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Bayesian Finite Element Model Updating of Reinforced Concrete Building Structures Under Seismic Excitation Part1: Fitting in Observed Space by TMCMC

Minoru Matsubara[1], Kenzo Kodera[2], Taro Yaoyama[3], Sangwon Lee[4], Takeshi Ugata[5]

[1]Manager, Taisei Corporation, Tokyo, Japan (mtbmnr00@pub.taisei.co.jp)
[2]Assistant Manager, Taisei Corporation, Tokyo, Japan (kdrknz00@pub.taisei.co.jp)
[3]Project Assistant Professor, The University of Tokyo, Tokyo, Japan(yaoyama@load.arch.t.u-tokyo.ac.jp)
[4]Project Researcher, The University of Tokyo, Tokyo, Japan(lee-sw@g.ecc.u-tokyo.ac.jp)
[5]Deputy Chief, Taisei Corporation, Tokyo, Japan(ugata@eng.taisei.co.jp)

ABSTRACT

Enhancing the probabilistic risk assessment of nuclear power plants requires the detailed response predictions of reactor buildings exposed to hazard-induced disturbances. Especially for seismic hazards, dynamic analysis of high-fidelity finite element (FE) models plays a critical role. However, non-negligible discrepancies often exist between model predictions and the observed behavior of actual buildings due to various factors, e.g., measurement noise, variabilities in material properties, uncertainties introduced during construction process, over-simplification in the modeling of joints and boundaries, and ageing and/or hazard-induced damages.

There are some case studies on model fitting for observation records using Bayesian updating (ex. Tomoo, S. (2013)). But many of them use simple models (e.g., lumped mass models). It is challenging to update the FE models because of their high computational loads.

In this paper, we propose an FE model Bayesian update method and study its applicability using the record of the shaking table test of 6-story RC building (Sugimoto et al., 2017).

INTRODUCTION

According to Bayes' theorem, the distribution of the parameter θ of a prediction model can be updated by the observation data D as follows:

$$p(\theta|\mathcal{D}) \propto p(\theta)L(\theta|\mathcal{D}) \tag{1}$$

where $p(\theta)$ is a prior distribution set from the information before the observation data are obtained, and $L(\theta|D)$ is a likelihood function of the observation data D.

Fig. 1 shows the flowchart we propose of the model updating and application of updated models. At the design phase, a seismic response analysis model based on nominal values of material properties is first created (the nominal model), which is used for deterministic and/or probabilistic safety evaluation. Later, when the building is constructed, data can be collected by measurements the microtremors, and the vibration properties can be evaluated. The analysis model can be updated using these data. Updating the model makes it possible to perform a more realistic PRA evaluation or to improve the accuracy of immediate damage assessment. The advantages of updating the model based on the measurements of microtremors, not earthquake-induced vibration, are as follows.

- Records can be actively collected at any time without waiting for an earthquake to occur.
- Because observatory installation is temporary, maintenance costs can be reduced. In addition, there will be present in the operation of the facility. It is also possible to temporarily rent and install many sensors to achieve high-density recordings.

Regarding Bayesian updating of building models, some methods have been reported (e.g., Saito ,2013). However, these methods dealt with lumped mass models. Typically, Bayesian updates require many analyses to calculate the posterior distribution; therefore updating finite element (FE) models with a high analytical load causes computation time problems.

