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PBMR LOOP ACOUSTICS

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ABSTRACT

The present paper provides a method to determine the acoustic pressure due to compressor, pumps and fans operations throughout a pipe loop of a reactor and specifically is indented for use in the Pebble Bed Modular Reactor Demonstration Power Plant (PBMR DPP). This is accomplished by combining simple one dimensional analytical pipe solutions with the output of a fluid dynamics computer code for both steady state and transient conditions. Simple one dimensional pipe models consisting of an acoustic wave equation with non-homogeneous boundary conditions are modeled via a transformation technique. It is demonstrated that these models can be used to properly couple various individual pipes, while still maintaining the proper physical acoustic behavior. A subroutine is developed whereby the geometry and temperature from a fluid dynamics computer code is used as input for the acoustic loop model for steady state conditions. A series of unknown constants required at the pipe/pipe interface, necessary to maintain pressure and pressure gradient continuity are solved for via matrix operations to obtain the required solution. This same procedure is followed via a time step for transient results. The resulting acoustic pressures can be then imported to a finite element pipe model to determine pipe vibration response.

1.0 INTRODUCTION

Acoustic response of reactor piping and reactor core support structures has been a subject at SMIRT since Penzes introduced his clever model in 1973. Several iterations to the Penzes model for PWR core support structure acoustic loading and additional models including pipe acoustics have been described in the attached bibliography. The intent of this paper is to utilize simple pipe acoustic models coupled with a fluid dynamic computer code output to obtain the acoustic pressure waves for a reactor circuit. It should be noted that the necessary geometry is part of the fluid dynamic computer code input, but more importantly, the output temperature from such a model is the needed information to determine the proper speed of sound. To accomplish this, the following studies will be performed:

1. Provide acoustic response to simple pipe models with various boundary conditions.
2. Utilizing a known solution for a simple pipe with a pump at one end and an open end on the other end, this same problem is then cut at an arbitrary location. Two simple pipe models are used to provide an alternate solution. It is demonstrated that the un-cut pipe and the cut pipe with two separate regions produce identical results. This provides assurance that:
    a. Proper acoustic boundary conditions are utilized for coupling pipes
    b. The use of the coupling of two regions implicitly contains the correct physical resonance conditions when applying the appropriate boundary conditions. This was challenged by Jong-sik Cheong et. al.
3. Perform a simple pipe loop to demonstrate how these simple models can achieve the appropriate acoustic response.

4. Utilize the above concepts to provide a sub-routine for fluid dynamic output to obtain both steady state and transient acoustic response for the PBMR primary loop.

2.0 MATHEMATICAL FORMULATION

Acoustic behavior in pipes and vessels can often be described as a one dimensional wave equation. To simulate the behavior of a pipe loop, four simple pipe models are necessary to obtain proper response. These four models consist of the interior being described by a simple one-dimensional wave equation with the boundary conditions as found in Table 1. The first entry, known as Case 1, represents a pipe of length L with a pump at the left hand side and an open end at the right. The second entry, Case 2, is the same as the first with a more arbitrary pressure gradient at the right. The third entry, Case 3, is the more general case of a pressure gradient at the both sides and case 4 is an arbitrary pressure gradient at the left with an open end at the right. This is illustrated in figure 1, for case 2 of a pump at the left and a general resistance, simulated by a spring at the right. The solution outline for these four models consists of:

Differential equation: $\Psi[P(x,t)] = 0$, where the operator $\Psi \equiv \frac{\partial^2}{\partial x^2} - \frac{1}{C_0^2}\frac{\partial^2}{\partial t^2}$

Boundary Conditions; A choice of two of the following four, as defined in Table 1:

$$P(0,t) = f(t) \qquad \text{(1-a)} \qquad \frac{\partial P(x,t)}{\partial x}\Big\rangle_{x=0} = f(t) \qquad \text{(1-b)}$$

$$P(L,t) = g(t) \qquad \text{(1-c)} \qquad \frac{\partial P(x,t)}{\partial x}\Big\rangle_{x=L} = g(t) \qquad \text{(1-d)}$$

