SMiRT-12 / K. Kussmaul (Editor)
1993 Elsevier Science Publishers B.V.

F04/1

EFFECTIVE ELASTIC METHOD FOR PERFORATED PLATES WITH IRREGULAR PENETRATION PATTERNS

M. Imaizumi[1], H. Kobayashi[1], Y. Koyama[1], H. Nuno[1], S. Kuga[2] and Y. Matsushima[2]

[1]Nuclear Development Corporation, 622-12 Funaishikawa, Tokaimura, Ibaraki 319-11, Japan
[2]Mitsubishi Atomic Power Industries Inc., Shiba Park Building 4-1, Shibakoen 2-chome, Minatoku, Tokyo 105, Japan

Abstract

There are some established methods for evaluating the design of perforated plates with equilateral penetration patterns, one of which is shown in ASME SecIII. But there is no established method for perforated plates with irregular penetration patterns. This paper proposes a new analytical method for irregular hole layouts which are often seen in reactor core internals etc.

The procedures of this method are as follows:

① The minimum repeating hole pattern (unit hole pattern) is taken out of the perforated plate to make a FEM model, from which the effective elastic constants (E^*, ν^*, G^*) are obtained.

② The above model is also used to analyze the detailed stresses in the unit hole pattern by loading it with 6 components of nominal stress (unit intensity) one at a time. From the results of the above 6 calculations, "stress multiplier matrices" [K] (6×6) are obtained for all the FEM elements in the unit hole pattern.

③ Then the nominal stresses $\{\sigma\}_{nom}$ are calculated for the total structural model composed of an equivalent solid material using E^*, G^*, ν^* obtained in ①.

④ The real stresses $\{\sigma\}_{real}$ of the perforated plate are obtained by multiplying the nominal stresses by the stress multiplier matrix obtained in ③ as,

$$\{\sigma\}_{real}=[K]\cdot\{\sigma\}_{nom}$$

Through the above procedures, the real stresses in all the FEM elements of the perforated plate are determined.

In order to perform the design calculations of perforated plates with an irregular hole layout, we made a new analytical system consisting of a series of computer programs, which handles all the calculations required for the design of perforated plates in accordance with the ASME SecIII code.

A scale model of a reactor core internal structure with an irregular penetration pattern was tested to verify the proposed evaluation method. The test results showed good agreement with the calculation results for the scale model, both for deformation and stress distributions.

1. Introducton

In the mechanical design of structures made of perforated plates, the effective elastic constants method is often used, which treats the perforated region as an equivalent solid material whose elastic constants are modified considering the softening effect of the penetrations. The sequence of procedures in this method is : ① to determine the effective elastic constants E^* & ν^*, ② to analyze the nominal stresses of the stuctures by using E^* & ν^*, and ③ to convert the nominal stresses into real stresses. The present status of this design technique seems to be as follows.

For perforated plates with triangular penetration patterns, the techniques for steps ① and ③ are not so complicated, since the elastic constants E^* & ν^* are isotropic; and the detailed design methods for these procedures have been established in ASME Sec III Article A-8000. Also for step ②, the conventional FEM analysis is applicable.

For perforated plates with square penetration patterns, things are a little complicated because of the orthotropic nature of E* & ν*. However, for procedures ① and ③ the necessary design knowledge is provided in references [1], [2], [3] etc. The orthotropy problem is concerned mainly with procedure ②, but this has been solved by introducing G* as a new effective elastic constant [4]. So designers can handle the problems with their own ingenuity.

Next, for perforated plates with irregular penetration patterns, step ① can be treated by FEM analysis on a minimum section consisting of the hole repeating pattern (denoted by unit hole pattern hereafter); and for step ② almost the same method can be applied as for the square penetration pattern. But for step ③, there is at present no established method.

The present study is mainly concerned with procedure ③ for irregular penetration patterns. When using the method developed here, the nominal stresses are converted into real stresses by simply multiplying the nominal stresses (6 components) by a matrix (6×6) which is composed of a group of stress multipliers. So the principle of the analysis is very simple, yet rigorous. As all the procedures from ① through ③ can be treated by computer programs, we made a total calculational system which follows all the procedural steps described above.

