On Yielding and Fracture of Damaged Materials

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Abstract

The theoretical model of the failure for the cracked solids subjected to the plane state of the stress is considered in the paper. To this end the various failure modes of the biaxially loaded cracked solid with a regular array of rectilinear cracks were discussed. This made is possible to formulate the appropriate failure criterion expressed in terms of the principal stresses in the homogenized equivalent material. The theoretical considerations were verified experimentally employing the uniaxially loaded models simulating the damaged solid. The results of the experiments corroborate the validity of the failure modes assumed and also the validity of the failure criterion derived.

1. Introduction

Strain induced damage in materials visualizes in formation and development of oriented cracks or voids appearing usually on the grain boundaries. Thus the damaging solid initially isotropic becomes anisotropic in its overall mechanical response. The characteristics of global anisotropy so produced changes with the damage growth and depends on patterns of damage. Mathematical modelling of a homogenized material response constitutes one of the most important concerns of the continuum damage mechanics. These models are formulated when describing the overall macroscopic response of the damaged solid involving now not only a material element but also a continuously distributed quantity representing damage. The mechanical properties of damaging material are governed by two groups of relations. In the first place we have stress-strain relations involving the damage variable assumed as a symmetric second rank tensor. The second group consists of evolution equation specyfying how damage increases in time or with respect to another time dependent quantity.

Various attempts to formulate such equations for damaged solids employing the concept of the damage tensor were presented by Vakulenko and Kachanov [1], Kachanov jr. [2], Murakami and Ohno [3], Krajcinovic [4] and Betten [5]. However the experimental results obtained by Litewka and Stanisławska [6] show that the yield criterion formulated as a scalar function of the stress tensor and the damage tensor cannot give the satisfactory description of the plastic behavior of the cracked material. In particular it concerns the uniaxial yield stress or failure stress determined for various loading orientations. As was pointed out in [7] the smallest value of the uniaxial yield stress measured experimentally for orthotropic equivalent solid is noticed for loading orientation defined by the angle $\alpha < \pi/2$. The further experimental datas confirming this statement can be found in the paper by Murakami and Imaizumi [8]. Thus the plastic anisotropy of the damaged material possessing three mutually perpendicular planes of symmetry is different to that observed in the elastic range where the extreme values of the material constants correspond to the principal directions of the material structure that means to the angles $\alpha = 0$ and $\pi/2$.

To clarify this problem and to obtain further datas necessary to formulate the constitutive equation the theoretical and experimental analysis of the failure modes for material with growing, simulated periodic damage was performed. To this end the theoretical models of failure for the cracked solid subjected to the plane state of stress were used. This enabled the analysis of the material behavior at failure and made it possible to formulate the suitable failure criterion. The validity of the theoretical approach to the problem was verified experimentally employing the models simulating the damaged material.

2. Theoretical model

An idealized theoretical model of the damaged solids with rectilinear cracks arranged in the square pattern assumed in the considerations presented in this note is shown in Fig. 1. The cracks with an irregular countour were approximated by the rectangular openings with constant width h and changing length l simulating the process of the damage evolution.

Considering the problem of failure in the microscale the stress distribution and the behaviour of the material at the level of the unit cell of the material structure was analysed. It was done under the following assumptions:

a) the rupture of the material appearing between adjacent cracks is the result of development of narrow plastic zones or rather slip lines,

b) the stress state in the plastic zones is determined by two components σ and τ shown in Fig. 1,

c) the matrix material yields according the Huber-Mises yield criterion.

The configuration of the plastic zones depends on the cracks arrangement and their dimensions and also on the loading direction. Two possible configurations of plastic zones associated with two different failure modes of the cracked solid are shown in Fig. 1.

The relations between the principal stresses σ_1 and σ_2 applied to the equivalent homogenized material subjected to the plane state of stress and the stresses σ and τ in the plastic zone developed in the matrix material at rupture were obtained considering two equilibrium conditions for the unit cell of the material structure. These equations and the Huber-Mises criterion of failure for matrix material written in the form

$$\sigma^2 + 3\,\tau^2 = \sigma_{FO}^2,$$

where σ_{FO} is the failure stress of the original material without cracks, were used to eliminate the unknown values of σ and τ . Thus a criterion of failure for cracked solid subjected to the plane state of the stress was derived in the form

