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Some Aspects of Stochastic Fracture Mechanics

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ABSTRACT

The authors will show the usefulness of the graphical presentation of the probabilistic and stochastic characteristics, and discuss the importance of the comunication with experts through the graphical presentation and the phisical aproach to the mechanism of fracture, in order to overcome the lack in experimental data.

1. INTRODUCTION

Although the method itself to assess the structural integrity or reliability through the probabilistic or stochastic Fracture Mechanics is rather simple and well developed, the result of the analysis often lacks in reality, because of insufficient data on the distributions of the probabilistic variables. To confront the problems of the lack in data, we might have to pay more attention to the following points: (i) communication with experts, and (ii) not phenomenological but physical approach to the mechanism of fracture.

To utilize expert-system, we must decompose the total system of assessment into indivisual sub-system corresponding to each field of experts, and ask effectively their opinions on the input and output of the sub-system. A graphical presentation on a probability paper of the input and output in each intermediate stage is very useful not only for the communication with experts but also for understanding by ourselves. Though the mean value and sometimes the variance can be estimated by the expert system, we have to extrapolate the distribution at the foot where the probability is low but essensial in the analysis. The physical approach to the characteristics of the stress and the failure mechanism is thought to be indispensable for a better extrapolation. The present authors would like to discuss some aspects of Stochastic Fracture Mechanics in these two phases.

2. STOCHASTIC PROCESS OF LIFE

Let the probability that the life T is less than t, or the distribution of the life be F(t), and 1-F(t) be $\overline{F}(t)$. Then $\overline{F}(t)$ means the probability of survival or the reliability and is given by the failure rate $\lambda(t)$, the probability of failure per unit time period as follows;

$$\overline{F}(t)=1-F(t)=\exp[-\int_0^t \lambda(t)dt] \tag{1}$$

Therefore the failure rate $\lambda(t)$ can be obtained by the slope of the curve $\overline{F}(t)$ on the semi-logarithmic graph, as is shown in Fig. 1. The curves A, B and C in the figure schematically present the examples of the failure of the machine, the static fatigue of glass[1] and the creep rupture of metal[2]-[3] respectively. When we know the failure rate

$\lambda(t)$ from $F(t)$, we can get some insight to the characteristics of the stochastic process. On the contrary, when we estimate $\lambda(t)$, we can evaluate the probability distribution.

When the fracture is caused by n kinds of independent failure modes, then the failure rate is given by the sum of the failure rate of each mode $\lambda_i(t)$, namely, $\lambda(t)=\sum_i \lambda_i(t)$. Fig.2 schematically shows a stochastic process of the structural failure during seismic excitation [4]. As is shown later, the failure rate is determined by the competition of stress and resistance in a broader sense.

3. APPLICATION OF SCALE OF WEIBULL'S PROBABILITY PAPER

Some probability papers are very useful for the communication as well as presentation in the analysis of probabilstic variables, especially the graph of the scale of the Weibull's probability paper for the analysis of extreme value. The location Y with reading y (where $0<y<1$) is given by the relation

$$Y=A\cdot\log[-\log(1-y)] \quad \text{(where A is a constant.)} \tag{2}$$

Here y is put by the distribution function $F(x)$ or $\bar{F}(x)=1-F(x)$ of the variable x (see Fig.3).

The failure rate $\lambda(t)$ of the failure mode of level-clossing type is determined by the competition of the resistance R and the largest extreme value of stress S per unit time. Let the values of S and R be x, the distribution functions of them be $F_S(x)$ and $F_R(x)$. It is convenient to plot $\bar{F}_S(x)=1-F_S(x)$ and $F_R(x)$ on the same probability paper because these functions are important near the intersecting point of them where both values are very small compared to 1. The distribution of the stress and the resistanse has more or less the characteristics of the smallest and the largest extreme respectively. As is well known, the asymptotic distribution of the smallest extreme of samples of large size is given by the distribution of the first or the third type:

$$1-F(x)=\exp\{-\exp[(x-x_0)/\eta]\}, \quad \text{(the smallest extreme of 1st type)} \tag{3}$$

$$1-F(x)=\exp\{-[(x-x_0)/\eta]^{\varepsilon}\}, \quad \text{(that of 3rd type, or Weibull)} \tag{4}$$

determined depending on the shape of the foot of the distribution function of the population which is proportional to $\exp[-(x-x_0)]$ or $(x-x_0)^{\varepsilon}$, where x_0, η and ε are constant parameters. Similarly, two kinds of the asymptotic distributions of the largest extreme are given by

$$1-\bar{F}(x)=\exp\{-\exp[-(x-x_0)/\eta]\}, \quad \text{(the largest extreme of 1st type)} \tag{5}$$

$$1-\bar{F}(x)=\exp\{-[(x-x_0)/\eta]^{-\varepsilon}\}, \quad \text{(the largest extreme of 2nd type)} \tag{6}$$

