TRANSIENT THERMAL STRESSES IN CIRCULAR CYLINDER UNDER INTERMITTENTLY SUDDEN HEAT GENERATION

Y. SUGANO, K. SAITO, Y. TAKEUTI

Department of Mechanical Engineering,
Faculty of Engineering, University of Osaka Prefecture, Mozu, Sakai, Osaka, Japan

SUMMARY

The thermal stress problems in a circular cylinder, probably for reasons of analytical convenience, have been solved by using of surface temperature as the thermal boundary conditions (see: C.K. Youngdahl, E. Sternberg, Trans. ASME,E,48(1961), 25).

Now the transient thermal stress problems in a circular cylinder subjected to intermittently sudden heat generation on the cylinder surface have never been solved as yet. They are, however, practically more useful in nuclear engineering.

In this paper, therefore, the thermal stresses associated with the transient temperature distribution arising in a circular cylinder under intermittently changing sudden heat generation over a finite band and with heat loss to a surrounding medium on the remainder of the cylinder surface are exactly analysed.

For the first time the temperature field in a circular cylinder under sudden heat generation over a finite band of the cylinder surface is determined by combined use of Fourier cosine, Laplace transforms in axial position and time, respectively.

Secondly we assume that the temperature fields in a circular cylinder subjected to heat generation $Q_i (i = 0, 1, 2, \ldots)$ independently over a finite band are given by $T_0(r, z, t)$, $T_1(r, z, t)$, $T_2(r, z, t), \ldots$ respectively.

Then, the temperature fields in a circular cylinder under intermittently sudden heat generation $Q_1 (i = 0, 1, 2, \ldots)$ for each time $t = 0 \sim t_0'$, $t_1 \sim t_1'$, $t_2 \sim t_2', \ldots$ are given as follows:
(1) for the $(i+1)$-th time $t = t_i \sim t_i'$ in heat generation,

$$T(r, z, t) = T_i^*(r, z, t) + T_i(r, z, t - t_i),$$

(2) for the next time $t = t_i' \sim t_{i+1}$ in non-heat generation,

$$T(r, z, t) = T_i^*(r, z, t) + T_i(r, z, t - t_i) - T_i(r, z, t - t_i').$$

Here $T_i^*(r, z, t)$ indicates the temperature field before the i-th heat generation Q_i. The thermal stresses associated with the temperature field described above are analysed by using the Hoyle stress functions.

Numerical calculations are carried out for the extensive case of the ratio of the heat-generating length to the diameter of cylinder.

It is found that the time in which the maximum stresses occur on the cylinder surface does not depend on the heat-generating length-to-diameter ratio.

1. Introduction

The thermal stress problems in a circular cylinder, probably for reasons of analytical convenience, have been solved by making use of surface temperature as the thermal boundary condition. For example, Youngdahl and Sternberg [1] give a exact solution for the transient stress field in a cylinder resulting from a sudden uniform change of surface temperature over a finite band. The thermal stresses in a finite solid cylinder due to a steady-state axisymmetric temperature field over one of its end surface are also analysed by K.T.S.R. Iyengar and K.Chandrashekhara [2]. Furthermore, J.Ignaczak [3] gives the thermal stresses in a infinite cylinder exposed to a discontinuous steady-state temperature over the cylinder surface. But the number of solved transient thermal stress problems in a cylinder subjected to sudden heat generation is quite small. The transient thermal stress problems in a cylinder subjected to intermittently sudden heat generation on the cylinder surface, as far as the authors are aware, have never been solved as yet. They are, however, practically more useful in nuclear engineering.

In this paper, therefore, the thermal stresses associated with the transient temperature distribution arising in a circular cylinder under intermittently changing sudden heat generation over a finite band and with heat loss to a surrounding medium on the reminder of the cylinder surface are exactly analysed. The cylinder is supposed to be homogeneous and isotropic with respect to both the thermal and mechanical properties are regarded to be independent of temperature. Furthermore, inertia effects are neglected along with the influence of thermoelastic coupling.

For the first time the temperature field in a cylinder under sudden uniform heat generation over a finite band and with heat loss to a surrounding medium on the reminder of the cylinder surface is determined by means of a combined Fourier cosine and Laplace-transform technique. Secondly by making use of the solution obtained, the temperature field in a cylinder under intermittently sudden heat generation is established. Once the temperature field is known, the associated field of thermal stress is analysed with a aid of the Hoyle stress functions [4]. All results are deduced in rapidly convergent series and integral form involving only exponential functions, Bessel functions, and the complementary error function. The temperature and stress variation along the axis and the surface of the cylinder are presented for two cases of the ratio of the heat-generating length to the cylinder diameter and for various combinations of the heat-generating time and cooling time.

2. Temperature Field

2.1 Temperature Field under Sudden Uniform Heat Generation

The heat conduction problems described in the introduction are conveniently reffered to cylindrical coordinates (r, θ, z), as shown in Fig.1. The governing equation for the transient heat conduction in a circular cylinder with no internal heat sources is given as follows

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t} \qquad (1)$$

It is assumed that the cylinder, initially at uniform temperature T_0, suddenly subjects to uniform heat generation at the constant rate Q^* over a finite band between $z=-b$ and $z=b$ of the cylinder surface, as shown in Fig.1 and Fig.2. On the reminder of the cylinder surface, heat is lost to a surrounding medium of constant temperature T_0 through a temperature-independent boundary conductance, h. From the above description, the initial and boundary conditions are given by:

$$(T)_{t=0} = T_0 \tag{2}$$

$$\lambda \frac{\partial T}{\partial r} + h(T-T_0) = Q^* H(t) H(b-|z|) \qquad at \quad r=a \tag{3}$$

where H( ): Heaviside function,

Q^* : heat generation rate per unit area unit time,

λ : thermal conductivity,

h : heat transfer coefficent.

In addition, the desired temperature distribution must meet the regularity requirement:

$$T \to T_0 \;, \quad \partial T/\partial z \to 0 \qquad as \quad |z| \to \infty \tag{4}$$

The temperature distribution is symmetrical about the plane z=0. Therefore, only one half of the cylinder ($0 \leqq z < \infty$) will be analysed. At this stage it is expedient to introduce dimensionless variavles by means of

$$\rho = r/a \;, \quad \zeta = z/a \;, \quad \tau = \kappa t/a^2 \;, \quad \beta = b/a \;, \quad m = ha/\lambda$$
$$\bar{Q} = \frac{Q^* a}{\lambda T_0} \;, \quad \bar{T}(\rho,\zeta,\tau) = [T(r,z,t) - T_0]/T_0\bar{Q} \;. \tag{5}$$

The equation of heat conduction eq.(1) then appears as

$$\nabla^2 \bar{T} = \frac{\partial \bar{T}}{\partial \tau} \tag{6}$$

where $\quad \nabla^2 = \partial^2/\partial\rho^2 + 1/\rho\,\partial/\partial\rho + \partial^2/\partial\zeta^2 \,,$

while eqs.(2),(3),and (4) become

$$(\bar{T})_{\tau=0} = 0 \tag{7}$$

$$\frac{\partial \bar{T}}{\partial \rho} + m\bar{T} = H(\beta - \zeta) \qquad at \quad \rho = 1 \,, \tag{8}$$

$$\bar{T} \to 0 \;, \quad \partial\bar{T}/\partial\zeta \to 0 \qquad as \quad \zeta \to \infty . \tag{9}$$

Let

$$\hat{T}(\rho,P,\tau) = \sqrt{\frac{2}{\pi}} \int_0^\infty \bar{T}(\rho,\zeta,\tau) \cos p\zeta \, d\zeta \tag{10}$$

Whence $\hat{T}(\rho,P,\tau)$ is the Fourier cosine transform of the temperature $\bar{T}(\rho,\zeta,\tau)$ with respect to ζ, p being the transform parameter. Because of the symmetry of the problem about the plane $\zeta=0$, i.e., $(\partial\bar{T}/\partial\zeta)=0$, and the condition eq.(9) that $\bar{T}$ and $\partial\bar{T}/\partial\zeta$ tend to zero as $\zeta \to \infty$, eq.(7) under the Fourier cosine transform goes into

$$\frac{\partial^2 \hat{T}}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \hat{T}}{\partial \rho} - P^2 \hat{T} = \frac{\partial \hat{T}}{\partial \tau} \,. \tag{11}$$

while the transformation of the initial and boundary conditions, eqs.(7) and (8) yields

$$(\hat{T})_{\tau=0} = 0 \,, \tag{12}$$

$$\frac{\partial \hat{T}}{\partial \rho} + m\hat{T} = \sqrt{\frac{2}{\pi}} \frac{\sin P\beta}{P} \qquad at \quad \rho = 1 \,. \tag{13}$$

Next, let

$$T^*(\rho,P,q) = \int_0^\infty \hat{T}(\rho,P,\tau) e^{-q\tau} d\tau \,, \tag{14}$$

so that $T^*(\rho,p,q)$ is the Laplace transform of $\hat{T}(\rho,p,\tau)$ with respect to τ, q being the new transform parameter. Applying eq.(14) to eqs.(11),(13), and using eq.(12), we obtain

$$\frac{d^2 T^*}{d\rho^2} + \frac{1}{\rho}\frac{dT^*}{d\rho} - (P^2 + q) T^* = 0 \,, \tag{15}$$

$$\frac{dT^*}{d\rho} + mT^* = \sqrt{\frac{2}{\pi}}\,\frac{\sin p\beta}{pq} \qquad \text{at} \quad \rho = 1. \tag{16}$$

The solution of eq.(15) which is finite at $\rho=0$ and satisfies the boundary condition eq.(16), is given by

$$T^* = \sqrt{\frac{2}{\pi}}\,\frac{\sin p\beta}{pq}\,\frac{I_0(\mu\rho)}{\{\mu I_0'(\mu)+mI_0(\mu)\}}\ , \tag{17}$$

where $\mu=\sqrt{p^2+q}$, and $I_n(\ )$ is the modified Bessel function of the first kind, of order n.