2. Analytical method

2.1 Total Procedure

As stated in the previous section, an analytical method was developed for the design evaluation of a perforated plate with an irregular hole layout. The sequence of the procedures in this method is as follows:

① Determination of the effective elastic constants
② Equivalent solid plate analysis of the total structure
③ Determination of the stress multiplier matrices
④ Transformation of nominal stresses into real stresses

The details of the above processes will be described in the following sections.

2.2 Determination of Effective Elastic Constants

To analyze the total structure as an equivalent solid plate, the effective elastic constants have to be determined. They are derived by requiring the equivalent solid material to have the same gross deformation as the perforated plate under the same loading and boundary conditions. In other words, the gross deformation is expressed by nominal strains and the loading by nominal stresses.

The nominal stress vs. nominal strain relation for a perforated plate can be expressed by simulating that for a general 3-dimensional anisotropic solid as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}_{nom} = \begin{bmatrix} 1/E_x^* & -\nu_{xy}^*/E_y^* & -\nu_{xz}^*/E_z^* & 0 & 0 & 0 \\ -\nu_{yx}^*/E_x^* & 1/E_y^* & -\nu_{yz}^*/E_z^* & 0 & 0 & 0 \\ -\nu_{zx}^*/E_x^* & -\nu_{zy}^*/E_y^* & 1/E_z^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{xy}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{yz}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{zx}^* \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}_{nom} \qquad (1)$$

This equation contains 12 effective elastic constants, but as is well known they can be reduced to 9 by using the following general relationship for elastic bodies.

$$\left.\begin{aligned} \nu_{yx}^*/E_x^* &= \nu_{xy}^*/E_y^* \\ \nu_{zy}^*/E_y^* &= \nu_{yz}^*/E_z^* \\ \nu_{zx}^*/E_x^* &= \nu_{xz}^*/E_z^* \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

Further, for perforated plates with no change in diameter of the holes with depth, the next relation applies.

$$\nu_{yz}^* = \nu_{xz}^* = \nu \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3)$$

where ν : Poisson's ratio for the unperforated solid material

By using eq.s (2) and (3) the independent effective elastic constants are reduced to the following:

$$E_x^*,\ E_y^*,\ E_z^*,\ \nu_{xy}^*,\ G_{xy}^*,\ G_{yz}^*,\ G_{zx}^* \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (4)$$

These constants are calculated using the models of a unit hole pattern as shown in

Fig.1. In the same calculation, the stress multiplier matrices [K] are also determined. These matrices will be discussed in sec 2.4. The procedures to determine the effective elastic constants from the above calculational models are given below.

1) E_x^*, E_y^*, ν_{xy}^*

From cal. on (a) of Fig.1 $\quad E_x^* = \sigma_x/\varepsilon_{x-nom} = \sigma_x/(\Delta l_x/l_x)$

From cal. on (b) $\quad E_y^* = \sigma_y/\varepsilon_{y-nom} = \sigma_y/(\Delta l_y/l_y)$

$\nu_{xy}^* = -\varepsilon_{x-nom}/\varepsilon_{y-nom} = -(\Delta l_x/l_x)/(\Delta l_y/l_y)$

2) E_z^* $\quad E_z^* = E \cdot A_{solid}/A_{nom}$

3) G_{xy}^* From cal. on (c) $\quad G_{xy}^* = \tau_{xy-nom}/\gamma_{xy-nom} = \sigma_{nom}/(\varepsilon_{x'-nom} - \varepsilon_{y'-nom}) = \sigma_{z-nom}/(\Delta l_{x'}/l_{x'} - \Delta l_{y'}/l_{y'})$

4) G_{yz}^* From cal. on (e) $\quad G_{yz}^* = \tau_{yz-nom}/\gamma_{yz-nom} = \tau_{yz-nom}/(u_z/l_y)$

5) G_{zx}^* From cal. on (f) $\quad G_{zx}^* = \tau_{zx-nom}/\gamma_{zx-nom} = \tau_{zx-nom}/(u_z/l_x)$ $\quad$ (5)

where $l_x, l_y, l_z, l_x', l_y', l_z'$: Dimensions of unit hole pattern model
- E : Young's modulus for unperforated solid material
- A_{solid} : Area of solid portion
- A_{nom} : Apparent contour square area
- σ_{nom} : Nominal stress to be applied to the model (c) to make the pure shear field
- u_z : Z-directional deflection of loading point

For the calculations shown in Figs. (e) and (f), care should be taken that the nominal stress forms a pure shear field without accompanying bending stresses and deflections. In order to achieve this condition, the size of l_z is intentionally made as follows :

$$l_z \gg l_x \text{ or } l_y \quad \cdots\cdots\cdots\cdots\cdots \quad (6)$$

if the original dimensions do not satisfy this condition.