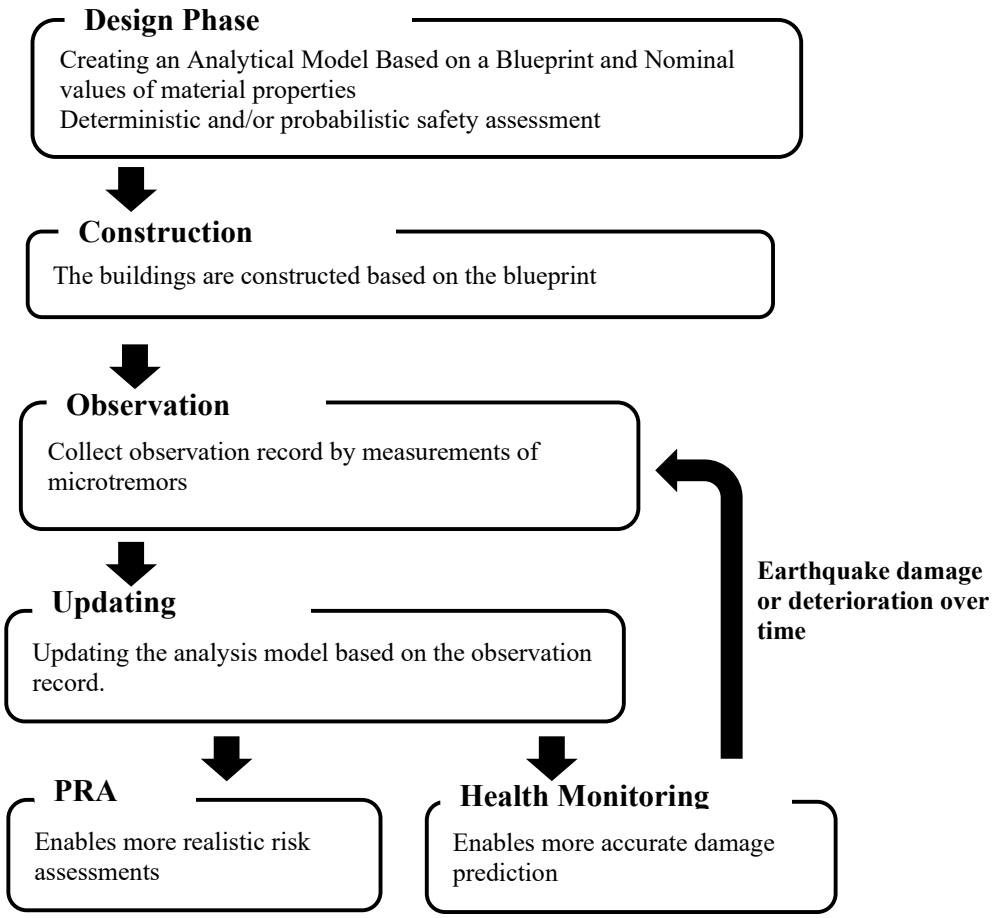


Figure 1. Flowchart of model updating and application of updated models.

METHODOLOGY

FE models Bayesian update method

In this paper, we propose a Bayesian update method using “TMCMC” as a sampling method and “Frequency domain likelihood function” as a likelihood function for updating FE models while considering the acceleration records observed in buildings.

Sampling method -TMCMC-

Response analysis for many samples of parameters θ is needed to evaluate the distribution of $L(\theta)$. It is necessary to devise a method of sampling θ because the response analysis of FE model is time consuming.

Markov Chain Monte Carlo (MCMC) simulation is used for Bayesian updating with a small number of samples even for high-dimensional problems. However, since MCMC calculates the posterior probability of one sample and then generates the next sample, and so on, it is usually not possible to parallelize the calculation. Therefore, the computing power of the computer cannot be utilized, and in the end, it takes a long calculation time.

Ching, J., Chen, Y.-C. et al. (2007) proposed Transitional MCMC (TMCMC) as a sampling method in Bayesian updating. This method allows computers to utilize their parallel computing power and shorten the analysis time associated with the model update. In recent years, some studies of the application of TMCMC to FE model updating have been reported.

TMCMC is a type of particle filter. x_i $(i = 1{\sim}N)$ satisfying (2) are samples (particles) representative of the prior distribution.

$$p(\boldsymbol{x}) \cong \frac{1}{N}\sum_i \delta(\boldsymbol{x} - \boldsymbol{x}_i) \tag{2}$$

In Bayesian updating, the particles are weighted as follow:

$$p(\boldsymbol{x}|\boldsymbol{y}) \propto p(\boldsymbol{x})L(\boldsymbol{x}|\boldsymbol{y}) \cong \frac{1}{N}\sum_i L(\boldsymbol{x}|\boldsymbol{y})\delta(\boldsymbol{x} - \boldsymbol{x}_i) = \frac{1}{N}\sum_i L(\boldsymbol{x_i}|\boldsymbol{y})\delta(\boldsymbol{x} - \boldsymbol{x}_i) \tag{3}$$

The distribution of the weighted particles is the posterior distribution.

In Equation (3), weighted particles are converted to posterior distribution at once. It is also possible to update the distribution gradually by the following equation using the monotonically increasing sequence $0 = q_0 < q_1 < q_2 < \cdots < q_M = 1$.

$$p_j(\boldsymbol{x}|\boldsymbol{y}) \propto p_{j-1}(\boldsymbol{x}|\boldsymbol{y})L(\boldsymbol{x}|\boldsymbol{y})^{q_j - q_{j-1}} \tag{4}$$

Here, $p_0(x|y) = p(x)$. This method is called the sequential Monte Carlo simulation. In particular, TMCMC proposed an algorithm for setting the sequence $\{q_j\}$ according to the trend of the jth posterior distribution $\{p_j\}$.