From the inversion theorem for the Laplace transform

$$\hat{T}(\rho,p,\tau) = \frac{1}{2\pi i}\sqrt{\frac{2}{\pi}}\,\frac{\sin p\beta}{p}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{I_0(\mu\rho)}{\{\mu I_0'(\mu)+mI_0(\mu)\}}\,e^{q\tau}\,\frac{dq}{q}\ , \tag{18}$$

in which the line of integration $\mathrm{Re}[q]=\gamma$ is to be chosen to the right of the singularities of the integrand. The integrand of eq.(18) is a single-valued function of q with poles at $q=0$, and at $q=-(p^2+\alpha_n^2)$, where $\pm\alpha_n$, n=1,2,---- are the positive ordered roots of

$$\alpha_n J_1(\alpha_n) - mJ_0(\alpha_n) = 0. \tag{19}$$

Using Cauchy's residue theorem, we find

$$\hat{T} = \sqrt{\frac{2}{\pi}}\,\frac{\sin p\beta}{p}\left\{\frac{I_0(p\rho)}{pI_0'(p)+mI_0(p)} - 2\sum_{n=1}^{\infty}\frac{\alpha_n^2 J_0(\alpha_n\rho)e^{-(p^2+\alpha_n^2)\tau}}{(m^2+\alpha_n^2)(p^2+\alpha_n^2)J_0(\alpha_n)}\right\}. \tag{20}$$

On the other hand, the inversion theorem for the Fourier cosine transformation yields,

$$\bar{T} = \sqrt{\frac{2}{\pi}}\int_0^{\infty}\hat{T}(\rho,p,\tau)\cos p\zeta\,dp\ . \tag{21}$$

From eqs.(20),(21) we have, formally,

$$\bar{T}(\rho,\zeta,\tau) = \bar{T}_1(\rho,\zeta) + \bar{T}_2(\rho,\zeta,\tau), \tag{22}$$

where

$$\bar{T}_1 = \frac{2}{\pi}\int_0^{\infty}\frac{\sin p\beta\cos p\zeta}{p}\,\frac{I_0(p\rho)}{pI_1(p)+mI_0(p)}\,dp\ , \tag{23}$$

$$\bar{T}_2 = -\frac{4}{\pi}\int_0^{\infty}\frac{\sin p\beta\cos p\zeta}{p}\sum_{n=1}^{\infty}\frac{\alpha_n^2 J_0(\alpha_n\rho)\,e^{-(p^2+\alpha_n^2)\tau}}{(p^2+\alpha_n^2)(m^2+\alpha_n^2)J_0(\alpha_n)}\,dp\ , \tag{24}$$

and $J_n(\ )$ is the Bessel function of the first kind, of order n.

Evidently, $\bar{T}_1$ is the steady-state temperature distribution, i.e., $\bar{T}(\rho,\zeta,\tau)\rightarrow\bar{T}_1(\rho,\zeta)$ as $\tau\rightarrow\infty$.

In order to carry out the numerical evaluation, it is first convenient to deduce a series representation for the steady-state part of the temperature field, given in integral form by eq.(23). For the purpose we consider the function

$$f(\xi) = \frac{1}{2\pi\xi}\,\frac{J_0(\xi\rho)}{\xi J_1(\xi)-mJ_0(\xi)}\,e^{-\xi\lambda_1}\ , \tag{25}$$

where λ_1 is real and positive and $\xi = R\,e^{-i\theta}$.

Let Γ be the closed contour in the ξ-plane consisting of the two semicircular arcs together with the straight-line segments, as shown in Fig.3.

Since the singularities of $f(\xi)$ which have a positive real part are at $\xi=\alpha_n$ (n=1,2,----), it is easily found that

$$\lim_{R_2\to\infty}\oint_{\Gamma} f(\xi)\,d\xi = i\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{\alpha_n\{\alpha_n J_0(\alpha_n)+mJ_1(\alpha_n)\}}\,e^{-\alpha_n\lambda_1}\ , \tag{26}$$

Observing that the residue of $f(\xi)$ at $\xi=0$ is $-1/2\pi m$, and by means of a well known result[5] in complex-variable integration, we have

$$\lim_{R_1\to 0}\left[\lim_{R_2\to\infty}\oint_{\Gamma} f(\xi)d\xi\right]=\frac{i}{2m}+\int_{i\infty}^{0} f(\xi)d\xi+\int_{0}^{-i\infty} f(\xi)d\xi . \tag{27}$$

With the aid of the respective changes of the variable of integration $\xi=ip$ and $\xi=-ip$, the first and second integrals appearing on the right-hand side of eq.(27) become

$$\int_{i\infty}^{0} f(\xi)d\xi=\frac{1}{2\pi}\int_0^\infty \frac{I_0(p\rho)}{p\{pI_1(p)+mI_0(p)\}}(\cos p\lambda_1-i\sin p\lambda_1)dp, \tag{28}$$

$$\int_{0}^{-i\infty} f(\xi)d\xi=-\frac{1}{2\pi}\int_0^\infty \frac{I_0(p\rho)}{p\{pI_1(p)+mI_0(p)\}}(\cos p\lambda_1+i\sin p\lambda_1)dp . \tag{29}$$

Substituting eqs.(26),(28) and (29) in eq.(27), we obtain

$$\frac{1}{\pi}\int_0^\infty \frac{I_0(p\rho)}{p\{pI_1(p)+mI_0(p)\}}\sin p\lambda_1 dp=\frac{1}{2m}-\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{\alpha_n\{\alpha_n J_0(\alpha_n)+mJ_1(\alpha_n)\}}e^{-\alpha_n\lambda_1} . \tag{30}$$

Assuming first $0\leqq\zeta<\beta$, we substitute $\lambda_1=\beta+\zeta$ and $\lambda_1=\beta-\zeta$ in eq.(30) and add the resulting equation. By substitution of the result into eq.(23), we obtain

$$\bar{T}_1=\frac{1}{m}-2\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(\alpha_n^2+m^2)J_0(\alpha_n)}e^{-\alpha_n\beta}\cosh\alpha_n\zeta . \qquad (0\leqq\zeta<\beta) \tag{31}$$

For $0<\beta<\zeta$, in turn, we substitute $\lambda_1=\zeta-\beta$ in eq.(30) and subtract the resulting equation from that obtained by putting $\lambda_1=\zeta+\beta$. By substitution of the result into eq.(23) once again, we obtain

$$\bar{T}_1=2\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(\alpha_n^2+m^2)J_0(\alpha_n)}e^{-\alpha_n\zeta}\sinh\alpha_n\beta . \qquad (0<\beta<\zeta) \tag{32}$$

Next, with a view toward reducing the transient part eq.(24) of the solution to a series representation which involves only tabulated functions, we consider the following integration

$$R_n(\zeta,\tau)=\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p(p^2+\alpha_n^2)}e^{-(p^2+\alpha_n^2)\tau}dp \tag{33}$$

Factorizing the integrand of eq.(33) and evaluating by reference to standard integral tables [6], thus substituting the resulting equation in eq.(24), we obtain finally

$$\begin{aligned}\bar{T}_2=-\frac{1}{2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}\Big[&e^{\alpha_n(\beta+\zeta)}\operatorname{erfc}\Big(\sqrt{\tau}\alpha_n+\frac{\beta+\zeta}{2\sqrt{\tau}}\Big)-e^{-\alpha_n(\beta+\zeta)}\operatorname{erfc}\Big(\sqrt{\tau}\alpha_n-\frac{\beta+\zeta}{2\sqrt{\tau}}\Big)+e^{\alpha_n(\beta-\zeta)}\operatorname{erfc}\Big(\sqrt{\tau}\alpha_n+\frac{\beta-\zeta}{2\sqrt{\tau}}\Big)\\&-e^{-\alpha_n(\beta-\zeta)}\operatorname{erfc}\Big(\sqrt{\tau}\alpha_n-\frac{\beta-\zeta}{2\sqrt{\tau}}\Big)+2e^{-\alpha_n^2\tau}\Big\{2-\operatorname{erfc}\Big(\frac{\beta+\zeta}{2\sqrt{\tau}}\Big)-\operatorname{erfc}\Big(\frac{\beta-\zeta}{2\sqrt{\tau}}\Big)\Big\}\Big]\end{aligned} \tag{34}$$

for the desired series representation of $\bar{T}_2$.