$$A\sigma_1^2 + B\ \sigma_2^2\ + C\,\sigma_1\ \sigma_2 = \sigma_{FO}^2 \tag{1}$$

where

$$\begin{aligned} A &= (1 - \chi)^{-2} \sin^2\alpha\ \sin^2\ \beta_1\ [\ 1 + 2\sin^2(\ \beta_1 - \alpha)] \\ B &= (1 - \chi)^{-2} \cos^2\alpha\ \ \sin^2\ \beta_1\ [\ 1 + 2\cos^2(\ \beta_1 - \alpha)] \\ C &= -\,(1 - \chi)^{-2} \sin 2\alpha\ \sin^2\ \beta_1\ \ \sin 2(\,\beta_1 - \alpha\,) \end{aligned} \tag{2}$$

for the failure mode 1 and

$$\begin{aligned} A &= \ (1 - \lambda)^{-2} \cos^2\alpha\ \ \cos^2\beta_2[1 + 2\sin^2(\alpha - \ \beta_2)] \\ B &= \ (1 - \lambda)^{-2} \sin^2\alpha\ \ \cos^2\beta_2[1 + 2\cos^2(\alpha - \beta_2)] \\ C &= -(1 - \lambda)^{-2} \sin 2\alpha\ \cos^2\beta_2\ \sin^2 2(\alpha - \ \beta_2) \end{aligned} \tag{3}$$

for the failure mode 2, where β_1 and β_2 are the angles shown in Fig.1 and $\lambda = l/P$, $\chi = h/P$ are the dimensionless cracks length and width respectively. It is seen from eqs.(1) , (2) and (3) that the behaviour of the cracked solid at failure is described by known geometrical parameters λ , χ, β_1, β_2, and the angle α indicating the loading orientation. The geometrical representation of the failure criterion (1) for $\lambda = 0.5$ and $\chi = 0.1$ and for the principal stress directions rotated with respect to the symmetry axes of the material structure through angles $\alpha = 0, \pi/12$, $\pi/6$ and $\pi/4$ is shown in Fig.2. The similar limit curves can be obtained for an arbitrary dimensionless crack length and for $\alpha < \pi/4$.

3. Experimental verification

The validity of the theoretical model assumed when analysing the cracked material behaviour and the result obtained in the form of the failure criterion (1) was verified experimentally employing the uniaxially loaded models simulating the damaged solid. The dimensions of the models cut out of the aluminium alloy metal sheet were as follows: total length 400 mm, length of the cracked portion 210 mm, width 70 mm, thickness 0.7 mm. The cracks were arranged as shown in Fig. 1 in the square pattern with the pith equal to 10 mm, crack length l = 2, 3, 4, 5, 6 and 7 mm and crack width h = 1 mm. The details concerning the models preparation and the experimental technique used are presented in [6]. The models were loaded up to fracture and the failure stress was recorded for various loading orientations represented by the angle $\alpha = 0$, $\pi/12$, $\pi/6$, $\pi/4$, $\pi/3$, $5\pi/12$ and $\pi/2$. The set of the photographs of the model after fracture for the crack length $\lambda = 0.5$ shown in

Fig. 3 confirmes the validity of the failure modes 1 and 2 assumed in the theoretical model. The same configurations of the plastic zones were obtained for others crack lengths thus the validity of the failure modes shown in Fig. 1 was confirmed experimentally.

The theoretical values of the uniaxial failure stress obtained from eqs. (1),(2) and (3) are compared with those measured experimentally in Figs. 4 and 5. Good quantitative and qualitative agreement of experimental and theoretical results seen in these figures corraborates the validity of the failure criterion derived. This agreement concerns not only the numerical values of the failure stress but also the ranges of the validity of the failure modes 1, 2.

4. Conclusions

Irregularity of the yield and failure stresses observed when determining experimentally their dependence on the loading orientation has been clarified theoretically. The lowest value of the failure stress noticed for loading orientation corresponding to the angle $\alpha < \pi/2$ is the result of the specific failure modes associated with the plastic zones configurations. The failure criterion derived theoretically for biaxially loaded damaged material made it possible to determine the suitable failure stress for various loading orientations, and for changing crack length.

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References

[1] Vakulenko,A.A., Kachanov,M.L.,"Continuum theory of medium with cracks", Izv. A.N. S.S.S.R.,M.T.T.,4,159-166 (1971),(in Russian).

[2] Kachanov,M., "Continuum models of medium with cracks", J. Eng. Div., Trans. ASCE, 106, 1039-1051 (1980).

[3] Murakami,S., Ohno,N., "A continuum theory of creep and creep damage", Creep in Structures, IUTAM Symp., Leicester 1980, A.R.S. Ponter, D.R. Hayhurst, ed., Springer, 422-443,(1981).