By gradually updating, the posterior distribution can be stably estimated for any likelihood function. In addition, since the likelihood of each particle can be calculated in parallel, the update of the parameters can be accelerated.

Likelihood function –Frequency domain function-

Saito (2013) proposed the following likelihood function for the Bayesian updating of the seismic response prediction model with parameter θ.

$$L(\theta|D) = \prod_i^{n_s}\prod_j^{n_t} \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left[-\frac{\left({}_o y_i(t_j) - {}_p\hat{y}_i(t_j|\theta)\right)^2}{2\sigma_y^2}\right] \tag{5}$$

$$\sigma_y^2 = \sum_i^{n_s} \sum_j^{n_t} \frac{1}{n_s n_t} \left({}_O y_i(t_j) - {}_p \hat{y}_i(t_j|\theta) \right)^2 \quad (6)$$

Here, i and j are indices of observation points and sample times, and n_s and n_t are the number of observation points and the number of sample times. ${}_O y_i(t_j)$ is the acceleration time history of the observation record. ${}_p \hat{y}_i(t_j|\theta)$ is the predicted value of the acceleration time history. ${}_p \hat{y}_i(t_j|\theta)$ is obtained by the dynamic time analysis.

But it is considered difficult to update models based on the measurement records of the microtremors using (5) because the records of microtremors have large noise. Due to the large noise, the amount of practical information obtained per measurement time is smaller than that of the seismic record. Therefore, when studying the vibration characteristics by measuring the microtremors, long time measuring, several tens of minutes or more, is needed to obtain enough amount of information. Thus, if the above equation is adopted for the records of microtremors, we must conduct a time history response analysis for long time data. It's difficult due to the computational cost of FE models.

Therefore, in this study, we propose the following frequency domain likelihood function:

$$L(\theta|D) = \prod_{i=1}^{n_s} \prod_{j=1}^{n_\omega} \frac{1}{\sqrt{2\pi\sigma_f(\omega_j)^2}} \exp\left[-\frac{\left| {}_O f_i(\omega_j) - {}_p f_i(\omega_j|\theta) \right|^2}{2\sigma_f(\omega_j)^2} \right] \quad (7)$$

Here, i and j are indices of observed points and frequency. $f(\omega)$ is the complex transfer function of each observed point with respect to the base input motion.

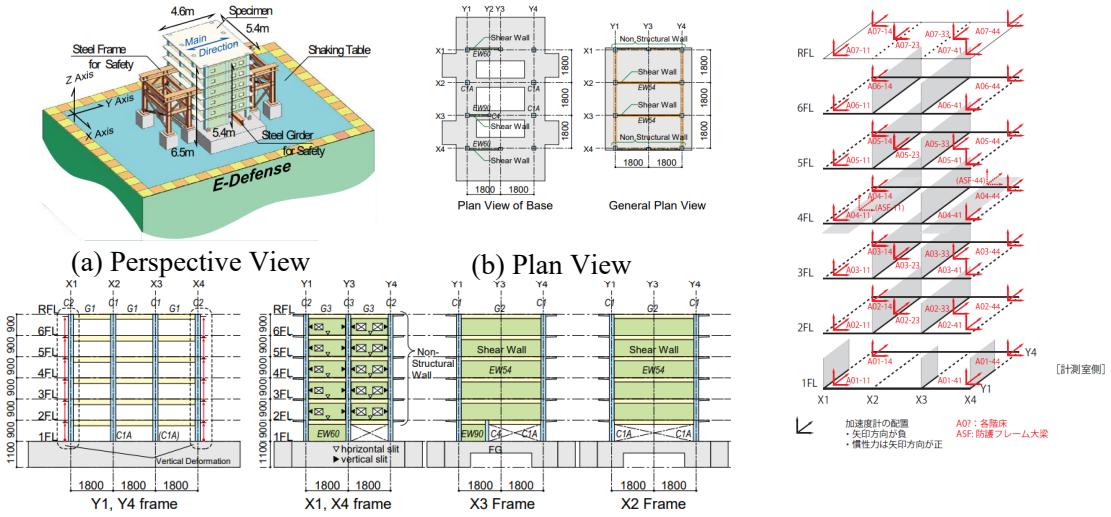
The above equation assumes an isotropic Gaussian distribution on the complex plane at each frequency. The mean ${}_O f_i(\omega_j)$ and the variance $\sigma_f(\omega_j)^2$ are evaluated from the record of White noise 2 on Day 1. Samples of transfer function are got by splitting the observation acceleration time history data in a time range and calculating the transfer function for each split time data.

