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AN EXPERIMENTAL STUDY OF VISCOUS DAMPING IN SLOSHING MODE OF CYLINDRICAL TANKS

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Abstract

The vibration tests of a cylindrical tank were carried out to investigate the viscous damping in sloshing mode. The effects of kinematic viscosity, liquid level and tank dimensions on the viscous damping were investigated.

The experimental results of the viscous damping in small wave height showed good agreement with calculated values due to the theoretical solution in an assumed laminar boundary layer by K.M. Case et al.. The viscous damping in sloshing mode depends on liquid viscosity, liquid level and tank dimensions. Especially the tank dimensions effect on the viscous damping, and the viscous damping decreases with the increase of tank diameter.

1. Introduction

Cylindrical tanks are used widely in nuclear power plants and chemical plants. Viscous damping is one of the important problems in the seismic design of cylindrical tanks.

Several experimental studies of the viscous damping in sloshing mode have been reported [1,2]. The theoretical solution for the viscous damping in the sloshing mode of cylindrical tanks has been reported by K.M. Case and W.C. Parkinson [1].

The subject of our study is to clarify the application of the theoretical solution for the viscous damping in the sloshing mode of cylindrical tanks. The vibration tests of a cylindrical tank were carried out to investigate the viscous damping in sloshing mode. The effects of kinematic viscosity, liquid level and tank dimensions on the viscous damping were investigated. The experimental results of the viscous damping were compared with theoretical ones. Moreover, the application limit of the theoretical solution was discussed.

2. Nomenclature

D : diameter of cylindrical tank
g : acceleration of gravity
H : liquid level
h : damping ratio
h_b : damping ratio on bottom
h_s : damping ratio on side wall
h_t : damping ratio within liquid
R : radius of cylindrical tank

δ : logarithmic decrement
δ_b : logarithmic decrement on bottom
δ_s : logarithmic decrement on side wall
δ_t : logarithmic decrement within liquid
ζ : wave height
ν : kinematic viscosity
ρ : liquid density
σ : surface tension
ω_m : natural circular frequency of m-th sloshing mode

3. Theoretical solution

The viscous damping in sloshing mode depends on liquid viscosity, liquid level and tank dimensions. The theoretical solution of the viscous damping in an assumed laminar boundary layer was reported in the case of small wave height [1].

The damping ratio in sloshing mode is expressed as follows;

$$h = h_t + h_s + h_b \tag{1}$$

$$h = \frac{\delta}{2\pi} = \frac{1}{2\pi}\left(\delta_t + \delta_s + \delta_b\right)$$

$$h_t = \frac{\delta_t}{2\pi} = \frac{1}{2\pi}\frac{4\pi\nu}{\omega_m}\left(\frac{\varepsilon_m}{R}\right)^2 \tag{2}$$

$$h_s = \frac{\delta_s}{2\pi} = \frac{1}{2\pi}\left(\frac{\nu}{2\pi\omega_m}\right)^{0.5}\left(\frac{\pi}{R}\right)\left\{\frac{1+\left(1/\varepsilon_m\right)^2}{1-\left(1/\varepsilon_m\right)^2} - \frac{2\varepsilon_m H/R}{\sinh\left(2\varepsilon_m H/R\right)}\right\} \tag{3}$$

$$h_b = \frac{\delta_b}{2\pi} = \frac{1}{2\pi}\left(\frac{\nu}{2\pi\omega_m}\right)^{0.5}\left(\frac{\pi}{R}\right)\frac{2\varepsilon_m}{\sinh\left(2\varepsilon_m H/R\right)} \tag{4}$$

$$\omega_m = \sqrt{g\cdot\varepsilon_m \ \tanh\left[\left(\varepsilon_m H/R\right)\left\{1+\sigma\left(\varepsilon_m/R\right)^2/\rho g\right\}\right]/R}$$

In fundamental mode,

$$e_1 = 1.841$$

The effect of surface tension on the natural frequency is small in the gravity field, and the term of surface tension can be neglected.

$$\omega_1 = \sqrt{1.841g\cdot\tanh(1.841H/R)/R} \tag{5}$$

4. Experiment

The vibration tests of a cylindrical tank were carried out to investigate the viscous damping in fundamental sloshing mode. An acrylic model was used in vibration tests. The model tank was set on a shaking table and excited using sinusoidal waves. Pressure and wave height were measured, and the damping ratio was determined from the result of the logarithmic decrement of free vibration.

Table 1 shows experimental conditions and the results of the damping ratio. Kinematic viscosity, liquid level and tank dimensions were changed, and the effects of these on the viscous damping were investigated. The liquids used in experiments were water and glycerin water. The kinematic viscosity of liquids were changed from 1cst to 560cst by the control of glycerin content.

5. Comparison of Theory and Experiment

The experimental results of the viscous damping were compared with theoretical ones. These results are shown in Fig.1,2 and 3. The experimental results of the viscous damping showed good agreement with calculated values due to the theoretical solution in an assumed laminar boundary layer.

5.1 Effect of Viscosity

Figure 1 shows the effect of kinematic viscosity on the viscous damping. The viscous damping increases with the increase of kinematic viscosity.

