DESIGN OF CONCRETE CONTAINMENTS FOR TANGENTIAL SHEAR LOADS

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SUMMARY

When containments are subjected to loads from winds, tornado or earthquake, both membrane and tangential (in-plane) shear forces result in the general shell. Due to the more complex nature of the loading and the greater magnitude, earthquake loading is primarily addressed. This paper presents the historical development of tangential shear considerations for concrete containments in the United States.

About ten years ago, the first major generation of concrete containments were under design in the United States. In general, the reinforced containments used both an orthogonal and inclined (diagonal) reinforcing system. The orthogonal reinforcing was designed to resist membrane loads and the inclined system resisted the tangential shear. Prestressed containments allowed a concrete principal tension stress of $3\sqrt{f_c}$ to $4\sqrt{f_c}$ for the various loading combinations which included earthquake loads. During this time period, one horizontal earthquake direction was combined with a vertical earthquake (2/3 horizontal) based on absolute summation of forces.

Containments designed within the last few years have had additional requirements. The first required that three earthquake components be considered (two horizontal and one vertical) and the maximum vertical ground motion be the same as the horizontal. However, all three components could be combined using the square root of the sum of the squares (SRSS). In addition, the (SRSS) technique was interpreted to mean that the maximum membrane and the maximum tangential shear force would combine directly when determining reinforcing. Another change for reinforced containments required that strain compatibility be considered and no yielding was permitted in the inclined reinforcing. During this time, the ASME Section III, Division 2 Containment Code was issued for use with provisions for considering tangential shear in reinforced containments, but no provisions were given for prestressed containments. The provisions for reinforced containments were extremely conservative. When the code was used with the requirement of three components of earthquake and no yielding of any reinforcing, large amounts of inclined reinforcement resulted.

Presently, there are several code changes that are in the process of being approved for reinforced containments and provisions are being added for prestressed containments.

Requirements for reinforced containments:

1. In general, inclined reinforcing is required, but the amount is reduced relative to past requirements and limited yielding is permitted.
2. Enough reinforcing must be supplied to satisfy equilibrium of the combined membrane and tangential shear loads.
3. Strain compatibility, which considers redistribution of forces due to concrete cracking, must be used to illustrate that the inclined reinforcing strain does not exceed code limits. When performing this analysis, a concrete compression strut may be used.

Requirements for prestressed containments:

1. Enough effective prestress must be supplied, so that concrete tension is not permitted for real loads other than earthquake, when considering a load combination which includes earthquake.
2. An orthogonal reinforcing system may be used to resist the earthquake loads.

In conclusion, there is an effort presently underway to introduce criteria in the ASME Section III, Division 2 code which will give adequate and reasonable design requirements for tangential shear.

1. Introduction

When containments are subjected to loads from winds, tornado or earthquake, both membrane and tangential (in-plane) shear forces result in the general shell. Due to the more complex nature of the loading and the greater magnitude, earthquake loading is primarily addressed. This paper presents the historical development of tangential shear considerations for concrete containments in the United States.

About ten years ago, the first major generation of concrete containments were under design in the United States. In general, the reinforced containments used both an orthogonal (hoop, vertical) and inclined (diagonal) reinforcing system. The orthogonal reinforcing was designed to resist membrane loads and the inclined system resisted the tangential shear. Prestressed containments allowed a concrete principal tension stress of $3\sqrt{f'_c}$ to $4\sqrt{f'_c}$ for the various loading combinations which included earthquake loads. During this time period, one horizontal earthquake direction was combined with a vertical earthquake (2/3 horizontal) based on absolute summation of forces.

Containments designed within the last few years have had additional requirements. The first required that three earthquake components be considered (two horizontal and one vertical) and the maximum vertical ground motion be the same as the horizontal. However, all three components could be combined using the square root of the sum of the squares (SRSS). In addition, the (SRSS) technique was interpreted to mean that the maximum membrane and the maximum tangential shear force would combine directly when determining reinforcing. During this time, the ASME Section III, Division 2 Containment Code [1] was issued for use with provisions for considering tangential shear in reinforced containments, but no provisions were given for prestressed containments. The provisions for reinforced containments were extremely conservative. Another change for reinforced containments required that strain compatibility be considered and no yielding was permitted in the inclined reinforcing.

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2. Description Of Containments and Loads

The following three containments will be used for illustrating the various criteria and design techniques.

(a) Prestressed containment with minimum prestress - Type 1, (prestress plus dead load equals 1.25 accident pressure).

(b) Prestressed containment with additional prestress above minimum - Type 2, (prestress plus dead load equals 1.50 accident pressure).

(c) Reinforced containment.

