



Analytical Displacement Solutions for Circumferential Through-wall Cracks in a Pipe

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ABSTRACT

A new bench mark test is proposed for linear finite element analysis. This test is originated during the course of a project to generate a finite element package which will be used for linear analysis. The problem is the displacement solutions of a pipe with a circumferential through-wall crack. As the theoretical and numerical results for this problem still have disparities, this example could be used for future reference.

INTRODUCTION

Recently a structural analysis software named Power Designer has been developed in order to enhance the design capabilities of small companies [1]. The main function is for linear analysis and devised to connect to remote large computers for nonlinear analysis. As the difficulties to develop finite element analysis for plates and shell analysis are well known and documented [2], we also had confronted same problem. The program has been checked by many bench mark tests during the course of the project and applied to the title problem. Although many fracture analysis and approximate calculation were performed, there was no conclusion for ovalization of the cross section of the cracked pipe. In this paper, several commercial finite element analysis packages are used and compared with Sanders solution. As there are still many differences between these results, the problems must be investigated by an analytical method. Also we suggest this problem as a new bench mark test for shell analysis.

DEFINITION OF THE PROBLEM

A schematic diagram of a cylindrical shell with a through-wall crack is shown in Fig.

1. This problem plays an important role in safety confirmation of the nuclear and chemical plants. The LBB(Leak-Before-Break) concept and the crack opening area are essential for evaluation of structural integrity of pipes. Many experimental works were carried out at EPRI, Battelle. But the thin shell formulation and perturbation solution of this problems is attributed to Sander and his pupil [3-4]. A number of subsequent works appeared after Sanders results. Yoo and Pan gave approximate formulas of COD by normalization of the second order asymptotic expansion of Sanders solution[5]. Recently fairly extensive studies were reported for this problem[6-8] But still the disparities between the finite element results with the analytical ones are not resolved.

ANALYTICAL RESULTS

Sanders provided analytical solution for cases of comparatively large crack by his shell governing equation which is considered as the best first order equation. Yoo and Pan presented fairly simple parabolic crack opening profile based on Sanders solution. Sanders results comprised of following four cases - remote bending and tension for infinite cylinder, remote bending and tension for semi-infinite cylinder with clamped ends.

BENCHMARK TEST FOR FINITE ELEMENT ANALYSIS

The finite element analysis model is $R/t > 20$ and $L/R = 10$ to prevent the effects of boundary ends. The length is 3m from the crack to the end, the radius is 0.3m, and the thickness is 0.01m. Since the model is symmetric, only half of cylindrical shell structure was meshed. The Young's modulus is 210 Gpa and the Poisson's ratio is 0.3. The general purpose package ABAQUS and NISA II and a newly developed Power Designer were used in benchmark test. The thin plate element S8R5 of ABAQUS, and NKTP=20 of NISA II were used. The loading was pure tension and bending. The applied nominal stress was $\sigma = 420 \times 10^6$ Pa since $\sigma / E = 0.002$ in analytic equation and the moment at the end was 231 N-m to match with the analytical solution. The model was composed of 2100 elements and 2196 nodes.

RESULTS AND DISCUSSIONS

The result from ABAQUS, NISA II, Power Designer and the analytical solution are compared with the perfect circle which is the original undeformed shape. The crack sizes have been changed from very small to moderately large range. Fig. 2. ~ Fig. 9. show the crack opening shapes in axial direction and deformed shape of circular cross section

including crack. In Fig. 2, the results from ABAQUS and NISA II are almost same and different from analytic solution for 180° . The Fig. 3,5,7,9 show the profile of the crack opening area in axial direction. The discrepancies between the results depend on the problems. Generally ABAQUS and Sanders are compared well with some exceptions. In Fig. 4 and 6, the analytical solution and ABAQUS results are very close but not others. In Fig. 7, ABAQUS and NISA II have same tendency for 180° and especially address crack closure phenomena which cannot be seen in the analytical solution. In Fig. 9, there is a big gap between the analytical solution and the finite element analyses, where ABAQUS and NISA II agree very well. In this research the size of crack was varied from 10° to 180° . Although there was no definite trend, it was found that ABAQUS and NISA II have almost same results when the crack size increase. But the analytical solution and the result from ABAQUS are close when the crack is small, but there appear big differences when the sizes of the crack grow. These observations evoke a contradiction about the general acceptance of the Sanders solution. The Sanders results are reported to have good agreements for comparatively large crack. So this problem has to be scrutinized by another analytical method and experiments. There seems to be many differences with Power Designer, which is considered not mature yet. Although the problem is restricted within linear elastic and thin shell region, there are still many things to be cleared up. Currently a new semi-analytical solution for this problem has been generated and will be presented.