Substituting eqs.(31),(32) and (34) in eq.(22), we obtain the solution to the heat-conduction equation (6) which satisfies eqs.(7),(8),(9), and is adequate for numerical evaluation of the temperature field.

2.2 Temperature Field under Intermittently Sudden Heat Generation

Now we consider the temperature fields in a circular cylinder under intermittently sudden heat generation Q_i (i=1,2,3,---) per unit time unit area for each time $t=0\sim t_0, t_1\sim t_1', t_2\sim t_2'$,---, as shown in Fig.4. To achieve this goal we assume first that the temperature fields in a circular cylinder subjected to heat generation $\bar{Q}_i$ (i=1,2,3,---) independently over a finite band of the cylinder surface are described by $\bar{T}_1(\rho,\zeta,\tau), \bar{T}_2(\rho,\zeta,\tau)$,----respectively, where $\bar{Q}_i, \bar{T}_i$ (i=1,2,3,---) are dimensionless variables as well as $\bar{Q}$, $\bar{T}$ in eq.(5), by use of eqs.(22), (31),(32),(34), $\bar{T}_i$ is given as follows:

$$\bar{T}_i(\rho,\zeta,\tau)=\frac{\bar{Q}_i}{\bar{Q}}\bar{T}(\rho,\zeta,\tau) \qquad (i=1,2,----)$$

Then the temperature field for the first cooling time $\tau_0'\sim\tau_1$ admits the representation

$$\bar{\bar{T}}(\rho,\zeta,\tau)=\bar{T}_1(\rho,\zeta,\tau)-\bar{T}_1(\rho,\zeta,\tau-\tau_0'), \tag{35}$$

and the temperature field for the second heat-generating time $\tau_1\sim\tau_1'$ is given as follows:

$$\bar{\bar{T}}(\rho,\zeta,\tau)=\bar{T}_1(\rho,\zeta,\tau)-\bar{T}_1(\rho,\zeta,\tau-\tau_0')+\bar{T}_2(\rho,\zeta,\tau-\tau_1) \tag{36}$$

Thus the desired temperature fields are given by

$$\bar{\bar{T}}(\rho,\zeta,\tau)=\bar{T}_i^{*}(\rho,\zeta,\tau)+\bar{T}_i(\rho,\zeta,\tau-\tau_{i-1}) \quad \text{for the i-th heat-generating time,} \tag{37}$$

$$\bar{\bar{T}}(\rho,\zeta,\tau)=\bar{T}_i^{*}(\rho,\zeta,\tau)+\bar{T}_i(\rho,\zeta,\tau-\tau_{i-1})-\bar{T}_i(\rho,\zeta,\tau-\tau_{i-1}') \quad \text{for the i-th cooling time,} \tag{38}$$

where $T_i^{*}(\rho,\zeta,\tau)$ denotes the temperature field before the i-th heat generation. For the sake of brevity, we consider the case of $\bar{Q}_i(i=1,2,---)=\bar{Q}$, as shown in Fig.5. Applying eqs.(31),(32),(34) to eqs.(37),(38), we find that the temperature fields for the i-th generating-time and the i-th cooling time are expressed as follows:

$$\begin{aligned}
\bar{\bar{T}}(\rho,\zeta,\tau)= & \left[\begin{array}{ll}\dfrac{1}{m}-2\displaystyle\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(\alpha_n^2+m^2)J_0(\alpha_n)}e^{-\alpha_n\beta}\cosh\alpha_n\zeta & (0\leqq\zeta<\beta)\\ 2\displaystyle\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(\alpha_n^2+m^2)J_0(\alpha_n)}e^{-\alpha_n\zeta}\sinh\alpha_n\zeta & (0<\beta<\zeta)\end{array}\right]-\frac{1}{2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}\\
& \Bigg[\Bigg[e^{\alpha_n(\beta+\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n+\frac{\beta+\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)-\sum_{j=1}^{i-1}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n+\frac{\beta+\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}\\
& -e^{-\alpha_n(\beta+\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n-\frac{\beta+\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)-\sum_{j=1}^{i-1}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n-\frac{\beta+\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}\\
& +e^{\alpha_n(\beta-\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n+\frac{\beta-\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)-\sum_{j=1}^{i-1}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n+\frac{\beta-\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}\\
& -e^{-\alpha_n(\beta-\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n-\frac{\beta-\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)-\sum_{j=1}^{i-1}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n-\frac{\beta-\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}\\
& +2\Big[\sum_{j=1}^{i}e^{-\alpha_n^2\{\tau-(j-1)(\tau_0+\tau_1)\}}\Big\{2-\operatorname{erfc}\Big(\frac{\beta+\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)-\operatorname{erfc}\Big(\frac{\beta-\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)\Big\}\\
& -\sum_{j=1}^{i-1}e^{-\alpha_n^2\{\tau-j\tau_0-(j-1)\tau_1\}}\Big\{2-\operatorname{erfc}\Big(\frac{\beta+\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)-\operatorname{erfc}\Big(\frac{\beta-\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}\Big]\Bigg]\Bigg] \quad (i\geqq 2)
\end{aligned}$$

for the i-th heat generating time, (39)

$$\begin{aligned}
\bar{\bar{T}}(\rho,\zeta,\tau)= & -\frac{1}{2}\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}\Bigg[\Bigg[e^{\alpha_n(\beta+\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n+\frac{\beta+\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)\\
& -\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n+\frac{\beta+\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}-e^{-\alpha_n(\beta+\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n-\frac{\beta+\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)\\
& -\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n-\frac{\beta+\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}+e^{\alpha_n(\beta-\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n+\frac{\beta-\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)\\
& -\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n+\frac{\beta-\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\Big)\Big\}-e^{-\alpha_n(\beta-\zeta)}\Big\{\sum_{j=1}^{i}\operatorname{erfc}\Big(\sqrt{\tau-(j-1)(\tau_0+\tau_1)}\,\alpha_n-\frac{\beta-\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\Big)
\end{aligned}$$

$$-\sum_{j=1}^{i} erfc\left(\sqrt{\tau-j\tau_0-(j-1)\tau_1}\,\alpha_n-\frac{\beta-\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\right)\Big\}$$

$$+2\left[\sum_{j=1}^{i} e^{-\alpha_n^2\{\tau-(j-1)(\tau_0+\tau_1)\}}\left\{2-erfc\left(\frac{\beta+\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\right)-erfc\left(\frac{\beta-\zeta}{2\sqrt{\tau-(j-1)(\tau_0+\tau_1)}}\right)\right\}\right.$$

$$\left.-\sum_{j=1}^{i} e^{-\alpha_n^2\{\tau-j\tau_0-(j-1)\tau_1\}}\left\{2-erfc\left(\frac{\beta+\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\right)-erfc\left(\frac{\beta-\zeta}{2\sqrt{\tau-j\tau_0-(j-1)\tau_1}}\right)\right\}\right]\Bigg] \qquad (i \geqq 1)$$

for the i-th cooling time. (40)

3. Stress Field

Now we solve the thermoelasticity problem associated with the temperature field described in section 2 with aid of a set of stress functions discussed by Hoyle[4].

Then, the complete determination of thermal stress field reduces to the determination of dimensionless stress functions $\bar{\Phi}(\rho,\zeta,\tau)$, $\bar{\Psi}(\rho,\zeta,\tau)$ satisfying

$$\nabla^2\bar{\Phi}=0, \qquad (41) \qquad\qquad \nabla^2\bar{\Psi}=\frac{1}{1-\nu}\frac{\partial^2\bar{\Phi}}{\partial\zeta^2}-\bar{T}, \qquad (42)$$

where the functions $\bar{\Phi},\bar{\Psi}$ are related to the stress function Φ,Ψ by

$$\bar{\Phi}=\frac{(1-\nu)\Phi}{a^2E\alpha T_0\bar{Q}}, \qquad \bar{\Psi}=\frac{(1-\nu)\Psi}{a^2E\alpha T_0\bar{Q}}. \qquad (43)$$

If the dimensionless stresses $\bar{\sigma}_{rr}$, $\bar{\sigma}_{\theta\theta}$, -- are related to the actual components by

$$\bar{\sigma}_{rr}=\frac{(1-\nu)\sigma_{rr}}{E\alpha T\bar{Q}}, \qquad \bar{\sigma}_{\theta\theta}=\frac{(1-\nu)\sigma_{\theta\theta}}{E\alpha T\bar{Q}}, \text{ ----------} \qquad (44)$$

the stress field is obtained from

$$\bar{\sigma}_{rr}=\frac{\partial^2\bar{\Psi}}{\partial\zeta^2}+\frac{1}{\rho}\left(\frac{\partial\bar{\Psi}}{\partial\rho}+\frac{\partial\bar{\Phi}}{\partial\rho}\right), \quad \bar{\sigma}_{\theta\theta}=\nu\nabla^2\bar{\Psi}-\frac{1}{\rho}\left(\frac{\partial\bar{\Psi}}{\partial\rho}+\frac{\partial\bar{\Phi}}{\partial\rho}\right)-(1-\nu)\bar{T},$$

$$\bar{\sigma}_{zz}=\frac{\partial^2\bar{\Psi}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\bar{\Psi}}{\partial\rho}, \qquad \bar{\sigma}_{rz}=-\frac{\partial^2\bar{\Psi}}{\partial\rho\partial\zeta}. \qquad (45)$$

Since we assume that the surface of the cylinder is free from traction, the boundary conditions on the stress field are

$$\bar{\sigma}_{rr}=0, \quad \bar{\sigma}_{rz}=0 \qquad at \quad \rho=1 \qquad (46)$$

and, evidently,

$$\bar{\sigma}_{rr},\ \bar{\sigma}_{\theta\theta},\ \bar{\sigma}_{zz},\ \bar{\sigma}_{rz}\longrightarrow 0 \qquad as \quad \zeta\longrightarrow\infty \qquad (47)$$

The thermal stress fields associated with the temperature fields obtained in section 2 require the determination of solutions $\bar{\Phi},\bar{\Psi}$ to eqs.(41),(42), such that the stresses of eq.(45) satisfy the conditions implied by eqs.(46),(47).