Therefore, the plot of all of the above $F(x)$ or $\bar{F}(x)$ versus x or $\log(x-x_0)$ becomes a straight line on the probability paper with Weibull's scale. If we plot $\bar{F}_S(x)$ of the stress and $F_R(x)$ of the resistance on such a probability paper (see Fig.3), we may often get so smooth curves that we can extrapolate them to the domain of lower probability.

4. EVALUATION OF FAILURE RATE

The failure rate $\lambda(t)$ is determined by the competition of stress and resistance. The direct understanding as follows is more important rather than the mathematical calculation. Let $\bar{F}_S(x)=1-F_S(x)$ and $F_R(x)$ be distributions of the stress per unit time and of the resistance, and then we can get F_R as a function of $\bar{F}_S$ from Fig.3, and plot the relation between them as is shown in Fig.4. The failure rate $\lambda(t)$ is given by the area P_f below the curve on Fig.4. Though the relation between F_R and $\bar{F}_S$ is known or estimated in general only near the intersecting point of these two curves, the upper and the lower bound of the failure rate will be clearly understood from the figure. Similar analysis of the failure rate can be done for confidence limits of distribution functions if necessary.

If these curves are nearly straight near the intersecting point in Fig.3 with the Weibull's scale of the ordinate and an arbitrary scale on the abscissa, the failure rate $\lambda(t)=P_f$ can be estimated on Fig.5 from the probability value P at the intersecting point together with the ratio of the slope of both curves ξ_R/ξ_S.

5. GRAPHICAL ESTIMATION OF THE EXTREME

The distribution of the smallest extreme F_n of the n samples with distribution F and the distribution of the largest extreme $\bar{F}_n=1-\bar{F}_n$ of the n samples with distribution $F=1-\bar{F}$ are given by the following relations respectively;

$$1-F_n=(1-F)^n \text{ (for the smallest)}, \qquad 1-\bar{F}_n=(1-\bar{F})^n \text{ (for the largest)} \tag{7}$$

So, it is clear from eq.(2) that the distribution of the extreme is obtained merely by the parallel shift of the original distribution upward on the probability paper with the Weibull scale not withstanding the shape of the curve and the scale of the variable (see Fig.3b). The amount of the shift is easily read by the scale where the reading y is smaller than 0.2, because the scale by eq.(2) can be approximated by Y=log y, namely by the logarithmic scale, in this range. It comes from this fact that we often observe the foot of the distribution can be presented by a straight line on the log-log paper or the semi-log paper.

According to the above procedure, we can estimate the distribution of the extreme very easily. For example, we can derive the distribution of the extreme stress for the total n years of its service from the distribution of anual extreme, the distribution of fatigue life of a structural member which contains n cracks from the distribution of the fatigue life determined by a single crack, the distribution of the size of the largest crack in a structure from the distribution of the size distribution of each crack, and so on.

Suppose that $F_S(x)$ in Fig.3a is the relation between the maximum stress distribution per unit time and the return priod $1/(1-F_S)$, and rotate the figure, and then we get a familiar figure as Fig.6 of the maximum expected stress for a given service period. We can approximately treat the level-crossing problem by using this figure.

6. EXAMPLES OF ANALYSIS IN SUB-SYSTEM

As mentioned in the introduction, a refinement of the analysis in sub-system is more important than a straightforward analysis from assumptions to the final result on the total risk of a big system, such as a nuclear power station against earthquakes for example. A few examples on colapse and fatigue will be shown here.

The fluctuation of responce factor of lightly damped structure to earthquakes is thought to be very large and difficult to be predicted. But as is shown in Fig.7 about an example of field data observed on the model structures of plant complex and vessels [5], we may get an engineering prediction of the largest extreme from the distribution on the probability paper, better than from the histogram. We can evaluate the probability of failure of level-crossing type by combining the distribution with that of the resistance of structural member.