CONCLUSION

In this research, a bench mark test was performed for deformation analysis of a cylindrical shell with a circumferential through-crack. This problem itself has great need to ensure safety of structures and also can be used for verifying the theory and finite element solution. From this investigation, the results of ABAQUS are most close to the analytical solution except cases when they are close to those from NISA II.

REFERENCES

1. Yoo, S. H., "Development of structural analysis library of virtual reality and its application," Final Report, SERI, 1997.
2. MacNeal, R. H., "Perspective on finite elements for shell analysis," *Finite Elements in Analysis and Design*, Vol. 30, 1998, pp. 175-186.
3. Sanders, J. L. Jr., "Circumferential through-cracks in cylindrical shell under tension," *Journal of Applied Mechanics*, Vol. 49, 1982, pp. 103-107.
4. Alabi, J. A. and Sanders, J. L., Jr., "Circumferential cracks at fixed end of pipes," *Engineering Fracture Mechanics*, Vol. 22-4, 1985, pp. 609-616.

5. Yoo, S. H. and Pan, J., "Approximate crack opening displacement solutions for long circumferential cracks in pipes subjected to bending and tension," *Journal of Pressure Vessel Technology*, Vol. 114-2, 1992, pp. 178-180.
6. Rahman, S., Brust, F.W., Ghadiali, N., and Wilkowski, G. M., "Crack-opening-area analyses for circumferential through-wall cracks in pipes – Part I : analytical models," *International Journal of Pressure Vessels and Piping*, Vol. 75-5, 1998, pp. 357-373.
7. Rahman, S., Brust, F.W., Ghadiali, N., and Wilkowski, G. M., "Crack-opening-area analyses for circumferential through-wall cracks in pipes – Part II : model validations," *International Journal of Pressure Vessels and Piping*, Vol. 75-5, 1998, pp. 375-396.
8. Rahman, S., Ghadiali, N., Wilkowski, G. M., Moberg, F. and Brickstad, B. "Crack-opening-area analyses for circumferential through-wall cracks in pipes – Part III : off-center cracks, restraint of bending, thickness transition and weld residual stresses," *International Journal of Pressure Vessels and Piping*, Vol. 75-5, 1998, pp. 397-415.

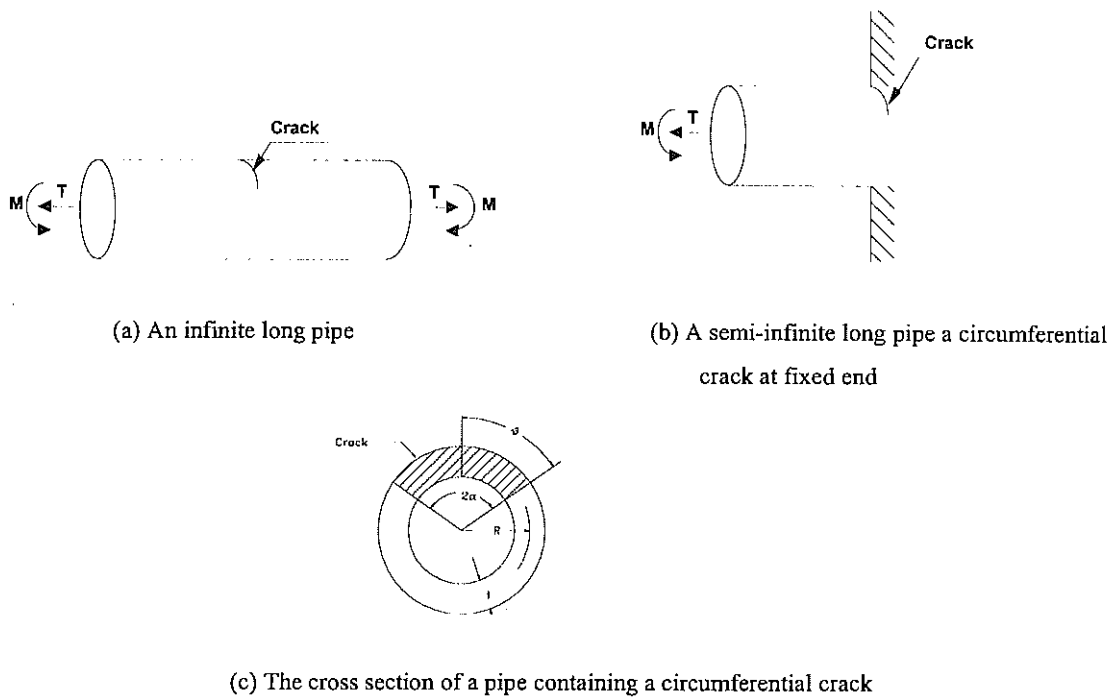
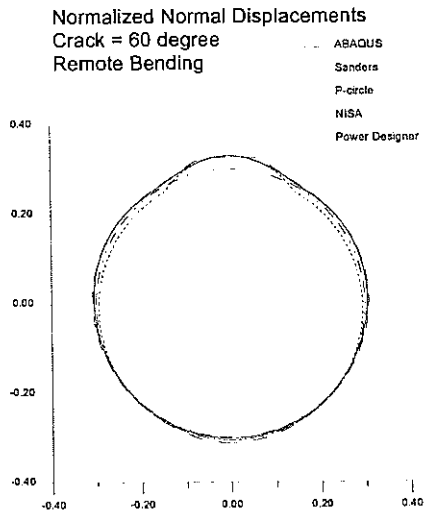
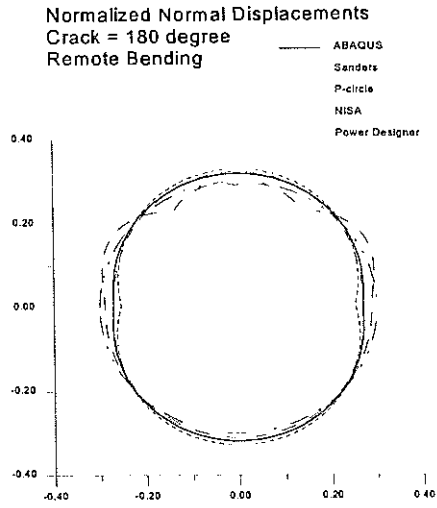