The method of solving this type of problem is found in the work of Fisher et al and illustrated in several papers by Cepkauskas. It consists of writing a transformation of the form:

$$P(x,t) = Q(x,t) + h(x)f(t) + i(x)g(t) \qquad (2)$$

The differential equation is transformed into a non-homogeneous differential equation:

$$\Psi[Q(x,t)] = -\Psi[Q(x,t) + h(x)f(t) + i(x)g(t)] \qquad (3)$$

The boundary conditions become:

$$P(0,t) = f(t) - [h(0)f(t) + i(0)g(t)] \qquad \text{(4-a)}$$

$$\frac{\partial P(x,t)}{\partial x}\Big\rangle_{x=0} = f(t) - \{\frac{\partial}{\partial x}[h(x)f(t) + i(x)g(t)]\}_{x=0} \qquad \text{(4-b)}$$

$$P(L,t) = g(t) - [h(L)f(t) + i(L)g(t)] \qquad \text{(4-c)}$$

$$\frac{\partial P(x,t)}{\partial x}\Big\rangle_{x=L} = g(t) - \{\frac{\partial}{\partial x}[h(x)f(t) + i(x)g(t)]\}_{x=L} \qquad \text{(4-d)}$$

By placing simple restrictions (0 or 1) on the auxiliary functions, h(x) and i(x), and their derivatives at the boundaries, the boundary conditions become homogeneous. This transformation makes the problem description more manageable. Fisher et al demonstrated that, contrary to previous authors, there is no need of defining the auxiliary functions on the interior of the problem. This is accomplished by after solving the problem in the transformed space, upon substituting back into the untransformed equation, terms appear that are the negative Fourier series expansions of the auxiliary functions and thus the auxiliary functions terms cancel. Following this procedure, the resulting solutions are found in Table 2. Details of case 2 are found in the SMIRT 5 paper by Cepkauskas and later addressed in Cepkauskas and Thanjekwayo.

Boundary conditions of fully acoustic closed end can be achieved by letting one of the two constants "A" and "B" equal zero. A very useful simplification of the required series is found in the book by L.B. W. Jolley. All series are reduced to a simple equation. It should be noted that Fisher et al., showed a numerical convergence difficulty at the boundaries. The use of the Jolley Series provides an exact solution at the interior and at the boundaries.

Case	**Left Boundary Condition** $@\,x=0$	**Right Boundary Condition** $@\,x=L$	**Natural Frequencies**	**Mode Shapes**	**Series indices "j"**
1	$P(x,t)=P_0\cos\omega$	$P(x,t)=0$	$\frac{j\pi C_0}{L}$	$\sin\frac{j\pi x}{L}$	1,2,3...
2	$P(x,t)=P_0\cos\omega_p$	$\frac{\partial P(x,t)}{\partial x}=B\cos\omega_p t$	$\frac{j\pi C_0}{2L}$	$\sin\frac{j\pi x}{2L}$	1,3,5...
3	$\frac{\partial P(x,t)}{\partial x}=A\cos\omega_p$	$\frac{\partial P(x,t)}{\partial x}=B\cos\omega_p t$	$\frac{j\pi C_0}{L}$	$\cos\frac{j\pi x}{L}$	0.1.2.3
4	$\frac{\partial P(x,t)}{\partial x}=A\cos\omega_p t$	$P(x,t)=0$	$\frac{j\pi C_0}{2L}$	$\cos\frac{j\pi x}{2L}$	1,3,5...

TABLE 1 Boundary Conditions, Natural Frequencies and Mode Shapes

Case	**Forced Response**	**Summation of Series per Jolley** **Let x*=x/L** **Let** $\alpha=L\omega_p/c_0$
1	$\sum_{j=1,2}^{\infty}\frac{2P_0c_0\omega_j}{L(\omega_j^2-\omega_p^2)}\sin\frac{j\pi x}{L}\cos\omega_p t$	$P_0\frac{\sin[\alpha(1-x^*)]}{\sin\alpha}$ (Jolley # 560)
2	$-\sum_{j=1,3}^{\infty}\frac{2c_0[P_0\omega_j+Bc_0\sin\frac{j\pi}{2}]}{L(\omega_j^2-\omega_p^2)}\sin\frac{j\pi x}{2L}\cos\omega_p t$	$\frac{P_0\cos[\alpha(1-x^*)]+(\frac{Bc_0}{\omega_p})\sin(\alpha x^*)}{\cos\alpha}$ (Jolley # 556 & 559)
3	$A\sum_{j=0,1,2}^{\infty}\frac{2c_0^2}{L(\omega_p^2-\omega_j^2)}\cos\frac{j\pi x}{L}\cos\omega_p t$ $-B\sum_{j=0,1,2}^{\infty}\frac{2c_0^2}{L(\omega_p^2-\omega_j^2)}\cos j\pi\cos\frac{j\pi x}{L}\cos\omega_p t$	$A\frac{c_0}{\omega_p}\{\frac{\cos[\alpha(1-x^*)]}{\sin\alpha}\}$ $-B\frac{c_0}{\omega_p}\{\frac{\cos[\alpha(x^*)]}{\sin\alpha}\}$ (Jolley # 559 & 558)
4	$A\sum_{j=1,3}^{\infty}\frac{2c_0^2}{L(\omega_j^2-\omega_p^2)}\cos\frac{j\pi x}{2L}\cos\omega_p t$	$\frac{-Ac_0}{\omega_p}\frac{\sin[\alpha(1-x^*)]}{\cos\alpha}$ (Jolley # 555)