All the containments have the following features (see Figure 1):

- Shape is a vertical cylinder with a hemispherical dome.
- Inside radius is 65' and the height from top of base slab to inside of the apex is 210'.
- Gross wall thicknesses are: reinforced, 4'-6"; prestressed Type 1, 3'-6"; prestressed Type 2, 4'-3". Net wall thickness for prestressed will be 6" less.
- Lined with 1/4" thick carbon steel plate.
- The concrete strength (f'_c) is 4000 psi for the reinforced containment and 6000 psi for

the prestressed containments. The reinforcement yield strength is 60,000 psi.

The load combination considered for illustration is:

$$1.0D + 1.25P_a + 1.25E_o + 1.0T_a \tag{1}$$

where D = Dead Load; P_a = Accident Pressure Load; E_o = Operating Basis Earthquake Load; T_a = Accident Thermal Load.

Only seismic will be considered herein since it is usually higher than wind and tornado.

The following outlines the ASME Section III, Division 2 code requirements, including some future code changes, for consideration of thermal effects.

Normal operation and the operating basis earthquake are considered as service loads (load factor of 1.0) and the design allowables are fairly low ($.5f_y$). When thermal effects are considered in combination with these loads, then the allowables are increased by 33%. When accident and environmental loads (operating basis and safe shutdown earthquake) are considered they are classified as factored loads and the reinforcing allowable is $.9f_y$, except for the inclined reinforcing which is permitted limited yielding. When thermal effects are combined with accident loads, then yielding is permitted due to the self-limiting nature of the thermal effects. An exception to this occurs when a prestressed containment resists tangential shear by providing enough prestress so that principal tension is not predicted. In this case, thermal gradients may cause concrete cracking and therefore it must be considered as an active load.

A summary of the loads used for illustration for the three containment types near the base is shown in Table I.

3. Design Criteria for Prestressed Containments

3.1 Present Criteria

The design of prestressed containments for tangential shear is presently not covered in the ASME Sec. III, Div. 2 code. Most containments designed in the past used a criteria similar to the following.

The tangential shear was limited to a value that, when combined with other membrane loads, the principal tension stress would not exceed $4\sqrt{f'_c}$. This condition is satisfied by the following equation where all terms are defined in Table II.

$$V_c = 4bt\sqrt{f'_c}\left[1 - \frac{(N_v+N_{ve})+(N_h+N_{he})}{4bt\sqrt{f'_c}} + \frac{(N_v+N_{ve})(N_h+N_{he})}{(4bt\sqrt{f'_c})^2}\right]^{1/2} \tag{2}$$

This equation is only valid if both (N_h+N_{he}) and (N_v+N_{ve}) are algebraically less than $+4bt\sqrt{f'_c}$. The design requires that $V_u \leq V_c$.

To illustrate this criteria, the Containment Type 2 defined in Section 2 will be used. Loading is shown in Table I.

$$t = (51-6) = 45 \text{ in}; \quad 4\sqrt{f'_c}\,bt = \frac{4\sqrt{6000}\;12\times45}{1000} = 167 \text{ k/ft}$$

$$N_v = -122 + 351 + 75 - 535 = -231 \text{ k/ft}; \quad N_{ve} = 232 \text{ k/ft} \tag{3}$$

$$N_h = 702 + 75 - 842 = -65 \text{ k/ft}; \quad N_{he} = 0; \quad V_u = 121 \text{ k/ft}$$

Thermal must be considered here since it can cause concrete cracking and affect shear capacity. The allowable shear is:

$$V_c = 167\left[1 - \frac{(-231+232)+(-65)}{167} + \frac{(-231+232)(-65)}{167^2}\right]^{1/2} = 196 \text{ k/ft} \quad (4)$$

Since the allowable of 196 k/ft is larger than V_u of 121 k/ft, the section would be acceptable.

3.2 Proposed Criteria

A code addition presently being processed through the ASME Sec. III, Div. 2 code states the following criteria:

(1) The applied tangential shear (V_u) shall not exceed $8.5bt\sqrt{f'_c}$.

(2) When both (N_h+N_{he}) and (N_v+N_{ve}) are compression, where N_h and N_v include thermal effects which cause membrane cracking, the allowable tangential shear force is:

$$V_c = \left[(N_h+N_{he})(N_v+N_{ve})\right]^{1/2} \quad (5)$$

In this expression, the shear is essentially resisted by membrane compression. When $V_u \leq V_c$ then bonded reinforcing is not necessary.