Fig. 1. A circular cylindrical shell with a circumferential crack

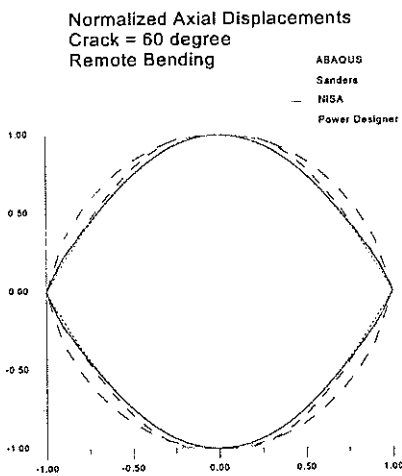


(a) Crack size ; 60°

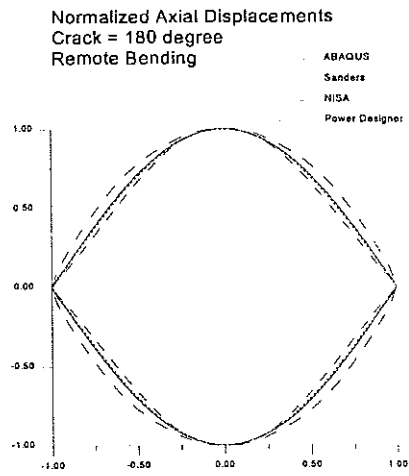


(b) Crack size ; 180°

Fig. 2. Normalized normal displacement of an infinite long pipe with a circumferential crack subjected to remote bending.

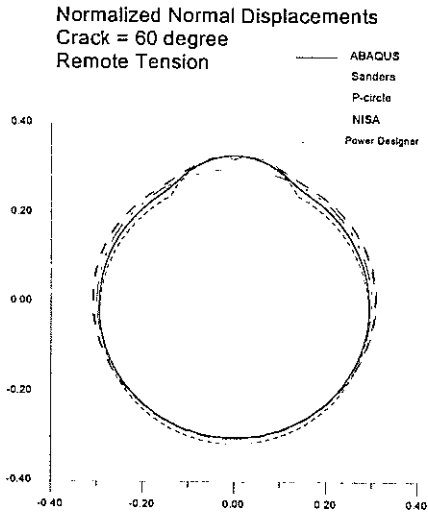


(a) Crack size ; 60°

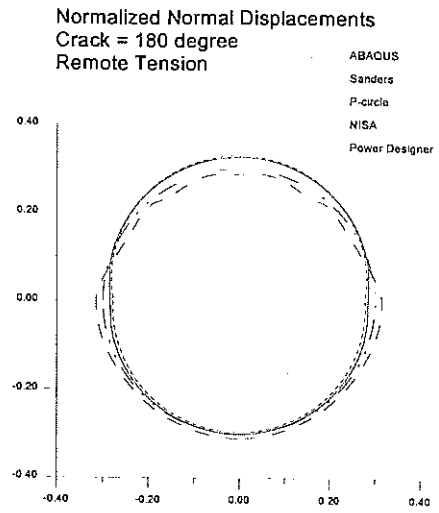


(b) Crack size ; 180°

Fig. 3. Normalized axial displacement of an infinite long pipe with a circumferential crack subjected to remote bending.

