NONLINEAR VISCOELASTIC BEHAVIOR OF SHELLS OF REVOLUTION UNDER ARBITRARY LOADING

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SUMMARY

The requirement of some structural components such as propulsion systems and gas turbines to operate at high temperatures and pressures make the accurate evaluation of the creep phenomenon exigent. In recent years, interest in detailed evaluation of creep effects has been increasing greatly because certain structures are required to operate under even higher temperatures than those mentioned above. The fast breeder reactor is one such example.

For the expected increases in operating temperatures and pressures, it becomes necessary to perform a thorough analysis of integral structural components of nuclear power plants throughout their life span. Hence, except for preliminary estimates, approximate methods will be deficient. On the other hand, with recent advances in the study of materials at elevated temperatures, constitutive equations have been derived which yield accurate description of the behavior of uniaxial specimens. However, the use of such laws would be prohibitive in computer storage and time if elaborate numerical methods such as the finite element method are used. With further generation of material data the hereditary creep theories can be established adequately so as to represent stress reversals. This would also require the use of additional parameters and thus the use of finite element method will become less practical as a tool to design the component.

Since a large class of structures operating at elevated temperatures are composed of rotationally symmetric shells, a solution technique can be developed which does not have the above drawbacks. One such technique involves the numerical integration of the governing shell equations. This method has been successfully applied for the static and dynamic analysis of thin elastic shells of revolution and for some cases of inelastic material behavior. It has been shown to render solutions efficiently and accurately, usually with only a fraction of computer time and storage requirements and data manipulation that is required for other numerical schemes such as the finite element method. Furthermore, the numerical integration method allows more flexibility for varying the integration step lengths than does the finite difference method and can provide uniform accuracy throughout the analysis. For nonlinear viscoelastic behavior the numerical integration technique is expected to provide similar efficiency to that obtained for the elastic problems.

The computer program developed can accept time variation of material properties. Since a single form for the material constitutive law cannot encompass all materials, provisions are made so that the analysis of a very large class of material behavior can be accomplished. In addition to incorporating general laws such as strain hardening and time hardening laws, actual experimentally determined strain-time curves can be presented to the computer for various stresses and temperatures. The solution is obtained step by step in time. The total solution is a collection of piece-wise linear quasi-elastic problems. The inelastic strain rates at every time step are calculated from stresses at a previous time step with account taken for the changes during the small time increment. Different criteria for automatically establishing time steps are discussed.

1. Introduction

The problem of creep, i.e. time-dependent deformations, can be critical in metals, concrete and other materials. The increased operating temperatures and pressure environments of some structures, such as power plants, has made it exigent to perform a thorough analysis of integral structural components throughout their life span. For metals at elevated temperatures test results indicate that creep is an intricate nonlinear time dependent phenomenon. For instance, a specimen under periodic loading would manifest some strain recovery. If the unloading period is short then the recovery is negligible. For unloading periods longer than loading periods the recovery is more significant. Upon reloading primary creep, i.e. creep at a higher rate, develops. The effect of this recovery manifests itself as a softening of the material. Hence in a refined analysis the recovery must be included even thought it is usually small. If linear time dependent (i.e. viscoelastic) behavior is assumed for such a material, it would exhibit strong strain recovery, a condition different from what it depicted in creep tests. The highly nonlinear behavior of such materials requires that alternative methods be sought: "Methods that are simple enough to be tractable and realistic enough to be useful" (1).

Experimental and analytical research into the behavior of materials and into the development of creep models has provided laws which govern the behavior of a wide class of materials. Since it is not possible to represent the behavior of all materials by a unique law, different empirical correlations have been developed for different classes of materials. References (2-5) discuss some of those laws. The two most widely used laws for metals assume either strain hardening or time hardening of the materials. The strain hardening law which assumes that creep strain rate is a function of stress, temperature and accumulated creep strain, is used where the primary stage of creep is dominant. The time hardening hypothesis, which assumes that the creep strain rate is a function of stress, time, and temperature, is assumed where secondary creep is dominant. These two models of creep behaviors are discussed in detail in a subsequent section.

A second categroy of creep research has been concerned with the development of solution techniques. Approximate methods such as the reference stress method (6-8) provide an estimate of creep strains suitable for a preliminary structural analysis. For detailed analyses of complex geometries, e.g. shell structures, recourse must be made to numerical methods.