The details of calculated results for the viscous damping against kinematic viscosity are shown in Fig.4. The calculation is conducted for the case of D=970mm and H=340mm, and kinematic viscosity is changed from 0.5cst to 1000cst.

The damping ratio within liquid,and that on bottom and side wall increase with the increase of kinematic viscosity. However,the damping ratio on the side wall by equation (3) is higher than that on the bottom by equation (4) and that within the liquid by equation (2). The damping ratio on the side wall is dominant for the total damping ratio by equation (1) below 1000cst. On the other hand, the influence of the damping ratio within liquid becomes large in the high viscosity level above 1000cst.

5.2 Effect of Liquid Level

The effect of liquid level(H/D) on the viscous damping is shown in Fig.2. The damping ratio at low liquid level is higher than that at high liquid level.

Table 1. Test Results of Damping Ratio

No.	Diameter D (mm)	Liquid Height H (mm)	H/D	Liquid	Kinematic Viscosity ν (cst)	Damping Ratio h (%)
1	970.0	340.0	0.351	Water	1.0	0.087
2	970.0	340.0	0.351	Glycerin 50%	7.0	0.190
3	970.0	340.0	0.351	Glycerin 62%	12.4	0.250
4	970.0	340.0	0.351	Glycerin 83%	56.0	0.540
5	970.0	340.0	0.351	Glycerin 91%	128.0	0.860
6	970.0	340.0	0.351	Glycerin 100%	560.0	1.900
7	970.0	150.0	0.155	Glycerin 62%	12.4	0.445
8	970.0	270.0	0.278	Glycerin 62%	12.4	0.280
9	970.0	400.0	0.412	Glycerin 62%	12.4	0.230
10	970.0	450.0	0.464	Glycerin 62%	12.4	0.222
11	970.0	200.0	0.206	Water	1.0	0.120
12	970.0	200.0	0.206	Glycerin 50%	7.0	0.260
13	970.0	200.0	0.206	Glycerin 62%	12.4	0.345
14	970.0	200.0	0.206	Glycerin 91%	128.0	1.280
15	970.0	200.0	0.206	Glycerin 100%	560.0	3.000
16*	3125.0	1093.8	0.350	Water	1.0	0.040

* : Reference [3]

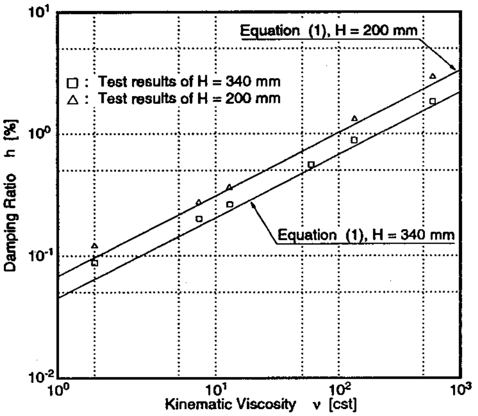

Fig. 1 Test Results of Damping Ratio against Kinematic Viscosity (D = 970 mm, H = 200, 340 mm)

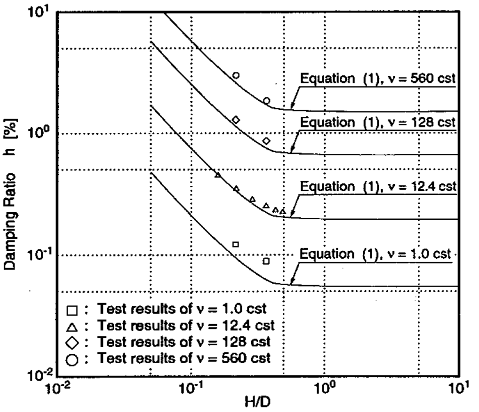

Fig. 2 Test Results of Damping Ratio against H/D (D = 970 mm, n = 1.0, 12.4, 128, 560 cst)

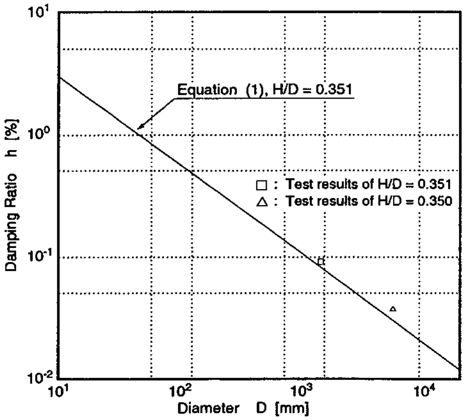

Fig. 3 Test Results of Damping Ratio against Tank Diameter (n = 1.0 cst, D = 970, 3125mm)

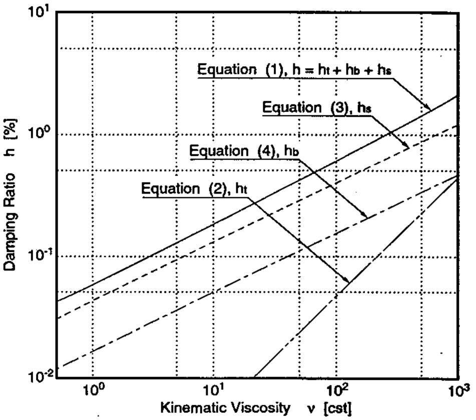

Fig. 4 Effect of Kinematic Viscosity on Damping Ratio (D = 970 mm, H = 340 mm, H/D = 0.351)