(3) When $V_u > V_c$ and N_h and N_v are compression, then bonded reinforcing shall be provided by the following:

$$A_{sh} = \frac{N_h+[N_{he}^2+V_u^2]^{1/2}}{.9f_y} \;;\quad A_{sv} = \frac{N_v+[N_{ve}^2+V_u^2]^{1/2}}{.9f_y} \quad (6)$$

In this case, thermal effects need not be considered.

The Type 1 containment in Section 2 will be used for this criteria.

$$t = 42-6 = 36''; \quad 4\sqrt{f'_c}\ bt = \frac{4\sqrt{6000}\ 12\times36}{1000} = 134 \text{ k/ft}$$

$$N_v = -102+351+75-446 = -122 \text{ k/ft}; \quad N_{ve} = 193 \text{ k/ft} \quad (7)$$

$$N_h = 702+75-702 = 75 \text{ k/ft}; \quad N_{he} = 0; \quad V_u = 101 \text{ k/ft}$$

$$(N_v+N_{ve}) = 193-122 = 71 \text{ k/ft}; \quad (N_h+N_{he}) = 75+0 = 75 \text{ k/ft (tension)}$$

Since both of these terms must be compression, then $V_c = 0$.

Determine the amount of reinforcing, but in this case it is not necessary to consider the thermal load. Therefore:

$$N_v = -197 \text{ k/ft}; \quad N_h = 0; \quad N_{ve} = 193 \text{ k/ft}; \quad N_{he} = 0; \quad V_u = 101 \text{ k/ft}$$

$$A_{sh} = \frac{0+[0^2+101^2]^{1/2}}{.9(60)} = 1.87 \text{ in}^2\text{/ft}; \quad A_{sv} = \frac{-197+[193^2+101^2]^{1/2}}{.9(60)} = .39 \text{ in}^2\text{/ft} \quad (8)$$

4. Design Criteria for Reinforced Containments

4.1 ACI 349 Criteria

ACI 349 [2] was the first published document defining criteria for a reinforced concrete containment. ACI 349 required that the earthquake shear be resisted by the inclined reinforcement and the vertical and hoop reinforcement resisted the membrane forces. Design equations for this criteria, neglecting a small amount of shear that could be resisted by the concrete are:

$$A_{si} = \frac{V_u}{.9f_y} \;;\quad A_{sh} = \frac{(N_h+N_{he})}{.9f_y} \;;\quad A_{sv} = \frac{(N_v+N_{ve})}{.9f_y} \quad (9)$$

Using the loading from Table I leads to the following requirements:

$$A_{si} = \frac{108}{.9(60)} = 2.0 \text{ in}^2/\text{ft}; \quad A_{sh} = \frac{702}{.9(60)} = 13.0 \text{ in}^2/\text{ft};$$

$$A_{sv} = \frac{(351-131)+206}{.9(60)} = 7.89 \text{ in}^2/\text{ft} \tag{10}$$

The total reinforcement per foot of wall is:

$$\sum A = 2(2.0) + 13.0 + 7.89 = 24.89 \text{ in}^2/\text{ft} \tag{11}$$

4.2 ASME, Section III, Division 2 Criteria

The ASME Sec. III, Div. 2 code has the following criteria: Diagonal reinforcing is only necessary if the applied shear stress v_u is greater than an allowable shear v_c which is determined as follows:

$$v_c = 12{,}000\rho \quad \text{for } \rho \leq 0.01$$

$$v_c = 93+2700\rho \quad \text{for } 0.01 \leq \rho \leq .025, \text{ but } v_c \leq 160 \text{ psi} \tag{12}$$

ρ is the lesser of the reinforcement steel ratio in the vertical or hoop direction. When v_u is less than v_c, then the shear may be resisted by vertical and hoop reinforcement. However, it must be increased by 1.5. The shear $(v_u - v_c)$ shall be resisted by inclined reinforcing.

The following example will illustrate application of the code:

$$\text{Let } d = t-6 = 48'' \tag{13}$$

Size the vertical and hoop reinforcement for the membrane loads. This will be the same as the previous example, i.e.,

$$A_{sv} = 7.89 \text{ in}^2/\text{ft}; \quad A_{sh} = 13.0 \text{ in}^2/\text{ft}; \quad \rho = \frac{7.89}{12(48)} = .0137 \text{ minimum}$$

$$v_c = 93+2700(.0137) = 130 \text{ psi}; \quad V_c = \frac{130(12)(48)}{1000} = 75 \text{ k/ft} \tag{14}$$

$$v_u = \frac{108(1000)}{12\times 48} = 188 \text{ psi}$$

The additional hoop and vertical reinforcing to resist V_c is:

$$A'_{sv} = A'_{sh} = \frac{(1.5)75}{.9(60)} = 2.08 \text{ in}^2/\text{ft} \tag{15}$$

and since $v_u > v_c$ then inclined reinforcing is required:

$$A_{si} = \frac{V_u - V_c}{.9f_y} = \frac{108-75}{.9(60)} = .61 \text{ in}^2/\text{ft} \tag{16}$$

$$\sum A = 2(.61)+(7.89+2.08)+(13.0+2.08) = 26.27 \text{ in}^2/\text{ft} \tag{17}$$