2.3 Equivalent Solid Plate Analysis of the Total Structure

The equivalent solid plate analysis of the total structure is performed using the effective elastic constants obtained in sec 2.2. In the case of analysis by FEM, a computer code which can deal with the general anisotropic elastic properties should be chosen.

By performing the above analysis, the nominal stresses in the total structure are obtained. The total structure generally consists of a pure solid plate region and an equivalent solid plate region. The stresses in the solid region are real stresses, and the nominal stresses in the equivalent solid region must be converted into real stresses by using the method which will be described in sec 2.5.

2.4 Determination of Stress Multiplier Matrices

The stress multiplier matrix is defined by the following equation for the transformation of nominal stresses into real stresses.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}_{nom} = \begin{bmatrix} K_{11} & K_{12} & \cdot & \cdot & \cdot & K_{16} \\ K_{21} & \cdot & \cdot & \cdot & \cdot & K_{26} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ K_{61} & K_{62} & \cdot & \cdot & \cdot & K_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}_{nom} \quad \cdots\cdots \quad (7)$$

or

$$\{\sigma\}_{real} = [K]\{\sigma\}_{nom}$$

If $\{\sigma\}_{real}$ is calculated by the unit hole pattern model with the loading of $\{\sigma\}_{nom}$, each element in [K] is determined by the next equation.

$$K_{ij} = \sigma_{i-real}/\sigma_{j-nom} \quad \cdots\cdots\cdots\cdots\cdots \quad (8)$$

The calculational models to determine each column of [K] in eq.(7) are shown in Fig.1. Fig.1-(a) is for determining the 1st column of [K], (b) for 2nd, (d) for 3rd, (c) for 4th, (e) for 5th, and (f) for 6th respectively. The magnitude of loading for each model is set so that the nominal stress will be unity, since this makes the calculation of eq. (8) simpler. The 6 real stress components from each calculation above are inserted into each corresponding column of [K]. The above process is repeated for all the FEM elements of the unit hole pattern model. Thus the [K] for each element, say $[K]_{elm}$, is determined.

$[K]_{elm}$ can be used to transform the nominal stresses from the total structural analysis, into real stresses at a specific point in the unit hole pattern. But in usual design practice, such as that of ASME SecIII, stresses in the minimun width of a ligament (average stress) are to be determined. For this purpose, other kinds of [K] are derived from $[K]_{elm}$. For instance, the $[K]_{ave}$ for average stresses across the width of a ligament is derived by the next equation.

$$[K]_{ave}=\Sigma([K]_{elm}(i)\cdot A(i))/\Sigma A(i) \quad \cdots\cdots\cdots\cdots\cdots \quad (9)$$

where A(i) : Area of the i-th element in a group of elements(i=1~n) consisting of minimum ligament width

The stresses at the surface of holes (peak stress) must also be evaluated in ASME SecIII. $[K]_{sur}$ for the stresses at the surface of holes is also determined using a similar procedure as for $[K]_{elm}$ described above. In this case, the stresses at the surface of holes are taken as σ_{i-real} in eq. (8).

2.5 Transformation of Nominal Stresses into Real Stresses

The transformation of nominal stresses into real stresses is performed by simply multiplying [K] by $\{\sigma\}_{nom}$ as shown in eq. (7). As previously mentioned in sec 2.4, several kinds of [K] can be made depending on the requirements of the design. ASME SecIII Article A-8000 specifies detailed design requirements for perforated plates. This article describes the relations applicable to perforated plates with an equilateral triangular hole layout whose elastic properties are isotropic, which might have facilitated establishing the very convenient design procedures. According to the procedures the real stresses (stress intensities) are calculated by using single valued stress multipliers "K"(for primary or secondary stresses), "Y" (for peak stresses) etc. In the present case of an irregular hole layout, "K", "Y" etc. are replaced by $[K]_{ave}$, $[K]_{sur}$ etc.

3. Total Analytical System

In order to perform the design evaluation of reactor core internals with irregular penetration patterns, we developed a new analytical system which consists of a series of computer programs and handles all the calculations required for the design of perforated plates including the calculations of stress intensities and fatigue analysis specified in ASME SecIII. The schematic diagram of this system is shown in Fig. 2.