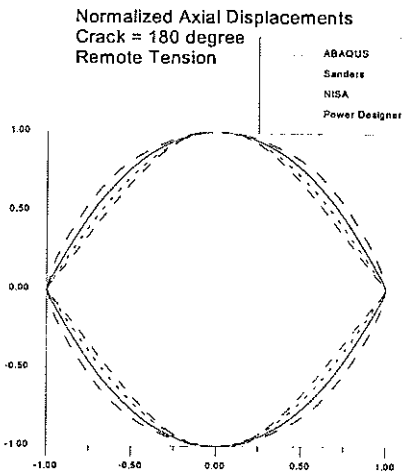


(a) Crack size ; 60°

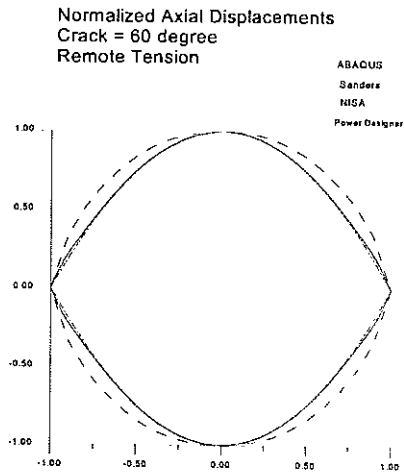


(b) Crack size ; 180°

Fig. 4. Normalized normal displacement of an infinite long pipe with a circumferential crack subjected to remote tension.

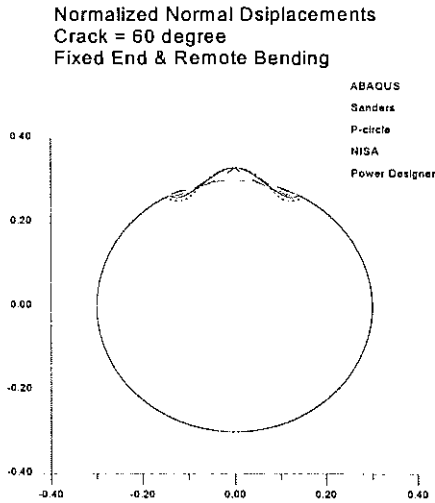


(a) Crack size ; 60°

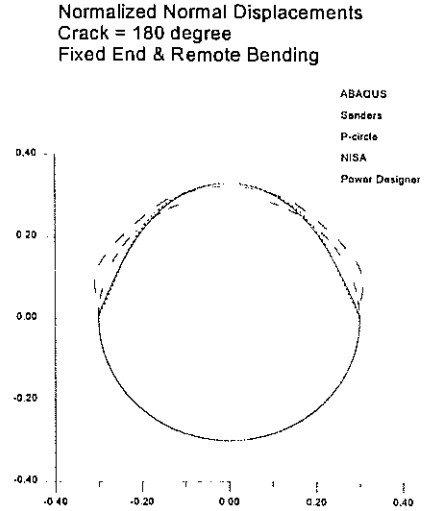


(b) Crack size ; 180°

Fig. 5. Normalized axial displacement of an infinite long pipe with a circumferential crack subjected to remote tension.

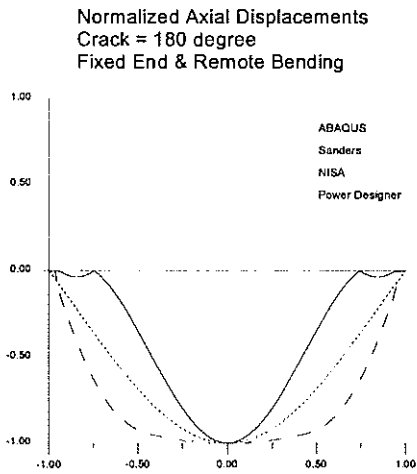


(a) Crack size ; 60°

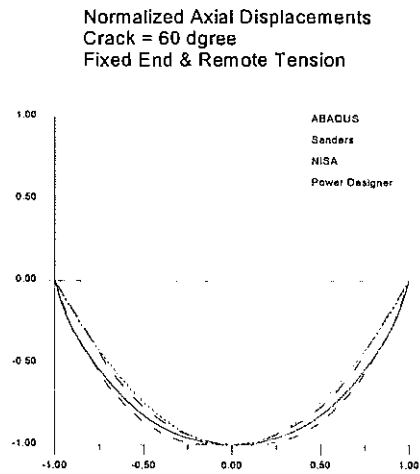


(b) Crack size ; 180°

Fig. 6. Normalized normal displacement of an infinite long pipe with a circumferential crack at the fixed end subjected to remote bending.