TABLE 2 Series Solution and Jolley Convergence

3.0 PIPE/PIPE INTERFACE CONDITION VERIFICATION

The use of a pipe/vessel interface condition of the form $\partial P(x,t)/\partial x = A\cos\omega_p t$ was utilized in the SMIRT 5 paper by Cepkauskas. This was based on physical reasoning. If the pump output is of the form: $P(x,t) = P_0 \cos\omega_p t$, then it is reasonable to assume the pressure gradient at the interface is an unknown constant times the same time dependency. The unknown constant "A" can then be found by also requiring pressure continuity at the interface. Prior to writing the SMIRT 5 paper, the following simple example was utilized to convince the author of this assumption. First the solution for a pipe of length L corresponding to case 1 is determined. Next this problem is cut into two regions at an arbitrary length L_2. The problem is then repeated using case 2 and case 4 and requiring pressure continuity at the junction of the two regions. This is illustrated in Figure 2, for a 10 meter pipe, arbitrarily cut at 7 meters, with 0.1 pump pressure, C_0=1200 m/s with a pump speed of 100 Hz. The curve identified as "SUM" is the results of case 2 for a 7 meter pipe and case 4 for the remaining 3 meter pipe. This solution is interesting in that the two regions now have different natural frequencies. This condition is criticized by Jong-sik Cheong et. al., but is seen to be appropriate.

4.0 SIMPLE PIPE SYSTEM EXAMPLE

Consider the simple example as shown in figure 3. This consists of a pump, two pipes and a long vessel. The pipes and vessel are numbered 1 through 4. Subscripts will be used to identify the pipe number. Pipe #1 can be represented by case 2 and has pressure distribution:

$$P_1(x_1^*,t) = \frac{P_0\cos[\alpha_1(1-x_1^*)] + (\frac{B_1 c_{0,1}}{\omega_p})\sin(\alpha x_1^*)}{\cos\alpha_1} \tag{5}$$

The vessel (pipe #2) and pipe #3 can be described with case # 3:

$$P_2(x_2^*,t) = A_2\frac{c_{0,2}}{\omega_p}\{\frac{\cos[\alpha_2(1-x_2^*)]}{\sin\alpha_2}\} - B_2\frac{c_{0,2}}{\omega_p}\{\frac{\cos[\alpha_2(x_2^*)]}{\sin\alpha_2}\} \tag{6}$$

$$P_3(x_3^*,t) = A_3\frac{c_{0,3}}{\omega_p}\{\frac{\cos[\alpha_3(1-x_3^*)]}{\sin\alpha_3}\} - B_3\frac{c_{0,3}}{\omega_p}\{\frac{\cos[\alpha_3(x_3^*)]}{\sin\alpha_3}\} \tag{7}$$

The fourth pipe can now be represented by case #4 (more on this choice will be discussed below):

$$P_4(x_4^*,t) = \frac{-A_4 c_{0,4}}{\omega_p}\frac{\sin[\alpha_4(1-x_4^*)]}{\cos\alpha_4} \tag{8}$$

Note that $B_1 = A_2$, $B_2 = A_3$ & $B_3 = A_4$, due to continuity of the pressure gradients at the pipe/pipe interfaces. Thus there are three unknown constants. These constants are determined by requiring pressure continuity at the pipe/pipe interfaces:

$$P_1(x_1^*=1,t) = P_2(x_2^*=0_1,t) \tag{9-a}$$

$$P_2(x_1^*=1,t) = P_3(x_2^*=0_1,t) \tag{9-b}$$

$$P_3(x_1^*=1,t) = P_4(x_2^*=0_1,t) \tag{9-c}$$

This leads to a set of three linear equations of the form:

$$a_{11}B_1 + a_{12}B_2 + a_{13}B_3 = b_1 \quad \text{(10-a)}$$

$$a_{21}B_1 + a_{22}B_2 + a_{23}B_3 = b_2 \quad \text{(10-b)}$$

$$a_{31}B_1 + a_{32}B_2 + a_{33}B_3 = b_3 \quad \text{(10-c)}$$

These equations are easily solved for the three unknowns using Cramer's rule. Thus the entire acoustic pressure distribution for figure 3 is determined and shown in figure 4 with L_1=10 m, L_2=15 m, L_3=20 m & L_4=25 m. The temperatures are T_1=100 °C, T_2=200 °C, T_3=300 °C & T_4=400 °C with the speed of sound determined by the equation $c_0 = \sqrt{\Gamma R(T + 273.3)/M}$, where R=8.314 kJ/kg.K. =5/3 & M=0.004. A pressure of 0.1 MPa is exiting the pump at 100 HZ. Figure 4 is a continuous plot of the four pipes, totaling 70 meters. Note the continuity of pressure and pressure gradient at the pipe/pipe interfaces. It should also be mentioned that this example is not realistic in that it has abrupt discontinuity of temperature but is merely used to demonstrate the use of the methodology.

5.0 APPLICATION TO PBMR

Flow models have been prepared for the entire PBMR pipe network. This model has the required geometry needed for the acoustic model and also provides the proper temperature for each pipe section. Thus, based on the flow model temperature, the required acoustic speed of sound can be determined. These models are provided for both steady state and transient conditions. The extension for steady state conditions is the same as found in the above example, with the exception of an increased number of equations to be solved. The transient is a repeat of the steady state case, except the unknown constants and pressure distributions are obtained at suitable time steps.

Typically the value of P_0 for a pump or a compressor is supplied by the vendor, either based on analysis or test. The input to the pump is assumed to be acoustically zero pressure. This condition in pipe #4 at the pump inlet in the example above is chosen as zero pressure. This needs to be confirmed from test results.

Vessels are three dimensional acoustic volumes. Typically one of the dimensions dominates. Thus it is acceptable, for a long vessel to be represented by a long pipe. However, added details can be achieved by modeling the vessel as a three dimensional volume. This can be accomplished utilizing the SMIRT 5 volume for a Pressurized Water Reactor coolant annulus and reducing the radius to zero or modeling the vessel as an equivalent annulus. If the SMIRT 5 solution is used, the Bessel Function terms that blow up at zero radius are set equal to zero. Coupling to a pipe is demonstrated in the SMIRT 5 paper.

A complex heat exchanger, such as the Recuperator in the PBMR design utilizes many tortuous flow paths. The first approximation is to use an equivalent pipe. Future analytical models or testing needs to be performed to better understand the acoustic behavior of the Recuperator. These vessels may actually help to dissipate the acoustic energy, this will need additional study. The graphite core reflectors and the fuel spheres may also be a means of dissipating acoustic energy. This needs further investigation also.

Past investigations have shown very little viscous damping in the acoustic equations at resonance. One needs to take precaution. If the acoustic pressure is large from these models, the original perturbation of pressure used to develop the acoustic wave equation is violated. Thus some of the

non-linear terms discarded in the development of the wave equation needs to be further examined.

The search for resonance in the piping is to determine pressure that causes the pipes to vibrate. A coupling of structural, acoustic and pump frequencies along with a contributing modal participation factor is needed for large displacements. The coupled problem is examined by Cepkauskas & Stevens. In reality the damping of the structure helps to dissipate the acoustic energy.

During heat up, the acoustic natural frequencies are constantly changing, thus resonance is sure to occur. The regions of resonance need to be determined and either the acoustic energy need to be dissipated via design or the heat up time needs to be accelerated to avoid fatigue damage.

A reactor coolant pump for a PWR comes to full speed very rapidly, thus the driving frequency starts at zero and arrives at its final frequency ω_p. This sweeps through any acoustic resonance quick enough to avoid high structural response. For the PBMR DPP the compressor start from zero to full speed occurs over a much larger time period than a PWR. This can be examined using the above methods by using time steps for changing the compressor speed. This will be a subject of a future investigation.

6.0 CONCLUSIONS

A model for calculating the acoustic pressure waves in a piping system utilizing simple pipe models coupled with flow model results is presented. This model relies on the output from a conventional fluid flow code for both the geometry and the temperature. The temperature is used to determine the instantaneous speed of sound for both the steady state and transient flow conditions. This methodology is currently being expanded on for three dimensional fluid cavities. The application to the PBMR reactor is being prepared. Additional studies are required for specific components, such as the Recuperator, and other phenomenon. It is also concluded that the interface condition utilized in the SMIRT 5 paper properly couples the pipe and annulus.