First, the "THERMOST", general purpose FEM code, analyzes the unit hole pattern FEM model shown in Fig. 1, to obtain the stress multipliers and the effective elastic constants. Next, the "KMAT" code calculates $[K]_{elm}$ and $[K]_{sur}$, and then $[K]_{ave}$ is derived from $[K]_{elm}$.

Then, the total structure is analyzed by the "THERMOST" code, using the effective elastic constants obtained above, to obtain the nominal stresses $\{\sigma\}_{nom}$ corresponding to the primary and secondary loadings.

The primary membrane and bending stresses are calculated by the "PRIM" code in which $[K]_{ave}$ at each ligament is multiplied by $\{\sigma\}_{nom}$ on the mid-plane and by $\{\sigma\}_{nom}$ on the surfaces of the plate respectively. The secondary stresses are calculated in the same way as the primary ones, except for taking a range of variation for $\{\sigma\}_{nom}$ to check for the occurrence of plastic ratchets. The peak stresses are calculated by $[K]_{sur}$ multiplied by the $\{\sigma\}_{nom}$ on the surfaces of the plate. The peak stresses are further used to evaluate fatigue usage factors. Both secondary stresses and peak stresses with fatigue evaluation are handled by the "SECOND" code.

4. Verification of the Analytical Method by Experiment

To evaluate the adequacy of the analytical system, an experiment was performed using a scale model; and the results of the experiment and the corresponding calculation were compared. The details will be described in the following sections.

4.1 Experiment using a Scale Model

Fig.3 shows the model test arrangement. A 1/5 scale model of a reactor core internal structure, made of perforated plate with an irregular penetration pattern, was used for this experiment. The model was installed on the test base, and a mechanical point load was applied at the center of plate, increasing the load in steps up to 8 tons. The strains in the main ligaments of the selected unit hole pattern of the perforated plate were measured by strain gages which were attached to the surface of the plate (Fig.4). Since the ligaments of the model were too narrow to use the rosette type strain gages, uniaxial type gages were used. So the calculational results were presented in the form of uniaxial strains for comparison with the test results. The deflections of the plate were also measured using DTF sensors mounted above the plate.

4.2 Comparison of Calculated and Experimental Results

The analysis which simulated the model test was performed using the total analytical system mentioned in sec 3. The deflections of the plate were calculated from the total structural analysis using the equivalent solid model, and the distributions of average stresses in the ligaments of the perforated plate were obtained by the "PRIM" code using $[K]_{ave}$. These stresses were further converted into strains, for comparison with the strains measured in the experiment.

Figs.5 and 6 show the comparison of experiment and analytical results for the deflection and strain distributions of the perforated region of model. They show very good agreement, both for deflections and strain distributions. From these results, the adequacy of this analytical method was verified.

5. Conclusion

For the design evaluation of perforated plates with irregular penetration patterns, an analytical system was developed which included the stress multiplier matrix method. The method was successfully verified by an experiment done on a scale model of a reactor core internal structure. This system will be brought into practical use for the design evaluation of perforated plates with an irregular hole layout.

