ABSTRACT

RAVINDRA APPAJI, SAMYUKTHA. Optimal Ordering Policy Characterizations in an Unreliable Supply Chain. (Under the direction of Dr. Russell E. King.)

We consider a single-product periodic-review inventory system for a retailer who has adopted a dual sourcing strategy to cope with potential supply process interruptions. Orders are placed either to a perfectly reliable supplier or to a less reliable supplier that offers a lower price. The inventory control problem for this supply chain is modeled as a discrete-time Markov Decision Process (MDP) in order to find the optimal ordering decisions from both suppliers. Through numerical experimentation, the structure of the optimal ordering policies under several cost scenarios as well as different supplier unreliability levels is determined. Four basic policy structures are found and are referred to as Case 1: Order only from the unreliable supplier; Case 2: Order simultaneously from both suppliers; Case 3: Order from one or the other but not both suppliers simultaneously; and Case 4: Order only from the reliable supplier. For all cases, (S,s)-like policies characterize well the optimal policy. Further experimentation is done to study the effects of the system parameters on the ordering decision, safety inventory levels and cost of the system. From the experiments it is seen that the ordering decision is influenced by the reliability of the supplier and the values of various cost parameters. The point at which the policy structure (case) changes is reported such as, given a set of parameters, at what reliability level does ordering from both suppliers (Case 2) become better than ordering only from the unreliable supplier (Case 1).

Optimal Ordering Policy Characterizations in an Unreliable Supply Chain

by
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BIOGRAPHY

Samyuktha R Appaji was born in Bangalore, India in 1984. She graduated from BMS College of Engineering, Bangalore in 2006 with a Bachelors degree in Mechanical Engineering. After graduation she worked with Cummins India Limited as an application engineer. In 2008, she joined North Carolina State University to pursue her Masters Degree in Industrial Engineering

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Chapter 1

Introduction

Uncertainties in the supply process should be considered when making ordering decisions to be able to manage the inventories in an effective way. Uncertainty might be present in the quantity, quality, or timing of orders delivered due to several internal or external factors, e.g. natural disasters, accidents, strikes, embargoes, or machine breakdowns. Proactive firms adopt several supply-side tactics to cope with potential supply chain disruptions such as sourcing from multiple suppliers or holding more inventory.

The deployment of optimal or near-optimal inventory policies can be vital for an enterprise. Firms can increase profits by increasing sales revenues or cutting down costs. When the market is not stable increasing revenue is almost impossible, hence cutting down costs becomes important. Managing inventory is a challenging task for any enterprise in a stochastic environment.

In our model we consider a single-product periodic-review inventory system for a retailer. To cope with any supply disruptions, the retailer has chosen two suppliers. One is reliable and the other unreliable but offers a lower cost. The orders are placed at the beginning of each period and lead times for both suppliers are one period (i.e delivered at the end of the period). The inventory control problem is modeled as a discrete-time Markov Decision Process (MDP). Through numerical study, the optimal ordering decisions for each possible system state are found by solving the MDP model. We then find an intuitive and relatively simple structure that characterizes well the optimal policy, i.e. that results in optimal or near optimal system cost. To do this, first the MDP model of the system is solved to find the list of the optimal decisions for every system state. Then, by a careful observation, this list of optimal decisions is transformed into an ordering policy with a few control parameters. This MDP-based approach allows us to find both the structure of the optimal policy (i.e. what control parameters are needed) as well as the optimal values of the control parameters. The effects of system parameters such as the unit cost of the product, backordering cost, and holding cost are studied to see how they influence the optimal ordering decisions.

The thesis is organized as follows: Chapter 2 presents a review of the literature on unreliable supply work. In Chapter 3 the problem is described and model is formulated as a discrete Markov Decision Process. In Chapter 4 characterization of the ordering decisions into policies is described. In Chapter 5, further experimentation is carried out by varying some of the system parameters and results are analyzed for changes in the ordering decisions, safety inventory and cost of the system. Finally, Chapter 6 presents the conclusions and a discussion of future work.

Chapter 2

Literature Review

Inventory control subject to supply interruptions has been studied by several authors. These studies can be categorized in a number of ways. In some of the studies, the inventory is periodically reviewed while others consider continuous-review inventory systems. Gullu *et al.* (1999), Ozekici and Parlar (1999), Parlar (1995) consider periodic-review inventory systems while Mohebbi (2004), Parlar and Perry (1995), and Qietal (2006) consider continuous-review inventory systems.

In some studies, a single unreliable supplier is assumed while others consider multiple unreliable suppliers with different reliability levels. Arreola-Risa and DeCroix (1998), Gullu *et al*. (1999), Ozekici and Parlar (1999), Song and Zipkin (1996), Mohebbi (2004), Parlar and Perry (1995), and Qietal (2006) consider a single–product, single-supplier problem. Swaminathan and Shanthikumar (1999), Tomlin (2006), Gurlur and Parlar (1997) consider two suppliers with uncertainty in their supply process.

Another distinction among studies of dual-source inventory systems is the manner in which the supplier's availability process is modeled. Arreola-Risa and DeCroix (1998) consider supply interarrival time and length of supply disruptions as random variables following exponential distributions. Parlar (1997) models the supplier availability as a semi-Markov process. The duration of on and off periods are modeled as Erlang and general distributions, respectively. Renewal theory is used to model the long-run average cost objective function. Gullu *et al*. (1999) model uncertainty as a three-point probability mass function under the assumption of a Bernoulli-type supply process where supply is completely available, partially available, or not available. They use a stochastic dynamic programming formulation to show optimality of order-up-to policies. Parlar and Perry (1995) consider supply availability as a two-state continuous Markov chain, i.e. “on” or “off”, random exponential fluctuations, and they construct the objective function using the renewal reward theorem as long-run average cost. They also analyze the problem when the yield is random. Gurlur and Parlar (1997) model both the suppliers’ availability as a semi-Markov process with the duration of the “on” period as having an Erlang distribution and the “off” duration as a general distribution. They employ the method of stages to transform the process into a Markovian process and use renewal theory to formulate the objective function, i.e., the long-run average cost. Mohebbi (2004) models the suppliers’ availability as an alternating renewal process in which “on” and “off” periods are independent random variables following general and hyper-exponential distributions respectively.

These studies also differ in the assumption of costs involved in the system. Song and Zipkin (1996), and Swaminathan and Shanthikumar (1999) consider all unsatisfied demand completely backlogged and do not consider lost sales in the total cost of the system, while Arreola-Risa and Decroix (1998) consider backorder and lost sales cost in the total cost calculation. Tomlin (2006) considers a system where the suppliers are capacity constrained while the reliable supplier has volume flexibility.

Two main approaches are observed in the literature regarding the inventory control problem subject to unreliable supply. One approach is to derive analytically the structure of the optimal policy such as in Anupindi and Akella (1993) and Swaminathan and Shanthikumar (1999). These papers derive the optimal ordering policy that minimizes the ordering cost, holding cost and penalty costs, and then give conditions relating marginal costs and supply uncertainty and show the optimality of the policy described. Most of the studies using this approach have made simplifying assumptions such as zero lead time and no set up costs, and usually, they do not discuss how to calculate the optimal values of the control parameters. A second approach is to find the optimal or near-optimal values of the parameters of a predetermined reasonable control policy such as in Parlar and Perry (1995) and Mohebbi (2004). In these papers, using renewal reward theory, a long run average cost objective function is formulated. However, in these papers, it is not indicated how the predetermined policy compares to the optimal policy. Also, it is not clear how the ordering decisions change with changes in the system parameters.

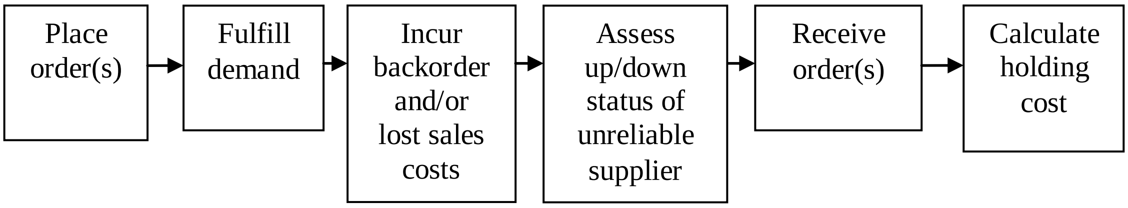
Our model solves to find the optimal ordering policy that minimizes the ordering cost, holding cost and penalty costs. We consider a fixed set up cost and lead time of one period. The optimal ordering decision for every state is found by solving the MDP. By careful observation, these decisions are characterized as a policy. Thus the MDP allows us to find the optimal structure for a scenario and also the parameters for that structure. In this thesis the model is run for various scenarios and the results analyzed to find the cost difference and ordering decisions for different scenarios. We carry out detailed analysis of the effects of changes in system parameters on the ordering decisions. We also analyze the cost improvement achieved when reliability is improved or cost of the reliable supplier is reduced.

Chapter 3

Problem Description

We consider a retailer that stocks inventory of some product to meet customer demand. Demand during a period is assumed to be stochastic and independent and identically distributed. Each unit carried in inventory during a period incurs a holding cost. For each unit demanded, a unit is deducted from inventory if there is stock on-hand. If not, the demand is backordered and met with future stock, incurring a backorder cost for each period the demand is unmet. However, there is a defined limit for the number of units that can be on backorder at any one time. Any demand beyond this limit is lost and incurs a lost sales cost for the retailer. In order to restock inventory, the retailer can make orders on either or both of two suppliers, one who is reliable and the other who is unreliable but offers a lower unit cost. The unreliability of this supplier is characterized in the following way. The state of the supplier can be “up” meaning they are available to accept orders or “down” where they do not accept orders. The details of the state transition process are described below.

The objective of the retailer is to determine an ordering strategy that minimizes total expected cost per period which includes the following: fixed ordering cost incurred for each order made on each supplier; variable ordering cost, assumed to be linear in the quantity ordered, where the unit cost is less for the unreliable supplier than the reliable one; holding cost, which is linear in the number of units in inventory; backorder penalty cost, which is linear in the number of units backordered; and lost sales cost, which is linear in the number of sales lost. The retailer orders the desired amount from either or both of the suppliers depending on the inventory position and status of the unreliable supplier. The orders are given at the beginning of each period. Lead times for both suppliers are one period, i.e. an order at the beginning of the period is delivered at the end of that period. The reliable supplier fills the ordered amount with certainty before the next period's demand is fulfilled. The retailer can order from the unreliable supplier only if the status is in the up state. The status of the unreliable supplier can change after the order is made. If the unreliable supplier's status goes from up to down, then the order is cancelled. Otherwise, the order is delivered at the end of the current period. If an order is made on the unreliable supplier, the fixed ordering cost is incurred whether the order is delivered or not. The unit purchasing cost is incurred only for units that are actually delivered. The timing of the events is described in Figure 3.1.



[---------Beginning of the period-----] [--------------------End of period---------------------]

Figure 3.1: Timeline of activity

In order to find the optimal order quantities from both suppliers, the problem is formulated as a discrete-time Markov decision process. A Markov decision process model is a stochastic, sequential-decision model that is defined by a set of system states, a set of alternative decisions to make, an immediate reward function and a transition probability matrix that defines the probability of going from one state to another in one transition under a selected decision (Howard, 1960). The MDP model for this system is described below.

State variables:

Let the state of the system at the beginning of the current period be *S*. *S* is composed of two random variables, i.e. $S=(I, J)$. *I* is the retailer inventory level and *J* is the unreliable supplier status where

$$J = \begin{cases} 0 & \text{if the unreliable supplier status is up at the beginning of the period} \\ 1 & \text{otherwise} \end{cases}$$

The unreliable supplier status is controlled by an underlying Markov process with transition probability matrix, *W*. Element (i,j) of *W*, W_{ij}, represents the probability of a transition of the unreliable status from *i* at the beginning of one period to *j* at the beginning of the next. Specifically, the unreliable supplier status is defined by α, the probability that the unreliable supplier stays in the up state from one period to the next, and β, the probability that this supplier transitions from the down state to up from one period to the next. Therefore,

$$W = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & \quad 1 \end{array} \\ \begin{bmatrix} \alpha & 1-\alpha \\ \beta & 1-\beta \end{bmatrix} \end{array}.$$

The retailer's inventory level *I* is limited to the range $I_{\min} \le I \le I_{\max}$, where $I_{\min} > -\infty$ and $I_{\max} < \infty$. To allow backorders, $-\infty < I_{\min} < 0$, but backorders beyond $|I_{\min}|$ reflect lost sales. These restrictions limit the state space to a range where the problem can be solved without excessive computational burden.

Decision variables and alternatives:

There are two decision variables: the order amount from the unreliable supplier, k_u, and the order amount from the reliable supplier, k_r. Let $k = (k_u, k_r)$ be a vector of the decision values (order quantities). For state $S=(I,J)$ the set of alternative order quantities, A_S is

$$A_S=\begin{cases}(k_u,k_r)\text{ such that }k_u\geq 0,k_r\geq 0,0\leq k_u+k_r\leq I_{\max}-I & \text{if } J=0\\ (k_u,k_r)\text{ such that }k_u=0,0\leq k_r\leq I_{\max}-I & \text{if } J=1\end{cases}$$

State transitions and transition probabilities:

In this model, there are two underlying Markov processes that define the state transitions: one associated with the status of the unreliable supplier and the other with the inventory position. The first is independent of the second.

Let the demand during a period, D, be stochastic according to some known distribution. Further, assume that the demand in each period is independent and identically distributed demand where $P_D(d)=P[\text{Demand during a period is } d \text{ units}]$.

Given the state of the system at the beginning of a period is $S=(I,J)$ and a decision $k=(k_u, k_r)$ is made (where k_u and k_r represent the order amounts from unreliable and reliable suppliers, respectively), and the demand during the period, D is d, then the next state $S'=(I',J')$ can be described as follows depending on the status of the unreliable supplier, J:

- $J = 0$: i.e. the current status for reliable supplier is up. Then,

$$J'=\begin{cases}0 & \text{if the unreliable supplier remains in up through the period}\\ 1 & \text{if the unreliable supplier goes down during the period}\end{cases}$$

$$I'=\begin{cases}\min\{I-d,I_{\min}\}+k_u+k_r & \text{if the unreliable supplier remains up through the period}\\ \min\{I-d,I_{\min}\}+k_r & \text{if the unreliable supplier goes to down during the period}\end{cases}$$

(Note that the order of k_u units is cancelled when the unreliable supplier status goes from up to down.)

- J = 1: i.e. the current status of the unreliable supplier is down

$$J' = \begin{cases} 0 & \text{if the unreliable supplier returns to up during the period} \\ 1 & \text{if the unreliable supplier stays down during the period} \end{cases}$$

$$I' = \min\{I - d, I_{\min}\} + k_r$$

(Note that when the unreliable supplier status is down, the retailer orders only from the reliable supplier, i.e. k_u = 0.)

Define $P_{SS'}^{k}$ to be the one period state transition probability from state S= (I, J) to state $S' = (I', J')$ when following alternative k. The value of $P_{SS'}^{k}$ can be put into 4 groups based on the status of the unreliable supplier given in Table 3.1 below.

Table: 3.1: State transitions and probabilities

Current State Unreliable Supplier Status, J	Next State Unreliable Supplier Status, J'	Next State Inventory, I'	Transition Probability, $P_{SS'}$
0	0	$\min\{I - d, I_{\min}\} + k_u + k_r$	$W_{00} P_D(d) = \alpha P_D(d)$
0	1	$\min\{I - d, I_{\min}\} + k_r$	$W_{01} P_D(d) = (1-\alpha) P_D(d)$
1	0	$\min\{I - d, I_{\min}\} + k_r$	$W_{10} P_D(d) = \beta P_D(d)$
1	1	$\min\{I - d, I_{\min}\} + k_r$	$W_{11} P_D(d) = (1-\beta) P_D(d)$

Cost function:

In this model, the aim is to minimize the expected cost per period. The cost function consists of the fixed ordering cost, material costs, holding costs, backordering costs, and lost sales cost. The following notation is used.

c_u - unit material cost from the unreliable supplier

c_r - unit material cost from the reliable supplier ($c_r > c_u$)

f - fixed cost of ordering from either supplier

h - unit holding cost per period

b - unit backordering cost per period

l - unit lost sales cost

B- backordered demand during the period

L- lost sales during the period

Given that the system is in state $S=(I, J)$, alternative $k = (k_u, k_r)$ is followed, and the demand during the period is $D = d$, the cost for the period is calculated as

$$C(S,k,d,J') = \delta(k_r) + \gamma(k_u) + h[I']^+ + bB + lL,$$

where the terms represent the costs of the orders placed to the reliable and unreliable suppliers, the inventory holding cost, backordering cost and the lost sales cost, respectively. These costs are calculated as follows.

$$\delta(k_r) = \begin{cases} f + c_r k_r \text{ for } k_r > 0 \\ 0 \text{ for } k_r = 0 \end{cases}$$

$$\gamma(k_u) = \begin{cases} f + c_u k_u \text{ for } k_u > 0 \text{ and } J' = 0 \\ f \text{ for } k_u > 0 \text{ and } J' = 1 \\ 0 \text{ for } k_u > 0 \end{cases}$$

$$[I']^+ = \max\{I', 0\}$$

$$B = \begin{cases} \text{- } \max\{I-d, I_{min}\} \text{ if } I-d < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$L = \begin{cases} I_{\min} - (I-d) \text{ if } I-d < I_{\min} \\ 0 \text{ otherwise} \end{cases}$$

Consider a state of the system $S=(I, J)$. The expected period cost for state *S* when following alternative *k*, q_S^k is computed as follows:

$$q_S^k = \sum_d P_D(d) \sum_{J'=0}^{1} W_{JJ'} C(S,k,J')$$

The MDP model is solved using a variant of Howard's policy iteration method (Howard, 1960) that is modified to include the fixed-policy successive approximation presented by Morton (1971). Morton's approach differs from Howard's in that relative values are computed by successive approximation, which eliminates the need for solving linear equations.