A large class of structures operating at elevated temperatures are composed of rotationally symmetric shells. Therefore, shells of revolution have been given considerable attention by different researchers. Although a closed form solution has been attempted using Novozhilov's complex variable formulation of shell equations (9), the most significant works on the creep of shells or revolution have employed either the finite element method (10-14) or the finite difference method (15-16). In the finite difference method it is difficult to select an appropriate mesh size which is simultaneously convergent and efficient. This selection can be made only partially on a trial and error basis. The finite element method is highly versatile and does not have the inherent stability problems of the finite difference method. A single computer program can include many different structural elements in its library. However, for the most practical problems, a large number of elements and numerous simultaneous equations are required. Since the problem of creep requires solution after every step in time, the solution to the large sets of simultaneous equations inherent to the finite element method becomes time consuming. This is particularly true if nonsymmetric loading conditions and/or time dependent properties are considered. Harmonic analysis and evaluation of the stiffness matrix for each time step are required in those instances. In such cases a numerical integration scheme presents an efficient alternative. Numerical integration techniques have been successfully used for the atatic and dynamic solutions of elastic problems (17-19). Reference (20) reports the solution of some creep problems of symmetrically loaded shells of revolution.

In the following sections a formulation and solution technique are presented for the creep analysis of shells of revolution subjected to arbitrary loads and temperature changes. Arbitrary creep laws are admitted in the formulation with specific attention given to the two common laws, i.e. strain hardening and time hardening. In the first part the governing equations for creep of shells of revolution are derived. The solution method requires the quasi-static linearization of the equations: linear incremental behavior is assumed during each time step. The incremental equations are expanded in Fourier series and solved by a numerical integration technique.

2. Creep Theory

It has been shown that creep strain ε^c is a function of stress σ, time t, temperature T and the accumulated creep strain. Additional parameters p can be included to account for factors such as hardening. Thus, in general

$$\varepsilon^c = f\,(\sigma, \varepsilon^c, t, T, p) \tag{1}$$

For numerical solutions as programmed for a digital computer, experimental values can be inputted to the computer and interpolations performed for other points. An alternative which is more often used is to seek a general formulation for a given class of materials and then calculate the constants from experimental data. The latter procedure has had relative success although it is generally agreed (3-4) that a single relationship that can encompass all materials is not possible. Thus, different formulas are suggested for different classes of materials (2-5). When a closed form or simplified solution is sought, usually a simple relationship such as a power law is used.

For more detailed analyses the most commonly used laws are time hardening and strain hardening. Time hardening assumes that the creep rate is a function of stress, temperature and time elapsed since creep began. This will provide good results when the secondary stage of creep is dominant. The creep rate then will be (a dot above a variable denotes its time derivative):

$$\dot{\varepsilon}^c = f\,(\sigma, t, T) \tag{2a}$$

This formulation has computational advantages, in that only the values at a previous time have to be saved. But it does not yield good results for varying loads (13). If the similarity of creep curves exists, as is the case for most metals, then

$$\dot{\varepsilon}^c = f_1\,(\sigma)\,\frac{df_2(t)}{dt}\,f_3\,(T) \tag{2b}$$

The strain hardening formulation is used when the primary stage of creep is the dominant part. It assumes that the creep rate is a function of stress, temperature and accumulated creep strain. Thus,

$$\dot{\varepsilon}^{c} = g\ (\sigma,\ \varepsilon^{c},\ T) \tag{3a}$$

or using the similarity of creep curves

$$\dot{\varepsilon}^{c} = g_1\ (\sigma)\ g_2\ (\varepsilon^{c})\ g_3\ (T) \tag{3b}$$

The strain hardening formulation is suitable for varying stress conditions. It does not, however, properly represent the material behavior under stress reversal. A modification is suggested in (5) . It can be shown that this relationship belongs to a more general formulation, namely equations of state and, thus, better variations of the former can be established (1).

The most commonly used method of generalizing the one-dimensiona case to multi-axial stress states is fashioned after the theory of plasticity. Thus, it is assumed that the hydrostatic stress does not effect the creep behavior. That is, only the deviatoric part of the stress tensor must be included in the analysis. Incompressibility is also assumed and flow rules such as the Prandtl-Reuss rule are used (23).