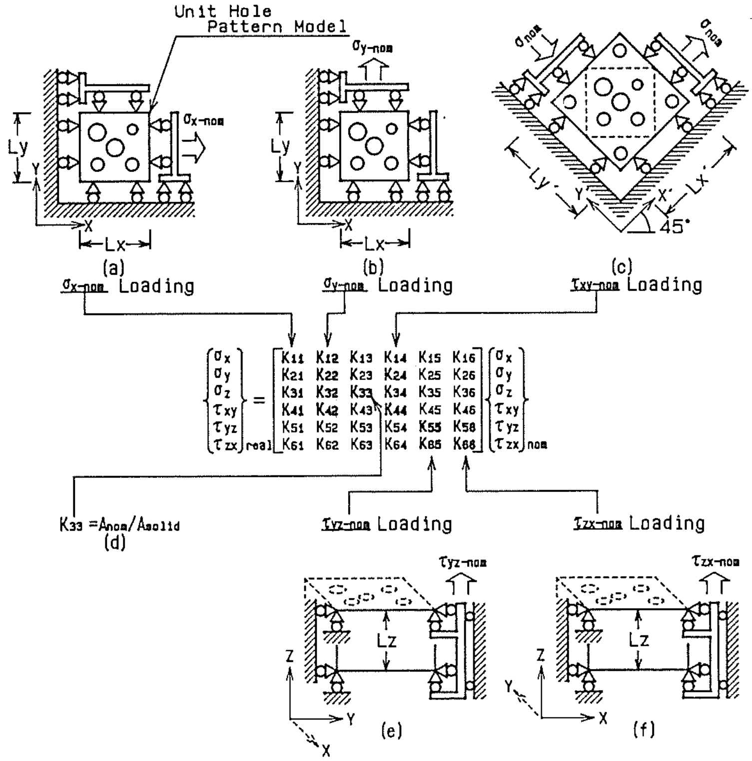
6. References

[1] O'Donnell, "A Study of Perforated Plates with Square Penetration Patterns", Welding Research Council Bulletin 124, 1967

[2] Slot & O'Donnell, "Effective Elastic Constants for Thick Perforated Plates with Square and Triangular Penetration Patterns", Journal of Engineering for Industry, 1971

[3] Jones & Gordon, "Fourier Series Evaluation of the Stress Multipliers Used in Perforated Plate Equivalent Solid Plate Analysis", ASME PVP Vol.98 1985

[4] O'Donnell, "Further Theoretical Treatment of Perforated Plates with Square Penetration Patterns", KPA-PVRC-69-342



$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix}_{real} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16}\\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26}\\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36}\\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46}\\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56}\\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66}\end{bmatrix} \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix}_{nom}$$

Fig.1 Principle of Determination of Stress Multiplier Matrix

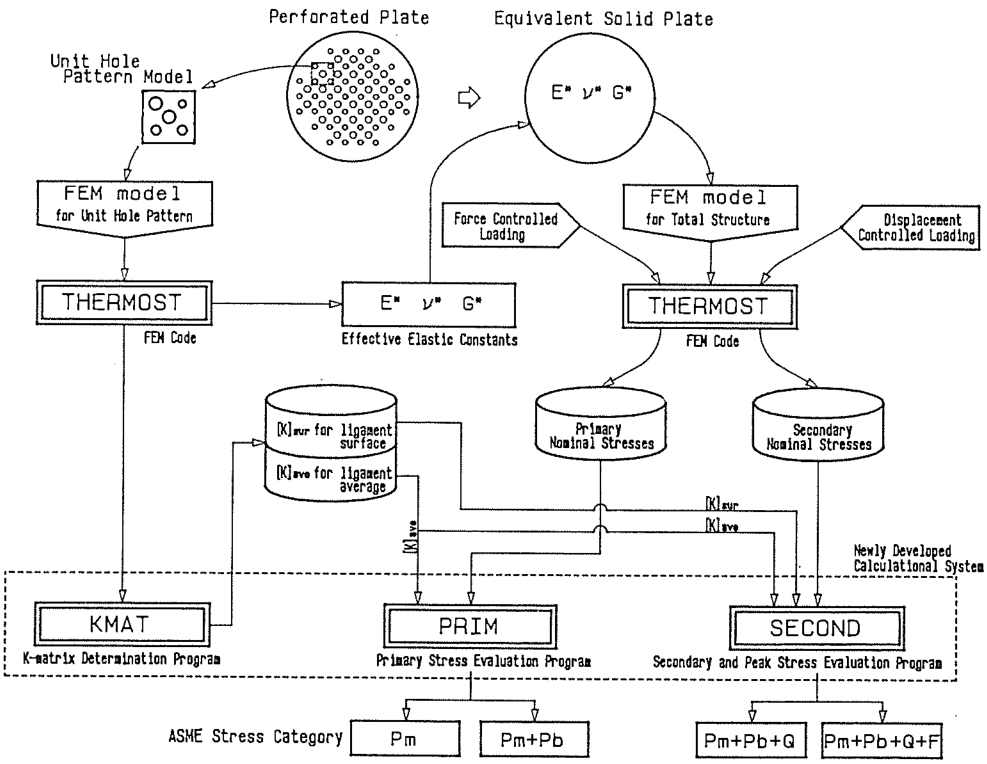


Fig.2 Total Analytical System

Pm : Primary Membrane Stresses
Pb : Primary Bending Stresses
Q : Secondary Membrane and Bending Stresses
F : Peak Stresses