The algorithm for the optimal per-period cost, i.e. the gain, *g*, is as follows.

Step 1. Initialize the relative value for each state *S*, v_s=0. Here, v_s is the cost of starting the process in state *S* relative to that of state *N* (defined below) as per the notation of Howard (1960).

Step 2. Arbitrarily let state *N* = (0,0). State *N* represents a base state used to determine the relative values, *v*'s (see step 4(d) below).

Step 3. Arbitrarily select an alternative *k* for each state *S*.

Step 4. **Value Determination**: Recursively evaluate the following equations *n* times (cheap iterations)

(a) $z_S = v_S$

(b) $y_S = q_S^k + \sum_{S'} P_{SS'}^k v_S \quad \forall S$

(c) $g = y_N$

(d) $v_S = y_S - y_N \quad \forall S$

Step 5. **Policy Improvement**: Evaluate the following

$$k = \arg\min_{k' \in A_S} \left\{ q_S^{k'} + \sum_{S'} P_{SS'}^{k'} v_S \right\} \quad \forall S$$

Step 6. Repeat Steps 4 and 5 until stopping criteria are met, i.e. no change in alternative for any recurrent state in Step 5 and $\sum_S (v_S - z_S)^2 < \varepsilon$ where ε is some arbitrarily small number.

The algorithm converges to the gain and optimal policy, i.e. optimal alternatives for the set of all states *S*.

Chapter 4

Policy Characterization

While the MDP finds the optimal policy for given input parameter values (demand distribution, costs, etc.), it does not directly provide intuition into the structure of the policy. The objective is to find a characterization of the optimal policy (find the structure of the optimal policy and the optimal parameter values). We seek a high-quality characterization that represents exactly, or very nearly, the optimal policy, yet requires few parameters and provides managerial insight into the policy. The measure used to evaluate the quality of the characterization is the percentage difference of the cost attained by following the characterization from the optimal cost of the MDP.

The process of determining high quality characterizations can vary from direct human observation to the use of sophisticated computer algorithms. Hodgson *et al.* (1987) use the direct observation approach to characterize optimal routing policies for automated guided vehicles systems as does Ahiska and King (2010a) to characterize production inventory decisions in a remanufacturing system. Ahiska and King (2010b) use neural networks to develop a functional relationship between the input parameters and the characterized policy for the same remanufacturing system problem.

In this thesis, an approach based on that of Ahiska and King (2010a) is used. The approach is as follows.

1. Determine the optimal policy, i.e. values of k_r and k_u, for each recurrent system state for every combination of input parameters in a defined experimental design.

2. For the recurrent states, interpret the order-up-to level for the reliable supplier, S_r and the unreliable supplier, S_u. This is done by summing of the inventory level and order quantities as shown below.

$$S_r = I + k_r$$
$$S_u = \begin{cases} I + k_u & \text{if ordering only from the unreliable supplier} \\ I + k_r + k_u & \text{if ordering from both suppliers} \end{cases}$$

3. The inventory level after which the ordered quantities are 0 is interpreted as the reorder points s_r for the reliable supplier and s_u for unreliable supplier.

4. The system is investigated under a variety of system input parameters by changing the value of one parameter at a time.

5. Based on the effects of changing an individual system parameter on the optimal policy structure as well as the policy parameter values, we characterize the resulting policy as belonging to one of four ordering cases, described below.

Characterization of the ordering decisions:

For the experimentation we assume the demand has some central tendency. Specifically, demand is assumed to have a mean of 5 and follow a discrete approximation to a Normal distribution as given below.

$$P_D(d) = \begin{cases} 0.02 & d = 0 \\ 0.05 & d = 1 \\ 0.08 & d = 2 \\ 0.11 & d = 3 \\ 0.15 & d = 4 \\ 0.18 & d = 5 \\ 0.15 & d = 6 \\ 0.11 & d = 7 \\ 0.08 & d = 8 \\ 0.05 & d = 9 \\ 0.02 & d = 10 \end{cases}$$

The values of the system parameters considered in the experiments are shown in Table 4.1.

Table 4.1: The system parameters and values considered in the experiments

System parameters	Values			
Unit ordering cost from reliable supplier, c_r	1.2	1.5	2.0	
Unit ordering cost from unreliable supplier, c_u	1			
Unit holding cost per period, h	0.2			
Unit backordering cost per period, b	2	4	6	
Fixed ordering cost, f	0	2	6	
Probability of staying up, α	0.12	0.5	0.7	1
Probability of going from down to up, β	0.4			

Given below are observations made regarding the optimal policy structure and characterization with example from the experimentation used to describe the characterization of the various policy structures:

1. When supplier is down, the optimal policy is to order only from reliable supplier according to an (s_r, S_r) policy.

Table 4.2: Policy characterization when the unreliable supplier is down

State of unreliable supplier	Inventory level	Ordered quantity from the unreliable supplier	Ordered quantity from the reliable supplier
1	1	0	18
1	2	0	17
1	3	0	16
1	4	0	15
1	5	0	14
1	6	0	13
1	7	0	12
1	8	0	11
1	9	0	10
1	10	0	9
1	11	0	0

In Table 4.2 the unreliable supplier status is down and thus no order is placed with the unreliable supplier. The policy can be characterized as follows:

- If the current inventory level is less than 11, order the difference between the current inventory level and 19 units from the reliable supplier.
- If the inventory level is greater than or equal to 11, order nothing.

2. When the unreliable supplier is up, the optimal policy structure is characterized as one of the following cases:

- **Case 1: Order only from the unreliable supplier according to an (S_u, s_u) policy.**

In Table 4.3 the unreliable supplier status is up and an order is placed with the unreliable supplier but no order is placed with the reliable supplier. The policy can be characterized as follows:

  - If the current inventory level is less than 12, order the difference between the current inventory level and 21 units from the unreliable supplier.

- If the inventory level is greater than or equal to 12, order nothing.

Table 4.3: Case 1 characterization

State of unreliable supplier	Inventory level	Ordered quantity from the unreliable supplier	Ordered quantity from the reliable supplier
0	0	21	0
0	1	20	0
0	2	19	0
0	3	18	0
0	4	17	0
0	5	16	0
0	6	15	0
0	7	14	0
0	8	13	0
0	9	12	0
0	10	11	0
0	11	10	0
0	12	0	0

- **Case 2: Order from the reliable and unreliable suppliers simultaneously or order only from the unreliable supplier depending on the inventory level, *I*.**
  - If $I < s_r$, first order from the reliable supplier up to S_r; then, order from the unreliable supplier up to S_u, $S_u > S_r$ (i.e. we order $S_u - S_r$ from the unreliable supplier).
  - If $s_r \leq I < s_u$, order from the unreliable supplier up to S_u ($S_u > S_r$).
  - If $I \geq s_u$, order nothing.

Table 4.4: Case 2 characterization

State of unreliable supplier	Inventory level	Ordered quantity from the unreliable supplier	Ordered quantity from the reliable supplier
0	1	19	10
0	2	19	9
0	3	19	8
0	4	19	7
0	5	19	6
0	6	24	0
0	7	23	0
0	8	22	0
0	9	21	0
0	10	20	0
0	11	19	0
0	12	18	0
0	13	17	0
0	14	16	0
0	15	0	0

In Table 4.4 the total order up to level is 30 (inventory level + ordered quantity from both the suppliers). For some inventory levels, orders are placed with both suppliers; therefore this is a case 2 type of policy. The policy can be characterized as follows:

- If the inventory level is less than 6, order the difference between the current inventory level and 11 (order up to 11 units) from the reliable supplier and 19 units from the unreliable supplier (order up to 30 units).
- If the inventory level is greater than or equal to 6 and less than 15, order the difference between the current inventory level and 30 units (order up to 30 units) from the unreliable supplier.
- If the inventory level is greater than or equal to 15, order nothing.

- **Case 3: Order from either the reliable or the unreliable supplier depending on the inventory level, but never order from both simultaneously.**
  - If $I < s_r$, order from the reliable supplier up to S_r.

- If $s_r \leq I < s_u$, order from the unreliable supplier up to S_u ($S_u \geq S_r$).
- If $I \geq s_u$, order nothing.

In Table 4.5 the status of the unreliable supplier is up. The policy can be characterized as follows:

- If the inventory level is less than 9, order the difference between the current inventory level and 19 units from the reliable supplier.
- If the current inventory level is greater than or equal to 9 and less than 13, order the difference between the current inventory level and 31 units from the unreliable supplier.
- If the inventory level is greater than or equal to 13, order nothing.

Notice that in no case is an order placed with both suppliers simultaneously.

Table 4.5: Case 3 characterization

State of unreliable supplier	Inventory level	Ordered quantity from the unreliable supplier	Ordered quantity from the reliable supplier
0	1	0	18
0	2	0	17
0	3	0	16
0	4	0	15
0	5	0	14
0	6	0	13
0	7	0	12
0	8	0	11
0	9	22	0
0	10	21	0
0	11	20	0
0	12	19	0
0	13	0	0

- **Case 4: Order only from the reliable supplier according to an (S_r, s_r) policy.**

In the Table 4.6 notice that if the inventory level is less than 11, some quantity is ordered from the unreliable supplier and none from the reliable supplier. The sum of inventory level and ordered quantity is order up to level which is 19 in this case. The policy can be defined as follows:

- If the inventory level is less than 11, order the difference between the current inventory level and 19 units, (order up to 19 units) from the unreliable supplier, and if the inventory level is greater than or equal to 11, order nothing for the given scenario.

Table 4.6: Case 4 characterization

State of unreliable supplier	Inventory level	Ordered quantity from the unreliable supplier	Ordered quantity from the reliable supplier
0	1	0	18
0	2	0	17
0	3	0	16
0	4	0	15
0	5	0	14
0	6	0	13
0	7	0	12
0	8	0	11
0	9	0	10
0	10	0	9
0	11	0	0

Chapter 5

Experimentation and Results

In this chapter we present a variety of computational experiments and describe the results from this extensive analysis of the model presented in the preceding chapters. In each section, one or more parameters are varied and the effect of these changes analyzed to observe how the ordering decision changes.

5.1 Effect of supplier recovery rate on system cost

This experiment is carried out to study the impact of changing β on the optimal policy case and reorder levels. The system parameters are held at the following values.

Table 5.1: The system parameters and values considered in section 5.1

System parameters	Values		
Unit ordering cost from reliable supplier, c_r	1.2	1.5	2
Unit ordering cost from unreliable supplier, c_u	1		
Unit holding cost per period, h	0.2		
Unit backordering cost per period, b	2	4	6
Fixed ordering cost, f	0	2	6
Probability of staying up, α	0.01:1 in increments of 0.01		
Probability of going from down to up, β	0.1:1 in increments of 0.1		

Out of the 27 cost scenarios from Table 5.1, three are selected to illustrate the effect on the optimal policy of changing β.

In the below Table 5.2, the threshold value of α at which the optimal ordering case changes from case i to i+1 is denoted by $\alpha_i^{\Delta}, i \in \{1,2,3\}$.

Table 5.2: Impact of β on optimal decision

Input Cost Parameters					Transition Thresholds			Decision				Resulting Policy Parameters									
c_u	c_r	f	b	β	α_1^Δ	α_2^Δ	α_3^Δ	0	1	2	3	$s_r(\alpha_1^\Delta)$	$s_u(\alpha_1^\Delta)$	$S_r(\alpha_1^\Delta)$	$S_u(\alpha_1^\Delta)$	$s_r(\alpha_2^\Delta)$	$s_u(\alpha_2^\Delta)$	$S_r(\alpha_2^\Delta)$	$S_u(\alpha_2^\Delta)$	$s_r(\alpha_3^\Delta)$	$S_r(\alpha_3^\Delta)$
1	1.2	0	2	1	0.9	0	–	1	2	4	–	5	15	5	15	14	–	14	–	–	–
1	1.2	0	2	0.7	0.9	0	–	1	2	4	–	5	15	5	15	14	–	14	–	–	–
1	1.2	0	2	0.4	0.9	0	–	1	2	4	–	5	15	5	15	14	–	14	–	–	–
1	1.2	0	2	0.1	0.9	0	–	1	2	4	–	5	15	5	15	14	–	14	–	–	–
1	1.5	2	4	1	0.79	0.45	0.21	1	2	3	4	3	15	10	26	10	16	19	31	12	21
1	1.5	2	4	0.7	0.79	0.44	0.21	1	2	3	4	3	15	10	26	10	16	19	31	12	21
1	1.5	2	4	0.4	0.79	0.44	0.21	1	2	3	4	3	15	10	26	10	16	19	32	12	21
1	1.5	2	4	0.1	0.79	0.44	0.21	1	2	3	4	3	15	10	27	10	16	19	32	12	21
1	2	6	6	1	0.72	0.43	0.17	1	2	3	4	2	17	10	37	8	19	23	46	12	28
1	2	6	6	0.7	0.72	0.41	0.17	1	2	3	4	2	17	10	38	8	20	23	47	12	28
1	2	6	6	0.4	0.7	0.37	0.17	1	2	3	4	3	18	11	41	9	20	24	49	12	28
1	2	6	6	0.1	0.7	0.34	0.17	1	2	3	4	3	18	11	46	9	20	25	50	12	28

Note that, since there are four possible ordering case outcomes, there are up to three transition points in the optimal ordering case as α decreases from 1 to 0, ceteris paribus. Moreover, note that when $\alpha = 1$, Case 1 is optimal by inspection, as Case 4 is similarly optimal when $\alpha = 0$. Therefore, all problem instances must have at least one transition, from Case 1 to Case 4, as α decreases from 1 to 0, and of course, some instances will reflect two or three transitions. In Table 5.2, Decision 0 corresponds to the optimal policy case for $1 \geq \alpha > \alpha_1^{\Delta}$, Decision 1 to the optimal policy case for $\alpha_1^{\Delta} \geq \alpha > \alpha_2^{\Delta}$, Decision 2 to optimal policy case for $\alpha_2^{\Delta} \geq \alpha > \alpha_3^{\Delta}$, and Decision 3 to optimal policy case for $\alpha_3^{\Delta} \geq \alpha$.

From Table 5.2 we observe that for a given set of input parameters (c_u, c_r, f, and b), the threshold α values change little, if any, as β is varied. Similarly, the policy parameters (s_r, s_u, S_r, S_u) are not influenced much by β. From the results in Table 5.2 and well as the other scenarios listed in Table 5.1 not shown in Table 5.2, we conclude that, for a given set of the key problem parameters, changing β appears to have no effect on the structure or the parameters of the ordering policy. Table 5.2 does seem to indicate, however, that as the various cost parameters change, the threshold values of α and policy parameters begin to change.

As expected the total cost increases as β reduces. This can be seen in Table 5.3 for the same subset of cost scenarios from Table 5.1. However, since the policy is unaffected by the value of β, for the rest of the experimentation in this chapter, β is held at a constant value of 1.

Table 5.3: Impact of β on total expected cost

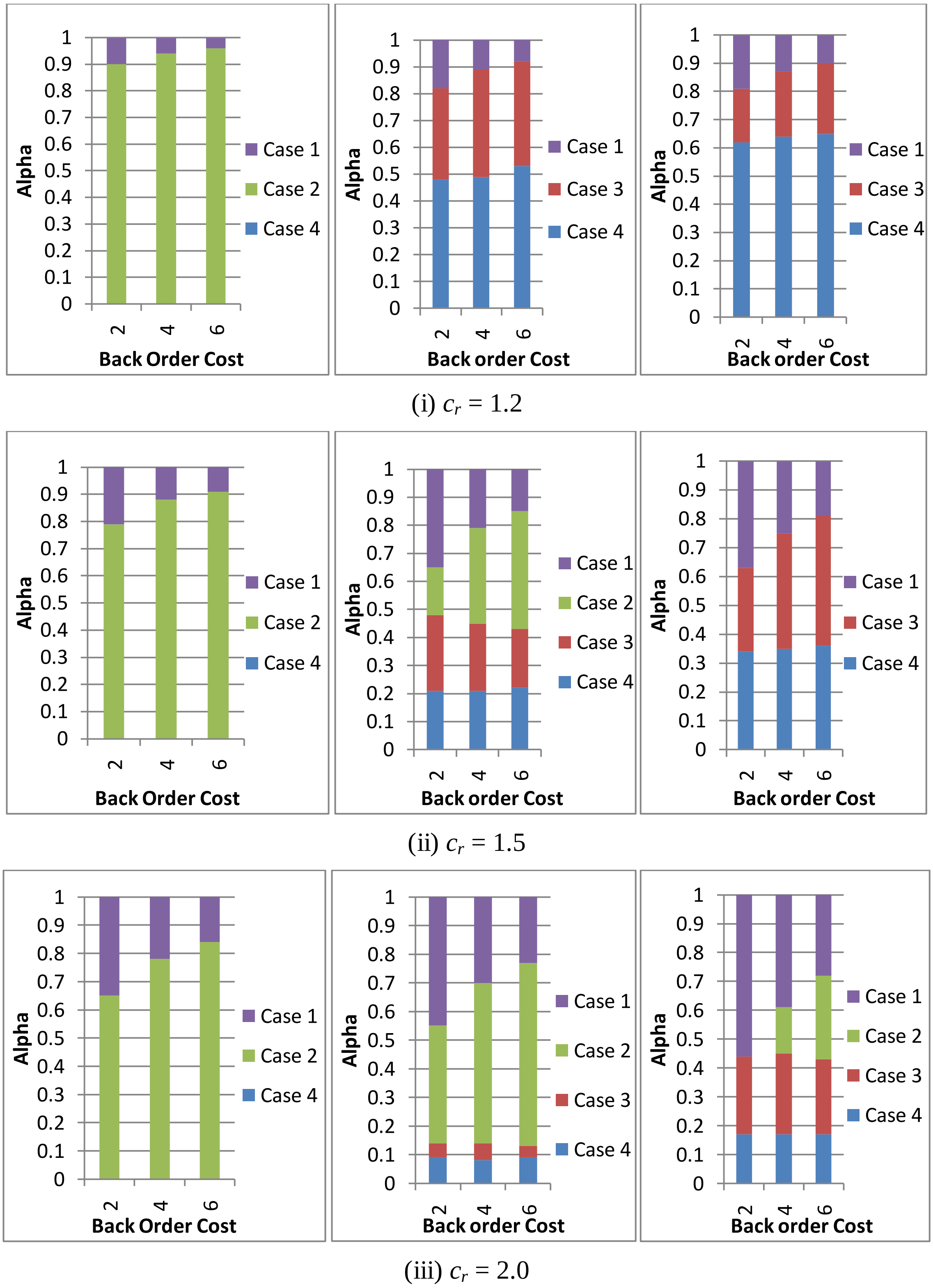
c_u	c_r	f	b	β	α_1^Δ	α_2^Δ	α_3^Δ	Cost at α_1^Δ	Cost at α_2^Δ	Cost at α_3^Δ
1	1.2	0	2	1	0.9	0	-	7.5058	8.1180	-
1	1.2	0	2	0.7	0.9	0	-	0%	0%	-
1	1.2	0	2	0.4	0.9	0	-	1%	0%	-
1	1.2	0	2	0.1	0.9	0	-	3%	0%	-
1	1.5	2	4	1	0.79	0.45	0.21	9.7340	10.7790	11.1709
1	1.5	2	4	0.7	0.79	0.44	0.21	1%	1%	0%
1	1.5	2	4	0.4	0.79	0.44	0.21	3%	2%	0%
1	1.5	2	4	0.1	0.79	0.44	0.21	9%	3%	0%
1	2	6	6	1	0.72	0.43	0.17	12.0649	13.7944	15.1079
1	2	6	6	0.7	0.72	0.41	0.17	2%	2%	0%
1	2	6	6	0.4	0.7	0.37	0.17	7%	5%	0%
1	2	6	6	0.1	0.7	0.34	0.17	17%	8%	0%

5.2 Effect of the reliability and costs on problem outcomes

In this section the impact of the backorder cost, fixed cost and reliable supplier cost is studied. For the experimental design of this section, the input parameters are given in Table 5.4.

