A Modified Energy-Based Method for Stress-Strain Analysis at Blunt Notches

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INTRODUCTION

Components operating at high temperature in power plants are exposed to cyclic loading conditions during service transients. Severe thermal gradients over thick sections cause plastic strains to develop at stress concentration regions, typically notch roots. Here crack initiation and subsequent propagation are primarily controlled by the local strain and stress fields.
The evaluation of the fatigue life of the component relies on the capability of correctly calculating strains and stresses in the plastic regions developed at notch roots. Simplified methods have been proposed, after the first well known approach of Neuber, to face the problem improving the accuracy of the results. The availability of such methods is valuable both for designers and users, who can avoid costly and time consuming numerical analyses, still obtaining reliable results.
In the present paper four methods from the literature will be therefore considered, namely the widely used Neuber approach and those proposed by Molski-Glinka, Polak, Kujawski-Ellyin. An evaluation and a comparison of their responses are readily obtained through a common energy-based interpretation. Still relying on the energy-based approach, an improvement is proposed, which offers the possibility of an accurate stress-strain analysis at notch roots, also under conditions of large scale yielding.

OVERVIEW AND DISCUSSION OF CURRENT METHODS

In this section the methods of Neuber, Molski-Glinka, Polak and Kujawski-Ellyin are outlined, showing their common features and how they have evolved from one into another.
The Neuber approach [Neuber,1961] states the relationship between the elastic stress concentration factor K_t and the product of the stress concentration factor, $K_\sigma=\sigma/S_n$, and the strain concentration factor, $K_\varepsilon=\varepsilon E/S_n$, S_n being the nominal stress, σ and ε the effective stress and strain at the notch root and E the material elastic modulus:

$$K_t^2 = K_\varepsilon K_\sigma \qquad (1)$$

This is easily rearranged into:

$$K_t^2 S_n^2/(2E) = S_e^2/(2E) = (1/2)\sigma\varepsilon \qquad (2)$$

$S_e=K_tS_n$ being the elastically calculated stress at the notch root.
Knowledge of K_t and S_n provides the value of the elastic stress S_e; the effective stress and strain at the notch root must be sought on the material tensile curve, satisfying Eqns.(2).
The Molski-Glinka method [Molski,Glinka,1981] relies on an energy-based interpretation of the terms in Eqns.(2). $S_e^2/(2E)$ is the elastic strain energy density due to the stress S_e, or, equivalently, K_t^2 times the nominal elastic strain energy density W_n due to the nominal stress S_n. $(1/2)\sigma\varepsilon$ may be regarded as an estimate of the elastic-plastic strain energy density W at the notch root. Hence:

$$K_t^2W_n = W \tag{3}$$

The assumption that the strain energy density W is governed, through the elastic factor K_t, by the nominal stress S_n may be held under small scale yielding conditions, where a large elastically strained volume surrounds the plastic zone developed at the notch root [Glinka,1985]. If the material tensile stress-strain behavior follows a Ramberg-Osgood law, $\varepsilon=\sigma/E+(\sigma/k)^{1/n}$, the correct formulation for W, obtained by integration along the σ-ε curve, will be:

$$W = \sigma^2/(2E) + [\sigma/(n+1)](\sigma/k)^{1/n} \tag{4}$$

A generalization of the energy-based approach has been proposed [Polak,1983], applicable also to large scale yielding conditions. The nominal strain energy density W_n is evaluated in the same way as the effective strain energy density W. If the Ramberg-Osgood model describes the material elastic-plastic behavior, then, integrating along the S_n-ε curve:

$$W_n = S_n^2/(2E) + [S_n/(n+1)](S_n/k)^{1/n} \tag{5}$$

The general formulation for the nominal strain energy density W_n, Eqn.(5), can be written so that it applies to each of the three methods:

$$W_n = S_n^2/(2E) + F\{[S_n/(n+1)](S_n/k)^{1/n}\} \tag{6}$$

where F is a factor, dependent on the method, affecting the plastic component of the total energy.
Still relying on an energy-based approach, a method has been recently proposed [Kujawski,Ellyin,1985], based on the application of the J-integral concept of Fracture Mechanics. The constancy of J along any path surrounding the notch tip leads to a formulation which can again be represented by Eqn.(6) with $F=1/K_t^2$.
In Fig.1 the typical response curves of the described methods are drawn in terms of the elastic notch stress S_e vs.the notch strain ε and Fig.2 (a to e) shows, for different situations, the response curves compared to numerical and experimental results. Molski-Glinka and Polak methods can be seen as two opposite ways of applying the energy approach: the first, by considering only the elastic contribution to the nominal strain energy density, F=0, should be used under small scale yielding conditions. The latter, by taking into account the whole plastic contribution, F=1, is intended to apply also to large scale yielding, where S_n approaches or even exceedes the yield value. The relative position of the response curves Se vs.ε follows immediately, i.e.

the Molski-Glinka curve overrides the Polak curve for every S_e beyond the yield value.
Application of the two methods has shown that the capability of approximating numerical and experimental results may be strongly dependent on the geometry and loading conditions [Glinka,1985]: under prevailing bending loading, as in the CT specimen geometry, where a higher constraint limits the plastic zone development, the Molski-Glinka method, which, with F=0, predicts lower strains, is more appropriate, whereas use of the Polak method may lead to largely overpredicted strains. Under prevailing tensile loading, where the plastic zone can more easily spread over the whole ligament, the Polak approach gives better approximations.
In the Neuber method, where again F=0, the approximation in the evaluation of the elastic-plastic energy term W (set equal to $(1/2)\sigma\varepsilon$) results in a curve lying below the Molski-Glinka curve in the whole range, and below the Polak curve in the small scale yielding regime, leading here to an overestimation of the numerically calculated strains. It should be noted that the Neuber curve overrides the Polak curve at the point where:

$$\sigma/S_e = (1/K_t)[2K_t^2/(1-n)]^{n/(n+1)} \quad (7a)$$

The Kujawski-Ellyin response curve lies in between Molski-Glinka and Polak curves for every $K_t>1$, being coincident with the latter for $K_t=1$ and approaching the former as K_t is increased. Further the Kujawski-Ellyin and Neuber response curves intersect each other at the point:

$$\sigma/S_e=(1/K_t)[2/(1-n)]^{n/(n+1)} \quad (7b)$$

Finally it is to be noted that the position of Molski-Glinka and Neuber curves in the S_e-ε, for a given material stress-strain curve, is independent of K_t, whereas the Polak and Kujawski-Ellyin curves are dependent on it.

THE PROPOSED METHOD

The analysis of the response curves of the methods here condidered and the evaluation of their capability of modeling FEM and experimental results (Fig.2) allow the conclusion that the energy-based approach can successfully be used to model actual situations, provided that the correct value is given to the factor F in Eqn.(6). As already stated, the modeling capability of the single approach varies in dependence of geometry, loading conditions, K_t values.
An alternative way of rationalizing the responses of the methods, allowing a more immediate evaluation of the effect of different expressions of the factor F, is to look directly at the calculated notch strains, i.e. by considering how the combination of S_n and K_t is translated into a notch tip strain.
Eqn.(3), with W and W_n given by Eqns.(4) (or (2), in case of the Neuber method) and (6), can be rearranged into:

$$\varepsilon_e^2 + a\varepsilon_e\varepsilon_p = (K_tS_n/E)^2 + 2F/((n+1)Ek^{1/n})\ S^{1/n+1} \quad (8)$$

where $\varepsilon_e=\sigma/E$ and $\varepsilon_p=(\sigma/k)^{1/n}$ are the elastic and plastic components of the notch strain ε, a and F are dependent on the method (see Table 1, rows 1 to 4).
A "notch equivalent strain" ε_q can be defined:

$$\varepsilon_q=\sqrt{(\varepsilon_e^2+a\varepsilon_e\varepsilon_p)} \quad (9)$$

which is directly related to the effective notch strain ε (which is in turn evaluated on the specific σ-ε curve).
Use of ε_q allows a more direct and immediate evaluation of the effect of any specific expression of F on the position of the response curve in the S_n-ε_q plane and hence in the S_e-ε plane.
Overview of Fig.2 indicates the need for a correction which allows the response curve to lie closer to the numerical and experimental results for the different situations which are characterized by different extents of the plastic flow at the notch tip.
A factor F is here proposed with the form:

$$F = [(n+1)/2](\sigma_y/S_n)^{K_t} \tag{10}$$

introducing a dependence on the ratio σ_y/S_n , σ_y being a suitable value representing the deviation from elastic behavior (σ_y has been assumed, for the Ramberg-Osgood curve, as the value where the plastic strain is 1/100 of the elastic strain; for a piecewise approximation of the stress-strain curve, σ_y is the last value of the elastic region). The correction allows the elimination of the divergencies encountered in the application of the Polak method, well reproducing the actual S_e-ε response, Figs.2a-b-c. Application of the method yields good results also in cases where the Polak approach was demonstrated to be fully applicable, Figs.2d-e.

CONCLUSIONS

Close examination of simplified methods for elastic-plastic notch strain evaluation confirms the adequacy of the energy-based approach, i.e. the notch tip strain energy density is directly related to the nominal strain energy density. The elastic stress concentration factor, which is readily obtainable for each specific geometry, should be used to relate the two energies strictly only under prevailing elastic conditions. Extension to large scale plasticity situations can still give satisfactory results: the degree of approximation is however dependent on geometry and loading conditions, e.g. tension vs. bending, and large overestimations of the notch strain may result.
A rationalization of the relative behavior of the response curves of different methods from the literature has been given. A modification to the energy-based approach, obtained by correcting the contribution of the plastic nominal strain energy density, makes the method capable of modeling the different situations (geometry/loading),also for large scale yielding conditions.

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Table 1 Values of the coefficients a and F in Eqn.(8) for different methods.

Method	a	F
Neuber	1	0
Molski-Glinka	2/(n+1)	0
Polak	"	K_t^2
Kujawski-Ellyin	"	1
Proposed	"	$[(n+1)/2](\sigma_y/S_n)^{K_t}$

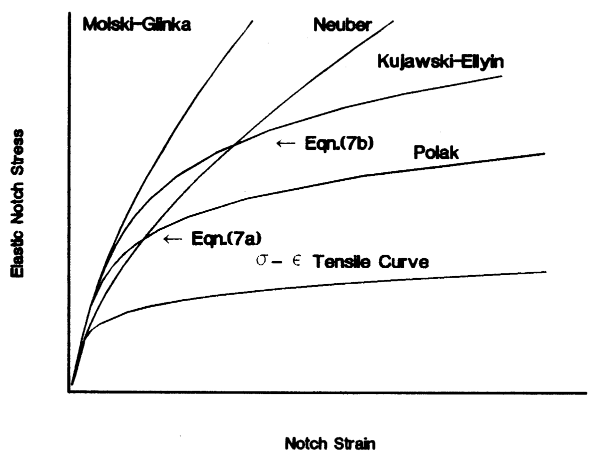
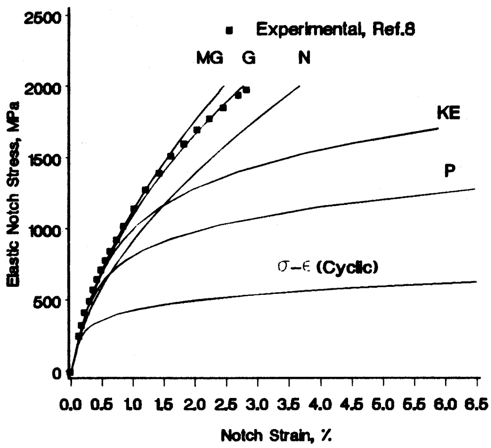
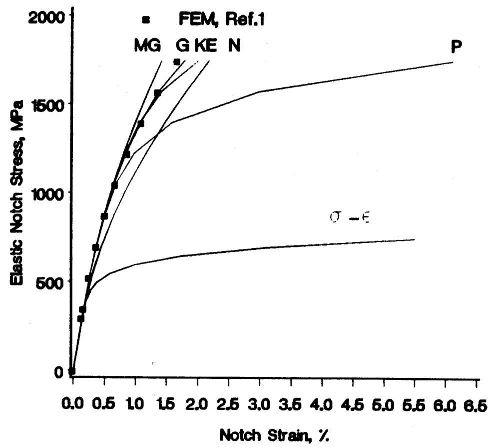