Applicability study of the proposed method using the record of Sugimoto et al. (2017)

As an applicability study of the proposed method, we created the FE model of a six-story RC building, a test specimen in Sugimoto et al. (2017). We updated the model using the acceleration records observed in the shaking table experiment. Fig. 2 shows the test specimen.

Table 1 shows the excitation history. JMA KOBE is seismic motion record observed in Japan. During excitation with JMA KOBE, white noise excitation is performed to check the vibration properties of the test piece.

In this paper, the FE model is updated using the record of White noise 2 of Day 1. After that, we conducted a response analysis for JMA KOBE (10%) and studied how accurately the updated model can predict the acceleration response.



(a) Perspective View (b) Plan View

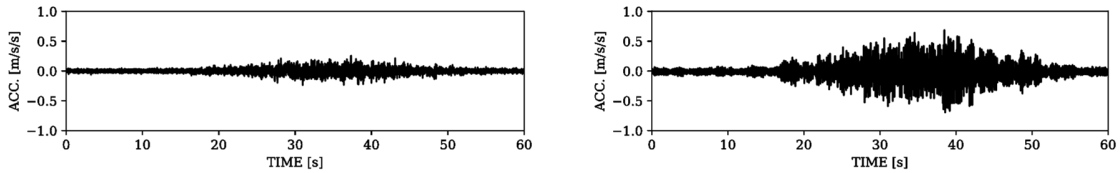
(c) Elevations (d) Observed Points

Figure 2. The test specimen (Sugimoto et al. ,2017 and ASEBI, https://asebi.bosai.go.jp/).

Table 1: The history of input waves（ASEBI, https://asebi.bosai.go.jp/）

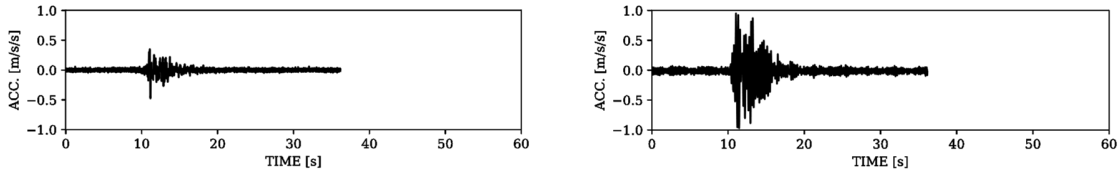
Day1	1	White noise 1	Day2	1	JMA KOBE (55%)	Day3	1	JMA KOBE (55%)
	2	White noise 2		2	White noise 1		2	White noise 1
	3	JMA KOBE (10%)		3	JMA KOBE (70%)		3	JMA KOBE (120%)
	4	White noise 3		4	White noise 2		4	White noise 2
	5	JMA KOBE (40%)		5	JMA KOBE (100%)		5	JMA KOBE (140%)
	6	White noise 4		6	White noise 3		6	White noise 3
	7	JMA KOBE (55%)					7	JR TAKATORI (120%)
	8	White noise 5						
	9	JMA KOBE (70%)						
	10	White noise 6						

*JMA KOBE：Record of a magnitude 7.3 earthquake observed on January 17, 1995 in Kobe, Japan
**JR TAKATORI：Records observed at JR Takatori Station (Kobe City) of the same earthquake



(1) Basemat (2) Roof

Figure 3 The X-dir. acceleration record of Day1 White noise2



(1) Basemat (2) Roof

Figure 4 The X-dir. acceleration record of Day1 JMA KOBE (10%).

Fig. 5 shows an FE model of the test specimen from Sugimoto et al. (2017) created for this study. The model is elastic. The roof, floor, and walls are modeled with shell elements. The columns and beams are modeled with beam elements. The lower end of the model was fixed. We use general-purpose 3-dimensional dynamic analysis program TDAP III for response analysis.

Eigenvalue analysis is performed using the created FE model with nominal values shown in Table 2. The eigen vectors of the modes with the largest effective mass ratio in the X, Y, and Z directions are shown in Fig. 6.