Table 5.4: The system parameters and values considered in section 5.2

System parameters	Values		
Unit ordering cost from reliable supplier, c_r	1.2	1.5	2
Unit ordering cost from unreliable supplier, c_u	1		
Unit holding cost per period, h	0.2		
Unit backordering cost per period, b	2	4	6
Fixed ordering cost, f	0	2	6
Probability of staying up, α	0.01:1 in increments of 0.01		
Probability of going from down to up, β	1		



(a) $f = 0$ (b) $f = 2$ (c) $f = 6$

Figure 5.1: Effect of α and b on optimal ordering case for various levels of c_r and f

Figure 5.1 shows the range of α values for which each case is best as the backorder cost is increased for different levels of fixed cost f, and c_r. The threshold α values are observed. The figure shows that when the fixed cost is 0 (Figure 5.1.a), policy case 3 is not observed, but as the fixed cost is increased (Figure 5.1.b), case 3 is observed and case 2 does not occur (Figure 5.1.b(i)) or diminishes (Figure 5.1.b(ii), 5.1.b(iii)). This is due to the fact that as the fixed cost increases, the cost for ordering from both suppliers simultaneously is very expensive.

We make the following other observations:

- as the backorder cost is increased, the policy changes from case 1 to either case 2 or case 3 at a higher reliability level of the supplier;
- the backorder cost is avoided by ordering only from the reliable supplier when α gets smaller;
- when the cost of the reliable supplier, c_r, is low, case 4 is more prominent;
- as c_r increases, case 4 becomes less prominent;
- the threshold α value at which the policy changes to case 4 is lower when the fixed cost is lowered;
- the threshold α value for case 1 increases as the backorder cost is increased; and
- the threshold α value for all case changes reduces as c_r is increased.

5.3 Effect of problem parameters on optimal ordering case

For the experimental design of this section, the input parameters are given in Table 5.5.

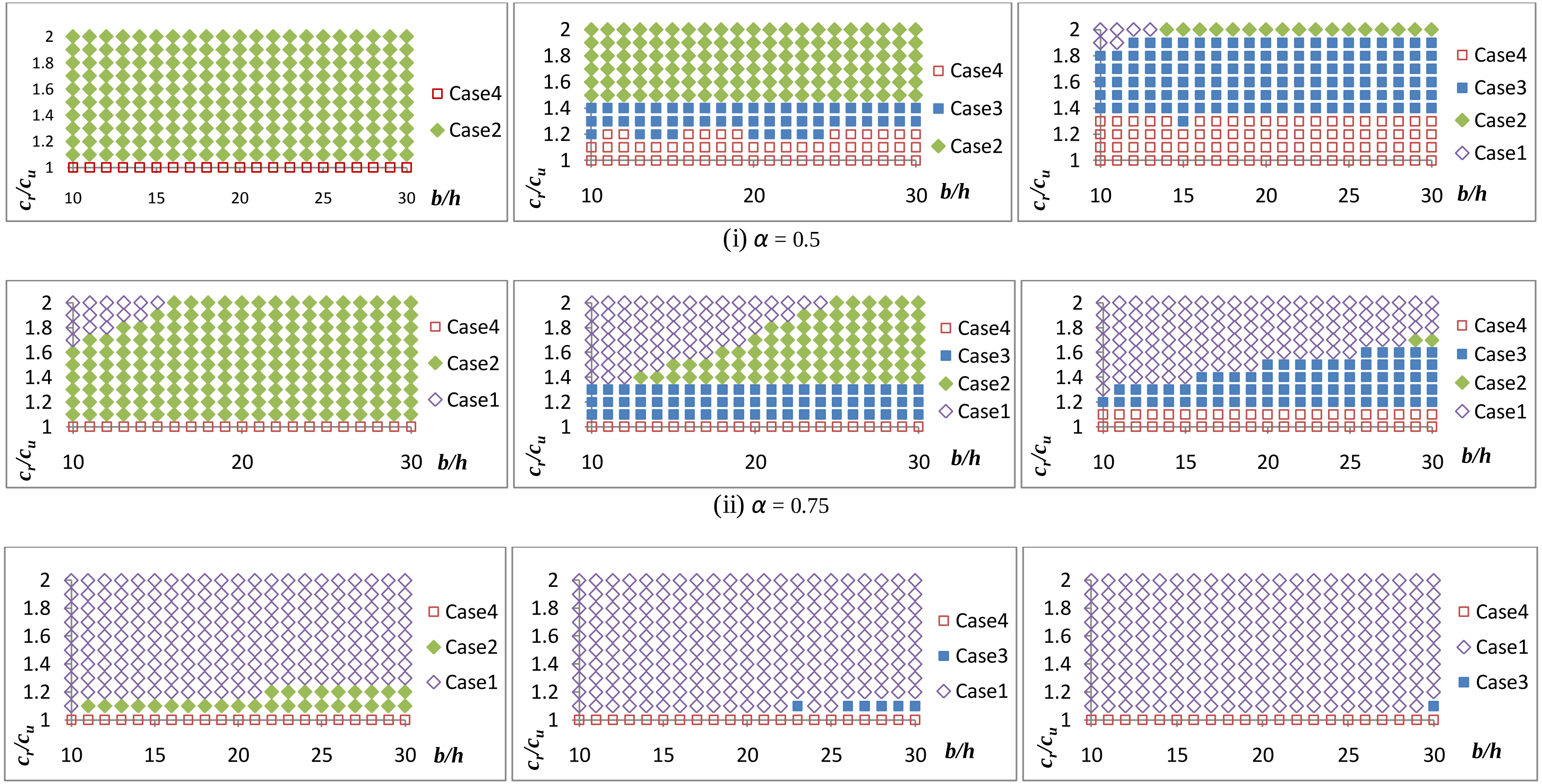
Table 5.5: The system parameters and values considered in section 5.3

System parameters	Values		
Unit ordering cost from reliable supplier, c_r	1:2 in increments of 0.1		
Unit ordering cost from unreliable supplier, c_u	1		
Unit holding cost per period, h	0.2		
Unit backordering cost per period, b	2:6 in increments of 0.2		
Fixed ordering cost, f	0	2	6
Probability of staying up, α	0.5	0.75	0.95
Probability of going from down to up, β	1		

From Figure 5.2.a, we can observe that when the fixed cost = 0; case 2 is best when α is 0.5 (Figure 5.2.a(i)). As α increases, case 1 first appears for the high c_r/c_u and low b/h scenarios when α=0.75 (Figure 5.2.b(ii)) and grows to dominate when α=0.95 (Figure 5.2.a(iii)). Case 3 does not appear when fixed cost = 0.

From Figure 5.2.b, as fixed cost increases, we observe that other cases appear. As α increases, case 1 appears (Figure 5.2.b(ii), 5.2.b(iii)). Also, case 4 becomes more prevalent as f increases for low values of c_r/c_u when α is low. Case 3 and case 4 appear for lower α levels and diminish as α increases, and case 1 and case 2 become more prominent.

When fixed cost is very high (Figure 5.2.c), case 2 appears only for a few values of α as seen in Figures 5.2.c(i) and 5.2.c(ii).



(i) α = 0.5

(ii) α = 0.75

(iii) α = 0.95

(a) f = 0

(b) f = 2

(c) f = 6

Figure 5.2: Policy change for α = [0.5, 0.75, 0.95]

We can make the following observations from the plots above:

- When $\alpha = 0.95$, case 1 is predominant. Case 2/case 3 are best only for a few scenarios and case 4 only occurs when the suppliers' costs are equal.
- When the unreliable supplier is fairly reliable, i.e. $\alpha = 0.75$, both case 1 and case 2/case 3 appear. However, when the fixed cost is high (Figure 5.2.c), case 2 is best only for a few scenarios.
- When α reduces to 0.5, case 2/case 3 are still present and in some scenarios case 4 also appears as seen in Figure 5.2(i). For higher fixed cost, case 1 is best only for a few scenarios (Figure 5.2.c(i)).

Safety stock was analyzed for different backorder cost, fixed cost and cost of the reliable supplier. The results are included in the Appendix C.

5.4 Effect of improving supplier parameters on system cost

In this section improvement in system costs are analyzed as the unreliable supplier's reliability is improved for different values of backorder cost, fixed cost and cost of the reliable supplier. For the experimental design of this section, the input parameters are given in Table 5.6.

Table 5.6: The system parameters and values considered in section 5.4

System parameters	Values				
Unit ordering cost from reliable supplier, c_r	1.2	1.5			
Unit ordering cost from unreliable supplier, c_u	1				
Unit holding cost per period, h	0.2				
Unit backordering cost per period, b	2	4	6		
Fixed ordering cost, f	2	6			
Probability of staying up, α	0.7	0.8	0.9	0.95	1
Probability of going from down to up, β	1				

In Figures 5.3-5.8, total cost improvement is defined as the percentage reduction in total cost from for a given set of input cost parameters and reliability level α versus the total cost for the same cost parameters and a reliability level of 0.7.   Figures 5.3, 5.4 and 5.5 show the cost reduction for fixed cost =2 and Figures 5.6, 5.7 and 5.8 represent cost reduction for fixed cost = 6. From all these figures we observe that as the reliability of the unreliable supplier improves from 0.7 to 1.0, the reduction in total cost increases, with greater improvement for the higher c_r/c_u ratio. When the reliability reaches 1.0, the total cost is reduced by about 14% for c_r/c_u = 1.5 and about 9% for c_r/c_u = 1.2. It can be also seen that the cost reduction is not influenced much as *b/h* increases (Figures 5.3 to 5.4 to 5.5 and Figures 5.6 to 5.7 to 5.8), however, we can see that optimal policy case changes. To avoid higher backorder cost, the optimal case changes from case 1 to case 2 or case 3. Also, as c_r/c_u increases, the optimal case changes from case 2/case 3 to case 1 to avoid higher reliable supplier cost. Fixed cost *f* has little impact of the cost improvement and on the policy case.