4.3 NRC Criteria

The United States Nuclear Regulatory Commission (NRC) took exception to the 160 psi for v_c and stated in Standard Review Plan Section 3.8.1 [3] that a v_c maximum of 40 psi was acceptable when considering the Operating Basis Earthquake and the value was 60 psi when considering the Safe Shutdown Earthquake. For a short time, the NRC also required that strain compatibility be considered for the reinforcing and none of the reinforcing be allowed to yield. If the hoop and vertical reinforcing are proportional to resist membrane loads,

$$A_{sv} = \frac{N_v + N_{ve}}{.9f_y} = \frac{(351-131)+206}{54} = 7.89 \text{ in}^2/\text{ft}$$

$$A_{sh} = \frac{N_h + N_{he}}{.9f_y} = \frac{702+0}{54} = 13.00 \text{ in}^2/\text{ft} \tag{18}$$

then strain compatibility (Section 5) requires the following inclined reinforcing when the concrete compression is neglected:

$$A_{si} = \frac{1}{1.8f_y}\left[V_u + \sqrt{V_u^2 + 8V_u\left[\frac{(N_v+N_{ve})(N_h+N_{he})}{(N_v+N_{ve})+(N_h+N_{he})}\right]}\right] \tag{19}$$

$$A_{si} = \frac{1}{1.8(60)}\left[108 + \sqrt{(108)^2+8(108)\left[\frac{(406)(702)}{426+702}\right]}\right] = 5.54 \text{ in}^2/\text{ft}$$

$\sum A = 2(5.54) + 7.89 + 13.0 = 31.97 \text{ in}^2/\text{ft}$

For this design, the inclined reinforcing will be stressed to $.9f_y$. However, the vertical and hoop will have a much lower stress.

A more optimum design using strain compatiblity (Section 5) is to have the following reinforcing:

$$A_v = 0; \quad A_h = 5.18 \text{ in}^2/\text{ft}; \quad A_{si} = 9.94 \text{ in}^2/\text{ft}; \tag{20}$$

$\sum A = 2(9.94)+5.18 = 25.06 \text{ in}^2/\text{ft}$

This is much lower than the previous, but the amount of inclined reinforcing has increased significantly.

4.4 Proposed Code Revision

The following code revision has been submitted to ASME Sec. III, Div. 2. However, at the time of writing, it was not yet passed. Based on committee discussions, this should pass in the near future. Strain compatibility of the reinforcing must be considered and the maximum strain in the inclined reinforcing is limited to .004 in/in.

The code case allows a v_c = 60 psi for conditions which combine earthquake and accident pressure loads. Reinforcing is required as defined by the following equations:

$$A_{si} = \frac{V_u - V_c}{.9f_y}; \quad A_{sv} = \frac{N_v + [N_{ve}^2 + V_c^2]^{1/2}}{.9f_y}; \quad A_{sh} = \frac{N_h + [N_{he}^2 + V_c^2]^{1/2}}{.9f_y} \tag{21}$$

Using the above leads to:

$$V_c = v_c bt = \frac{(60)12(54)}{1000} = 39 \text{ k/ft}; \quad A_{si} = \frac{108-39}{.9(60)} = 1.28 \text{ in}^2/\text{ft}$$

$$A_{sv} = \frac{351-131+[206^2+39^2]^{1/2}}{.9(60)} = 7.96 \text{ in}^2/\text{ft} \tag{22}$$

$$A_{sh} = \frac{702+[0^2+39^2]^{1/2}}{.9(60)} = 13.72 \text{ in}^2/\text{ft}; \quad \sum A = 2(1.28)+7.96+13.72 = 24.24 \text{ in}^2/\text{ft}$$

This proposed code revision is used as an example in the strain compatibility analysis in Section 5.

The preceding code revision is a combination of absolute summation and SRSS and is therefore slightly conservative. The presence of inclined reinforcing should not affect the total amount of reinforcing. The following is a new technique that will give consistent results.