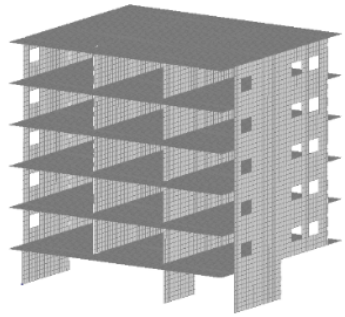
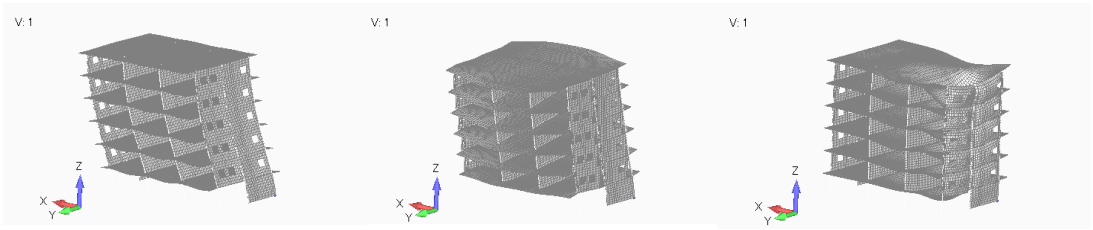


Figure 5. FE model.

Table 2: The Nominal Value of concrete status

Concrete strength	30N/mm^2
Young's module	2.44×10^4N/mm^2
Poisson ratio	0.2



(a) X-dir.(3.6Hz) (b) Y-dir.(9.6Hz) (c) Z-dir. (20.6Hz)

Figure 6. Eigen values and eigen vectors of the Nominal model.

Parameters to be updated and their prior distribution

In this paper, we updated each layer's Young's module of the RC material, E_i. So θ in (1) is 6 dimensional vector $\theta = \{E_1, E_2, \dots, E_6\}$.

To set the prior distribution of parameters, the first-mode frequency and damping ratio were estimated by ARX for the acceleration record of White noise 2 on Day 1. The result is shown in Table 3.

Table 3: Frequency and Damping factor estimated by ARX

X-dir.		Y-dir.		Z-dir.	
Frequency [Hz]	Damping Ratio [-]	Frequency [Hz]	Damping Ratio [-]	Frequency [Hz]	Damping Ratio [-]
4.4Hz	0.02	10.2Hz	0.02	24.0Hz	0.01

Focusing on the X direction, the ratio between the first-mode frequency of the modal analysis result of the FE model is shown in Fig. 6 and the frequency of the ARX shown in Table 3 is 4.4/3.58 = 1.23. Considering this, we assume the prior distribution of Young's coefficients for each layer, as shown in Table 4. Here, we assume that there is a correlation between the Young's modulus of each layer and, give a correlation coefficient of 0.7.

We used the Rayleigh damping. Considering the ARX result, the Rayleigh damping coefficients α and β are set such that the damping ratio is 0.02 at 4.4 Hz and 15 Hz.

Table 4 Prior distribution

	Median	C.O.V.	Notes
Young's module	3.7×10^4	0.5	Log normal distribution

RESULTS

Case1 Numerical experiment (time domain)

We performed a response analysis of the FE model with a certain parameter θ_{true}. Considering the analysis results as observational records, we attempted to update the FE model using the likelihood function (5) by following steps.

i. We performed response analysis on an analysis model with θ_{true}. The input waveform is the observation record of Day 1-White noise 2 of Sugimoto et al. (2017) shown in Fig. 3(1).
ii. White Gaussian noise $N(0,0.02)$ is added to the acceleration time history of the analysis result. The noise-added data is treated as a target observation record.
iii. Based on the prior distribution of Table 4 and the likelihood function of equation (5), the parameters are updated by TMCMC.

Figure 7(1) shows the comparison between θ_{true} and the updated parameter distribution. The updated parameters consist with θ_{true}.

Case2 Numerical experiment (frequency domain)

Updating parameters using the likelihood function (7) is also conducted for the same problem with Case1. The result is shown in Figure 7(2).

The results are slightly different from the results of Case 1, but they roughly consist with the true value θ_{true}. Therefore, the method proposed in this paper is likely applicable to the FE model updating.

Case3 Updating by Observed Record

Using the frequency domain likelihood function of Equation (7), the parameters are updated using the actual observation record from Sugimoto et al. (2017). The updated parameters are shown in Fig. 7(3). As a result of the update, the Young's coefficient of the first layer is slightly smaller, and those of the fourth and sixth layers are higher. Fig. 8 shows a comparison between the transfer function of the updated parameters and the transfer function of the Day 1-White noise 2 recording. The updated transfer function fits better with the observation record than the nominal model.

Next, a response analysis is performed for the JMA KOBE (10%) record using the updated parameters and compared with the observed record of the shaking test. Fig. 9 shows the comparison results. The response analysis results of the updated parameters are closer to the observation records than the nominal model.