[4] Krajcinovic,D., "Constitutive equations for damaging materials", J.Appl. Mech., 50, 2, 355-360 (1983).

[5] Betten,J., "Damage tensors in continuum mechanics", J.Méc.Théor.Appl., 2, 1, 13-32 (1983).

[6] Litewka,A.,Stanisławska,J., "Experimental simulation of anisotropic damage", Mech.Teoret.Stos., 21, 3/2,361-370 (1983).

[7] Litewka,A.,Sawczuk,A., "Experimental evaluation of the overall anisotropic material response on continuous damage", Mechanics of Material Behavior, Drucker Anniversary Volume, G.J.Dvorak,R.T.Shield,ed.,(1984), 239-252.

[8] Murakami,S.,Imaizumi,T.,"Mechanical description of creep damage state and its experimental verification", J.Méc.Théor.Appl.,1, 5, 743-761 (1982).

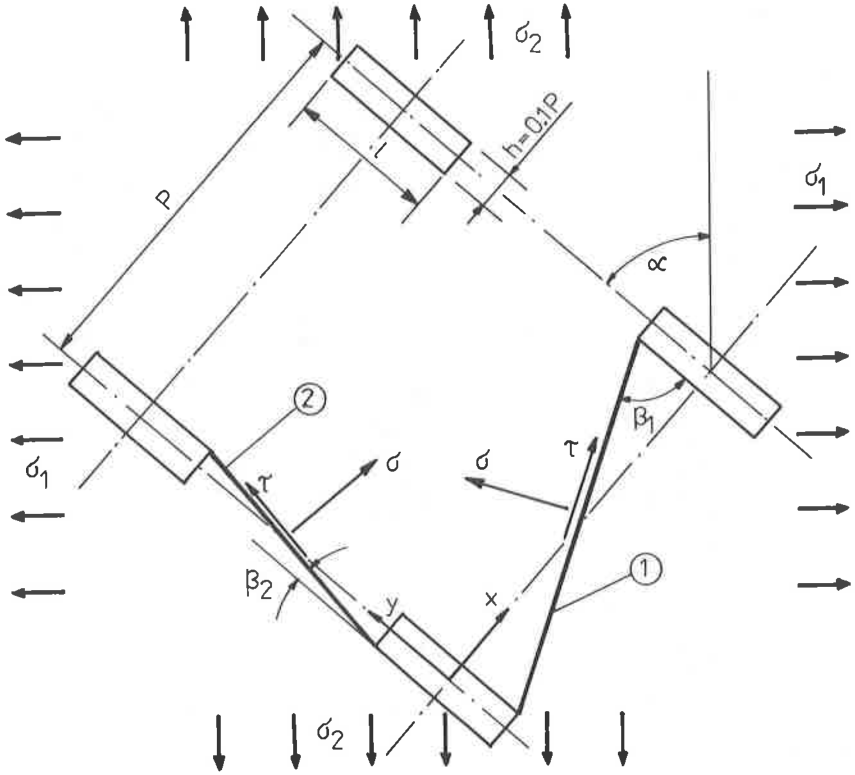

Fig.1. Failure modes for the model of the cracked solid.

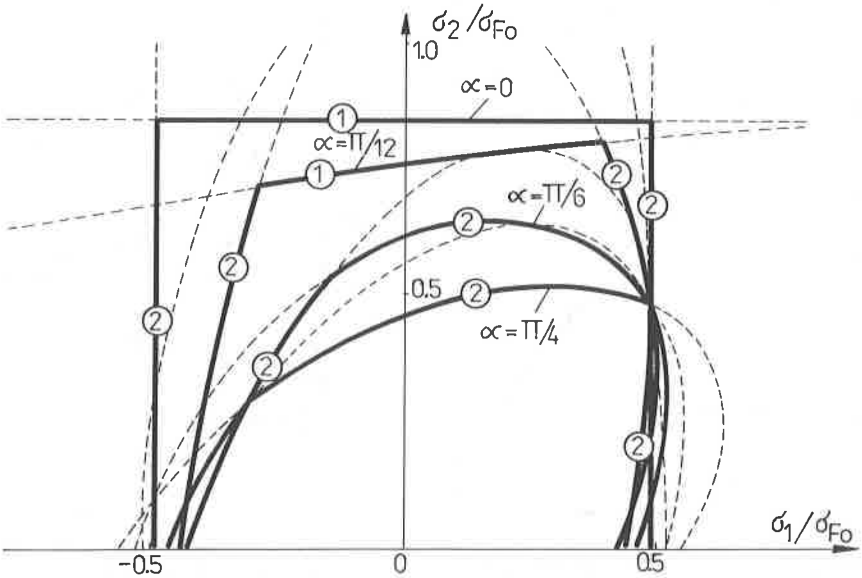

Fig.2. Failure surfaces for cracked solid with crack length λ = 0.5.