The general solution of eq.(42) may be written as

$$\bar{\Psi}=\bar{\Psi}_0(\rho,\zeta,\tau)+\bar{\Psi}_1(\rho,\zeta)+\bar{\Psi}_2(\rho,\zeta,\tau)+\bar{\Psi}_3(\rho,\zeta,\tau), \qquad (48)$$

where $\bar{\Psi}_0$ is harmonic, while $\bar{\Psi}_1$, $\bar{\Psi}_2$ and $\bar{\Psi}_3$ stand for the particular solutions to the differential equations

$$\nabla^2\bar{\Psi}=-\bar{T}_1, \qquad (49)$$

$$\nabla^2\bar{\Psi}=-\bar{T}_2, \qquad (50)$$

$$\nabla^2\bar{\Psi}=\frac{1}{1-\nu}\frac{\partial^2\bar{\Phi}}{\partial\zeta^2}. \qquad (51)$$

3.1 Sress Field Associated with Case 2.1 in Section 2

The harmonic functions $\bar{\Phi},\bar{\Psi}_0$ are assumed in the form

$$\bar{\Phi}=\frac{2}{\pi}\int_0^\infty A_1(p,\tau)I_0(p\rho)\sin p\beta\cos p\zeta\,dp, \qquad (52)$$

$$\bar{\Psi}_0=\frac{2}{\pi}\int_0^\infty B_1(p,\tau)I_0(p\rho)\sin p\beta\cos p\zeta\,dp, \qquad (53)$$

in which $A_1(p,\tau)$, $B_1(p,\tau)$ are as yet unknown functions to be determined consistent with the boundary conditions, eqs. (46), (47).

Substituting from eqs. (23), (24) into eqs. (49), (50) respectively, in turn, we obtain the appropriate particular solutions to eqs. (49), (50) as follows:

$$\bar{\Psi}_1 = -\frac{1}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p^2} \frac{\rho I_1(p\rho)}{\{pI_1(p)+mI_0(p)\}} dp \tag{54}$$

$$\bar{\Psi}_2 = -\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p} \sum_{n=1}^{\infty} \frac{\alpha_n^2 J_0(\alpha_n \rho) e^{-(p^2+\alpha_n^2)\tau}}{(p^2+\alpha_n^2)^2(m^2+\alpha_n^2)J_0(\alpha_n)} dp \tag{55}$$

Substituting from eq. (52) into eq. (51), we also obtain

$$\bar{\Psi}_3 = -\frac{1}{\pi(1-\nu)}\int_0^\infty A_1(p,\tau) p\rho I_1(p\rho) \sin p\beta \cos p\zeta \, dp. \tag{56}$$

Applying eqs. (48), (52), (53), (54), (55), (56) to the formulas for $\bar{\sigma}_{rr}$ and $\bar{\sigma}_{rz}$ in eq. (45), evaluating the resulting expressions at $\rho=1$, and applying eq. (46), we arrive at a pair of simultaneous linear algebraic equation in $A_1(p,\tau)$ and $B_1(p,\tau)$. Solving the resulting equation, and substituting $A_1(p,\tau)$, $B_1(p,\tau)$ in eqs. (52), (53), (56), we obtain

$$\bar{\Phi} = \frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p\{pI_1(p)+mI_0(p)\}\nabla(p)} \left[I_0^2(p) - I_1^2(p) - 4\{pI_1(p)+mI_0(p)\}^2 C_1(p,\tau) \right] dp, \tag{57}$$

$$\begin{aligned} \bar{\Psi} = {} & \frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p^2\{pI_1(p)+mI_0(p)\}\nabla(p)} \{ I_0(p)I_1(p)I_0(p\rho) - \rho I_1^2(p)I_1(p\rho)\} dp \\ & - \frac{4}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p^2} \frac{\left[\{p^3 I_0(p) + m(p^2+2(1-\nu))I_1(p)\}I_0(p\rho) - \{pI_1(p)+mI_0(p)\}p^3\rho I_1(p\rho)\right] C_1(p,\tau)}{\nabla(p)} dp \\ & - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\alpha_n^3 J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)} S_n(\zeta,\tau), \end{aligned} \tag{58}$$

where

$$\nabla(p) = \{p^2 + 2(1-\nu)\}I_1^2(p) - p^2 I_0^2(p), \tag{59}$$

$$C_1(p,\tau) = \sum_{n=1}^{\infty} \frac{\alpha_n^2 e^{-(p^2+\alpha_n^2)\tau}}{(p^2+\alpha_n^2)^2(m^2+\alpha_n^2)}, \tag{60}$$

and since the improper integral of the infinite series in eq. (55) is inconvenient for numerical work, we reserve the order of integration and summation for $\bar{\Psi}_3$ and consider a integration

$$S_n(\zeta,\tau) = \int_0^\infty \frac{\sin p\beta \cos p\zeta}{p(p^2+\alpha_n^2)^2} e^{-(p^2+\alpha_n^2)\tau} dp \tag{61}$$

With aid of eq. (33), we obtain a following series representation for $S_n(\zeta,\tau)$ which involves only the exponential function and the complementaly error function.

$$\begin{aligned} S_n(\zeta,\tau) = \frac{\pi}{8}\alpha_n^{-4} \Big[& \Big\{1 - \frac{\alpha_n(\beta+\zeta)}{2} - \alpha_n^2\tau\Big\} e^{\alpha_n(\beta+\zeta)} \operatorname{erfc}\Big(\sqrt{\tau}\alpha_n + \frac{\beta+\zeta}{2\sqrt{\tau}}\Big) - \Big\{1 + \frac{\alpha_n(\beta+\zeta)}{2} - \alpha_n^2\tau\Big\} e^{-\alpha_n(\beta+\zeta)} \operatorname{erfc}\Big(\sqrt{\tau}\alpha_n - \frac{\beta+\zeta}{2\sqrt{\tau}}\Big) \\ & + \Big\{1 - \frac{\alpha_n(\beta-\zeta)}{2} - \alpha_n^2\tau\Big\} e^{\alpha_n(\beta-\zeta)} \operatorname{erfc}\Big(\sqrt{\tau}\alpha_n + \frac{\beta-\zeta}{2\sqrt{\tau}}\Big) - \Big\{1 + \frac{\alpha_n(\beta-\zeta)}{2} - \alpha_n^2\tau\Big\} e^{-\alpha_n(\beta-\zeta)} \operatorname{erfc}\Big(\sqrt{\tau}\alpha_n - \frac{\beta-\zeta}{2\sqrt{\tau}}\Big) \\ & + 2e^{-\alpha_n^2\tau}\Big\{2 - \operatorname{erfc}\Big(\frac{\beta+\zeta}{2\sqrt{\tau}}\Big) - \operatorname{erfc}\Big(\frac{\beta-\zeta}{2\sqrt{\tau}}\Big)\Big\}\Big] \end{aligned} \tag{62}$$