4.5 New Technique

First, calculate the amount of steel if inclined reinforcing were not necessary:

$$A_{sv} = \frac{N_v+[N_{ve}^2+V_u^2]^{1/2}}{.9f_y} = \frac{351-131+[206^2+108^2]^{1/2}}{.9(60)} = 8.38 \text{ in}^2/\text{ft}$$

$$A_{sh} = \frac{N_h+[N_{he}^2+V_u^2]^{1/2}}{.9f_y} = \frac{702+[0^2+108^2]^{1/2}}{.9(60)} = 15.0 \text{ in}^2/\text{ft} \tag{23}$$

$\sum A = 8.38 + 15.0 = 23.38 \text{ in}^2/\text{ft}$

This is the same amount of reinforcing required regardless of the orientation. Therefore, if the required amount of inclined reinforcing is:

$$A_{si} = \frac{V_u - V_c}{.9f_y} = 1.28 \text{ in}^2/\text{ft} \tag{24}$$

then the required horizontal and vertical reinforcing is:

$$A_{sv} = 8.38 - 1.28 = 7.10 \text{ in}^2/\text{ft}; \quad A_{sh} = 15.0 - 1.28 = 13.72 \text{ in}^2/\text{ft} \tag{25}$$

5. Strain Compatibility Analysis for Reinforced Containments

. Figure 1 shows the model used for the strain compatibility analysis of a typical wall element of a reinforced concrete containment structure. The element contains both orthogonal and inclined reinforcement and is subject to membrane loads only. An idealized crack pattern is shown with parallel cracks oriented perpendicular to the direction of maximum principal strain.

The following assumptions are used for the analysis of the cracked reinforced concrete element.

(a) Concrete has zero tensile strength - concrete acts only as compressive struts parallel to the direction of minimum principal strain.

(b) Shear resistance of cracked concrete and steel reinforcing is neglected.

(c) Reinforcing and concrete struts act as concurrent truss system at the center of the element.

The theoretical development of this mathematical model and a discussion of the applicability of the assumptions for a reinforced concrete containment can be found in a paper by Duchon [4].

The following equations represent equilibrium, stress-strain and strain compatibility for the cracked element with typical inclined reinforcement, $\rho_3 = \rho_4$, oriented at 45° relative to the hoop and vertical reinforcing.

$$\text{Equilibrium:} \quad N_{vt} = \sigma_1 \rho_1 A_g + \frac{\sigma_3 \rho_3 A_g}{2} + \frac{\sigma_4 \rho_3 A_g}{2} + f_c A_g \sin^2\beta$$

$$N_{ht} = \sigma_2 \rho_2 A_g + \frac{\sigma_3 \rho_3 A_g}{2} + \frac{\sigma_4 \rho_3 A_g}{2} + f_c A_g \cos^2\beta \tag{26}$$

$$V_t = \frac{\sigma_3 \rho_3 A_g}{2} - \frac{\sigma_4 \rho_3 A_g}{2} = f_c A_g \sin\beta\cos\beta$$

where N_{vt}, N_{ht} and V_t are the total membrane loads and other terms are defined in Table II.

$$\text{Stress-Strain:} \quad \sigma_i = E_s \varepsilon_i; \quad f_c = E_c \varepsilon_{II} \text{ (compression only)} \tag{27}$$

$$\text{Strain Compatibility:} \quad \varepsilon_1 = \varepsilon_I \cos^2\beta + \varepsilon_{II} \sin^2\beta; \quad \varepsilon_2 = \varepsilon_I \sin^2\beta + \varepsilon_{II} \cos^2\beta$$

$$\varepsilon_3 = \frac{\varepsilon_I}{2}(1 + 2\sin\beta\cos\beta) + \frac{\varepsilon_{II}}{2}(1 - 2\sin\beta\cos\beta) \tag{28}$$

$$\varepsilon_4 = \frac{\varepsilon_I}{2}(1 - 2\sin\beta\cos\beta) + \frac{\varepsilon_{II}}{2}(1 + 2\sin\beta\cos\beta)$$

These governing equations reduce to a single fourth order equation in terms of the crack angle β.

$$\tan^4\beta + C_1 \tan^3\beta + C_2 \tan\beta + C_3 = 0 \tag{29}$$

where $C_1 = [\alpha_{11}(\frac{N_{vt}}{V_t}) + \alpha_{12}(\frac{N_{ht}}{V_t})]/\eta$; $C_2 = [\alpha_{21}(\frac{N_{vt}}{V_t}) + \alpha_{22}(\frac{N_{ht}}{V_t})]/\eta$; $C_3 = -\frac{\gamma}{\eta}$

and $\alpha_{11} = \rho_3(\frac{1}{2} + n\rho_3 + n\rho_2) + \rho_2;\quad \alpha_{12} = -\rho_3(\frac{3}{2} + n\rho_1 + n\rho_3)$

$$\alpha_{21} = \rho_3(\frac{3}{2} + n\rho_2 + n\rho_3);\quad \alpha_{22} = -\rho_3(\frac{1}{2} + n\rho_1 + n\rho_3) - \rho_1 \tag{30}$$

$$\eta = \frac{\rho_3}{2}(1+n\rho_1+n\rho_2)+\rho_2(1+n\rho_1);\quad \gamma = \frac{\rho_3}{2}(1+n\rho_1+n\rho_2)+\rho_1(1+n\rho_2)$$