In addition, an eigenvalue analysis of the updated FE model was performed to investigate the distribution of first-mode frequencies. Fig. 10 shows the analysis results. The variation in the natural frequencies was not as large as the variation in the parameters. Figure 11 shows the relationship between the Young's modulus of the first and second layers. There seems to be a negative correlation between the two.

In addition, we conducted a static push over analysis for the updated FE model and the nominal model. evaluated the stiffness of each layer. The results are shown in Fig. 12. The figure also shows the shear force–deformation relationship of each layer obtained from the measurement record of JMA KOBE

(10%). For the measurement record, a bandpass filter of 3 ~ 8 Hz was applied to extract the primary mode component. The comparison shows that the stiffness of the updated FE model is consistent with that of the test specimen.

Table 5: Time required for analysis of Case 3

The Num. of particles N	*The Num. of generations M*	*Analysis time For each generation*	*Total time*
1000	***24***	***60 min.***	***About 24 hours***

※CPU：Xeon Gold6246 (3.3GHz, 12cores×2)

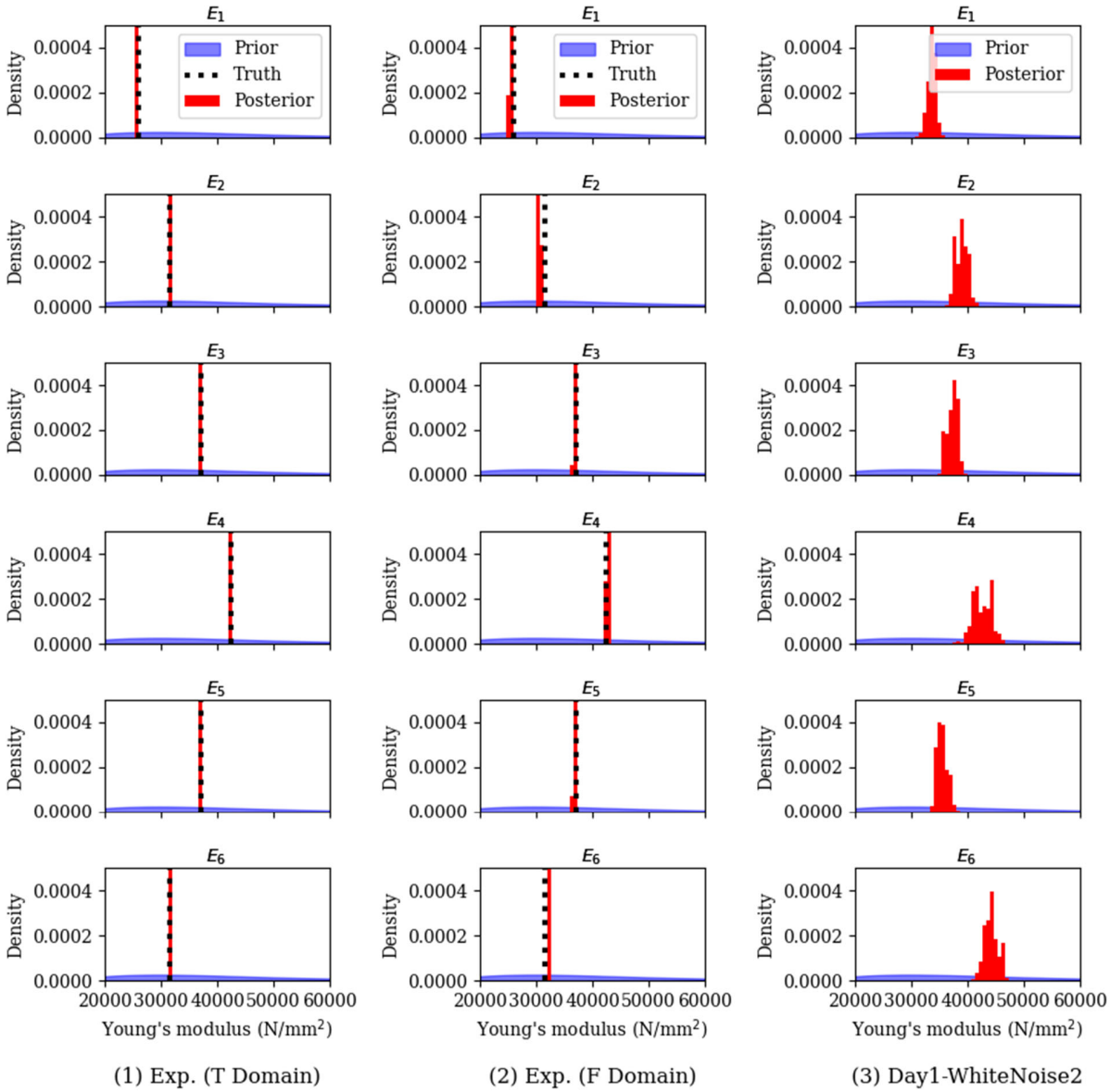


Figure 7 Updated parameters（Young's module）.