By use of eqs. (45), (57), (58), the thermal stresses are

$$\bar{\sigma}_{rr} = \frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p\{pI_1(p)+mI_0(p)\}\nabla(p)} \left[\{p\rho I_1^2(p) + \rho^{-1}(pI_0^2(p) - pI_1^2(p) + I_0(p)I_1(p))\}I_1(p\rho) - \{I_1^2(p) + pI_0(p)I_1(p)\}I_0(p\rho) \right] dp$$

$$+\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\xi}{V(p)}\Big[\!\Big[\{p(m+p^2)I_0(p)+[2m(1-\nu)+p^2(m+1)]I_1(p)\}I_0(p\rho)-\{[2m(1-\nu)+p^2]pI_0(p)$$

$$+[2m(1-\nu)+p^2(m+2(1-\nu))]I_1(p)\}p^{-1}\rho^{-1}I_1(p\rho)-\{pI_1(p)+mI_0(p)\}p^2\rho I_1(p\rho)\Big]\!\Big]C_1(p,\tau)dp$$

$$+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{(m^2+\alpha_n^2)J_0(\alpha_n)}\left[\alpha_n\rho^{-1}J_1(\alpha_n\rho)S_n(\xi,\tau)+J_0(\alpha_n\rho)\{R_n(\xi,\tau)-\alpha_n^2S_n(\xi,\tau)\}\right], \qquad (63)$$

$$\bar{\sigma}_{\theta\theta}=\frac{2(1-\nu)}{\pi}\int_0^\infty\frac{\sin p\beta\cos p\xi}{p\{pI_1(p)+mI_0(p)\}V(p)}\left[\{pI_1^2(p)-pI_0^2(p)-I_0(p)I_1(p)\}\rho^{-1}I_1(p\rho)+\{p^2I_0^2(p)-(1+p^2)I_1^2(p)\}I_0(p\rho)\right]dp$$

$$+\frac{4}{\pi}\int_0^\infty\frac{\sin p\beta\cos p\xi}{V(p)}\Big[\!\Big[\{[p^2+2m(1-\nu)]pI_0(p)+[m(p^2+2(1-\nu))+2(1-\nu)p^2]I_1(p)\}p^{-1}\rho^{-1}I_1(p\rho)$$

$$-(1-2\nu)\{pI_1(p)+mI_0(p)\}pI_0(p\rho)\Big]\!\Big]C_1(p,\tau)dp$$

$$-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{(m^2+\alpha_n^2)J_0(\alpha_n)}\{\alpha_n\rho^{-1}J_1(\alpha_n\rho)S_n(\xi,\tau)-J_0(\alpha_n\rho)R_n(\xi,\tau)\}, \qquad (64)$$

$$\bar{\sigma}_{zz}=\frac{2(1-\nu)}{\pi}\int_0^\infty\frac{\sin p\beta\cos p\xi}{p\{pI_1(p)+mI_0(p)\}V(p)}\left[\{pI_0(p)I_1(p)-2I_1^2(p)\}I_0(p\rho)-p\rho I_1^2(p)I_1(p\rho)\right]dp$$

$$+\frac{4}{\pi}\int_0^\infty\frac{\sin p\beta\cos p\xi}{V(p)}\Big[\!\Big[\{p(2m-p^2)I_0(p)-[2m(1-\nu)+p^2(m-2)]I_1(p)\}I_0(p\rho)+\{pI_1(p)+mI_0(p)\}p^2\rho I_1(p\rho)\Big]\!\Big]C_1(p,\tau)dp$$

$$+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^4J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}S_n(\xi,\tau), \qquad (65)$$

$$\bar{\sigma}_{rz}=\frac{2(1-\nu)}{\pi}\int_0^\infty\frac{\sin p\beta\cos p\xi}{\{pI_1(p)+mI_0(p)\}V(p)}\{I_0(p)I_1(p)I_1(p\rho)-\rho I_1^2(p)I_0(p\rho)\}dp$$

$$-\frac{4}{\pi}\int_0^\infty\frac{\sin p\beta\sin p\xi}{V(p)}\Big[\!\Big[\{p^3I_0(p)+m[p^2+2(1-\nu)]I_1(p)\}I_1(p\rho)-\{pI_1(p)+mI_0(p)\}p^2\rho I_0(p\rho)\Big]\!\Big]C_1(p,\tau)dp$$

$$+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^3J_1(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}T_n(\xi,\tau), \qquad (66)$$

where

$$T_n(\xi,\tau)=\frac{\pi}{16\alpha_n^3}\Big[\{1-2\alpha_n^2\tau+\alpha_n(\beta-\xi)\}e^{-\alpha_n(\beta-\xi)}\mathrm{erfc}\Big(\alpha_n\sqrt{\tau}-\frac{\beta-\xi}{2\sqrt{\tau}}\Big)+\{1-2\alpha_n^2\tau-\alpha_n(\beta-\xi)\}e^{-\alpha_n(\beta-\xi)}\mathrm{erfc}\Big(\alpha_n\sqrt{\tau}+\frac{\beta-\xi}{2\sqrt{\tau}}\Big)$$

$$-\{1-2\alpha_n^2\tau+\alpha_n(\beta-\xi)\}e^{-\alpha_n(\beta+\xi)}\mathrm{erfc}\Big(\alpha_n\sqrt{\tau}-\frac{\beta+\xi}{2\sqrt{\tau}}\Big)-\{1-2\alpha_n^2\tau-\alpha_n(\beta+\xi)\}e^{\alpha_n(\beta+\xi)}\mathrm{erfc}\Big(\alpha_n\sqrt{\tau}+\frac{\beta+\xi}{2\sqrt{\tau}}\Big)$$

$$+\frac{4\alpha_n\sqrt{\tau}}{\sqrt{\pi}}e^{-\alpha_n^2\tau}\Big\{e^{-\left(\frac{\beta-\xi}{2\sqrt{\tau}}\right)^2}-e^{-\left(\frac{\beta+\xi}{2\sqrt{\tau}}\right)^2}\Big\}\Big]. \qquad (67)$$

3.2 Stress Field Associated with Case 2.2 in Section 2

The harmonic functions $\bar{\Phi}$, $\bar{\Psi}_0$, and the particular solution $\bar{\Psi}_3$ are given by eqs. (52), (53) and (56) as well as case 3.1.

In order to obtain the solution of the thermal stress problem associated with the temperature field of case 2.2 in section 2 easily, eqs. (39), (40) are required to express as the integrals. By eqs. (23), (24), (37), (38), we obtain

$$\bar{\bar{T}} = \frac{2}{\pi}\int_0^\infty \frac{\sin p\beta \cos p\zeta}{p} \frac{I_0(p\rho)}{pI_1(p)+mI_0(p)}dp - \frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p}\sum_{n=1}^\infty \frac{\alpha_n^2 J_0(\alpha_n\rho)}{(p^2+\alpha_n^2)(m^2+\alpha_n^2)J_0(\alpha_n)}\left\{\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_a^2} - \sum_{j=1}^{i-1} e^{-(p^2+\alpha_n^2)\tau_b^2}\right\}dp$$

$(i \geqq 2)$ for the i-th heat generating-time, (68)

$$\bar{\bar{T}} = -\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p}\sum_{n=1}^\infty \frac{\alpha_n^2 J_0(\alpha_n\rho)}{(p^2+\alpha_n^2)(m^2+\alpha_n^2)J_0(\alpha_n)}\left\{\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_a^2} - \sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_b^2}\right\}dp$$

$(i \geqq 1)$ for the i-th cooling time, (69)

where $\tau_a = \sqrt{\tau-(j-1)(\tau_0+\tau_1)}$, $\tau_b = \sqrt{\tau - j\tau_0 - (j-1)\tau_1}$.

In order to deduce a series representation for $\bar{\bar{T}}(\rho,\zeta,\tau)$ which involves only tabulated functions, we first reverse the order of integration and summation in the second integral of eq. (68), and the integral of eq. (69) and consider the integrals

$$R_{1n}(\zeta,\tau) = \int_0^\infty \frac{\sin p\beta\cos p\zeta}{p(p^2+\alpha_n^2)}\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_a^2}dp, \tag{70}$$

$$R_{2n}(\zeta,\tau) = \int_0^\infty \frac{\sin p\beta\cos p\zeta}{p(p^2+\alpha_n^2)}\sum_{j=1}^{i-1} e^{-(p^2+\alpha_n^2)\tau_b^2}dp, \tag{71}$$

$$R_{3n}(\zeta,\tau) = \int_0^\infty \frac{\sin p\beta\cos p\zeta}{p(p^2+\alpha_n^2)}\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_b^2}dp, \tag{72}$$

By reference to eq. (33), we obtain

$$R_{1n}(\zeta,\tau) = \sum_{j=1}^{i}\frac{\pi}{8\alpha_n^2}\Big[e^{\alpha_n(\beta+\zeta)}\mathrm{erfc}\Big(\tau_a\alpha_n+\frac{\beta+\zeta}{2\tau_a}\Big) - e^{-\alpha_n(\beta+\zeta)}\mathrm{erfc}\Big(\tau_a\alpha_n-\frac{\beta+\zeta}{2\tau_a}\Big) + e^{\alpha_n(\beta-\zeta)}\mathrm{erfc}\Big(\tau_a\alpha_n+\frac{\beta-\zeta}{2\tau_a}\Big)$$
$$- e^{-\alpha_n(\beta-\zeta)}\mathrm{erfc}\Big(\tau_a\alpha_n-\frac{\beta-\zeta}{2\tau_a}\Big) + 2e^{-\alpha_n^2\tau_a^2}\Big\{2-\mathrm{erfc}\Big(\frac{\beta+\zeta}{2\tau_a}\Big)-\mathrm{erfc}\Big(\frac{\beta-\zeta}{2\tau_a}\Big)\Big\}\Big], \tag{73}$$

$$R_{2n}(\zeta,\tau) = \sum_{j=1}^{i-1}\frac{\pi}{8\alpha_n^2}\Big[e^{\alpha_n(\beta+\zeta)}\mathrm{erfc}\Big(\tau_b\alpha_n+\frac{\beta+\zeta}{2\tau_b}\Big) - e^{-\alpha_n(\beta+\zeta)}\mathrm{erfc}\Big(\tau_b\alpha_n-\frac{\beta+\zeta}{2\tau_b}\Big) + e^{\alpha_n(\beta-\zeta)}\mathrm{erfc}\Big(\tau_b\alpha_n+\frac{\beta-\zeta}{2\tau_b}\Big)$$
$$- e^{-\alpha_n(\beta-\zeta)}\mathrm{erfc}\Big(\tau_b\alpha_n-\frac{\beta-\zeta}{2\tau_b}\Big) + 2e^{-\alpha_n^2\tau_b^2}\Big\{2-\mathrm{erfc}\Big(\frac{\beta+\zeta}{2\tau_b}\Big)-\mathrm{erfc}\Big(\frac{\beta-\zeta}{2\tau_b}\Big)\Big\}\Big]. \tag{74}$$

On the other hand, $R_{3n}(\zeta,\tau)$ is obtained by replacing $\sum_{j=1}^{i-1}$ in eq. (74) with $\sum_{j=1}^{i}$.