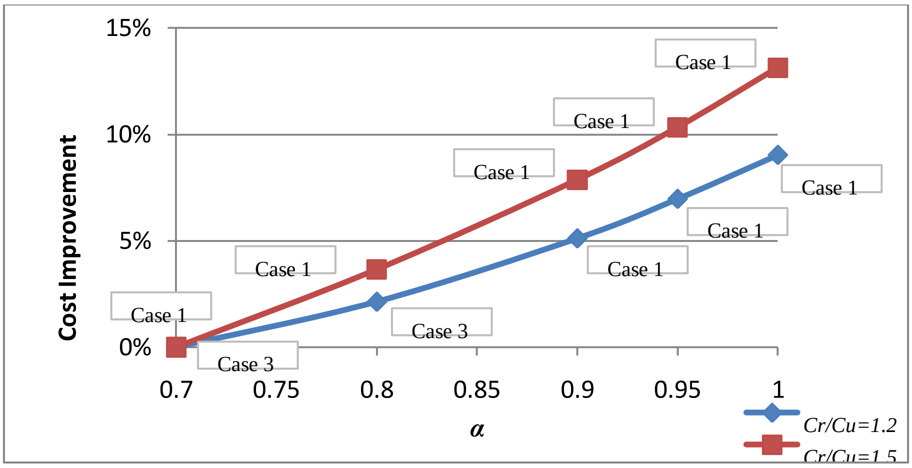


Figure 5.3: Cost improvement when *f* = 2, *b/h* = 10

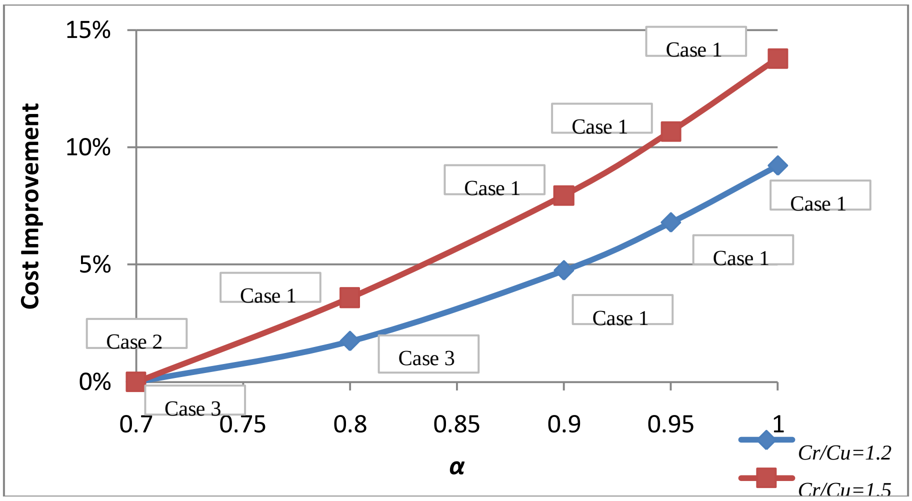


Figure 5.4: Cost improvement when *f* = 2, *b/h* = 20

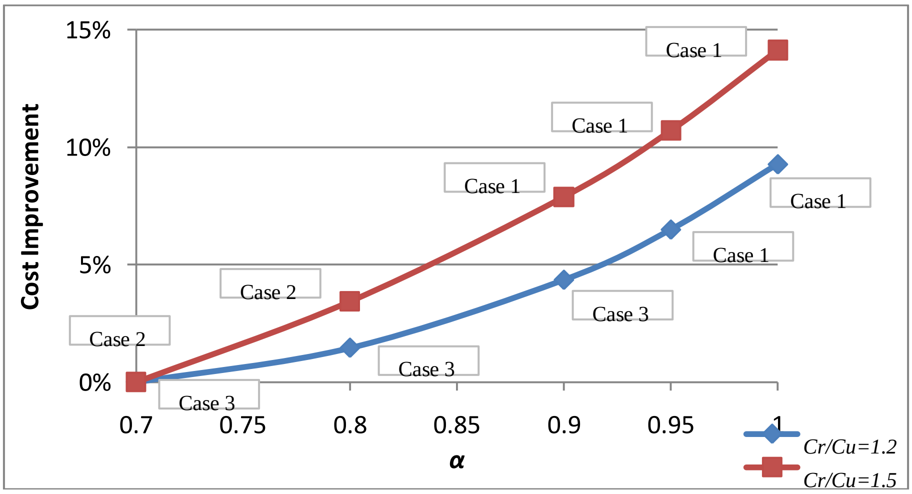


Figure 5.5: Cost improvement when *f* = 2, *b/h* = 30

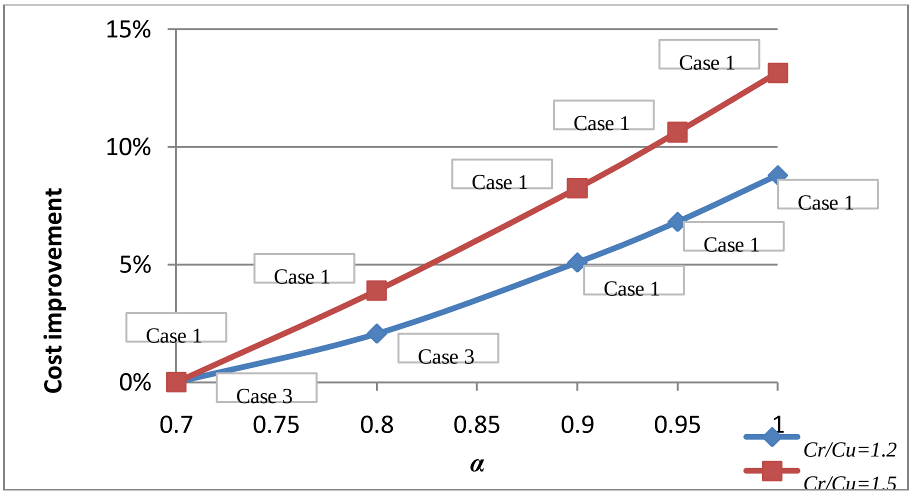


Figure 5.6: Cost improvement when $f = 6$, $b/h = 10$

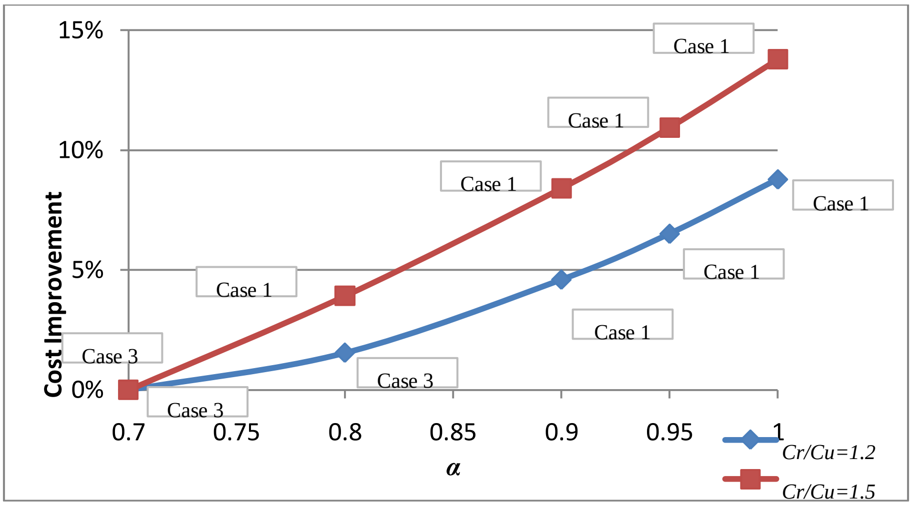


Figure 5.7: Cost improvement when $f = 6$, $b/h = 20$

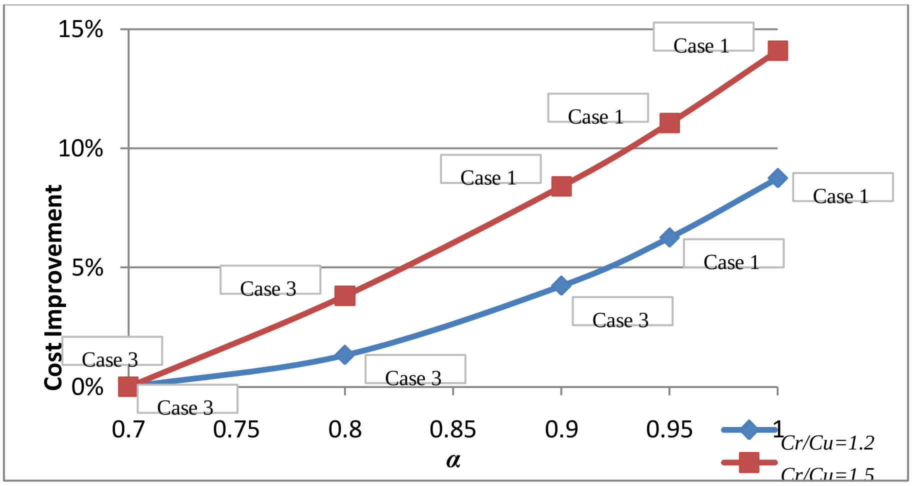


Figure 5.8: Cost improvement when $f = 6$, $b/h = 30$

Further analysis was carried out for additional levels of the c_r/c_u ratio (by changing the reliable supplier's cost). Additionally the backorder cost and fixed cost are ranged as in the previous experiment. For Figures 5.9 – 5.14, we plot the percentage cost improvement for a specified unit cost from the reliable supplier and other cost parameter values versus the same case when the reliable supplier unit cost is 1.5. This is done for two different unreliable supplier α values of 0.7 and 0.9. For the experimental design of this section, refer to Table 5.7.

Table 5.7: The system parameters and values considered in section 5.4

System parameters	Values					
Unit ordering cost from reliable supplier, c_r	1	1.1	1.2	1.3	1.4	1.5
Unit ordering cost from unreliable supplier, c_u	1					
Unit holding cost per period, h	0.2					
Unit backordering cost per period, b	2	4	6			
Fixed ordering cost, f	2	6				
Probability of staying up, α	0.7	0.9				
Probability of going from down to up, β	1					

From Figures 5.9-5.14 we observe that the cost improvement increases as the unit cost of the reliable supplier is reduced. For all values of b/h ratio and fixed cost f, lowering the unit cost from 1.5 to 1.o yields a cost reduction of about 14% when $\alpha = 0.7$ and 6% for $\alpha = 0.9$. However, as in the previous set of figures the fixed cost and backorder cost has only minimal impact of cost reduction.

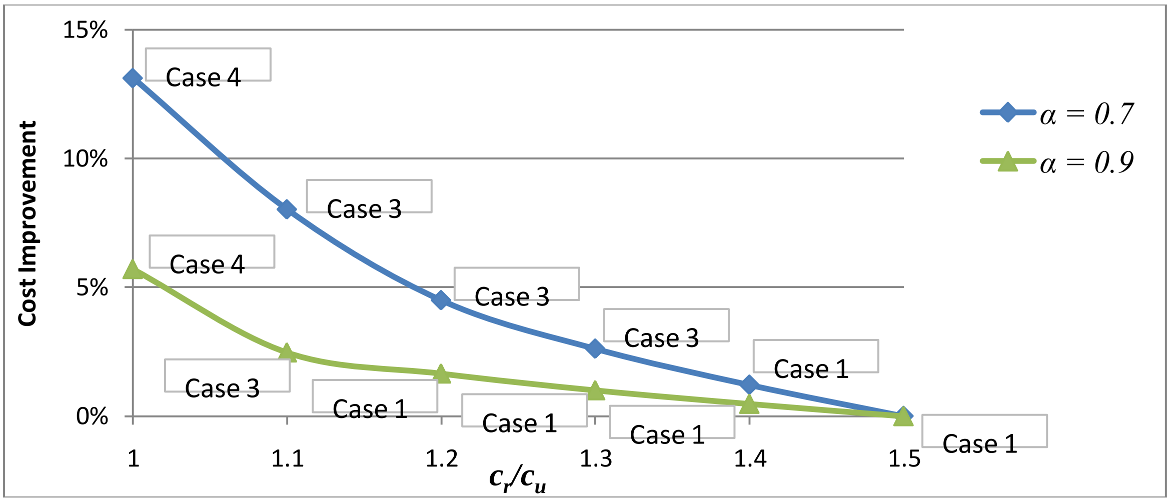


Figure 5.9: Cost improvement when $f = 2$, $b/h = 10$

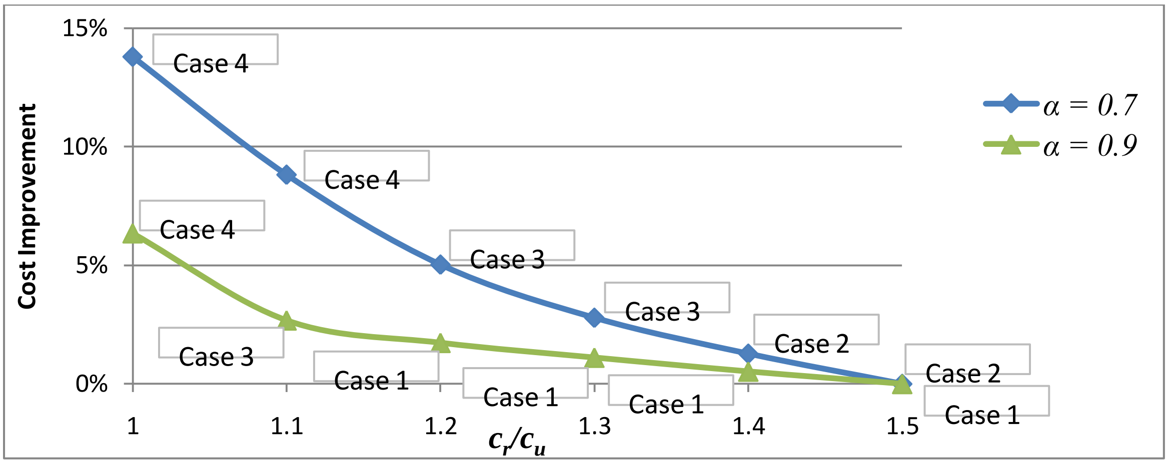


Figure 5.10: Cost improvement when $f = 2$, $b/h = 20$

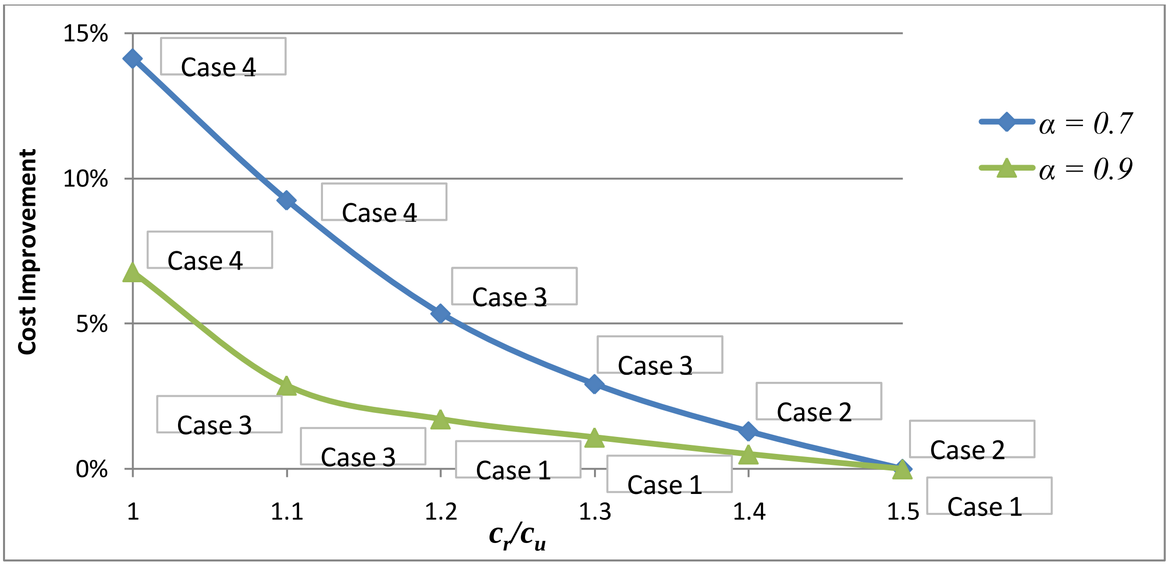


Figure 5.11: Cost improvement when $f = 2$, $b/h = 30$

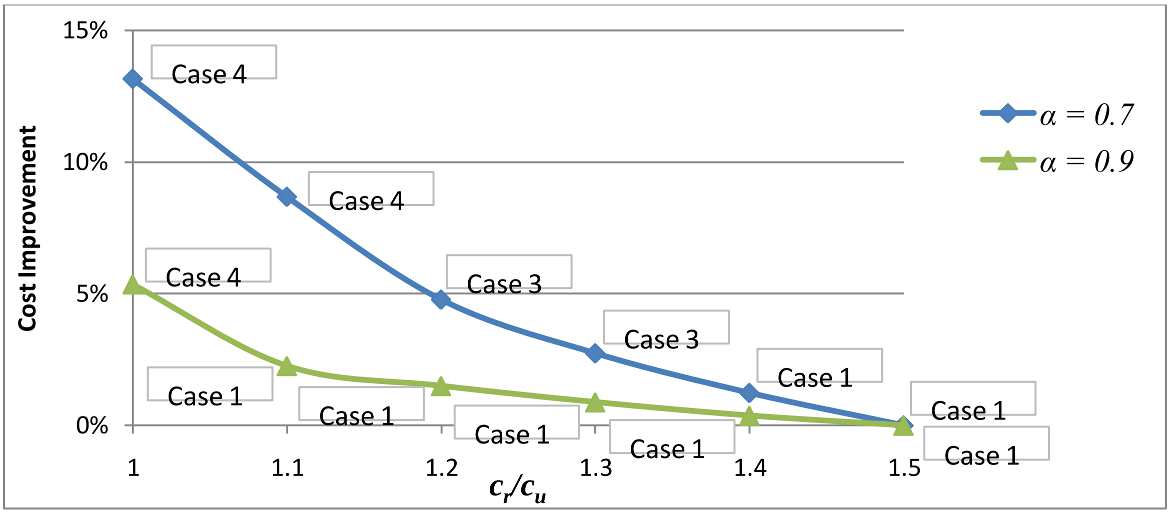


Figure 5.12: Cost improvement when $f = 6$, $b/h = 10$

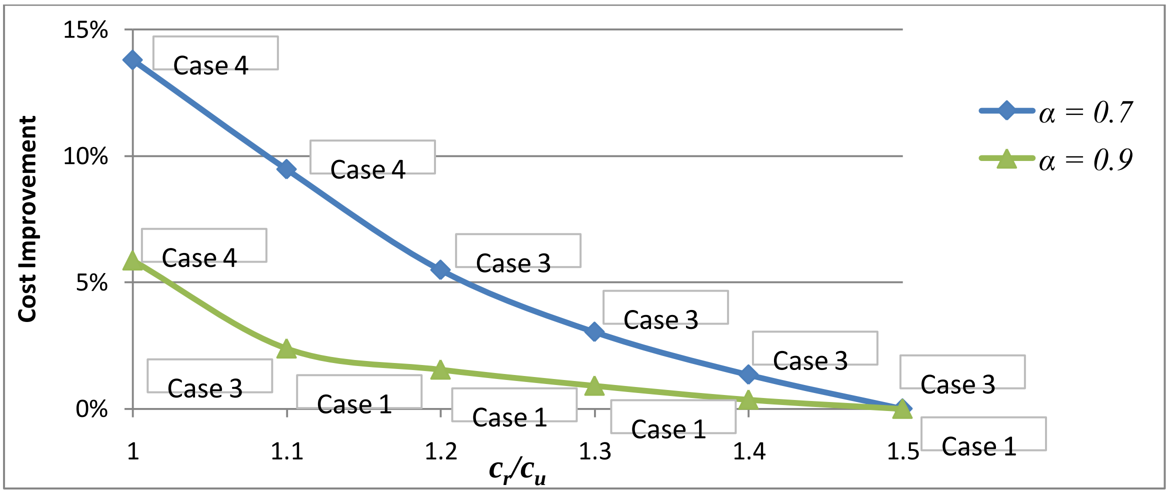


Figure 5.13: Cost improvement when $f = 6$, $b/h = 20$

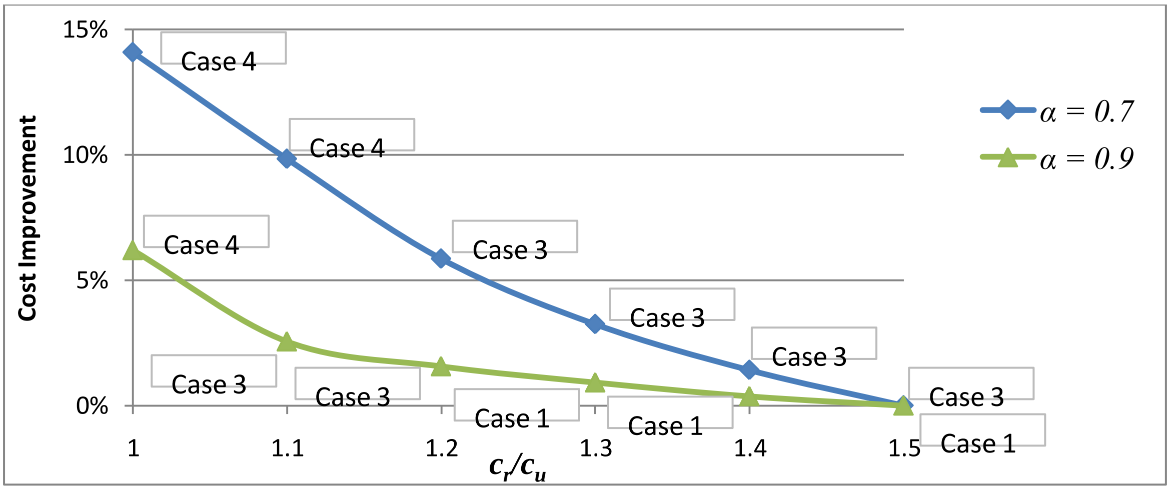
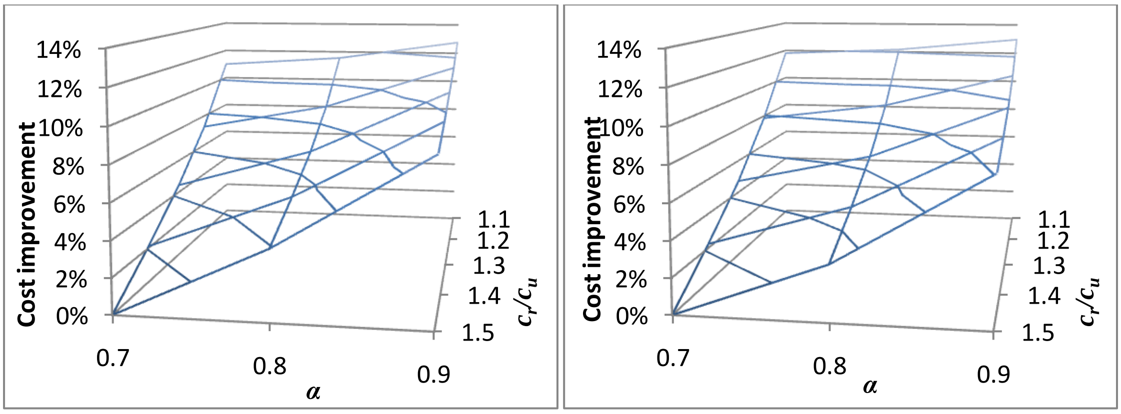
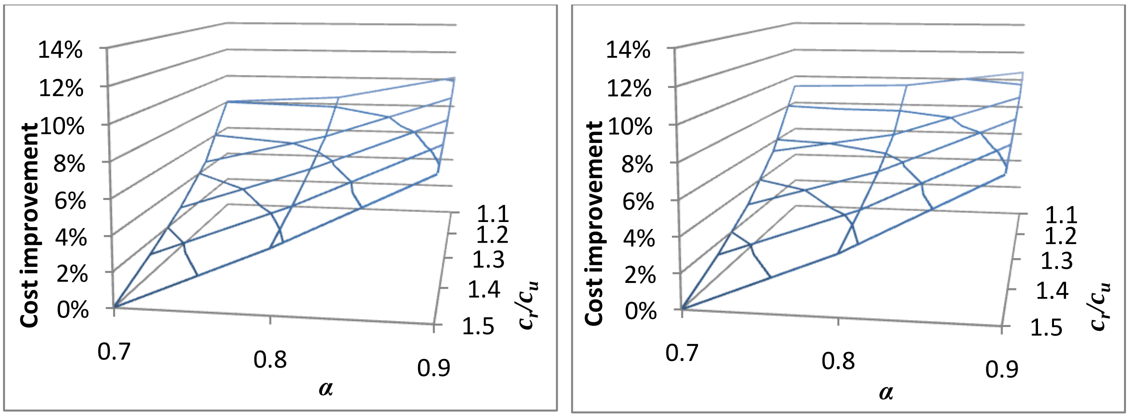