3. Formulation of Equations

It is possible to formulate the governing field equations for a shell in terms of curvilinear coordinates x_1, x_2 on the middle surface of the shell if all pertinent three dimensional variables are integrated with respect to the coordinate z through the thickness h to obtain equivalent surface variables. For a surface of revolution, let the meridional coordinate x_1 be the angle formed between the normal to the surface and the axis of revolution, and let x_2 be the azimuth angle which the meridian plane makes with a reference meridian. The radius of curvature of a meridional line is R_1, and R_2 is the radius of curvature of a cross-sectional circle with the radius r. Since the shell geometry is axially-symmetric all geometric quantities are independent of x_2. Partial derivatives with respect to x_1 and x_2 will be denoted respectively by a comma followed by subscripts 1,2 and a dot above a variable indicates time rate. The governing equilibrium, strain-displacement and constitutive equations can be obtained from Ref. 22 with the addition of creep forces.

The differential equations for shells of revolution can be set up as eight first-order equations in which the derivatives of the fundamental variables with respect to the arc length are expressed in terms of other variables. The fundamental variables are those variables that appear on the boundaries. The fundamental variables are the displacement components u, v, w, θ_1, N_{11}, M_{11}, Q, N where $N = N_{12} + M_{12}/r\ \sin\ (x_1)$ (4)

and $$Q = Q_1 + \frac{M_{12,2}}{r} \tag{5}$$

The final form of the equations suitable to numerical integration can be obtained by algebraic manipulation. After rearranging, and forming derivatives with respect to arc length, s, the variables are subsequently expanded as Fourier series in x_2 as follows:

$$\left\{p_1,\ p_3,\ T_0,\ T_1\right\} = \left\{p_1^n,\ p_3^n,\ T_0^n,\ T_1^n\right\} \begin{Bmatrix} \cos\ (nx_2) \\ \sin\ (nx_2) \end{Bmatrix} \tag{6a}$$

$$p_2 = p_2^n \begin{Bmatrix} \sin\ (nx_2) \\ \cos\ (nx_2) \end{Bmatrix} \tag{6b}$$

$$\left\{\Delta N_{11},\ \Delta N_{22},\ \Delta M_{11},\ \Delta M_{22}\right\} = \left\{\Delta N_{11}^{n},\ \Delta N_{22}^{n},\ \Delta M_{11}^{n},\ \Delta M_{22}^{n}\right\} \begin{Bmatrix}\cos\ (nx_2)\\ \sin\ (nx_2)\end{Bmatrix} \tag{7a}$$

$$\left\{\Delta N_{12},\ \Delta M_{12}\right\} = \left\{\Delta N_{12}^{n},\ \Delta M_{12}^{n}\right\} \begin{Bmatrix}\sin\ (nx_2)\\ \cos\ (nx_2)\end{Bmatrix} \tag{7b}$$

where p_i are tractions, ΔN_{ij}, ΔM_{ij} are creep forces and moments.

The equations are then written for every harmonic, n, and after their time rates are taken the final form of the differential equations are obtained. In the derivations no assumptions are made as to the form of creep law and the general case of time-dependent loads is considered. The final form of the equations is:

$$\epsilon^{n} = 2\epsilon_{12}^{n} - v_{,s}^{n} \tag{8a}$$

$$\kappa^{n} = 2\kappa_{12}^{n} - v_{,s}^{n}\ \frac{\sin\ (x_1)}{r} \tag{8b}$$

$$\dot{\epsilon}_{22}^{n} = \frac{n\ \dot{v}}{r} + \frac{\dot{u}}{r}\cos\ (x_1) + \frac{\dot{w}^{n}}{r}\sin\ (x_1) \tag{8c}$$

$$\dot{\beta}_2^{n} = \frac{\dot{v}^{n}}{r}\sin\ (x_1) + \frac{n}{r}\dot{w}^{n} \tag{8d}$$

$$\dot{\kappa}_{22}^{n} = \frac{n}{r}\dot{\beta}_2^{n} + \frac{\dot{\beta}_1^{n}}{r}\cos\ (x_1) \tag{8e}$$

$$\dot{\epsilon}^{n} = -\frac{n}{r}\dot{u}^{n} - \frac{\dot{v}^{n}}{r}\cos\ (x_1) \tag{8f}$$