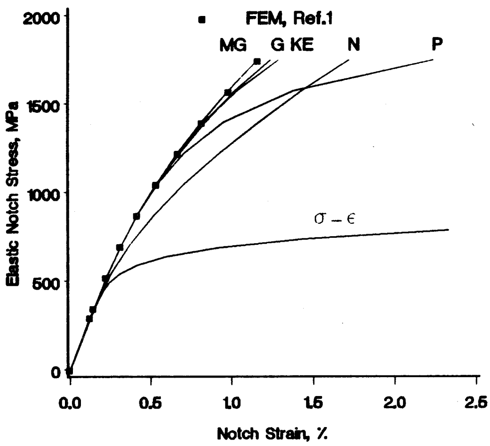
Fig.1 Typical response curves of simplified methods



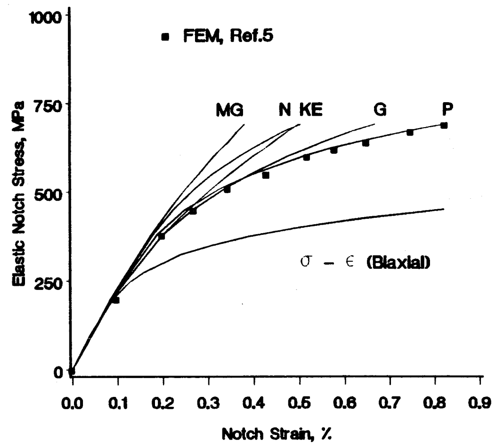
a) Keyhole CT Specimen - Kt=2.88 - Plane Stress



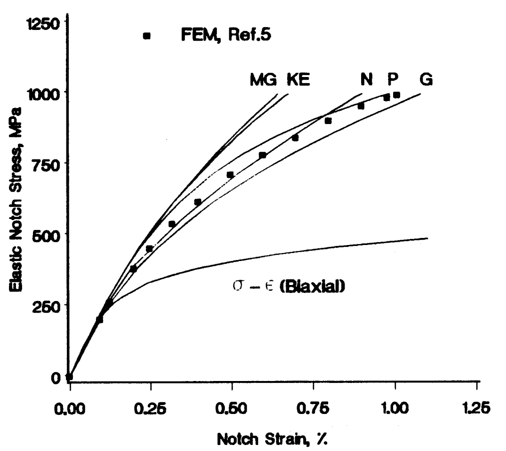
b) Bluntly Notched CT Specimen - Kt=2.93 - Plane Stress



c) Bluntly Notched CT Specimen - Kt=2.93 - Plane Strain



d) Notched Round Bar (Tension) - Kt=1.97 - Plane Strain



e) Notched Round Bar (Tension) - Kt=3.46 - Plane Strain

Fig.2 Response curves of simplified methods compared to experimental and numerical (FEM) results for different geometries

G : Proposed Method

KE: Kujawski-Ellyin

MG: Molski-Glinka

N : Neuber

P : Polak