Figure 5.14: Cost improvement when $f = 6$, $b/h = 30$

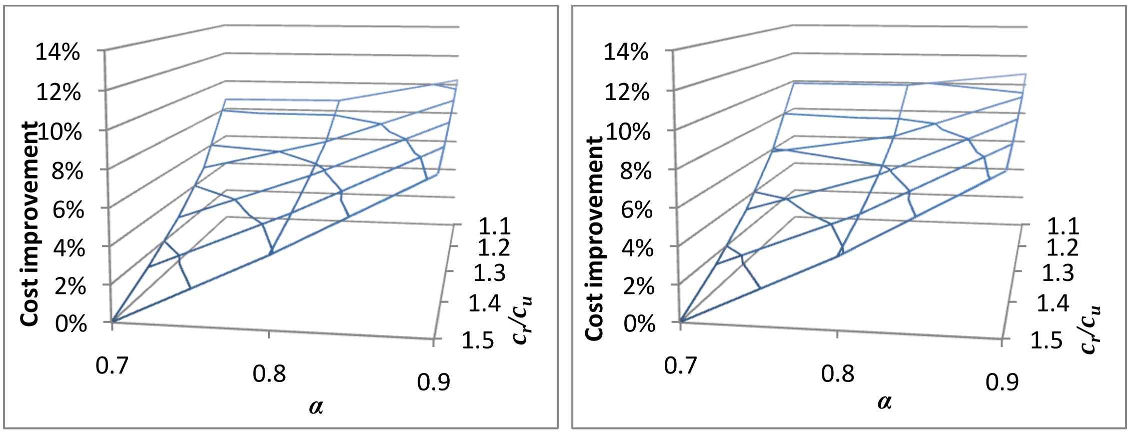
Figure 5.15 is a combined plot of reliability improvement and cost reduction of the reliable supplier. It gives the cost improvement achieved when either reliability is improved and/or cost is reduced. We can observe that the pattern of the plot remains the same when fixed cost and/or backorder cost is increased.



(i) f = 0



(ii) f = 2



(iii) f = 6

(a) b/h = 10 (b) b/h = 30

Figure 5.15: Cost improvement

5.5 Comparison of pure ordering strategies

In this section the impact of backorder cost and fixed cost on the difference between the total cost of pure ordering strategies, i.e. either ordering only from the unreliable supplier or ordering only from the reliable supplier, is analyzed. This experiment was carried out to analyze the difference in cost in cases of ordering only from one of the supplier. The definition of the terminologies used is given below:

1. Modified Case 1 - order only from the unreliable supplier if the unreliable supplier status is up and order nothing if the unreliable supplier status is down
2. Pure Case 4 - order only from the reliable supplier
3. Δcost – the change in the cost between a modified case 1 and pure case 4 relative to pure case 4

For the experimental design of this section, the input parameters are given below.

Table 5.8: The system parameters and values considered in section 5.5

System parameters	Values		
Unit ordering cost from reliable supplier, c_r	1.2	1.5	2
Unit ordering cost from unreliable supplier, c_u	1		
Unit holding cost per period, h	0.2		
Unit backordering cost per period, b	2	4	6
Fixed ordering cost, f	0	2	6
Probability of staying up, α	0.01:1 in increments of 0.01		
Probability of going from down to up, β	1		

By limiting the alternatives for each state in the MDP, the optimal modified case 1 and pure case 4 policies and their costs can be determined. For each parameter setting, the value of Δcost is calculated as

Δcost = (cost of modified case 1 - cost of pure case 4)/cost of pure case 4.

Figures 5.16, 5.17 and 5.18 show the results for different backorder and fixed costs.

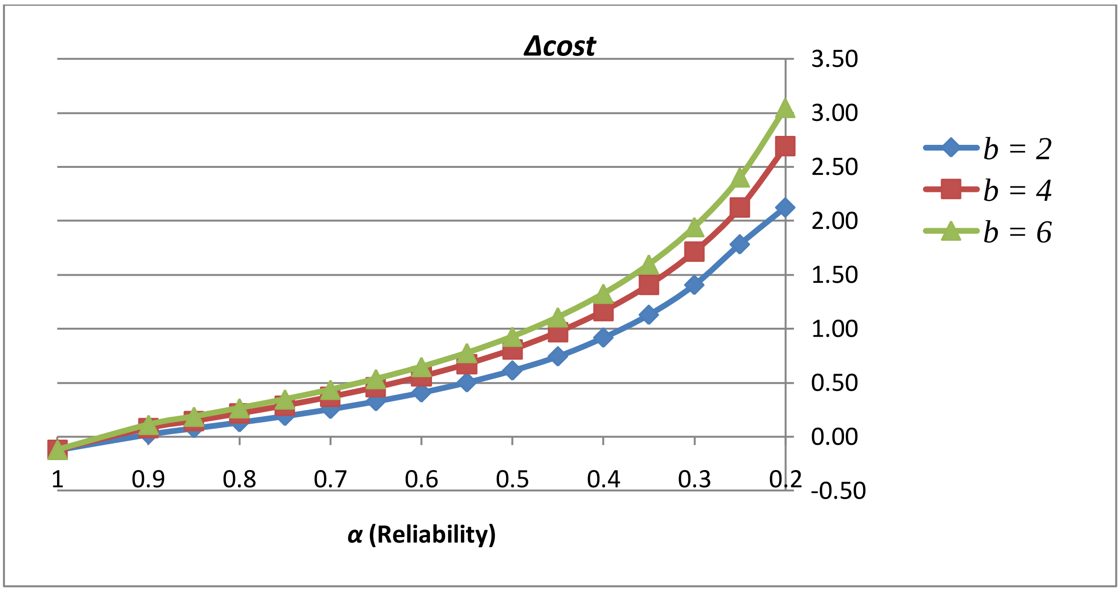


Figure 5.16: Δcost versus reliability (f=0, c_r=1.2)

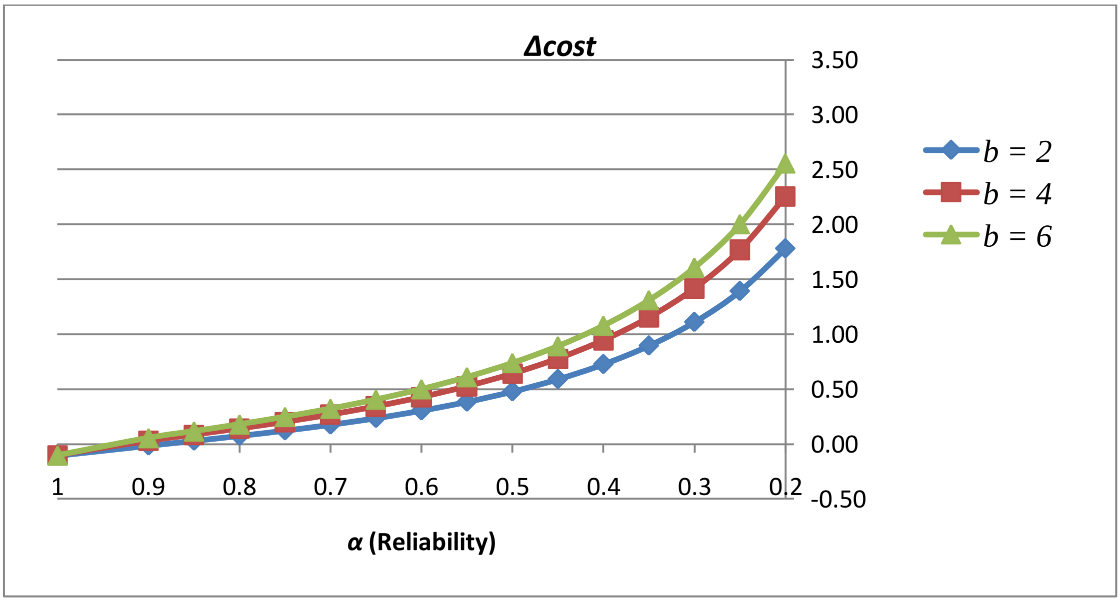


Figure 5.17: Δcost versus reliability (f=2, c_r=1.2)

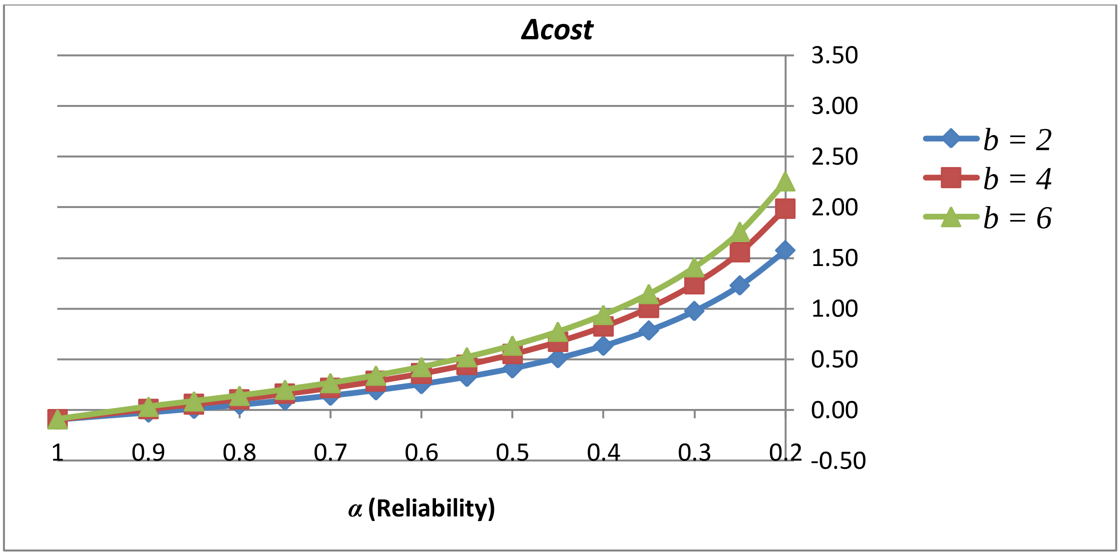


Figure 5.18: Δcost versus reliability (f=6, c_r=1.2)

From Figures 5.16, 5.17 and 5.18 we observe that Δcost increases as backorder cost increases and reduces as fixed cost increases. For the same experiment, individual cost components for the modified case 1 is recorded and analyzed. Figure 5.19 below shows a typical graph of the various cost components as α is reduced for a modified case 1.

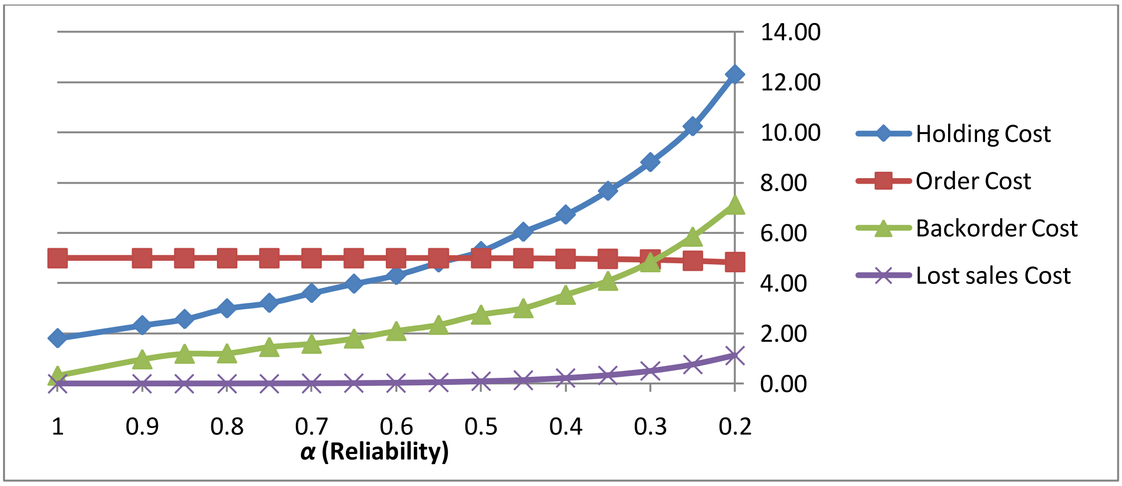


Figure 5.19: Individual cost components versus reliability for modified case 1 policy

The detailed results of the individual cost components are included in Appendix B.

Finally, the experiment was repeated for c_r = 1.5 and c_r = 2. Δcost reduces as the cost of the reliable supplier increases and increases as backorder cost increases and reduces as fixed cost increases. The detailed results are included in Appendix C.

Further experimentation was performed in order to understand better potential options for the retailer in order to improve performance. Two logical ways for the retailer to improve their performance are to (1) negotiate a lower price from either supplier or (2) negotiate an improved reliability from the unreliable supplier. The tables below address comparisons of these choices.

First, the retailer may be interested in determining how much must the unreliable supplier improve their reliability or lower their unit cost to achieve a specified percentage improvement in total cost. Table 5.9 below indicate the α and c_u required to achieve approximately 1%, 5% and 10% cost improvements for a modified case 1 policy for different backorder cost and fixed cost. The left column gives the improvement in total cost for a given increase in α from a value of 0.75 while the right column gives the unit cost required to approximately match this same improvement. For example, in Table 5.9(b) when the fixed ordering cost is 6 and the backorder cost is 2, increasing the supplier reliability from 0.75 to 0.89 leads to a 10.19% reduction in total cost while reducing unit cost from 1.00 to 0.76 (with α fixed at 0.75) yields about the same improvement (10.25%).

An alternative question is "how much improvement in reliability is equivalent to a 5, 10, and 20% reduction in unit cost from the unreliable supplier. Table 5.10 shows the reliability required by the unreliable supplier to meet the same cost improvement as obtained when c_u is reduced by 5%, 10% and 20% for a modified case 1 policy.

Table 5.9: Cost improvement analysis for modified case 1

(a) $b = 2, f = 2$

α	Cost	Cost improvement
0.75	10.579	
0.76	10.4847	0.89%
0.81	10.0367	5.13%
0.87	9.5308	9.91%

c_u	Cost	Cost improvement
1	10.5790	
0.98	10.4791	0.94%
0.89	10.0291	5.20%
0.8	9.5792	9.45%

(b) $b = 2, f = 6$

α	Cost	Cost improvement
0.75	11.707	
0.76	11.6099	0.83%
0.82	11.0771	5.38%
0.89	10.5143	10.19%

c_u	Cost	Cost improvement
1	11.7070	
0.98	11.6070	0.85%
0.87	11.0571	5.55%
0.76	10.5071	10.25%

(c) $b = 6, f = 2$

α	Cost	Cost improvement
0.75	12.2177	
0.76	12.0797	1.13%
0.8	11.554	5.43%
0.85	10.9373	10.48%

c_u	Cost	Cost improvement
1	12.2177	
0.97	12.0677	1.23%
0.87	11.5677	5.32%
0.75	10.9677	10.23%

(d) $b = 6, f = 6$

α	Cost	Cost improvement
0.75	13.3441	
0.76	13.2068	1.03%
0.8	12.6838	4.95%
0.86	11.9546	10.41%

c_u	Cost	Cost improvement
1	13.3441	
0.97	13.1941	1.12%
0.87	12.6941	4.87%
0.72	11.9441	10.49%

Table 5.10: Reliability required for cost reduction

(a) $b = 2, f = 2$

c_u	Cost	Cost improvement
1	10.5790	
0.95	10.3291	2.36%
0.9	10.0791	4.73%
0.8	9.5792	9.45%

α	Cost	Cost improvement
0.75	10.5790	
0.78	10.3024	2.61%
0.8	10.1256	4.29%
0.87	9.5308	9.91%

(b) $b = 2, f = 6$

c_u	Cost	Cost improvement
1	11.7070	
0.95	11.4570	2.14%
0.9	11.2071	4.27%
0.8	10.7071	8.54%

α	Cost	Cost improvement
0.75	11.7070	
0.78	11.4279	2.38%
0.81	11.1633	4.64%
0.87	10.6667	8.89%

(c) $b = 6, f = 2$

c_u	Cost	Cost improvement
1	12.2177	
0.95	11.9677	2.05%
0.9	11.7177	4.09%
0.8	11.2177	8.18%

α	Cost	Cost improvement
0.75	12.2177	
0.77	11.9453	2.23%
0.79	11.6822	4.38%
0.83	11.1785	8.51%

(d) $b = 6, f = 6$

c_u	Cost	Cost improvement
1	13.3441	
0.95	13.0941	1.87%
0.9	12.8441	3.75%
0.8	12.3441	7.49%

α	Cost	Cost improvement
0.75	13.3441	
0.77	13.0728	2.03%
0.79	12.8112	3.99%
0.83	12.3147	7.71%

Also of interest is to examine unit cost reduction from the reliable and improvement in reliability for the unreliable supplier. In Table 5.11 the total cost is given as the unit cost of the reliable supplier is reduced from 1.4 to 1.1 in increments of 0.1 (left columns) for different values of backorder and fixed costs. The right columns so the reliability required for an unreliable supplier (with unit cost 1.0) to achieve approximately the same total cost. For example, in Table 5.11(b), when the backorder cost is 2 and fixed cost is 6, approximately the same total cost for the retailer is achieved by the reliable supplier offering a unit cost of 1.1 or the unreliable supplier having a reliability of 93%.