(i) For the i-th heat-generating time

Substituting the steady-state part and transient part of eq. (68) into eqs. (49), (50), we obtain the particular solutions of eqs. (49), (50)

$$\bar{\Psi}_1 = -\frac{1}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p^2}\frac{\rho I_1(p\rho)}{\{pI_1(p)+mI_0(p)\}}dp, \tag{75}$$

$$\bar{\Psi}_2 = -\frac{1}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p}\sum_{n=1}^\infty \frac{\alpha_n^2 J_0(\alpha_n\rho)}{(p^2+\alpha_n^2)^2(m^2+\alpha_n^2)J_0(\alpha_n)}\left\{\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_a^2} - \sum_{j=1}^{i-1} e^{-(p^2+\alpha_n^2)\tau_b^2}\right\}.$$

$(i \geqq 2)$ (76)

Substituting eqs.(48),(52),(53),(75),(76),(56) into the formulas for $\bar{\sigma}_{rr}$ and $\bar{\sigma}_{rz}$ in eq.(45), evaluating the resulting expressions at $\rho=1$, and applying eq.(46), we obtain a pair of simultaneous linear algebraic equations in $A_1(p,\tau)$ and $B_1(p,\tau)$, the solution of which is given by

$$A_1(p,\tau) = \frac{(1-\nu)\left[\{I_0^2(p)-I_1^2(p)\}-4\{pI_1(p)+mI_0(p)\}^2\mathcal{G}(p,\tau)\right]}{p\{pI_1(p)+mI_0(p)\}\,V(p)} \tag{77}$$

$$B_1(p,\tau) = \frac{(1-\nu)I_0(p)I_1(p)-2\{pI_1(p)+mI_0(p)\}\left[p^3I_0(p)+m\{p^2+2(1-\nu)\}I_1(p)\right]\mathcal{G}(p,\tau)}{p^2\{pI_1(p)+mI_0(p)\}\,V(p)} \tag{78}$$

With the substitution from eqs.(77),(78) into eqs.(52),(53) and (56), the stress functions are written as

$$\bar{\Phi} = \frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\cdot I_0(p\rho)\left[\{I_0^2(p)-I_1^2(p)\}-4\{pI_1(p)+mI_0(p)\}^2\mathcal{G}(p,\tau)\right]}{p\{pI_1(p)+mI_0(p)\}\,V(p)}\,dp \tag{79}$$

$$\begin{aligned}\bar{\Psi} = &-\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\left[I_0(p\rho)\{p^3I_0(p)+m[p^2+2(1-\nu)]I_1(p)\}-p^2\rho I_1(p\rho)\{pI_1(p)+mI_0(p)\}\right]\mathcal{G}(p,\tau)}{p^2V(p)}\\
&+\frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\left[I_0(p)I_1(p)I_0(p\rho)-\rho I_1^2(p)I_1(p\rho)\right]}{p^2\{pI_1(p)+mI_0(p)\}\,V(p)}\\
&-\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p}\sum_{n=1}^{\infty}\frac{\alpha_n^2J_0(\alpha_n\rho)}{(p^2+\alpha_n^2)^2(m^2+\alpha_n^2)J_0(\alpha_n)}\left\{\sum_{j=1}^{i}e^{-(p^2+\alpha_n^2)\tau_a^2}-\sum_{j=1}^{i-1}e^{-(p^2+\alpha_n^2)\tau_b^2}\right\}dp \quad (i\geq 2)\end{aligned} \tag{80}$$

where

$$\mathcal{G}(p,\tau) = \sum_{n=1}^{\infty}\frac{\alpha_n^2}{(p^2+\alpha_n^2)^2(m^2+\alpha_n^2)}\left\{\sum_{j=1}^{i}e^{-(p^2+\alpha_n^2)\tau_a^2}-\sum_{j=1}^{i-1}e^{-(p^2+\alpha_n^2)\tau_b^2}\right\} \tag{81}$$

Applying eqs.(68),(73),(74),(79),(80) to eq.(45), we obtain finally

$$\begin{aligned}\bar{\sigma}_{rr} = &\frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\left[I_1(p\rho)\{p\rho I_1^2(p)+\rho^{-1}(pI_0^2(p)-pI_1^2(p)+I_0(p)I_1(p))\}-I_0(p\rho)\{I_1^2(p)+pI_0(p)I_1(p)\}\right]}{p\{pI_1(p)+mI_0(p)\}\left[\{p^2+2(1-\nu)\}I_1^2(p)-p^2I_0^2(p)\right]}\,dp\\
&+\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{V(p)}\Big[\Big[I_0(p\rho)\left[p(m+p^2)I_0(p)+\{2m(1-\nu)+p^2(m+1)\}I_1(p)\right]-(p\rho)^{-1}I_1(p\rho)\left[p\{2m(1-\nu)+p^2\}I_0(p)\right.\\
&\left.+\{2m(1-\nu)+p^2(m+2(1-\nu))\}I_1(p)\right]-p^2\rho I_1(p\rho)\{pI_1(p)+mI_0(p)\}\Big]\Big]\,\mathcal{G}(p,\tau)\,dp\\
&+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{(m^2+\alpha_n^2)J_0(\alpha_n)}\left[\alpha_n\rho^{-1}J_1(\alpha_n\rho)\{S_{1n}(\zeta,\tau)-S_{2n}(\zeta,\tau)\}+J_0(\alpha_n\rho)\{R_{1n}(\zeta,\tau)-\alpha_n^2S_{1n}(\zeta,\tau)-R_{2n}(\zeta,\tau)+\alpha_n^2S_{2n}(\zeta,\tau)\}\right]\end{aligned} \tag{82}$$

$$\begin{aligned}\bar{\sigma}_{\theta\theta} = &\frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\left[\rho^{-1}I_1(p\rho)\{pI_1^2(p)-pI_0^2(p)-I_0(p)I_1(p)\}+I_0(p\rho)\{p^2I_0^2(p)-(1+p^2)I_1^2(p)\}\right]}{p\{pI_1(p)+mI_0(p)\}\,V(p)}\,dp\\
&+\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\Big[\Big[-(1-2\nu)pI_0(p\rho)\{pI_1(p)+mI_0(p)\}+(p\rho)^{-1}I_1(p\rho)\left[pI_0(p)\{p^2+2m(1-\nu)\}+I_1(p)\{m(p^2+2(1-\nu))+2(1-\nu)p^2\}\right]\Big]\Big]}{V(p)}\cdot\mathcal{G}(p,\tau)\,dp\\
&-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{(m^2+\alpha_n^2)J_0(\alpha_n)}\left[\alpha_n\rho^{-1}J_1(\alpha_n\rho)\{S_{1n}(\zeta,\tau)-S_{2n}(\zeta,\tau)\}-J_0(\alpha_n\rho)\{R_{1n}(\zeta,\tau)-R_{2n}(\zeta,\tau)\}\right]\end{aligned} \tag{83}$$

$$\bar{\sigma}_{zz} = \frac{2(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta\left[I_0(p\rho)\{pI_0(p)I_1(p)-2I_1^2(p)\}-p\rho I_1^2(p)I_1(p\rho)\right]}{p\{pI_1(p)+mI_0(p)\}\,V(p)}\,dp$$

$$+\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{V(p)}\Big[\!\Big[I_0(p\rho)\big[p(2m-p^2)I_0(p)-\{2m(1-\nu)+p^2(m-2)\}I_1(p)\big]+p^2\rho I_1(p\rho)\{pI_1(p)+mI_0(p)\}\Big]\!\Big] G'(p,\tau)\,dp$$

$$+\frac{4}{\pi}\sum_{n=1}^\infty \frac{\alpha_n^4 J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}\{S_{1n}(\zeta,\tau)-S_{2n}(\zeta,\tau)\} \tag{84}$$