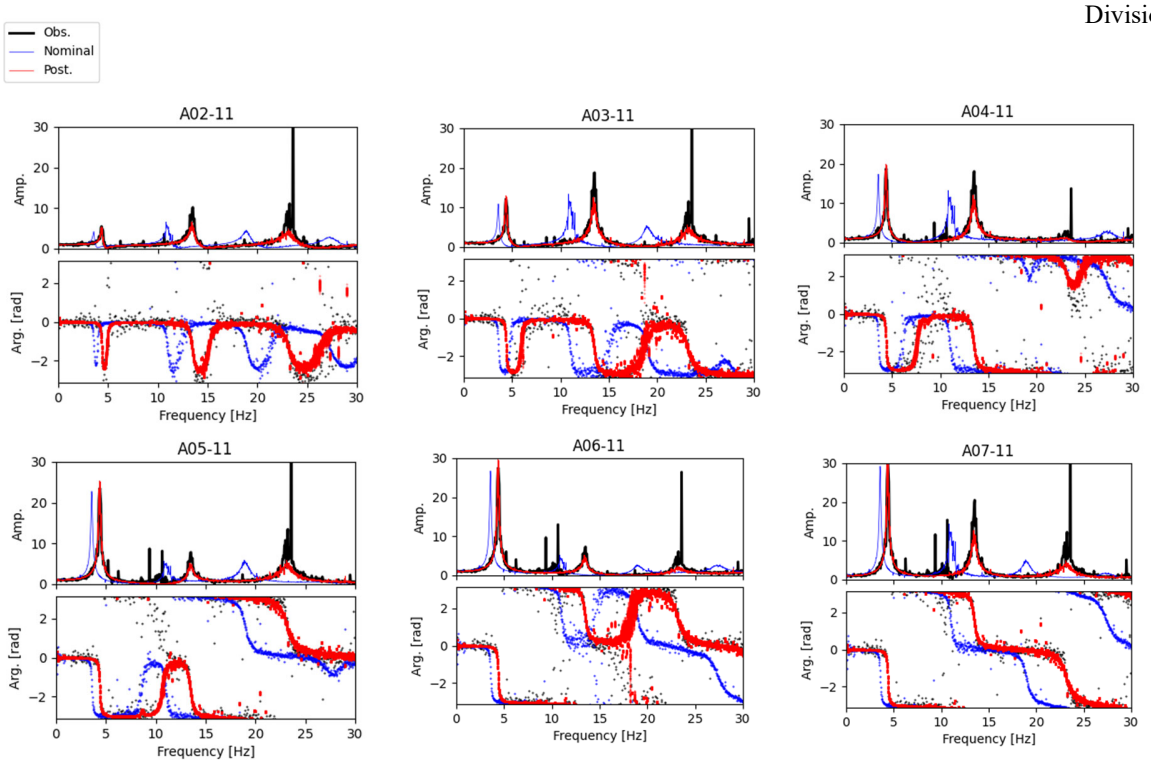


Figure 8. Comparing of transfer function (Obs.:Observed records, Nominal: the nominal model, Post.: Case3 updated model).

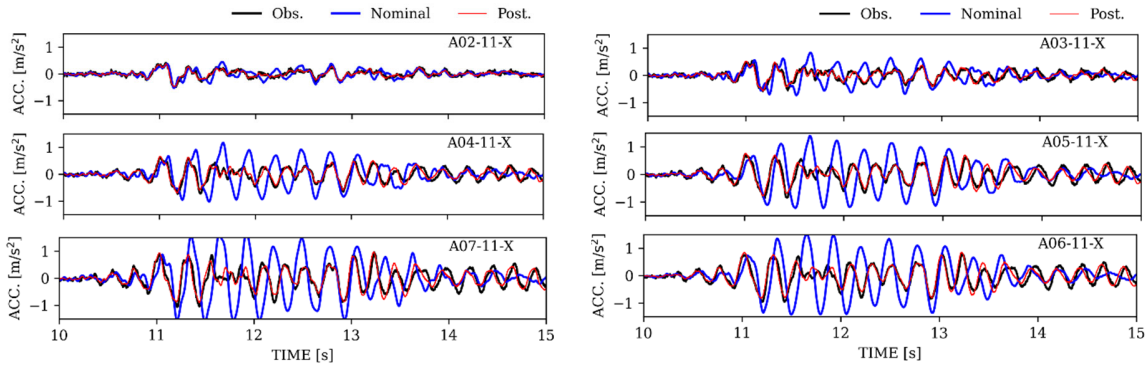


Figure 9. Acceleration response for JMA KOBE (10%).