The principal strains and reinforcing and concrete stresses are:

Principal Strains: $\varepsilon_I = \frac{1}{E_s}\left[\frac{wN_{vt}-xN_{ht}}{\xi}\right];\ \varepsilon_{II} = \frac{1}{E_s}\left[\frac{zN_{ht}-yN_{vt}}{\xi}\right]$

where $w = (n\rho_2+1)\cos^2\beta + \frac{n\rho_3}{2};\quad x = (n\rho_1+1)\sin^2\beta + \frac{n\rho_3}{2};\quad y = n\rho_2\sin^2\beta + \frac{n\rho_3}{2}$ (31)

$$z = n\rho_1\cos^2\beta + \frac{n\rho_3}{2}$$

$$\xi = \frac{A_g}{n}\left[(n^2\rho_1\rho_2 + \frac{n^2\rho_1\rho_3}{2} + \frac{n^2\rho_2\rho_3}{2} + \frac{n\rho_3}{2})(\cos^2\beta-\sin^2\beta) + n\rho_1\cos^4\beta - n\rho_2\sin^4\beta\right]$$

Stresses:

$$\sigma_1 = (a_{11}N_{vt}+a_{12}N_{ht})/\xi;\quad \sigma_2 = (a_{21}N_{vt}+a_{22}N_{ht})/\xi$$

$$\sigma_3 = (a_{31}N_{vt}+a_{32}N_{ht})/2\xi;\quad \sigma_4 = (a_{41}N_{vt}+a_{42}N_{ht})/2\xi;\quad f_c = E_c\varepsilon_{II}$$

where $a_{11} = (\cos^2\beta-\sin^2\beta)n\rho_2 + (\cos^2\beta-\sin^2\beta)n\rho_3/2 + \cos^4\beta$

$$a_{12} = (\sin^2\beta-\cos^2\beta)n\rho_3/2 - \sin^2\beta\cos^2\beta$$

$$a_{21} = (\sin^2\beta-\cos^2\beta)n\rho_3/2 + \sin^2\beta\cos^2\beta$$

$$a_{22} = (\cos^2\beta-\sin^2\beta)n\rho_1 + (\cos^2\beta-\sin^2\beta)n\rho_3/2 - \sin^4\beta$$

$$a_{31} = \cos^2\beta(1 + \sin2\beta)(n\rho_2+1) - \sin^2\beta(1-\sin2\beta)n\rho_2+n\rho_3\sin2\beta \tag{32}$$

$$a_{32} = \cos^2\beta(1-\sin2\beta)n\rho_1-\sin^2\beta(1+\sin2\beta)(n\rho_1+1)-n\rho_3\sin2\beta$$

$$a_{41} = \cos^2\beta(1-\sin2\beta)(n\rho_2+1)-\sin^2\beta(1+\sin2\beta)n\rho_2-n\rho_3\sin2\beta$$

$$a_{42} = \cos^2\beta(1+\sin2\beta)n\rho_1-\sin^2\beta(1-\sin2\beta)(n\rho_1+1)+n\rho_3\sin2\beta$$

The solution is found when the real root and resulting reinforcing and concrete stresses satisfy the equations of equilibrium.

If the effect of the concrete compression strut is neglected, or if there is a general tensile strain field in the model, the concrete stress is taken as zero. The governing equation remains the same, except $f_c = 0$ and strain compatibility becomes:

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_3 + \varepsilon_4 = \varepsilon_I + \varepsilon_{II} \tag{33}$$

These equations are independent of the crack angle β, and can be solved directly for the reinforcing stresses and strains.

5 1 Inclined Yielding

The preceding equations considered linear stress-strain material properties. If the inclined reinforcing (i = 3) is yielding, the equilibrium and stress-strain equations for the cracked concrete element can be modified to simulate a yielding condition as follows.