where

$$S_{1n}(\zeta,\tau)=\frac{\pi}{8\alpha_n^4}\sum_{j=1}^{i}\Big[\!\Big[\Big[1-\tau_a^2\alpha_n^2-\frac{(\beta+\zeta)\alpha_n}{2}\Big]e^{\alpha_n(\beta+\zeta)}\operatorname{erfc}\Big(\tau_a\alpha_n+\frac{\beta+\zeta}{2\tau_a}\Big)-\Big[1-\tau_a^2\alpha_n^2+\frac{(\beta+\zeta)\alpha_n}{2}\Big]e^{-\alpha_n(\beta+\zeta)}\operatorname{erfc}\Big(\tau_a\alpha_n-\frac{\beta+\zeta}{2\tau_a}\Big)$$
$$+\Big[1-\tau_a^2\alpha_n^2-\frac{(\beta-\zeta)\alpha_n}{2}\Big]e^{\alpha_n(\beta-\zeta)}\operatorname{erfc}\Big(\tau_a\alpha_n+\frac{\beta-\zeta}{2\tau_a}\Big)-\Big[1-\tau_a^2\alpha_n^2+\frac{(\beta-\zeta)\alpha_n}{2}\Big]e^{-\alpha_n(\beta-\zeta)}\operatorname{erfc}\Big(\tau_a\alpha_n-\frac{\beta-\zeta}{2\tau_a}\Big)$$
$$+2e^{-\alpha_n^2\tau_a^2}\Big\{2-\operatorname{erfc}\Big(\frac{\beta+\zeta}{2\tau_a}\Big)-\operatorname{erfc}\Big(-\frac{\beta-\zeta}{2\tau_a}\Big)\Big\}\Big]\!\Big] \tag{85}$$

$$S_{2n}(\zeta,\tau)=\frac{\pi}{8\alpha_n^4}\sum_{j=1}^{i-1}\Big[\!\Big[\Big[1-\tau_b^2\alpha_n^2-\frac{(\beta+\zeta)\alpha_n}{2}\Big]e^{\alpha_n(\beta+\zeta)}\operatorname{erfc}\Big(\tau_b\alpha_n+\frac{\beta+\zeta}{2\tau_b}\Big)-\Big[1-\tau_b^2\alpha_n^2+\frac{(\beta+\zeta)\alpha_n}{2}\Big]e^{-\alpha_n(\beta+\zeta)}\operatorname{erfc}\Big(\tau_b\alpha_n-\frac{\beta+\zeta}{2\tau_b}\Big)$$
$$+\Big[1-\tau_b^2\alpha_n^2-\frac{(\beta-\zeta)\alpha_n}{2}\Big]e^{\alpha_n(\beta-\zeta)}\operatorname{erfc}\Big(\tau_b\alpha_n+\frac{\beta-\zeta}{2\tau_b}\Big)-\Big[1-\tau_b^2\alpha_n^2+\frac{(\beta-\zeta)\alpha_n}{2}\Big]e^{-\alpha_n(\beta-\zeta)}\operatorname{erfc}\Big(\tau_b\alpha_n-\frac{\beta-\zeta}{2\tau_b}\Big)$$
$$+2e^{-\alpha_n^2\tau_b^2}\Big\{2-\operatorname{erfc}\Big(\frac{\beta+\zeta}{2\tau_b}\Big)-\operatorname{erfc}\Big(\frac{\beta-\zeta}{2\tau_b}\Big)\Big\}\Big]\!\Big] \tag{86}$$

(ii) For the i-th cooling time

Substituting eq. (69) into eq. (50) once again, we obtain

$$\bar{\Psi}_1=0 \tag{87}$$

$$\bar{\Psi}_2=-\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p}\sum_{n=1}^\infty \frac{\alpha_n^2 J_0(\alpha_n\rho)}{(p^2+\alpha_n^2)^2(m^2+\alpha_n^2)J_0(\alpha_n)}\Big\{\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_a^2}-\sum_{j=1}^{i} e^{-(p^2+\alpha_n^2)\tau_b^2}\Big\}dp \quad (i\geqq 1) \tag{88}$$

Furthermore we obtain the following stress functions for i-th cooling time in the same way as case (i).

$$\bar{\Phi}=-\frac{8(1-\nu)}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p}\,\frac{I_0(p\rho)\{pI_1(p)+mI_0(p)\}M(p,\tau)}{V(p)}dp \tag{89}$$

$$\bar{\Psi}=\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{p^2V(p)}\Big[\!\Big[p^2\rho I_1(p\rho)\{pI_1(p)+mI_0(p)\}-I_0(p\rho)\big[m\{p^2+2(1-\nu)\}I_1(p)+p^3I_0(p)\big]\Big]\!\Big]M(p,\tau)\,dp$$
$$-\frac{4}{\pi}\sum_{n=1}^\infty \frac{\alpha_n^2 J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}\{S_{1n}(\zeta,\tau)-S_{3n}(\zeta,\tau)\} \qquad (i\geqq 1) \tag{90}$$

where $M(p,\tau)$, $S_{3n}(\zeta,\tau)$ are given by replacing $\sum_{j=1}^{i-1}$ in eqs. (81), (86) with $\sum_{j=1}^{i}$ respectively.

The thermal stresses then appear as

$$\bar{\sigma}_{rr}=\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{V(p)}\Big[\!\Big[I_0(p\rho)\big[p(m+p^2)I_0(p)+\{2m(1-\nu)+p^2(m+1)\}I_1(p)\big]-(p\rho)^{-1}I_1(p\rho)\big[p\{2m(1-\nu)+p^2\}I_0(p)$$
$$+\{2m(1-\nu)+p^2(m+2(1-\nu))\}I_1(p)\big]-p^2\rho I_1(p\rho)\{pI_1(p)+mI_0(p)\}\Big]\!\Big]M(p,\tau)\,dp$$
$$+\frac{4}{\pi}\sum_{n=1}^\infty \frac{\alpha_n^2}{(m^2+\alpha_n^2)J_0(\alpha_n)}\Big[\alpha_n\rho^{-1}J_1(\alpha_n\rho)\{S_{1n}(\zeta,\tau)-S_{3n}(\zeta,\tau)\}+J_0(\alpha_n\rho)\{R_{1n}(\zeta,\tau)-\alpha_n^2S_{1n}(\zeta,\tau)-R_{3n}(\zeta,\tau)+\alpha_n^2S_{3n}(\zeta,\tau)\}\Big] \tag{91}$$

$$\bar{\sigma}_{\theta\theta}=\frac{4}{\pi}\int_0^\infty \frac{\sin p\beta\cos p\zeta}{V(p)}\Big[\!\Big[-(1-2\nu)pI_0(p\rho)\{pI_1(p)+mI_0(p)\}+(p\rho)^{-1}I_1(p\rho)\big[pI_0(p)\{p^2+2m(1-\nu)\}+I_1(p)\{m(p^2+2(1-\nu))+2(1-\nu)p^2\}\big]\Big]\!\Big]M(p,\tau)\,dp$$

$$-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{(m^2+\alpha_n^2)J_0(\alpha_n)}\left[\alpha_n\rho^{-1}J_1(\alpha_n\rho)\{S_{1n}(\zeta,\tau)-S_{3n}(\zeta,\tau)\}-J_0(\alpha_n\rho)\{R_{1n}(\zeta,\tau)-R_{3n}(\zeta,\tau)\}\right] \tag{92}$$

$$\bar{\sigma}_{zz}=\frac{4}{\pi}\int_0^{\infty}\frac{\sin p\beta\cos p\zeta}{V(p)}\left[\left[I_0(p\rho)\left[p(2m-p^2)I_0(p)-\{2m(1-\nu)+p^2(m-2)\}I_1(p)\right]+p^2\rho I_1(p\rho)\{pI_1(p)+mI_0(p)\}\right]\right]M(p,\tau)dp$$

$$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\alpha_n^4J_0(\alpha_n\rho)}{(m^2+\alpha_n^2)J_0(\alpha_n)}\{S_{1n}(\zeta,\tau)-S_{3n}(\zeta,\tau)\} \tag{93}$$

4. Numerical results

The numerical evaluation of the solution for the temperature distributions, in eqs.(31),(32),(34),(39),(40), and for the associated thermal stresses, given by eqs.(63)-(66),(82)-(84),(91)-(93), necessitates the computation of certain improper integrals and infinite series.

Numerical calculations were carried out for the space and time dependence of the temperature and of the stress components along the axis, and on the surface of the cylinder. In all of the computations, the heat-generating length, $\beta=b/a$ was chosen to be 0.1, and the values of Poisson's ratio and Biot number were assumed to be given by $\nu=0.3$, $m=0.02$ respectively. In the numerical evaluations of the temperature field and the resulting stresses in a cylinder subjected to intermittently changing heat generation, furthermore, the heat-generating time and the cooling time were taken as $\tau_0=0.05$, $\tau_1=0.50$ respectively.

The axial variation of temperature at the surface and the axis of the cylinder under constant heat generation at various times, is illustrated in Fig.6. The graph for $\bar{\sigma}_{rr}$ along the axis and $\bar{\sigma}_{\theta\theta}$ on the surface of the cylinder subjected to constant heat generation, as function of ζ for various times, appears in Fig.7. As is apparent from Fig.7, $\bar{\sigma}_{rr}$ along the axis and $\bar{\sigma}_{\theta\theta}$ on the cylinder surface for $\tau=0.01$ and steady-state undergo a reversal in sign. From the computations of $\bar{\sigma}_{\theta\theta}$ for various times between $\tau=0.1$ and $\tau=1.0$, we found that the circumferential stress, $\bar{\sigma}_{\theta\theta}$ at $\rho=1,\zeta=0$ reaches a negative maximum at approximately $\tau=0.25$ and then decreases toward its steady-state value. Finally, Fig.8 and Fig.9 depict the time dependence of the temperature $\bar{\bar{T}}$ and the thermal stress components $\bar{\sigma}_{\theta\theta}$ at $\rho=1,\zeta=0$ and $\bar{\sigma}_{rr}$ at $\rho=\zeta=0$ in the cylinder subjected to intermittently sudden heat generation. With a increase of the number of heat generation, a negative maximum of $\bar{\sigma}_{\theta\theta}$ which occurs at the end of heat-generating time, decreases and a maximum of temperature at the same time increases slightly.