The damage of cummurative type by earthquakes has a strong corelation with the maximum response and the duration of time. Therefore the cumulative damage U^* which is normalized by maximum responce and means the effective number of cycles is useful to predict the failure rate [6]. Fig.8 shows examples of the distribution of U^*, obtaied from time-history analysis for a lightly damped structure under artificial earthquakes generated from stationally random waves modulated by two types of envelope functions which correspond to inpulsive and short earthquakes (Type I) and long earthquakes of large scale (Type IV). As is shown, the dispersion is small, and we can obtain the cumurative damage from Fig.7 and Fig.8.

On the other hand, the usage factor or the cummulative damage U at fracture fluctuates around unity. Fig.9 shows the distribution of U in low-cycle fatigue under stationary and non-stationary loads for the materials of nuclear pressure vessels. The failure rate by fatigue can be predicted from such figures.

7. EXAMPLES OF CRACK PROBLEMS

Fig.10 shows an example of the initial crack size, and the curve is again smooth enough for an engineering evaluation on a probability paper with a linear or logarithmic scale in abscissa. Such a smooth curve is slso expected for the distribution of the size of undetectable cracks after non-destructive testing and that of the time to crack initiation. The fluctuation of the parameters in the Paris-Erdogan's crack growth law and that of the crack length under random loads will be also discussed in the lecture.

8. APPROACH TO PHYSICAL MECHANISM OF FRACTURE

An elementary process of micro-mechanism of fracture may be activated by thermal fluctuation and stress cycle, and it is a stochastic process in time domain. A microscopic stress concentration may also fluctuate place by place, and it is a stochastic process in space domain. Along this line, the stochastic approach has been extensively developed by Yokobori[2]-[3] and his co-workers, to the problems of brittle fracture, corrosion cracking, creep, fatigue, their interaction and so on.

Some illustrative examples[8] will be shown here. Suppose that the elementary process is a rate process of Arrhenius type, or Poisson process. Then the distribution of the time of occurrence of the event which is caused by n-times of seccessive occurrence of the elementary proccesses is expressed by Gamma-distribution. Fig.11 shows the distribution of the time t devided by its mean $\bar{t}$. If a macroscopic event is caused by the first occurrence of many nuclei in material, the distribution of its occurrence is obtained by parallel upward shift of the curve, and reaches asymptotically to the Weibull distribution as the number of the nuclei increases. Smooth or nearly straight curves on the Weibull probability paper (Fig.11) have been derived by Yokobori et al. for various combination of stochastic processes. This kind of approach to the phisical mechanisms of fracture is not only indespensable in extraporating a distribution from very limited data by experiments and experts into the range of lower probability, but also hopeful to get in near future the absolute value of the probability at the foot of the distribution.

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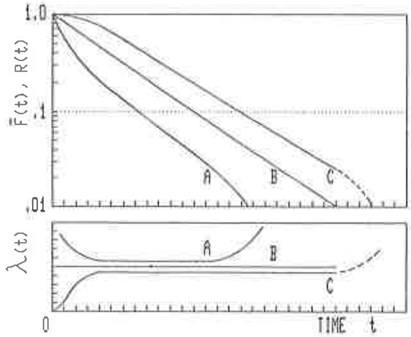