$$\dot{\kappa}^{n} = -\frac{2n}{r^2}\dot{w}^{n}\cos\ (x_1) + \frac{n\dot{u}^{n}}{rR_1} + \frac{\cos\ (x_1)}{r}\left[\frac{1}{R_1} - 2\,\frac{\sin\ (x_1)}{r}\right]\dot{v}^{n} - \frac{2n}{r}\dot{\beta}_1^{n} \tag{8g}$$

$$A_1 = C_{11}D_{11} + K_{11}^{2} \tag{8h}$$

$$\dot{\epsilon}_{11}^{n} = \frac{1}{A_1}\left(\dot{N}_{11}^{n} - C_{12}\dot{\epsilon}_{22}^{n} - K_{12}\dot{\kappa}_{22}^{n} - H_{11}\dot{T}_0^{n} - H_{12}\dot{T}_1^{n} + \Delta\dot{N}_{11}^{n}\right) D_{11} - \frac{1}{A_1}\left(\dot{M}_{11}^{n} - D_{12}\dot{\kappa}_{22}^{n} - K_{12}\dot{\epsilon}_{22}^{n} - H_{12}\dot{T}_0^{n} - F_{12}\dot{T}_1^{n} + \Delta\dot{M}_{11}^{n}\right) K_{11} \tag{8i}$$

$$\dot{\kappa}_{11}^{n} = \frac{1}{A_1}\left(\dot{M}_{11}^{n} - D_{12}\dot{\kappa}_{22}^{n} - K_{12}\dot{\epsilon}_{22}^{n} - H_{12}\dot{T}_0^{n} - F_{12}\dot{T}_1^{n} + \Delta\dot{M}_{11}^{n}\right) C_{11} - \frac{1}{A_1}\left(\dot{N}_{11}^{n} - C_{12}\dot{\epsilon}_{22}^{n} - K_{12}\dot{\kappa}_{22}^{n} - H_{11}\dot{T}_0^{n} - H_{12}\dot{T}_1^{n} + \Delta\dot{N}_{11}^{n}\right) K_{11} \tag{8j}$$

$$\dot{N}_{22}^{n} = C_{12}\dot{\epsilon}_{11}^{n} + C_{22}\dot{\epsilon}_{22}^{n} + K_{12}\dot{\kappa}_{11}^{n} + K_{22}\dot{\kappa}_{22}^{n} + H_{22}\dot{T}_0^{n} + H_{21}\dot{T}_1^{n} - \Delta\dot{N}_{22}^{n} \tag{8k}$$

$$\dot{M}_{22}^{n} = D_{12}\dot{\kappa}_{11}^{n} + D_{22}\dot{\kappa}_{22}^{n} + K_{12}\dot{\epsilon}_{11}^{n} + K_{22}\dot{\epsilon}_{22}^{n} + H_{21}\dot{T}_0^{n} + F_{21}\dot{T}_1^{n} - \Delta\dot{M}_{22}^{n} \tag{8l}$$

$$A_2 = C_{66} + 2K_{66}\,\frac{\sin\ (x_1)}{r} + D_{66}\,\frac{\sin^2\ (x_1)}{r^2} \tag{8m}$$

$$\dot{v}_{,s}^{n} = \frac{1}{A_2}\left\{\dot{N}^{n} - \left[C_{66} + K_{66}\,\frac{\sin\ (x_1)}{r}\right]\dot{\epsilon}^{n} - \left[K_{66} + D_{66}\,\frac{\sin\ (x_1)}{r}\right]\dot{\kappa}^{n} + \Delta\dot{N}_{12}^{n} + \frac{\sin\ (x_1)}{r}\,\Delta\dot{M}_{12}^{n}\right\} \tag{8n}$$

$$\dot{M}^n_{12} = K_{66} (\dot{\epsilon}^n + \dot{v}^n_{,1}) + D_{66} \left[\dot{\kappa}^n + \frac{\dot{v}^n_{,s}}{r} \sin (x_1) \right] - \Delta\dot{M}^n_{12} \tag{8o}$$