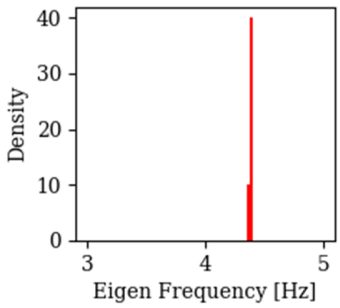


Figure 10. The distribution of the eigen frequency of Case3 updated model.

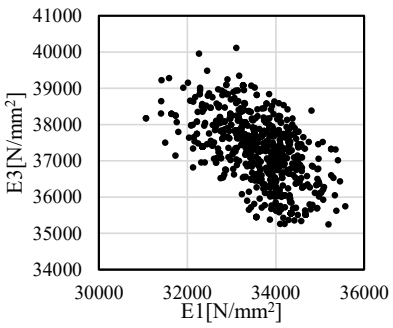


Figure 11. Correlation between E1 and E3 of Case3 updated model.

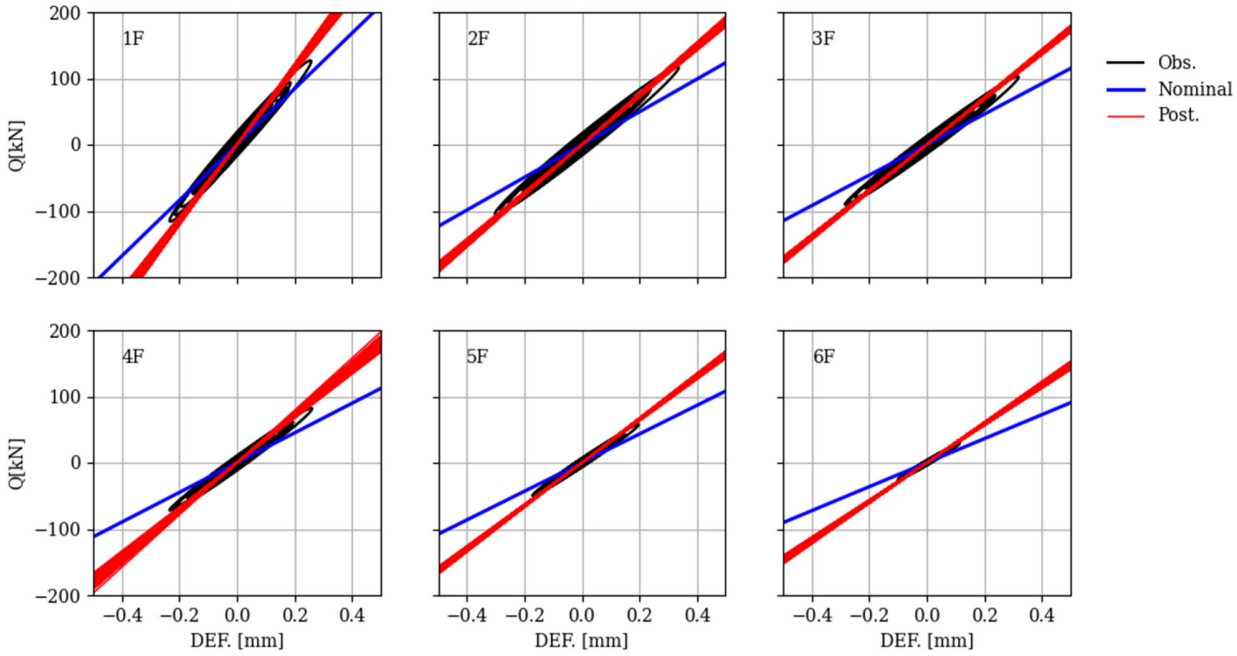


Figure 12. Layer stiffness of the updated FE model.

CONCLUSION

- We propose a Bayesian updating method for FE models using TMCMC and the frequency domain likelihood function.
- We verified the applicability of the proposed method through a numerical experiment.
- We attempted to update the FE model using the actual observed record of Sugimoto et al. (2017). It was found that the FE model updated using the method proposed in this paper can accurately predict the acceleration response during an earthquake.

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