Table 5.11: Cost for modified case 1 versus pure case 4

(a) $b = 2, f = 2$

c_r	Cost	Cost improvement	α	Cost	Cost improvement
1.4	10.4258		0.77	10.3927	
1.3	9.9258	4.80%	0.82	9.9520	4.24%
1.2	9.4258	9.59%	0.9	9.2820	10.69%
1.1	8.9258	14.39%	0.95	8.8637	14.71%

(b) $b = 2, f = 6$

c_r	Cost	Cost improvement	α	Cost	Cost improvement
1.4	11.6905		0.75	11.7070	
1.3	11.1905	4.28%	0.81	11.1633	4.64%
1.2	10.6905	8.55%	0.87	10.6667	8.89%
1.1	10.1905	12.83%	0.93	10.2074	12.81%

(c) $b = 6, f = 2$

c_r	Cost	Cost improvement	α	Cost	Cost improvement
1.4	10.7894		0.87	10.7000	
1.3	10.2894	4.63%	0.9	10.3392	3.37%
1.2	9.7894	9.27%	0.94	9.8153	8.27%
1.1	9.2894	13.90%	0.97	9.3498	12.62%

(d) $b = 6, f = 6$

c_r	Cost	Cost improvement	α	Cost	Cost improvement
1.4	12.1079		0.85	12.0745	
1.3	11.6079	4.13%	0.89	11.5984	3.94%
1.2	11.1079	8.26%	0.93	11.1020	8.05%
1.1	10.6079	12.39%	0.97	10.5555	12.58%

Chapter 6

Summary and Conclusions

In this thesis, a single retailer facing stochastic demand with two suppliers, one reliable and the other unreliable but with a better unit cost was considered. The unreliable supplier can be in either of two states: “up” and available to satisfy orders or “down” and unavailable to satisfy orders. The process of transitioning between states is assumed to be a simple Markov process defined by two parameters α (the probability of staying in the “up” state) and β (the probability of transitioning from the “down” state to the “up” state. Product from the two suppliers is indistinguishable, with all units incurring the same holding cost. Each order from either supplier incurs the same fixed ordering cost. The system is modeled as a discrete time, discrete state Markov Decision Process with one supplier being unreliable. Four possible optimal policy structures are identified: Order exclusively from the unreliable supplier; Order from one or the other but not both suppliers; Order from both suppliers simultaneously; and Order exclusively from the reliable supplier. The optimal structure is identified under various input parameter values.

From the experiments in Section 5.1 we conclude that recovery rate, β (the probability that the unreliable supplier returns to the up state from being in the down state in the previous period) has very little impact on the ordering policies, although the cost does increase as β decreases.

As the fixed cost declines from larger values, the system transitions from single-source ordering, to “one or the other” ordering, and ultimately to ordering from both suppliers. The extent of "both" ordering, however, is conditional on the reliability of the unreliable supplier (Figure 5.2). Moreover, for moderate fixed ordering costs, the "one or other" case is preferred over the "both" case when the reliable supplier cost is closer to that of the unreliable supplier (Figure 5.2). However, when the backorder costs are extremely high, it is optimal to order from both suppliers. When fixed cost is negligible the optimal decision to order only from the reliable supplier occurs only when the probability of delivery of the cheaper supplier is zero. From the analysis we also found that even when the cost of the reliable supplier, backorder costs and fixed costs are high, it is optimal to order from both suppliers simultaneously.

From our analysis, we find that when the reliability of the cheaper supplier is low, unless the suppliers charge similar unit costs, it is optimal to order from both suppliers. However, we noticed that, when the fixed cost and cost of the reliable supplier is high, the optimal policy is to order only from the cheaper supplier. But when backorder costs also increase, it is better to order from both of them. When the cheaper supplier is fairly reliable, it is optimal to order from cheaper supplier when backorder cost is low. When backorder costs increase it is optimal to order from both of them. We also found that when the cheaper supplier is highly reliable, it still may be optimal to "dual-source" to avoid backorder costs despite the additional fixed ordering costs (Figure 5.2, bottom row of graphs).

Analysis shows that the system costs decline as the reliability of the cheaper supplier improves and when the cost of the reliable supplier is reduced relative to the unreliable supplier's cost. The pattern of the plot (refer to Figure 5.15) remains the same when fixed cost and/or backorder cost is increased. But for negligible fixed cost, the cost reduction is higher.

An extension for this model would be to consider different fixed cost and holding cost for the two suppliers and analyzing the effect of these costs on the policy structures. Other possible extensions would be to consider multiple unreliable suppliers and different demand characteristics to check the effect on policy structures. Extensive analysis could be carried out to check for interaction effects between the system parameters.

References

Abboud, N.E., (2001), “A discrete time Markov production-inventory model with machine breakdowns”, *Computers & Industrial Engineering,* 39, 95-107.

Ahiska, S.S. and King, R.E., (2010a) “Inventory optimization in a one product recoverable manufacturing system,” *International Journal of Production Economics,* 124(1), 11-19.

Ahiska, S.S. and King, R.E., (2010b) “Inventory policy characterization methodologies for a single-product recoverable manufacturing system,” *working paper*.

Arreola-Risa, A. and DeCroix, G., (1998), “Inventory management under random supply disruptions and partial backorders,” *Naval Research Logistics,* 45, 687-703

Gullu, R., Onel, E., and Erkip, N., (1999), “Analysis of an inventory system under supply uncertainty,” *Int. J. Production Economics,* 59, 377-385

Gurlur, U. and Parlar, M., (1996), “An inventory problem with two randomly available suppliers,”

Hodgson, T.J., King, R.E., Monteith, S.K. and Schultz, S.R., (1987) “Developing control rules for an AGVS using Markov decision processes,” *Material Flow,* 4(1), 87-98.

Howard, R.A., (1960), Dynamic Programming and Markov Processes, The MIT Press: Cambridge.

Minner, S., (2003), “Multiple-supplier inventory models in supply chain management: A review,” *International Journal of Production Economics,* 81-82(3), 265-279

Mohebbi, E., (2004), “A replenishment model for the supply uncertainty problem,” *Int. J. Production Economics,* 87, 25-37

Ozekici, S. and Parlar, M., (1999), “Inventory models with unreliable suppliers in a random environment,” Annals of Operations Research, 91, 123-136

Parlar, M. and Perry, D., (1995), “Analysis of a (Q,r,T) Inventory Policy with Deterministic and Random Yields when Future Supply is Uncertain,” *European Journal of Operational Research* 84, 431-443.

Parlar, M. and Perry, D., (1996), “Inventory Models of Future Supply Uncertainty with Single and Multiple Suppliers,” *Naval Research Logistics* 43(2), 191-210.

Parlar, M., Wang, Y., and Gerchak, Y., (1995), “A periodic review inventory model with Markovian supply availability,” *International Journal of Production Economics*, 42, 131-136

Parlar, M., (1997), “Continuous-review inventory problem with random supply interruptions,” *European Journal of Operational Research* 99, 366-385

Qi, L., Shen, Z.M., and Snyder, L.V., (2006), “A continuous-review inventory model with disruptions at both supplier and retailer,”

Swaminathan, J.M. and Shanthikumar, J.G., (1999), “Supplier diversification: effect of discrete demand,” *Operations Research Letters* 24, 213-221.

Song, J. and Zipkin, P.H., (1996), “Inventory control with information about supply conditions,” *Management Science*, 42(10), 1409-1419

Tomlin, B., (2006), “On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks”, *Management Science*, 52(5), 639–657.

Tomlin, B. and Wang, Y., (2005), “On the value of mix flexibility and dual sourcing in unreliable newsvendor networks,” *Manufacturing and Service Operations Management*, 7(1), 37-57

Yang, J., Qi, X., and Xia, Y., (2005), “A production-inventory system with Markovian capacity and outsourcing option,” *Operations Research*, 53(2), 328-349

APPENDICES

APPENDIX A

Impact of backorder cost

In this section the impact of the backorder cost is studied in more detail. This is done to understand how the policy changes as the backorder cost is changed. For the experimental design of this section, the input parameters are c_u = 1, h = 0.2, l = 6, f = [0.1, 0.25, 0.5], b =2:6 in increments of 0.1, c_r= 1.2, α = 0:1 in increments of 0.01 and β = 1.

In Figure A.1, the optimal case is shown for combinations of α and backorder cost. Notice that for a given value of the backorder cost, we see how the optimal case goes from case 1 to case 4 as α is reduced. Notice that the threshold α's (point at which the color changes on the graph) vary as the backorder cost is increased forming a wave-like pattern.

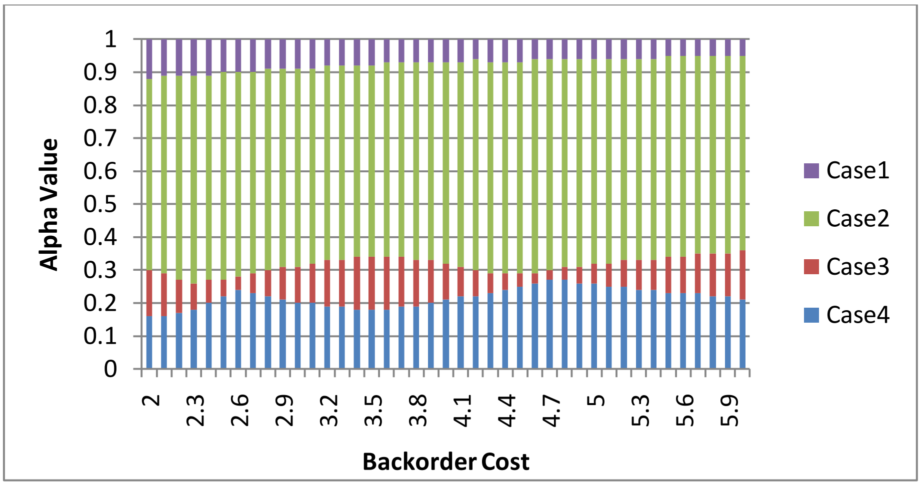


Figure A.1: The change pattern of the policy structure as backorder cost is changed for f = 0.1

The wave height is greatest between case 3 and case 4. Therefore, the policy parameters for case 3 and case 4 are analyzed. We observe from Figure A.2 that when a reorder level changes, a wave is formed. When the reorder point of case 3 changes, the wave curves up and when the reorder point of case 4 changes, the wave curves down. It is conjectured that the wave is caused by the fact that the policy parameters are constrained to be integer.

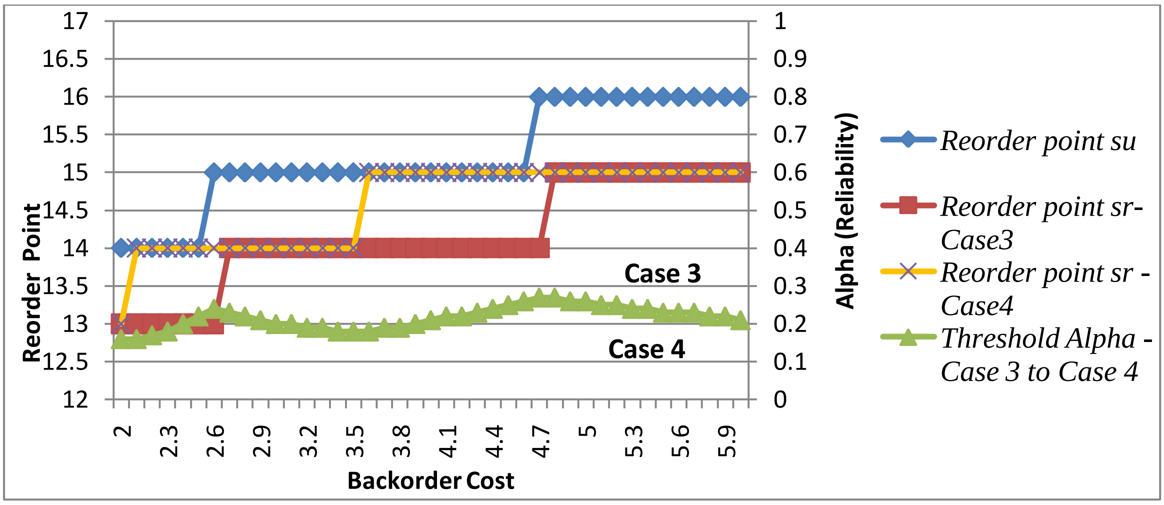


Figure A.2: Reorder point for threshold alpha between Case 3 and Case 4 (f =0.1)

Similar experimentation with fixed cost f = [0.25, 0.5] was carried but the results so the same effect. Details of these experiments are given below. Figure A.3 and A.4 represents wave pattern for f = 0.25 and 0.5. Figure A.5 and A.6 represent the reorder point change for f = 0.25 and 0.5.

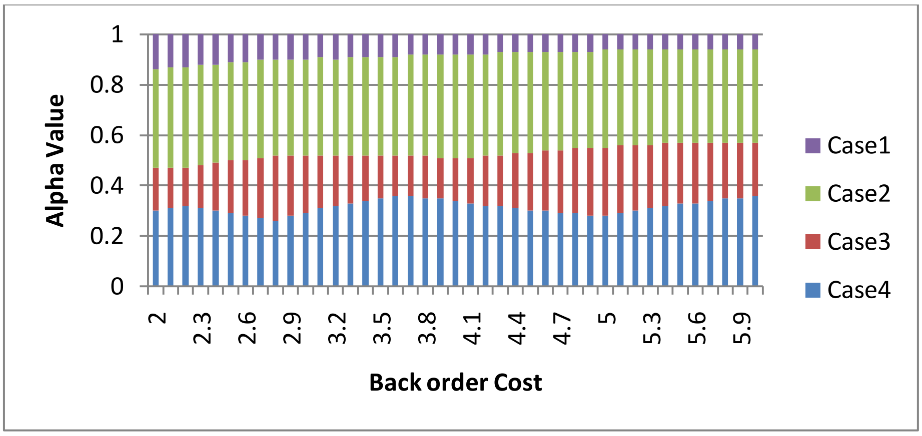


Figure A.3: The change pattern of the policy structure as backorder cost is changed for f = 0.25

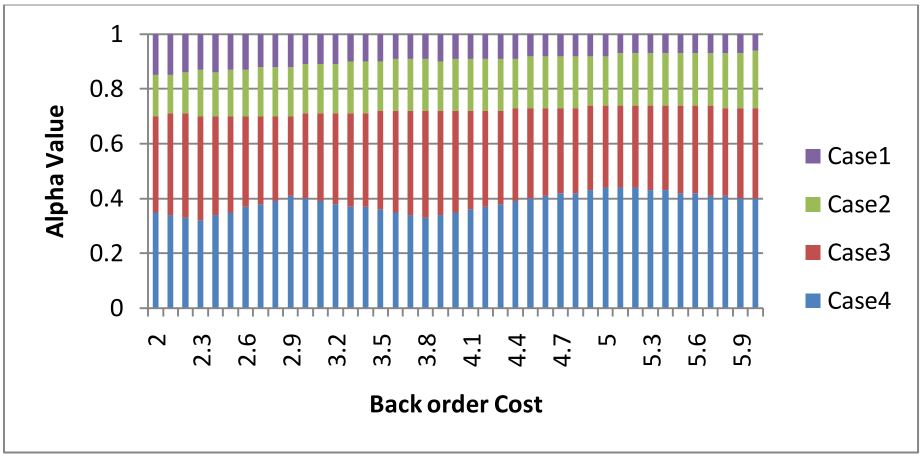


Figure A.4: The change pattern of the policy structure as backorder cost is changed for *f* = 0.5

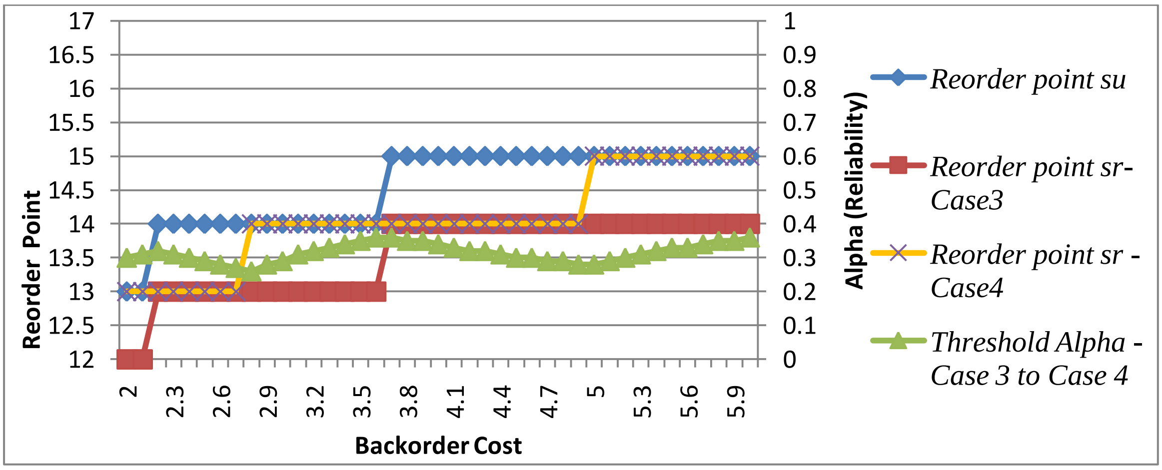


Figure A.5: Reorder point for threshold alpha between Case 3 and Case 4 (*f* =0.25)

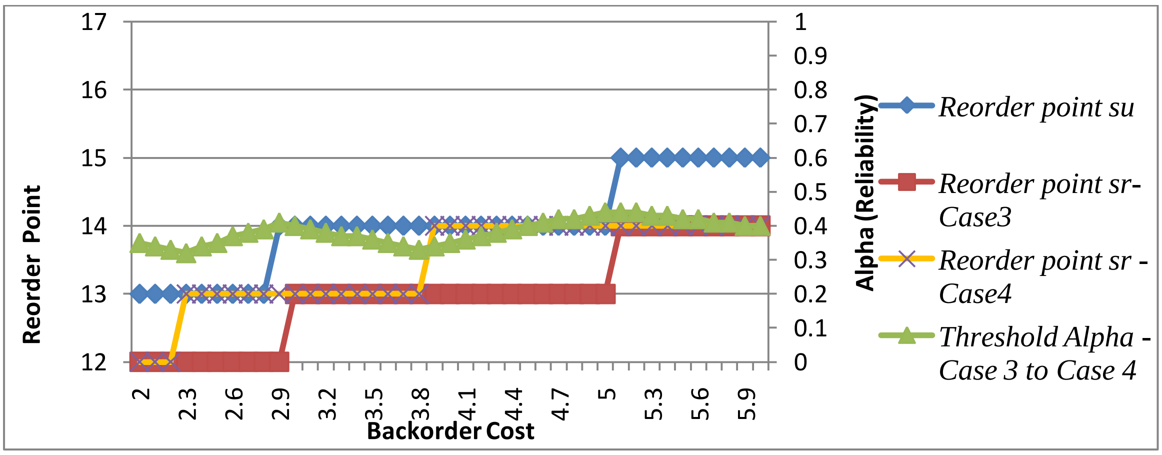


Figure A.6: Reorder point for threshold alpha between Case 3 and Case 4 (*f* =0.5)

APPENDIX B

Detailed results of the individual cost components for a modified case 1 and pure case 4 policies

In figure B.1 it is observed that the lost sales cost increases as α is reduced regardless of the fixed cost of ordering.