$$\dot{w}^n_{,s} = \frac{\dot{u}^n}{R_1} - \dot{\beta}^n_1 \tag{8p}$$

$$\dot{u}^n_{,s} = \dot{\epsilon}^n_{11} - \frac{\dot{w}^n}{R_1} \tag{8q}$$

$$\dot{\beta}^n_{1,1} = \dot{\kappa}^n_{11} \tag{8r}$$

$$\dot{N}^n_{11,s} = - \frac{n\dot{N}^n}{r} + n \left[\frac{1}{R_1} + \frac{\sin (x_1)}{r}\right] \frac{\dot{M}^n_{12}}{r} + \frac{\cos (x_1)}{r} \dot{N}^n_{22} - \frac{\cos (x_1)}{r} \dot{N}^n_{11} - \frac{\dot{Q}}{R_1} - \dot{p}^n_1 \tag{8s}$$

$$\dot{M}^n_{11,s} = \frac{\cos (x_1)}{r} \dot{M}^n_{22} - \frac{2n}{r} \dot{M}^n_{12} - \frac{\cos (x_1)}{r} \dot{M}^n_{11} + \dot{Q}^n \tag{8t}$$

$$\dot{Q}^n_{,s} = -2n \frac{\dot{M}^n_{12}}{r^2} \cos (x_1) + \dot{Q}^n \frac{\cos (x_1)}{r} + \frac{\dot{N}_{22}}{r} \sin (x_1) + \frac{\dot{N}^n_{11}}{R_1} + \frac{n}{r^2} \dot{M}^n_{22} - \dot{p}^n_3 \tag{8u}$$

$$\dot{N}^n_{,s} = \left[\frac{1}{R} - \frac{\sin (x_1)}{r}\right] \frac{\cos (x_1)}{r} \dot{M}^n_{12} - 2\dot{N}^n \frac{\cos (x_1)}{r} + \frac{n}{r} \dot{N}^n_{22} + \frac{n\dot{M}^n_{22}}{r^2} \sin (x_1) - \dot{p}^n_2 \tag{8v}$$

where κ_{ij} are curvature changes, ϵ_{ij} are middle surface strains, C_{ij}, D_{ij}, K_{ij}, H_{ij} and F_{ij} are material constants defined on page 52 of Ref. 22, and T_0, T_1 are integrated temperature effects. If material properties vary with time, then additional terms will appear in the above relations. Such general equations are derived in (24). For isotropic materials, the creep forces are

$$\Delta N_{11} = \frac{K}{h} \int_z (\bar{\epsilon}^c_{11} + \upsilon\, \bar{\epsilon}^c_{11}) \left(1 + \frac{z}{R_2}\right) dz \tag{9a}$$

$$\Delta N_{22} = \frac{K}{h} \int_z (\bar{\epsilon}^c_{22} + \upsilon\, \bar{\epsilon}^c_{11}) \left(1 + \frac{z}{R_1}\right) dz \tag{9b}$$

$$\Delta N_{12} = \frac{K}{h} (1 - \upsilon) \int_z 2\bar{\epsilon}^c_{12} \left(1 + \frac{z}{R_2}\right) dz \tag{9c}$$

$$\Delta M_{11} = \frac{K}{h} \int_z (\bar{\epsilon}^c_{11} + \upsilon\, \bar{\epsilon}^c_{22}) \left(1 + \frac{z}{r_2}\right) zdz \tag{10a}$$

$$\Delta M_{22} = \frac{K}{h} \int_z (\bar{\epsilon}^c_{22} + \upsilon\, \bar{\epsilon}^c_{11}) \left(1 + \frac{z}{R_1}\right) zdz \tag{10b}$$

$$\Delta M_{12} = \frac{K}{h} (1 - \upsilon) \int_z 2\bar{\epsilon}^c_{12} \left(1 + \frac{z}{R_2}\right) zdz \tag{10c}$$

where $K = \frac{Eh}{1 - \upsilon^2}$, E is Young's modulus, υ is Poisson's ratio, and the components of total strain ϵ_{ij} are the sums of elastic and creep strain, i.e.

$$\bar{\epsilon}_{ij} = \bar{\epsilon}^e_{ij} + \bar{\epsilon}^c_{ij} \tag{11a}$$

where $$\bar{\epsilon}_{ij} = \epsilon_{ij} + \kappa_{ij} z \tag{11b}$$

4. Solution Technique

The governing differential equations for creep of rotationally symmetric shells derived in the form of eq. (8) are suited to numerical integration. The steps in the solution of the problem are as detailed below.