The stress-strain curve for the reinforcing is elastic-perfectly-plastic with a code allowable of $.9f_y$ assumed for yield. The stress and strain in the inclined reinforcing (3) are taken as:

$$\sigma_3 = .9f_y;\quad \varepsilon_3 \geq .9\varepsilon_y \tag{34}$$

The force in inclined (3) at the yield stress of $.9f_y$ is:

$$F_3 = \rho_3 A_g \sigma_3 = .9f_y A_{si} \tag{35}$$

The inclined yield force (F_3) is a constant and when substituted in Eq. (26), the modified equations of equilibrium become:

$$N_{vt} = \sigma_1 \rho_1 A_g + \frac{F_3}{2} + \frac{\sigma_4 \rho_3 A_g}{2} + f_c A_g \sin^2\beta; \quad N_{ht} = \sigma_2 \rho_2 A_g + \frac{F_3}{2} + \frac{\sigma_4 \rho_3 A_g}{2} + f_c A_g \cos^2\beta$$

$$V_t = \frac{F_3}{2} - \frac{\sigma_4 \rho_3 A_g}{2} - f_c A_g \sin\beta\cos\beta \quad \text{or, rewritten as:} \tag{36}$$

$$N'_{vt} = N_{vt} - \frac{F_3}{2} = \sigma_1 \rho_1 A_g + \frac{\sigma_4 \rho_3 A_g}{2} + f_c A_g \sin^2\beta$$

$$N'_{ht} = N_{ht} - \frac{F_3}{2} = \sigma_2 \rho_2 A_g + \frac{\sigma_4 \rho_3 A_g}{2} + f_c A_g \cos^2\beta \tag{37}$$

$$V'_t = V_t - \frac{F_3}{2} = -\frac{\sigma_4 \rho_3 A_g}{2} - f_c A_g \sin\beta\cos\beta$$

These modified equations apply to a cracked concrete element where the inclined reinforcing (3) has been removed and replaced with an equivalent yield force. The solution for this inclined yielding case is found using the previously defined governing equations with $\rho_3 = 0$ and the modified loads N'_{vt}, N'_{ht} and V'_t.

<u>Example</u>: An example of the strain compatibility analysis with inclined yielding is presented using the reinforcing determined for the proposed code revision for reinforced containments, Section 4.4.

$A_{si} = 1.28\ in^2/ft$; $A_{sv} = 7.96\ in^2/ft$; $A_{sh} = 13.72\ in^2/ft$;

$E_c = 3.75\times10^3$ ksi; $E_s = 30\times10^3$ ksi

Since strain compatibility requires an absolute summation of normal and shear loads, the following adjustments must be made to the loads.

$$N_{vt} = N_v + [N_{ve}^2 + V_c^2]^{1/2} - V_c = 220 + [206^2 + 39^2]^{1/2} - 39 = 390 \text{ k/ft}$$

$$N_{ht} = N_h + [N_{he}^2 + V_c^2]^{1/2} - V_c = 702 + [0 + 39^2]^{1/2} - 39 = 702 \text{ k/ft} \tag{38}$$

$$V_t = V_u = 108 \text{ k/ft}$$

A strain compatibility analysis with these loads assuming elastic material properties gives $\sigma_3 = 98.0$ ksi which indicates yielding of the inclined reinforcing. A strain compatibility analysis considering inclined yielding would use the following modified loads:

$$N'_{vt} = N_{vt} - \frac{F_3}{2} = 390 - \frac{69}{2} = 355.5 \text{ k/ft}$$

$$N'_{ht} = N_{ht} - \frac{F_3}{2} = 702 - 34.5 = 667.5 \text{ k/ft} \tag{39}$$

$$V'_t = V_t - \frac{F_3}{2} = 108 - 34.5 = 73.5 \text{ k/ft}$$

where $F_3 = A_{si}(.9f_y) = 1.28\times54 = 69.0$ k/ft

These modified loads applied to the cracked concrete element with $\rho_3 = 0$ and $\sigma_3 = .9f_y$ give the following:

Stresses: $\sigma_1 = \sigma_2 = \sigma_3 = 54$ ksi; $\sigma_3 = -1.8$ ksi; $f_c = -.2$ ksi (40)

Crack angle: $\beta = 45°$

All reinforcing, except the compression inclined is at yield ($.9f_y$) with the strain in inclined (3) being:

$\varepsilon_3 = \varepsilon_I = .00366$ in/in < .004 in/in

6. General Remarks and Conclusions

Reinforced containments have used inclined reinforcing for many years to resist tangential shear. The amount of inclined reinforcing has varied depending upon the particular criteria in use. The necessity of inclined reinforcing has not really been established and the future ASME Sec. III, Div. 2 code change is conservative. Testing should be done to either verify the need for inclined reinforcement or provide information for the elimination of the inclined reinforcement.

Prestressed containments will have two options in the future for resisting tangential shear. The first is to provide enough prestress force so that principal tension is not predicted due to all loads in a load combination including earthquake, wind, tornado and thermal. The other is to provide supplemental bonded reinforcing steel to resist loads in excess of the prestress when considering load combinations which include earthquake, wind and tornado.