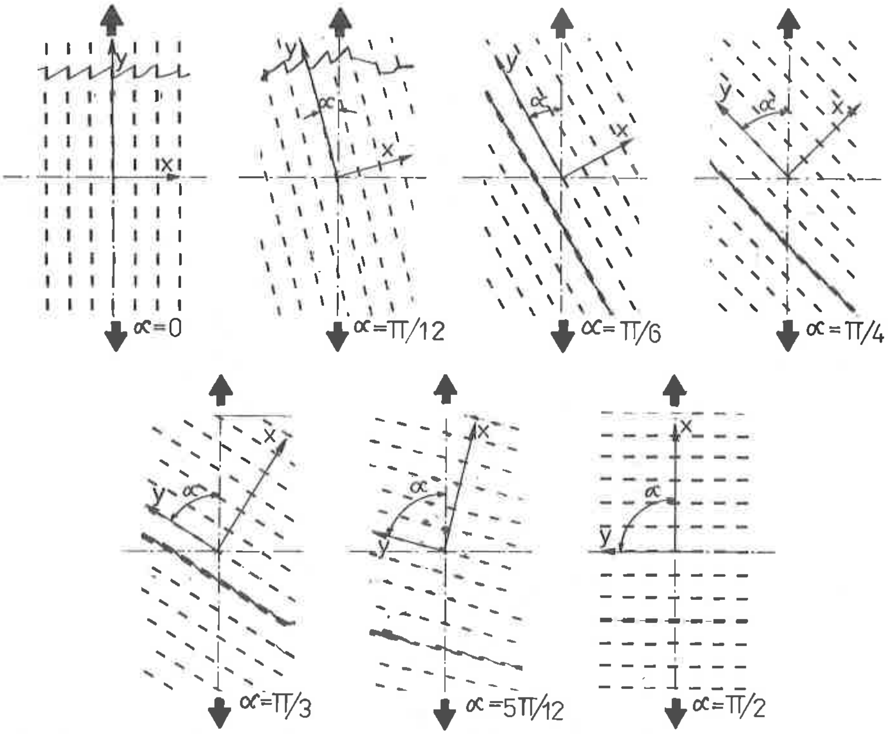

Fig.3. Models simulating the cracked solid after fracture ($\lambda = 0.5$).

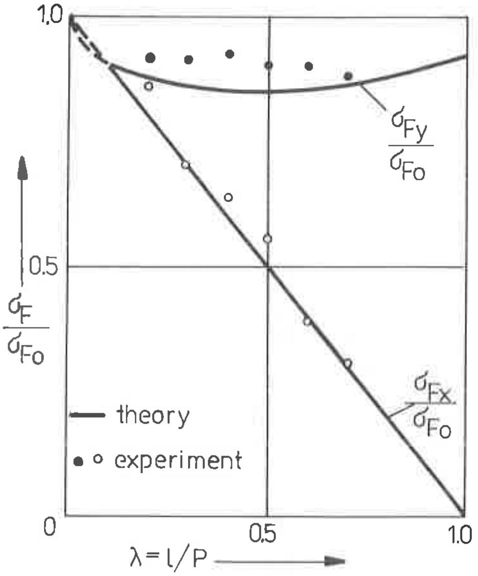

Fig.4. Uniaxial failure stress for loading in x and y directions versus the dimensionless crack length λ.

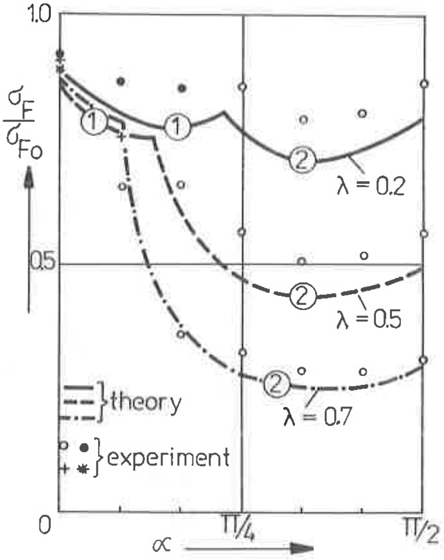

Fig.5. Uniaxial failure stress σ_n versus the angle α.