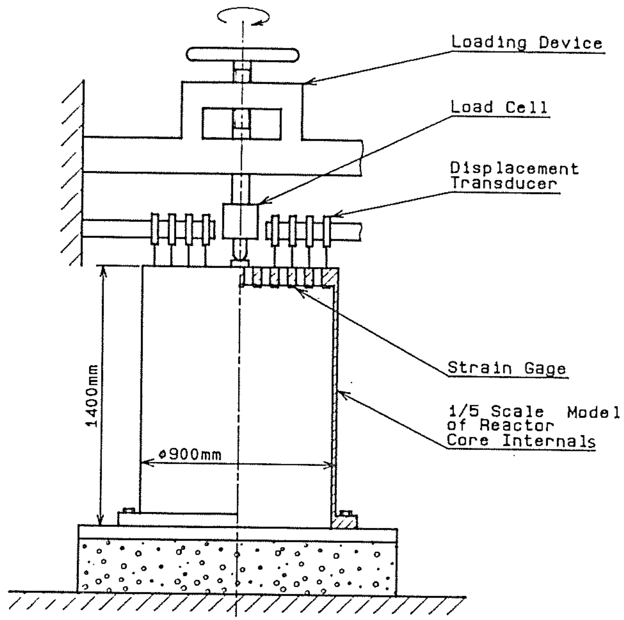


Fig.3 Model Test Arrangement

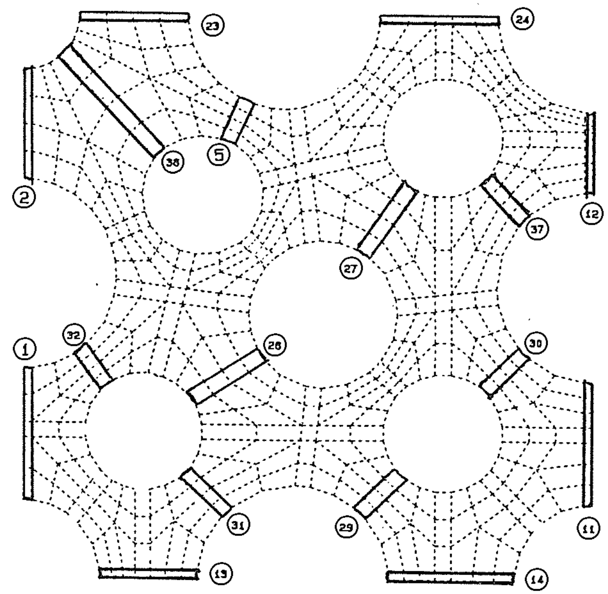


Fig.4 Strain Measurement Points on Unit Hole Pattern

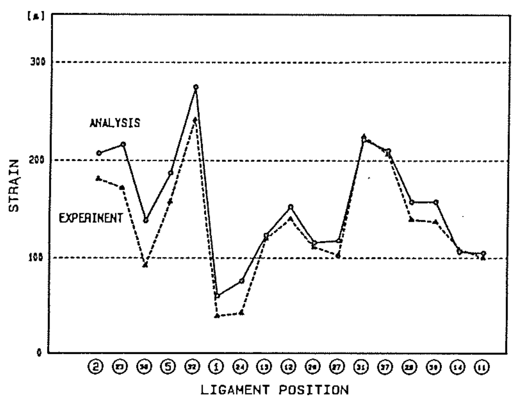


Fig.5 Comparison of Experimental and Analytical Ligament Strains

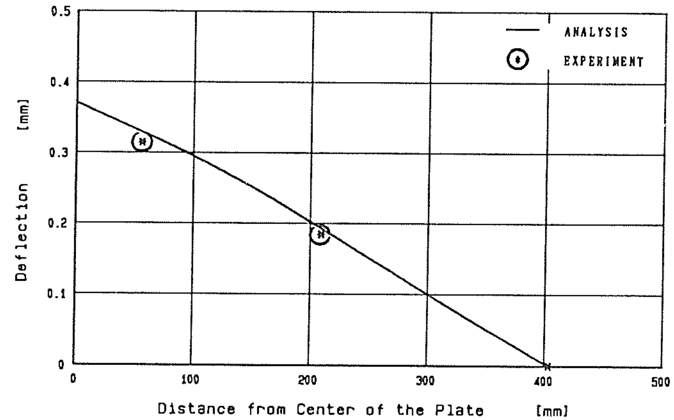


Fig.6 Comparison of Experimental and Analytical Deflection