REFERENCES

[1] YOUNGDAHL,C.K.,STERNBERG,ELI., "Transient Thermal Stresses in a Circular Cylinder," Trans. ASME, E, 48, 25(1961)

[2] IYENGAR,K.T.S.R.,CHANDRASHEKHARA,K., "Thermal Stresses in a Finite Solid Cylinder due to an Axisymmtric Temperature Field at the End Surface," Nucl. Eng. Design, 3, 21(1966)

[3] IGNACZAK,J., "Thermal Stresses in a Long Cylinder Heated in a Discontinuous Manner over the Lateral Surface," Arch. Mech. Stosow., 10, 25(1958)

[4] HOYLE,R.D., "Transient Temperature Stresses in Axially Symmetrical Systems with Special Application to a Solid Rotor of a Steam Turbin," Proc. Inst. Mech. Engrs., 169, 553(1955)

[5] MACROBERT,T.M., "Functions of a Complex Variable," Macmillan Company, London, (1947)

[6] ERDELYI,A., "Tables of Integral Transforms," vol.1, McGraw-Hill, New York, (1954)

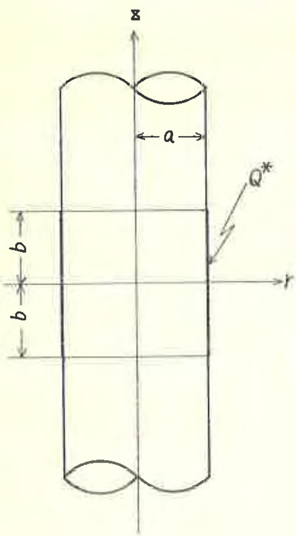

Fig.1 Coordinate system

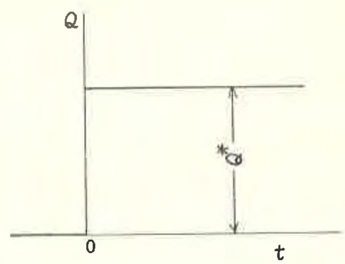

Fig.2 Heat generation at the constant rate

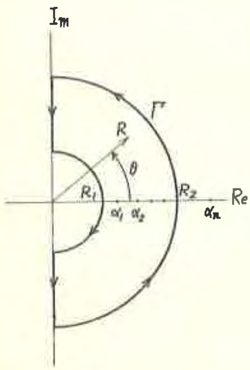

Fig.3 Integral path

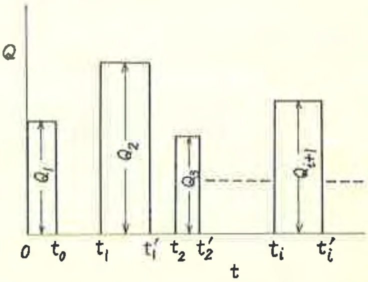

Fig.4 Intermittently changing heat generation

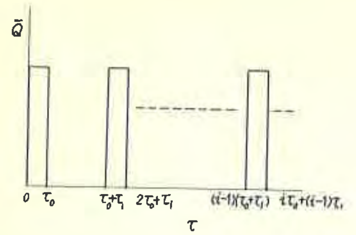

Fig.5 Intermittently changing heat generation

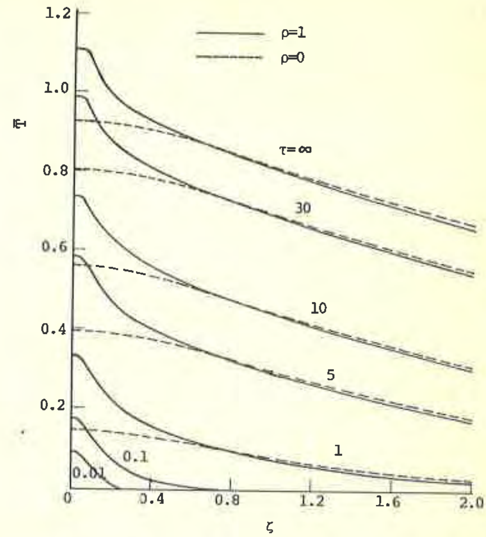

Fig.6 Temperature at ρ=0 and ρ=1 as a function of ζ for various values of τ (β=0.1)

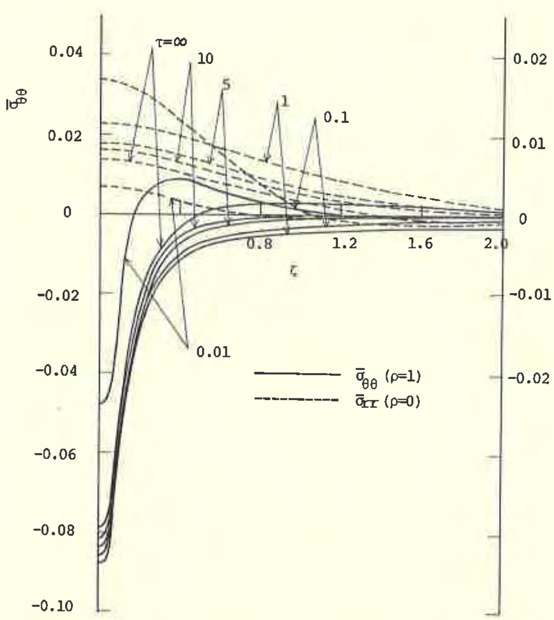

Fig.7 Radial and circumferential stresses at ρ=0 or ρ=1 as a function of ζ for various values of τ (β=0.1)

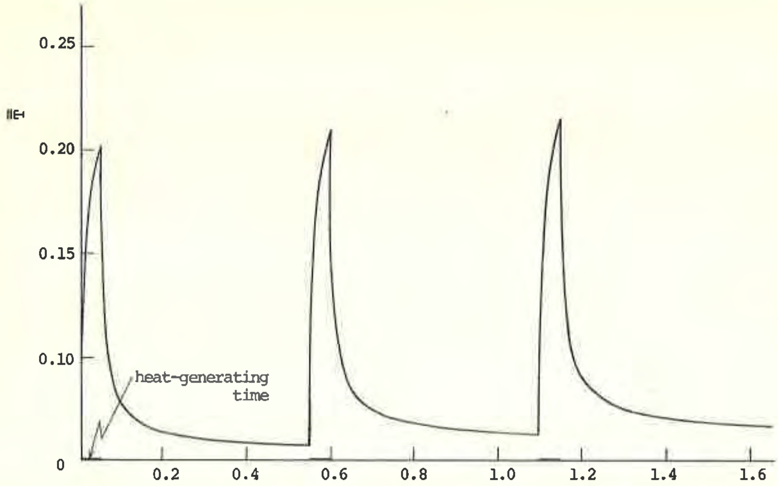

Fig.8 Temperature at ρ=1, ζ=0 as a function of τ (τ_0=0.05, τ_1=0.50, β=0.1)

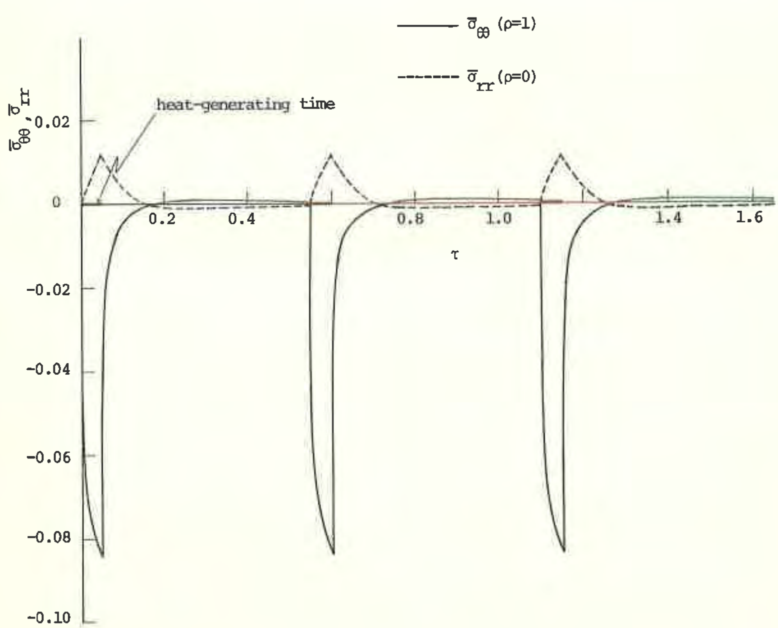

Fig.9 Radial and circumferential stresses at ρ=0 or ρ=1 and ζ=0 as a function of τ (τ_0=0.05, τ_1=0.50, β=0.1)