Starting with the elastic solution, the elastic differential equations, which are a simplified version of eq. (8), are numercially integrated. Strains and stresses are evaluated that yield the creep strains. The creep forces are then calculated at the integration points along the meridian of the shell. A time step is subsequently calculated either internally or from the values of time provided.

The rate equations, eqs. (8), are integrated using the creep strain rates which are in turn found by interpolating between values of stress obtained during the previous time step. These yield the creep forces and the integration can be pursued to obtain the fundamental variables as well as strains. In those calculations, the stresses and creep strains extant at the beginnning of that step are included.

Again, using the stresses from the previous step, the creep strains and their rate are calculated and the new values of the variables are found by an intergration scheme, e.g. Newmark's method. If necessary, the value of stresses at selected time steps are iterated on to obtain more accurate values. In most cases, one iteration is desirable and sufficient. This above process is repeated for the length of the time span or until the steady state is reached.

The differential equations of shells of revolution along with the boundary conditions constitute a boundary-value problem. As stated before, these differential equations can be set up as eight first-order equations each one expressing the derivative of one fundamental variable, with respect to the arc length, in terms of the other variables.

In order to integrate these equations, the boundary-value problem is first cast into a set of initial-value problems (25,17,18). To this end the unknown variables at the beginning of the shell are assigned values. When the integration of the initial value problems has reached the end of the shell, the boundary conditions at the end of the shell are imposed and the assumed values at the beginning of the integration are corrected. This process is possible if the shell is elastic. Hence, the solution of the creep problem must be obtained as a collection of piecewise linear elastic problems. The integration can be performed by any of the available numerical schemes. The so called predictor-corrector methods (26) present more efficient methods. However, the use of Runge-Kutta method (25,26,27) can provide for a solution with uniform accuracy throughout the integration span by adjusting the size of the integration steps.

The integration scheme as discussed previously, can proceed without difficulty for very short shells. For shells of any practical length the solution grows very fast as the integration is carried out along the meridian of the shell. Such erroneous growth in the numerical solution exists in equations whose solution has an exponential form. In the closed-form solutions the coefficients of the exponential solutions with positive exponents are equated to zero. Due to the approximate nature of the numerical integration schemes such coefficients of the rapidly growing terms turn out to be a small number instead of exactly zero. As the length of integration increases the exponential function grows rapidly and its product with the small coefficient becomes so significant that it dominates other terms in the solution of the equation. To alleviate this situation, the coefficient of such independent solutions must be recalculated as the solution starts to grow, thus keeping the product close to zero. Two methods have been devised for this purpose.

In the first method (18), the so called multi-segment method, the solution is obtained for a combination of short segments in which the solution is stable. Every segment is treated as an initial-value problem for which eight complementary solutions and a particular solution are found. These segments are then combined by assigning the compatibility between the two adjacent segments and the boundary conditions. Thus, the method involves the solution of a system of simultaneous equations. The number of equations involved here is much less than that encountered in the finite element or finite difference methods. The implementation of the multisegment method is simple, even in the presence of branch shells, because of the availability of equation solving routines (28).

In the second method, the so called suppression method (29-32); four complementary solutions and one particular solution are obtained. Other variables are found from the values thus determined. As the extraneous solutions start to become significant, the assumed values in the initial value problems are readjusted so that the fast growing solutions are suppressed before becoming significant.

In creep problems a solution is required at every time step. Therefore, the choice between the above two techniques becomes critical. If the material properties do not vary with time, the complementary solution obtained at the beginning of the process can be stored and used again. Only the particular solution has to be recomputed at every time step. In that case, the multi-segment and the suppression method present comparable efficiency. However, if the material properties vary with time, the suppression method becomes more efficient since it requires only four independent solutions and the additional calculations are less time consuming compared with the solution of a system of simultaneous equations.

5. Conclusion

This paper has discussed the solution of nonsymmetric creep behavior of shells of revolution by means of a numerical integration scheme. The formulation is general and any correlation or experimentally obtained creep data can be used. Preliminary verifications of the program indicate that more efficiency can be achieved than with the finite element method. The analysis can be carried out with uniform accuracy throughout the solution by adjusting the length of integration step. This is an advantage over the finite difference method. Furthermore since the computer time and storage is less than in other numerical methods more parameters can be used for better representation of materials, and time varying material properties and loads are tractable.

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