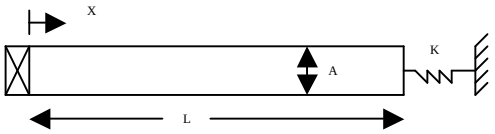

Figure 1

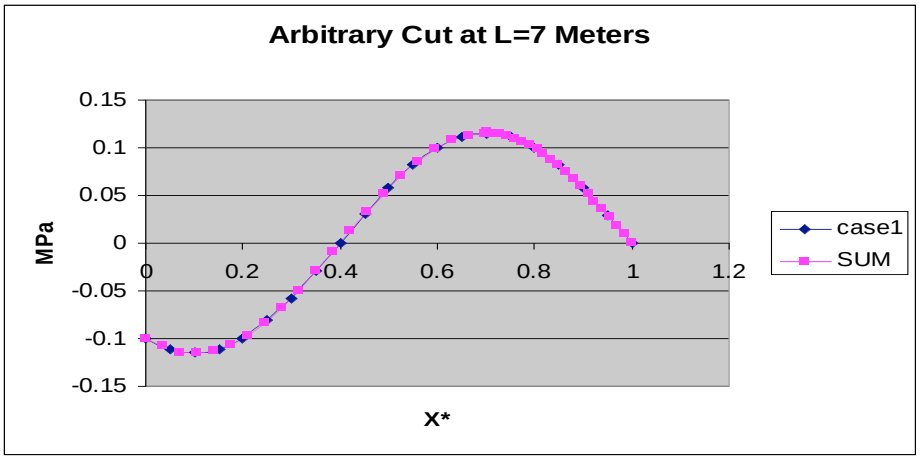

Figure 2

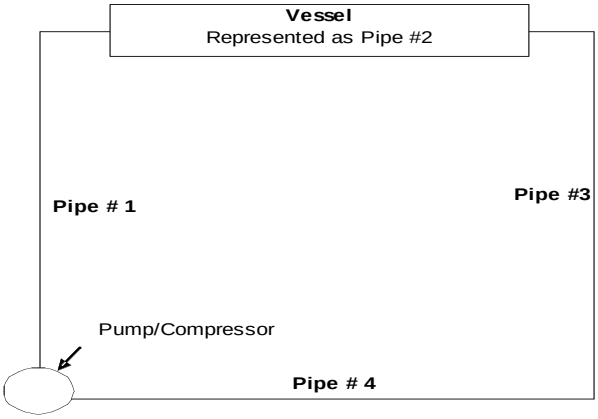

Figure 3

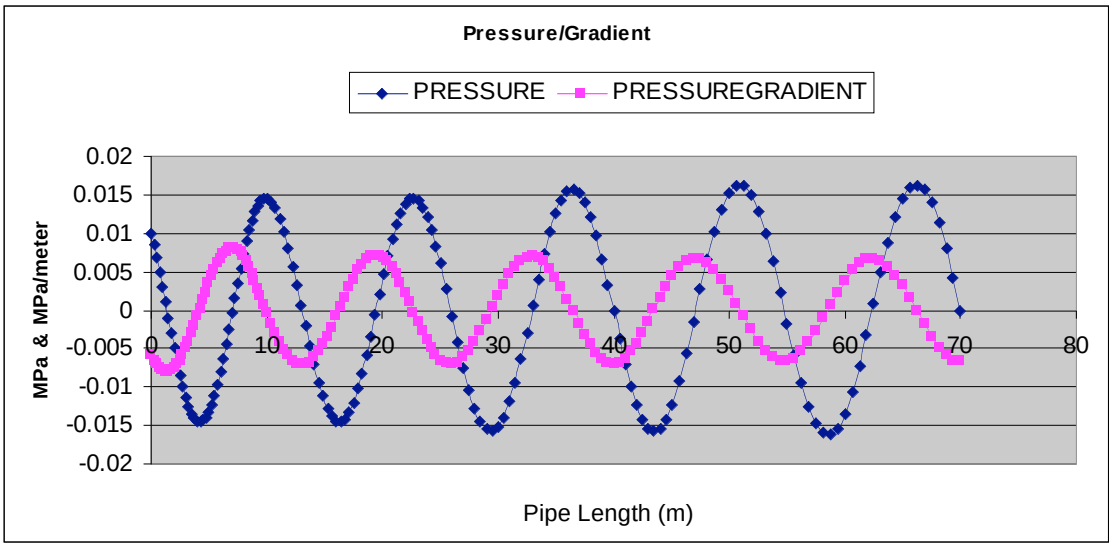

Figure 4

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