Even with the inclusion of the proposed code changes, the design is very conservative based on the following:

(1) Predicted seismic forces are very high and should constitute extreme upper limits.

(2) When considering seismic loads, the worst possible point in the structure is considered on an elastic basis and no redistribution of load is considered.

(3) The governing case is usually the factored condition of 1.25 accident pressure and 1.25 operating basis earthquake with a peak factored earthquake combined with peak accident conditions which is highly improbable.

References

[1] ASME Boiler and Pressure Vessel Code, Section III, Division 2, 1975 edition.

[2] ACI Committee 349, "Criteria for Reinforced Concrete Nuclear Power Containment Structures", ACI Journal, pp. 2-28, January, 1972.

[3] U.S. Nuclear Regulatory Commission, Standard Review Plan, Section 3.8.1, "Concrete Containments", October, 1975.

[4] Duchon, N.B., "Analysis of Reinforced Concrete Membrane Subject to Tension and Shear", ACI Journal, pp. 578-583, September, 1972.

Table I Summary Of Loads For The Three Containment Types Near The Cylinder Base

	Reinforced Containment Loads (k/ft)		
	Vertical	Hoop	Tangential Shear
1.0D	-131	---	---
$1.25E_o$	206	---	108
$1.25P_a$	351	702	---
$1.0T_a$	75	75	---

	Prestressed Containment Loads (k/ft)					
	Type 1			Type 2		
	Vertical	Hoop	Tangential Shear	Vertical	Hoop	Tangential Shear
1.0D	-102	---	---	-122	---	---
$1.25E_o$	193	---	101	232	---	121
$1.25P_a$	351	702	---	351	702	---
$1.0T_a$	75	75	---	75	75	---
$1.0F_e$*	-446	-702	---	-535	-842	---

*F_e = effective prestress force at end of plant life.
Positive is tension and negative is compression.

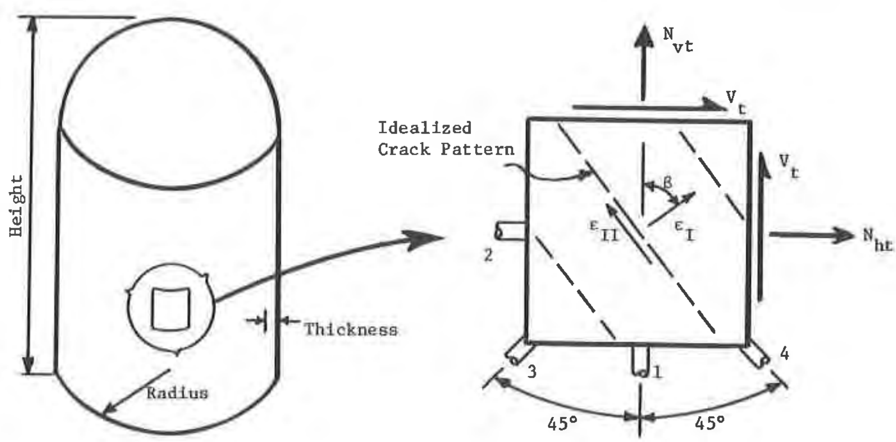

Figure 1 Containment And Typical Wall Element

Table II Nomenclature

A_g = total concrete cross-sectional area

A_{si} = reinforcing steel in the inclined direction; will be oriented at ±45° to vertical for the criteria presented

A_{sv} = reinforcing steel in the vertical direction

A_{sh} = reinforcing steel in the horizontal direction

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of reinforcing steel

i = index referring to direction in plane of membrane; thus: 1 = vertical; 2 = horizontal; 3 = inclined; 4 = inclined; I = maximum principal (largest tensile); II = minimum principal

N_h = loads in the horizontal direction other than earthquake; includes P_a, F_e and T_a when applicable

N_{he} = maximum loads in the horizontal direction due to earthquake (E_o); will be assumed as zero herein

N_v = loads in the vertical direction other than earthquake; includes D, F_e, P_a and T_a when applicable

N_{ve} = maximum loads in the vertical direction due to earthquake (E_o)

n = modulus of elasticity ratio: E_s/E_c

ρ_i = ratios of area of reinforcement in direction i to total concrete area of section perpendicular to i

V_c = an allowable shear force determined from the allowable shear stress ν_c by $V_c = \nu_c bt$ where b = unit width of section; t = thickness of section

V_u = maximum tangential shear load due to earthquake (E_o)

β = angle between directions i = 1 and i = I

ε_i = normal strain in direction i

σ_i = normal stress in direction i