Fig.1 Reliability and failure rate

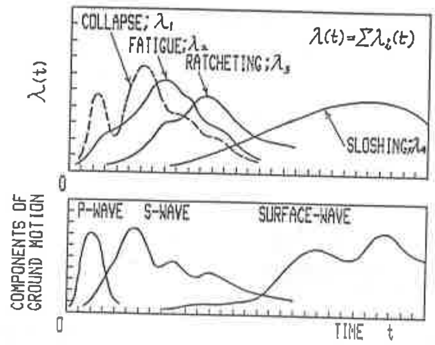

Fig.2 Failure rate during earthquakes

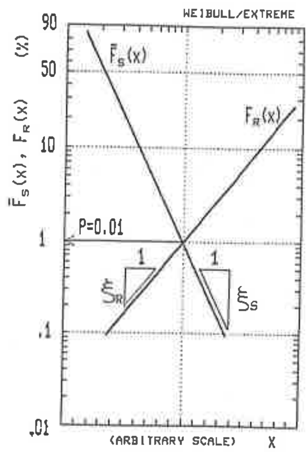

Fig.3a Stress and resistance

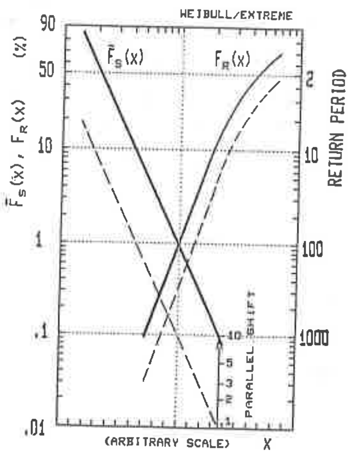

Fig.3b Estimation of extreme

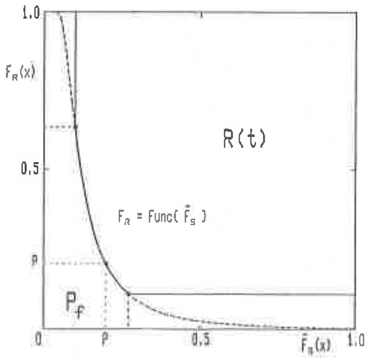

Fig.4 Lower and upper bounds

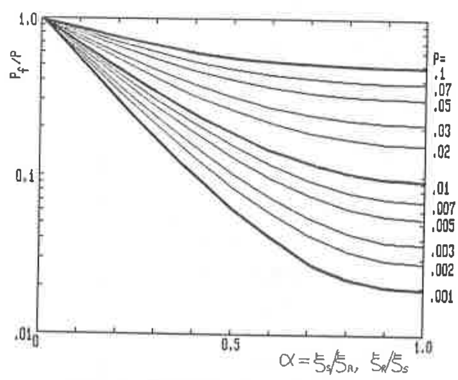

Fig.5 Probability of failure

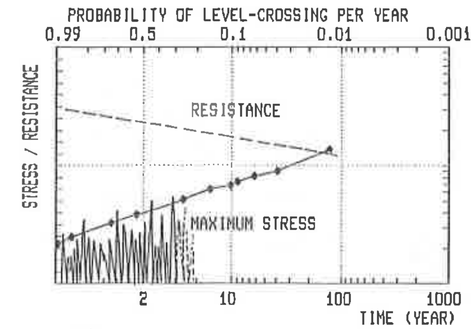

Fig.6 Level-crossing

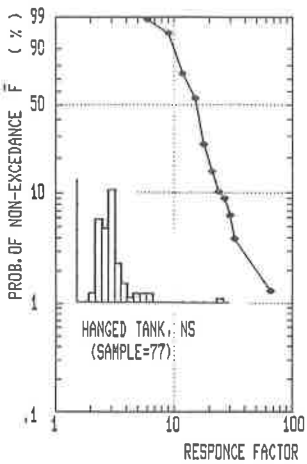

Fig.7 Responce factor

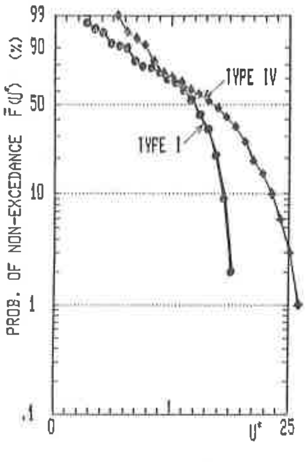

Fig.8 Effective number of cycles

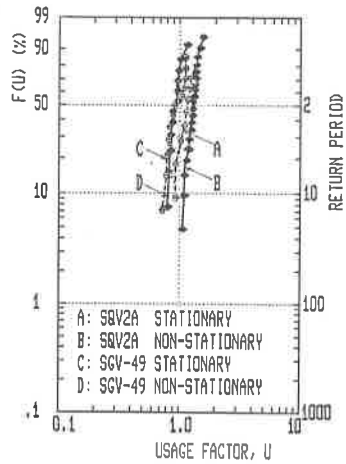

Fig.9 Usage factor at fracture

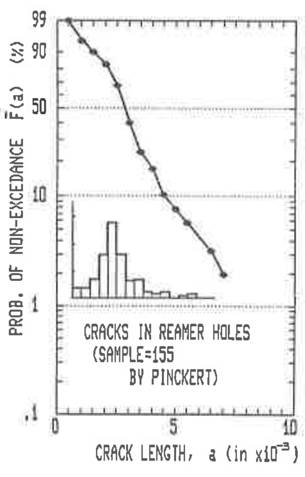

Fig.10 Crack length

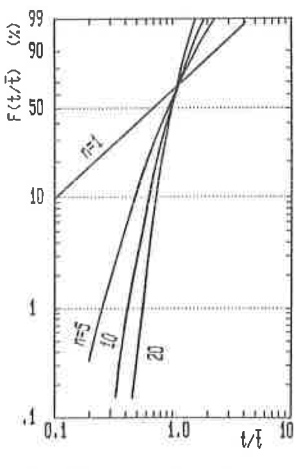

Fig.11 Life distribution

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