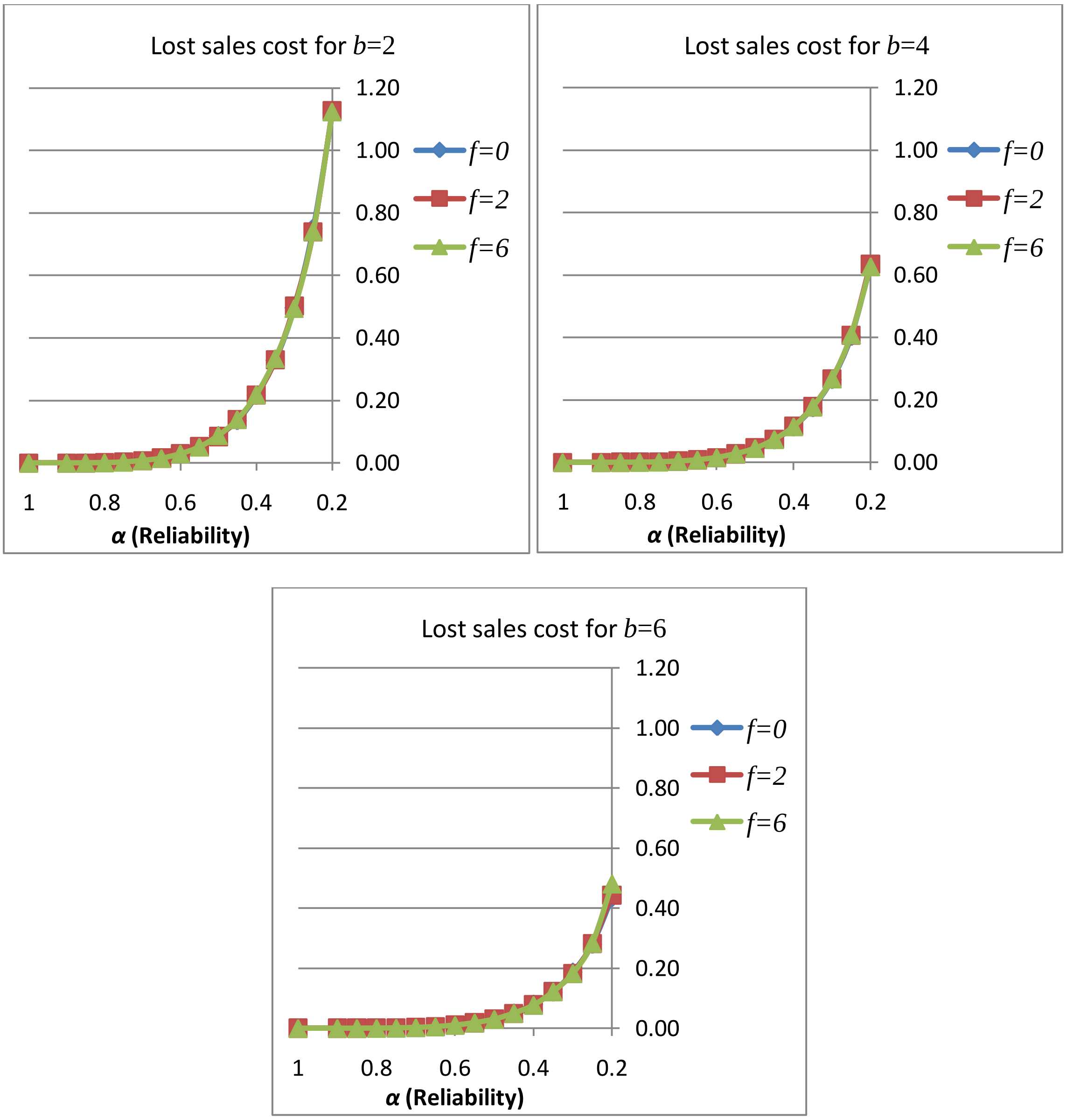


Figure B.1: Expected lost sales cost versus reliability for pure case 1 policy (c_r = 1.2)

In Figure B.2 it is observed that the expected backorder cost increases as α is reduced regardless of the fixed cost of ordering. While from Figure B.3, we observe that as the holding cost increases as the backorder cost increases. The system carries more inventory to avoid higher backorder cost.

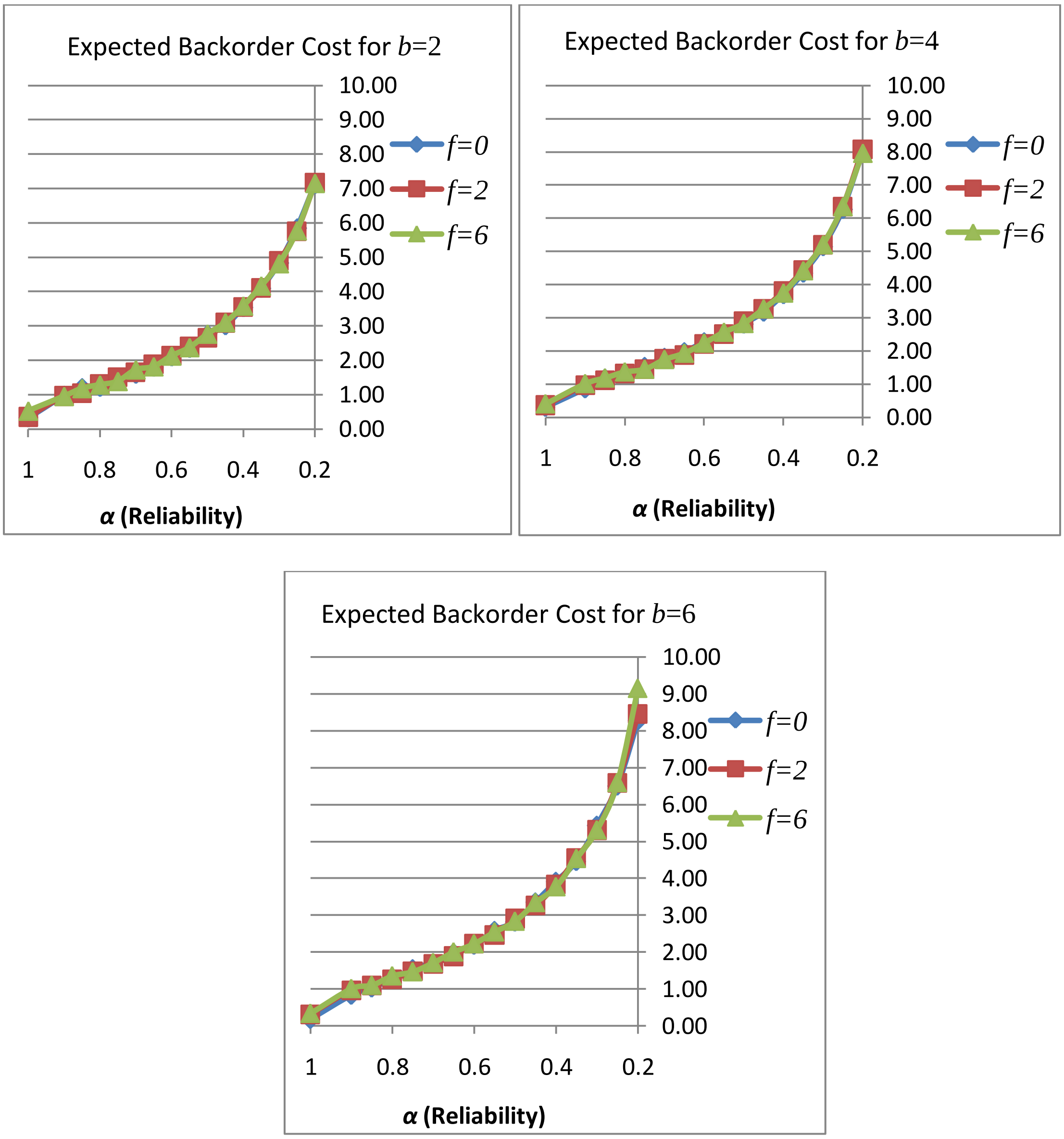


Figure B.2: Expected backorder cost versus reliability for pure case 1 policy (c_r = 1.2)

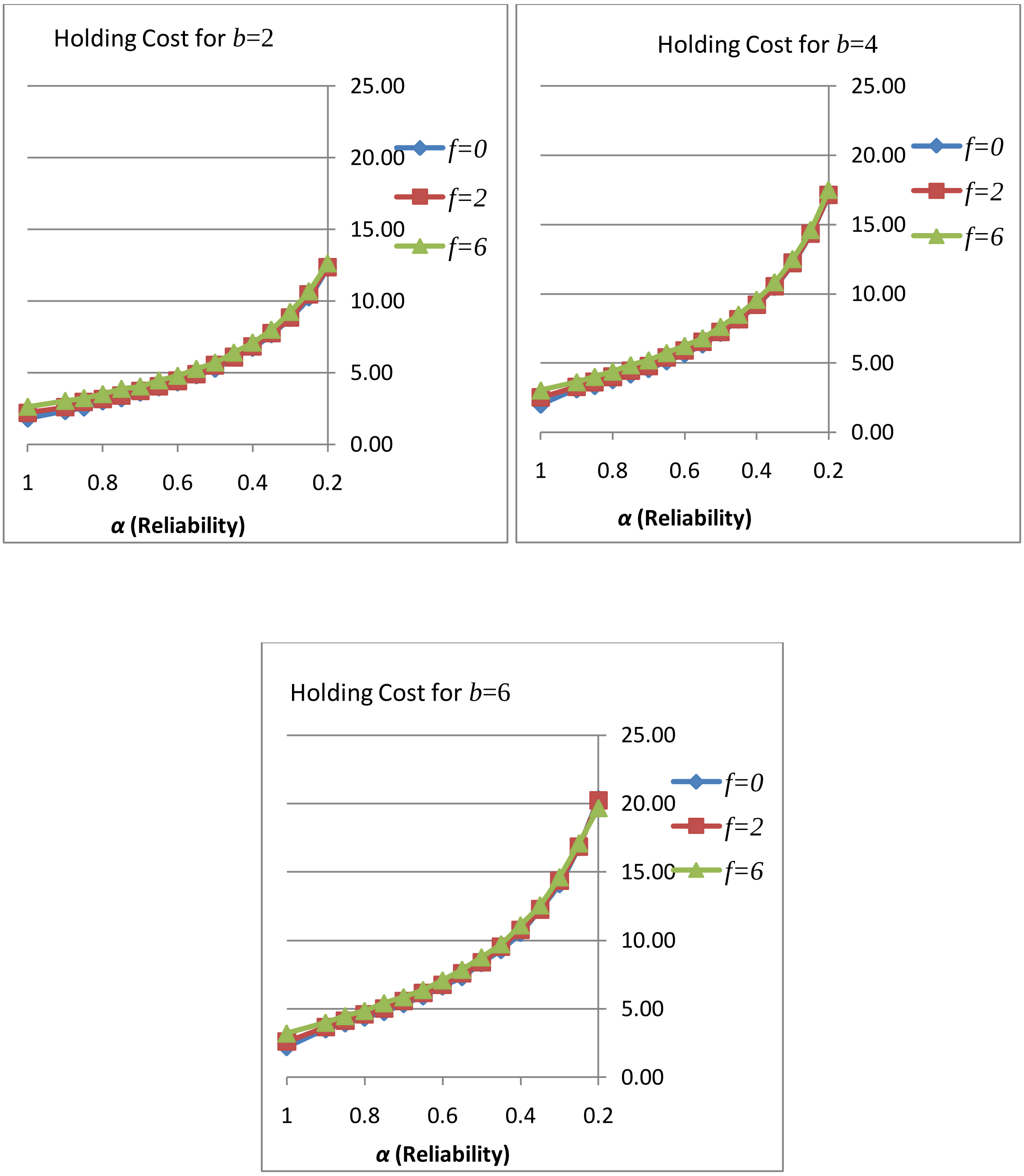


Figure B.3: Expected holding cost versus reliability for pure Case 1 policy (c_r = 1.2)

Order cost increases as the fixed cost increases (Figure B.4). It remains nearly the same as the backorder cost increases. For lower fixed cost (f = [0,2]) and for lower α levels, order cost is lesser than that for higher α levels. When the fixed cost is very high (f

= 6), the order cost also increases for lower α levels. This is due to the system continuing to order more to avoid backorder and lost sales cost.

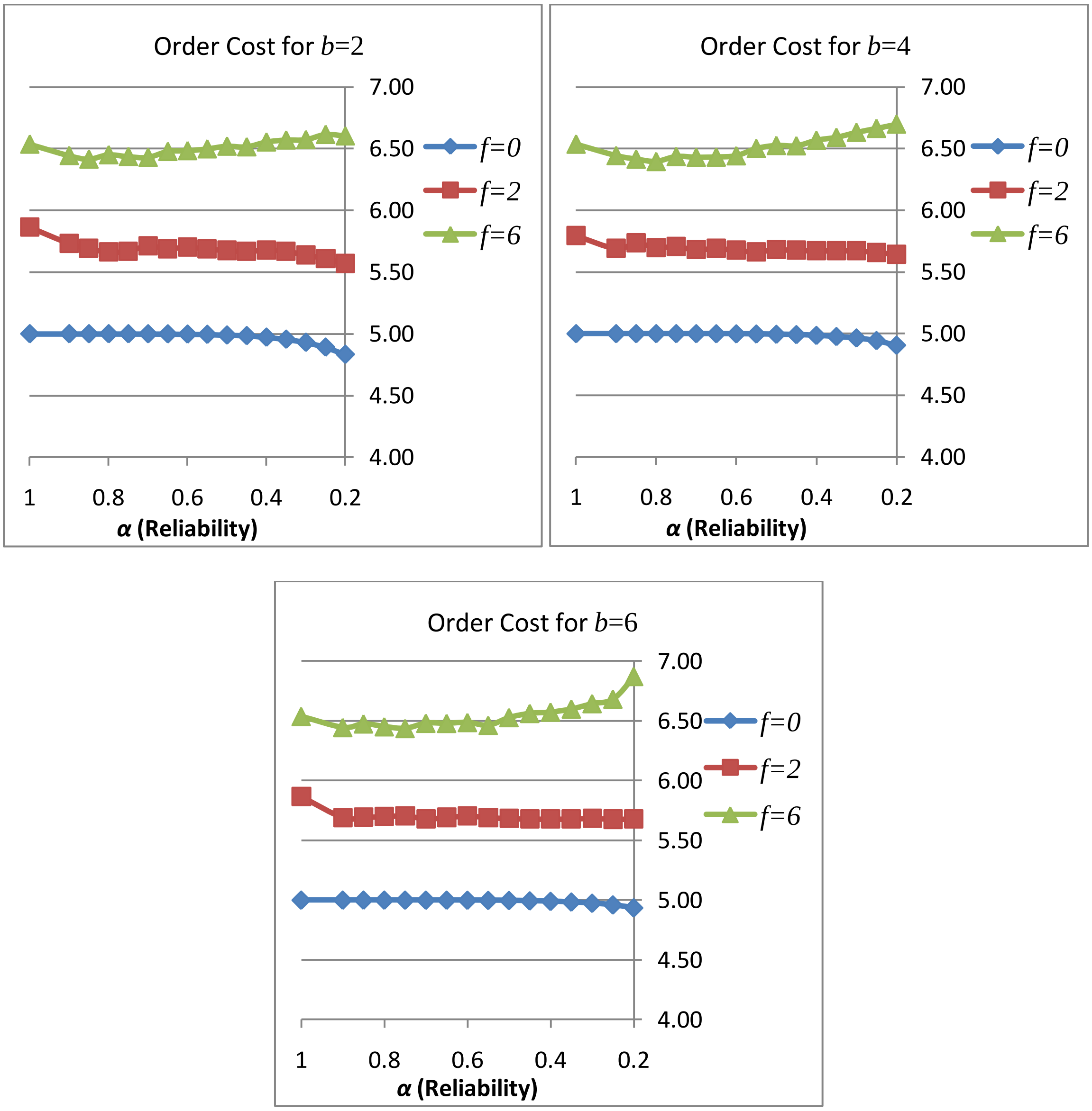


Figure B.4: Expected ordering cost versus reliability for pure Case 1 policy (c_r = 1.2)

Δcost for c_r=1.5 and c_r=2:

Figures B.5, B.7 and B.9 represent results for c_r=1.5 and figures B.6, B.8 and B.10 represent results for c_r=2.

