the system possesses two resonant modes, two nonresonant modes with a frequency of the primary system, and a nonresonant mode with a frequency of the secondary one. Hence, considering only the first four modes of the combined system, and since in this case $\beta_1 = 0.250$, $\beta_2 = -0.125$, $\{df\} = \{0.0625 \quad 0.3750 \quad 0.5625\}^T$, and the elements of the matrix of $\Phi_o(i,j)$ factors, reading from left to right and from top to bottom, are 0.750, 0.375; 0.150, 0.525; 0.100, 0.100; the equipment's maximum modal distortions are as follows:

For the first mode, I=1 and J=1; therefore, from equations 13 to 17 one obtains $\{d\psi_p\}_1 = \{0.5 \; 0.5 \; 0.5\}^T$, $x_{y1} = 0.0266$m, SD(1.0,0.05,0.0266) = 9.82cm, and $\{X_s\}_1 = \{2.10 \quad 5.60 \quad 2.11\}^T$. For the second and third, resonant modes, I=2, J=1, and thus, from equations 3 to 9, D=0.09889, $\{d\psi_p\}_m = \{0.00022 \quad -0.00011 \quad -0.00045\}^T$; $\{d\psi_p\}_n = \{0.40022 \quad -0.20011 \quad -0.80045\}^T$, $x_{ym} = 0.0111$m; $x_{yn} = 0.0266$m, SD(2.0, 0.00106, 0.0111) = 3.66cm, SD(2.0, 0.09995, 0.0266) = 4.49cm, $\rho_{mn} = 1.118$, $\alpha_{mn} = 0.607$, and $\{X_s\}_2 = \{1.55 \quad 3.11 \quad -4.66\}^T$ cm. For the fourth mode, I=3 and J=2. Then from equations 18 to 21, one gets $\mu_{s3} = 0.606$, $\{d\psi_p\}_3 = \{1.873 \quad 5.618 \quad 5.618\}^T \times 10^{-5}$, $x_{y2} = 0.0052$m, SD($2\sqrt{2}$, 0.0014, 0.0052) = 3.95cm and $\{X_s\}_3 = \{1.20 \quad -2.40 \quad 1.20\}^T$ cm.

Combining all these vectors of maximum modal distortions on the basis of the square root of the sum of the squares, estimates of the maximum distortions in the secondary system result thus as $\{2.87\ 6.84\ 5.25\}^T$ cm.

5 COMPARATIVE STUDY

The accuracy of the proposed approximate procedure is evaluated by comparing the approximate and direct numerical integration solutions of nine different 2-DOF secondary systems attached to a basic 3-DOF elasto-plastic structure, when the base of the structure is subjected to four different earthquake ground motions. Various frequency distributions, points of attachment, and attachment configurations are considered. The details of the study are reported elsewhere (Villaverde 1986), and the results summarized in Tables 3. This table gives the average for the four earthquakes considered of the error percentages obtained when comparing the approximate with the direct integration solutions.

Table 3. Error percentages for secondary systems in comparative study

Element	System								
	A1	A2	A3	B1	B2	B3	C1	C2	C3
1	4.97	7.05	12.8	5.94	-6.86	10.5	29.3	-20.0	-5.28
2	1.76	11.4	7.73	10.0	-14.3	5.53	19.0	-34.5	-2.33
3	-	-	-	-	-	-	32.3	-42.7	2.10

ACKNOWLEDGMENTS

The support of this study by the National Science Foundation through Grant No. CEE-830754 is herein gratefully acknowledged.

REFERENCES

Kawakatsu, T., K. Kitada, T. Takemory, Y. Kuwabara & Y. Okiwara 1979. Floor response spectra considering elasto-plastic behavior of nuclear facilities. Trans. 5th Int. Conf. on Struc. Mech. in Reactor Tech., Berlin. K9/4.

Lin, J. and S.A. Mahin 1985. Seismic response of light subsystems on inelastic structures. J. Struc. Eng., ASCE. 111: 400-417.

Villaverde, R. 1980. Earthquake response of systems with nonproportional damping by the conventional response spectrum method. Proc. 7th World Conf. on Earthq. Eng., Istanbul. 5:467-474.

Villaverde, R. 1984. On Rosenblueth's rule to combine the modes of systems with closely spaced natural frequencies. Bull. Seism. Soc. Am. 74:325-338.

Villaverde, R. 1986. Nonlinear seismic response of nonstructural elements attached to buildings. Report No. SM86-5, Univ. of California, Irvine, Calif.

Villaverde, R. 1987. A new approach for the modal analysis of nonlinear multistory structures. Proc. 5th Canadian Conf. Earth. Eng., Ottawa.

Viti, G., M. Olivieri, & S. Travi 1981. Development of nonlinear floor response spectra. Nuclear Eng. and Design. 64:33-38.