(a) Crack size ; 60°



(b) Crack size ; 180°

Fig. 7. Normalized axial displacement of an infinite long pipe with a circumferential crack at the fixed end subjected to remote bending.

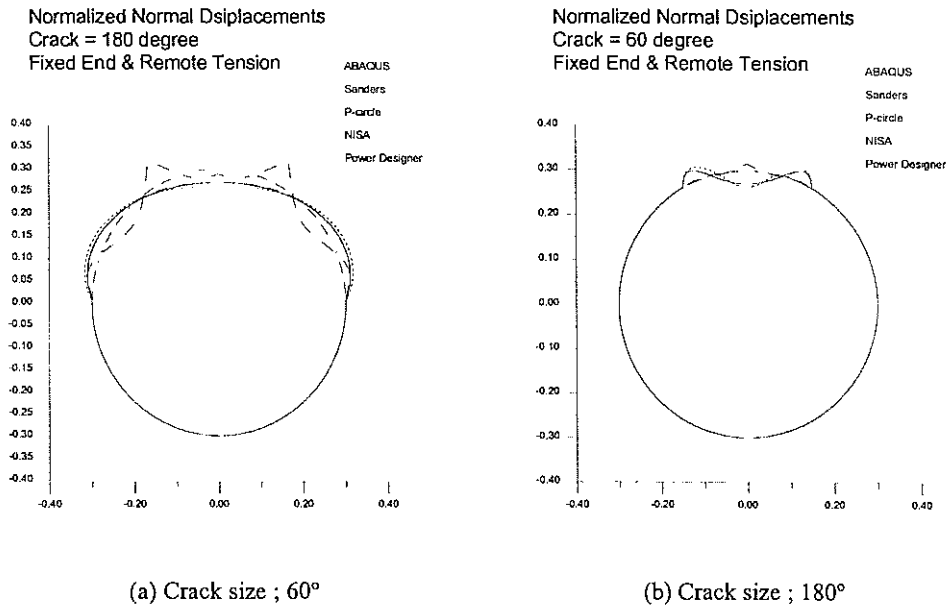


Fig. 8. Normalized normal displacement of an infinite long pipe with a circumferential crack at the fixed end subjected to remote tension.

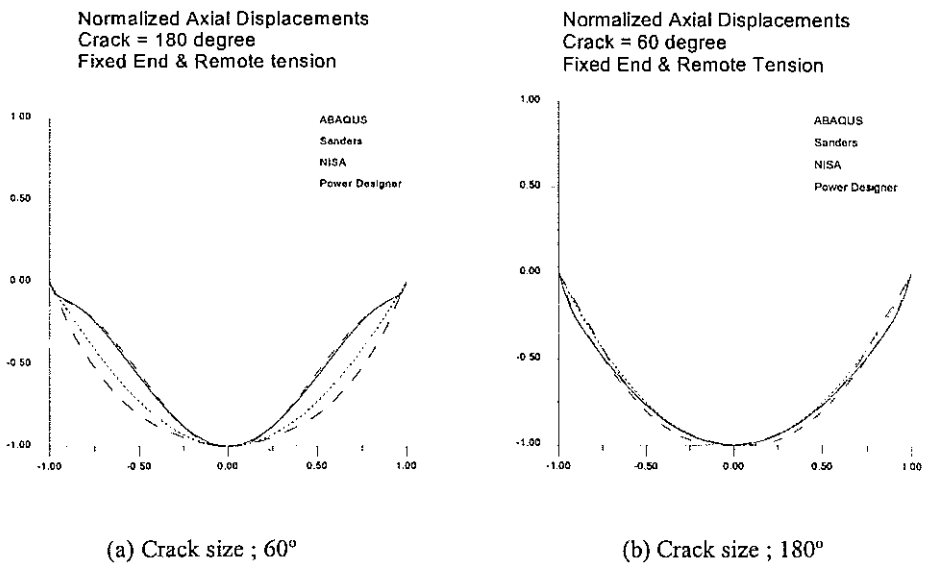


Fig. 9. Normalized axial displacement of an infinite long pipe with a circumferential crack at the fixed end subjected to remote tension.