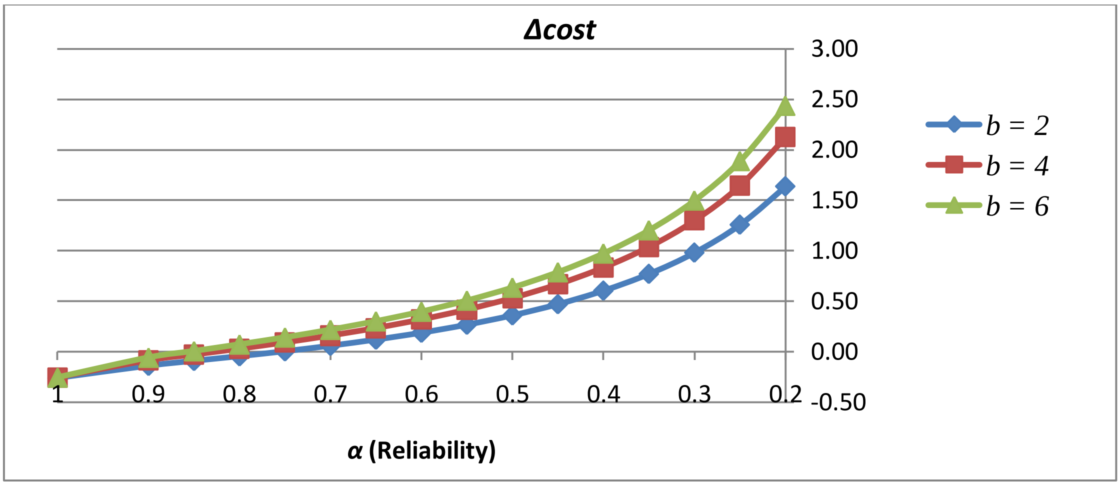


Figure B.5: Δcost versus supplier reliability (f=0, c_r=1.5)

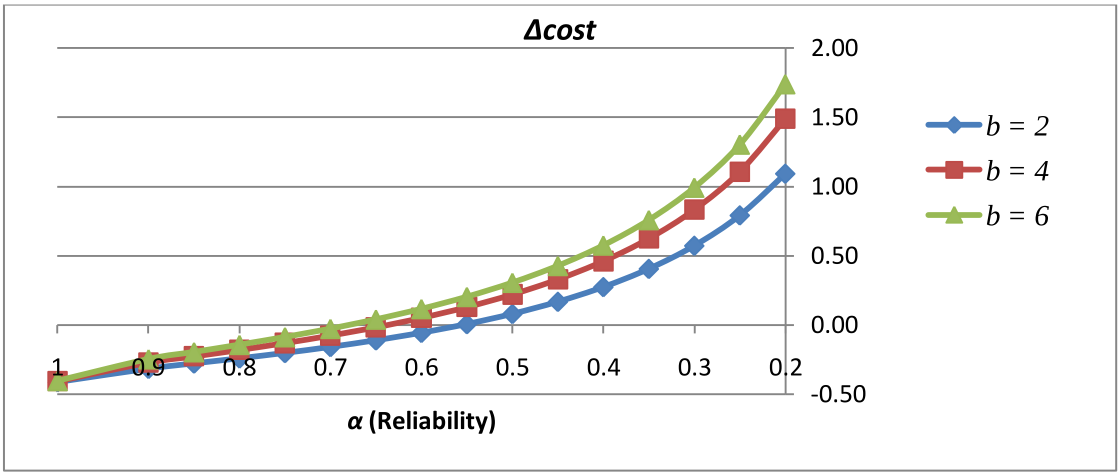


Figure B.6: Δcost versus supplier reliability (f=0, c_r=2)

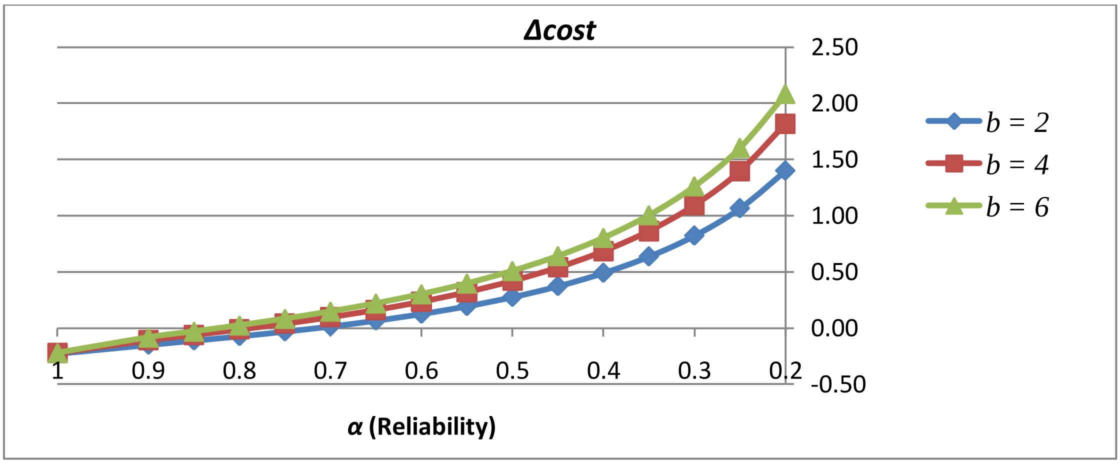


Figure B.7: Δcost versus supplier reliability (f=2, c_r=1.5)

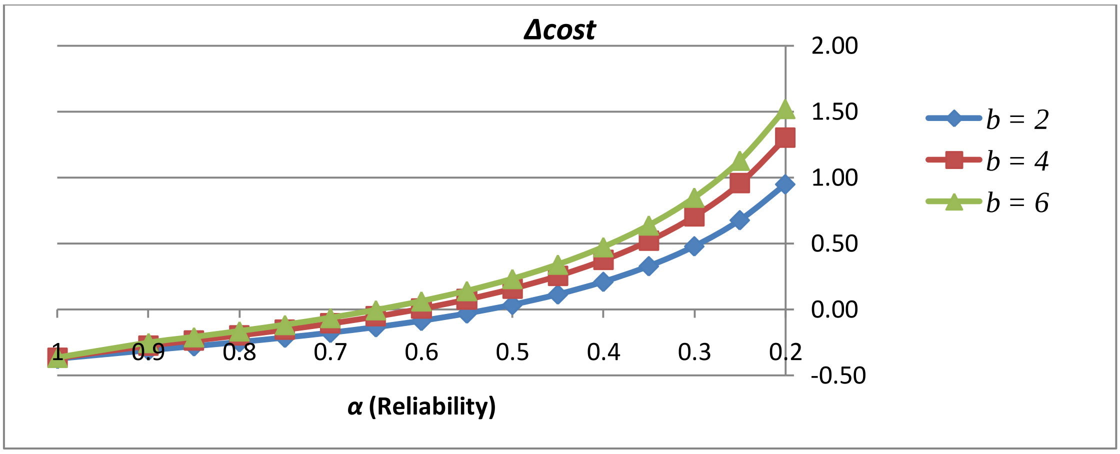


Figure B.8: Δcost versus supplier reliability (f=2, c_r=2)

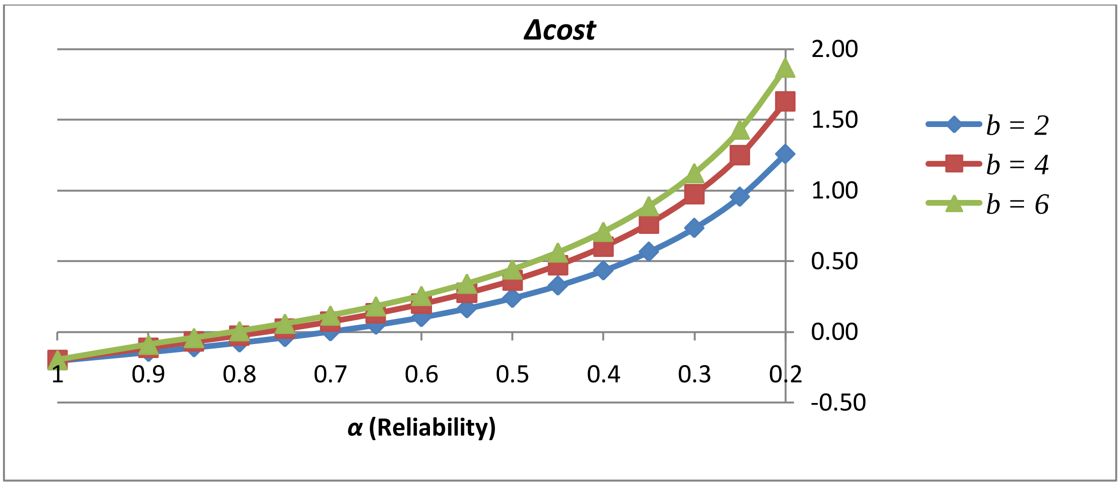


Figure B.9: Δcost versus supplier reliability (f=6, c_r=1.5)

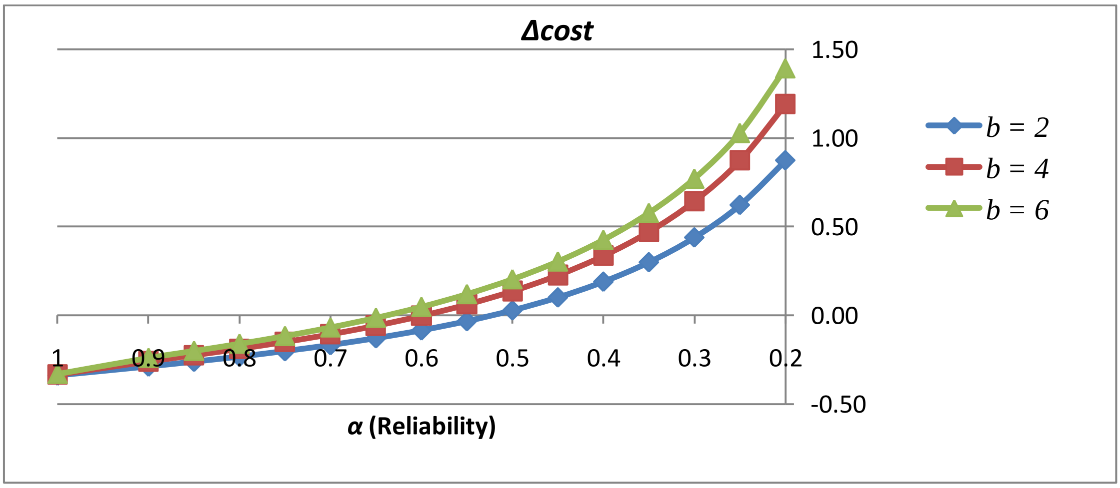


Figure B.10: Δcost versus supplier reliability (f=6, c_r=2)

APPENDIX C

Safety stock analysis:

In this section safety stock is analyzed for different backorder cost, fixed cost and cost of reliable supplier. The results from the experiments in section 5.2 are referred to in this section. For the experimental design of this section, the input parameters are $c_u = 1$, $h = 0.2$, $l = 6$, $f = [2, 6]$, $b = [2, 4, 6]$, $c_r = [1.2, 1.5, 2]$, $\alpha = 0.75$ and $\beta = 1$.

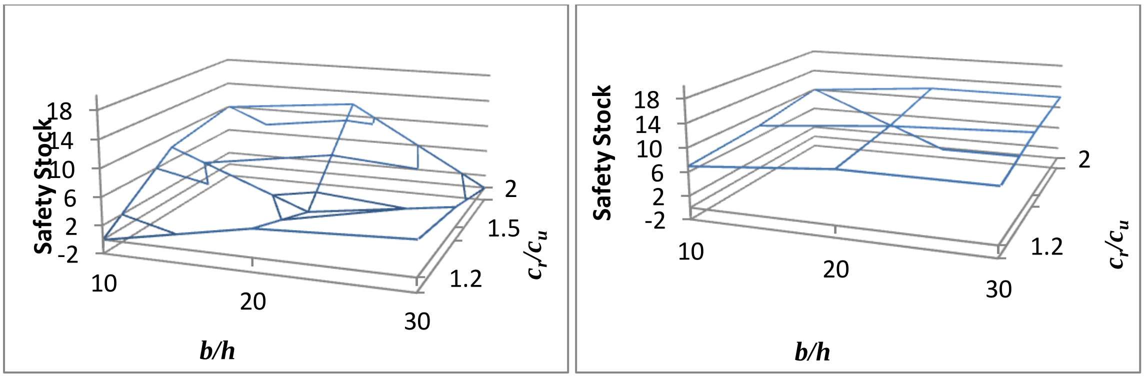
Safety Stock is calculated as (Reorder point – Expected Demand * leadtime)

For evaluating safety stock an upper and lower bound is considered. This is due to case 2 and case 3 having 2 reorder points.

Lower Bound: The general trend is that the safety stock increases as *b/h* ratio increases and case 1 has the highest safety stock and case 2 and case 3 the lowest.

Upper Bound: The general trend is that the safety stock increases as *b/h* ratio increases. Here case 2 and case 3 safety stock level is as high as case 1 safety stock.

From Figure C.1 and Table C.1 below we can observe that in lower bound when the policy changes to case 1 the safety stock is high. In the upper bound in these scenarios, case 2 and case 3 safety stock also increases to meet the safety stock level of case 1. Therefore we see that the safety stock is not only dependent on *b/h* ratio and c_r/c_u ratio but also case dependent. Consider the case when alpha = 0.75 and $f = 2$.



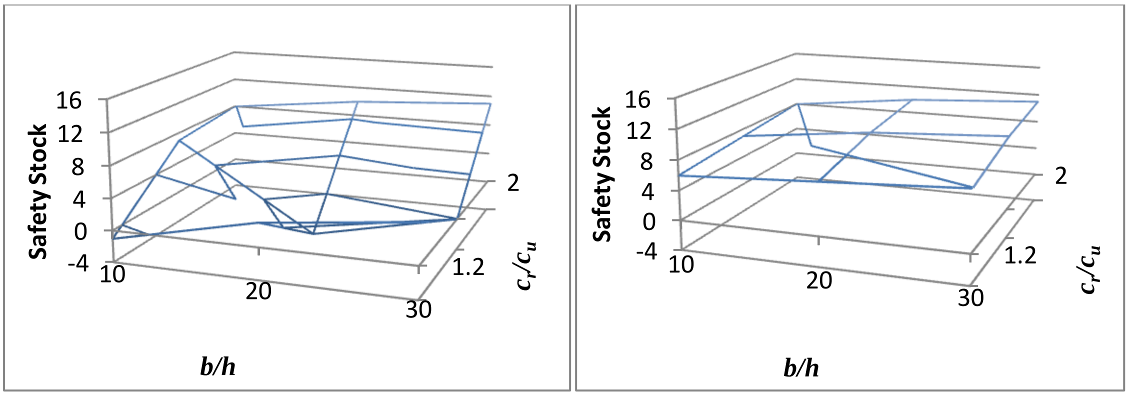
(a)Lower bound (b)Upper Bound

Figure C.1: Safety stock (f = 2)

Table C.1: Safety stock for α = 0.75, f = 2

c_r/c_u	b/h	Case	Safety Stock (Lower Bound)	Safety Stock (Upper Bound)
1.2	10	3	0	7
1.2	20	3	4	9
1.2	30	3	5	9
1.5	10	1	8	8
1.5	20	2	0	10
1.5	30	2	3	11
2	10	1	10	10
2	20	1	12	12
2	30	2	0	12

In Figure C.2(a) (Lower bound) the surface is uneven because of low safety stock levels when case 2 or case 3 is optimal whereas in Figure C.2(b) (upper bound) the surface is flattened because of high safety stock level for case 2 and case 3.



(a) Lower bound (b) Upper Bound

Figure C.2: Safety stock (f = 6)

Table C.2: Safety stock for α = 0.75, f = 6

c_r/c_u	b/h	Case	Safety Stock (Lower Bound)	Safety Stock (Upper Bound)
1.2	10	3	-1	6
1.2	20	3	3	7
1.2	30	3	5	8
1.5	10	1	7	7
1.5	20	3	-4	9
1.5	30	3	0	10
2	10	1	8	8
2	20	1	10	10
2	30	1	11	11

From Figure C.2 and Table C.2 above we see that when case 3 is best the lower bound safety stock is very low and upper bound safety stock is as high as case 1 safety stock level.

From the above analysis we observe that the expected safety stock for a case 2 and case 3 has a wide range between its upper and lower bounds. And that for case 1 it is always high, due to higher unreliability of the supplier. However in case 2 and case 3, orders are placed with the reliable supplier therefore one need not always carry as high safety stock as case 1. Also when the fixed cost is increased safety stock decreases. By this the system allows to order less frequently to avoid the high fixed cost.