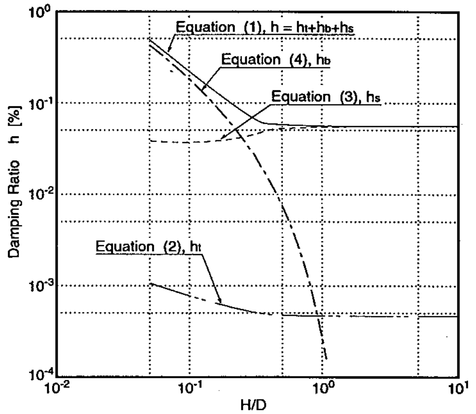

Fig. 5 Effect of Liquid Level on Damping Ratio (n = 1.0 cst, D = 970 mm)

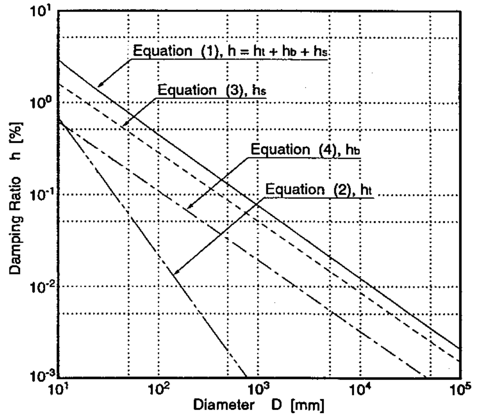

Fig. 6 Effect of Tank Diameter on Damping Ratio (n = 1.0 cst, H/D = 0.351)

The details of calculated results of the viscous damping against the aspect ratio H/D are shown in Fig.5. The calculation is conducted for the case of ν=1cst(water) and D=970mm, and the aspect ratio H/D is changed from 0.05 to 10.

The damping ratio on the bottom by equation (4) is dominant for the total damping ratio by equation (1) below H/D=0.1. However, as the increase of the aspect ratio H/D, the damping ratio on the side wall by equation (3) increases and that on the bottom decreases, the damping ratio on the side wall becomes to be dominant for the total damping ratio above H/D=0.5.

5.3 Effect of Tank Dimensions

The effect of tank dimensions on the viscous damping is shown in Fig.2. The damping ratio decreases with the increase of tank diameter. The details of calculated results of the viscous damping against the tank diameter are shown in Fig.6. The calculation is conducted for the case of ν=1cst(water) and H/D=0.351, and tank diameter is changed from 10mm to 100000mm(100m).

The damping ratio within liquid, and that on bottom and side wall decrease with the increase of tank diameter. However, the damping ratio on the side wall by equation (3) is higher than that on the bottom by equation (4) and within the liquid by equation (2), and the damping ratio on the side wall by equation (3) is dominant for the total damping ratio by equation (1) in all tank diameter. On the other hand, the influence of the kinematic viscosity of liquid itself on the damping ratio is larger in case of small diameter, however, as the increase of tank diameter, the influence of that becomes smaller.

6. Discussion

The experimental results of the damping ratio have good agreement with the calculated values due to the theoretical solution in an assumed laminar boundary layer.

The above mentioned experimental results of damping ratios were in the case that the wave height was about 1/30 of tank diameter.

In this section, the application limit of the theoretical solution is discussed.

Fluid flows are classified in three pattern with used Reynolds number Re [4].

$Re < 5 \times 10^5$: Laminar Flow
$5 \times 10^5 < Re < 5 \times 10^6$: Transition Flow
$5 \times 10^6 < Re$: Turbulent Flow

Reynolds number Re can be presented as follows ;

$$Re = VL/\nu \qquad (6)$$

Where, L is the typical length that is represented by a tank diameter D, and V is the velocity that can be represented by wave height and natural circular frequency.

$$V = \zeta\omega_1$$

$$Re = \zeta\omega_1 D/\nu \qquad (7)$$

$$= nD^2\omega_1/\nu \qquad (8)$$

where ζ is,

$$\zeta = nD \qquad (9)$$

Substituting the conditions of wave height, tank dimensions and kinematic viscosity in equation (8), the value of Reynolds number can be obtained.

Figure 7 shows the application limit of the theoretical solution for damping ratio. The maximum wave height ratio $n=\zeta/D$ based on the recorded seismic data of oil storage tanks in Japan is shown in Fig.7. The calculated result of equation (8) shows that the application limit of wave height decreases with the increase of tank diameter, and the fluid flow in a tank changes from laminar flow to transition flow at D=25m.

This result shows that the theoretical solution in an assumed laminar boundary layer can be applied to the estimation of the viscous damping of cylindrical tanks under 25m in diameter. However, it is necessary to take notice of the estimation of viscous damping of the tanks over 25m in diameter by the theoretical solution. The damping ratio in transition and turbulent flow will increase because of the occurrence of eddy.

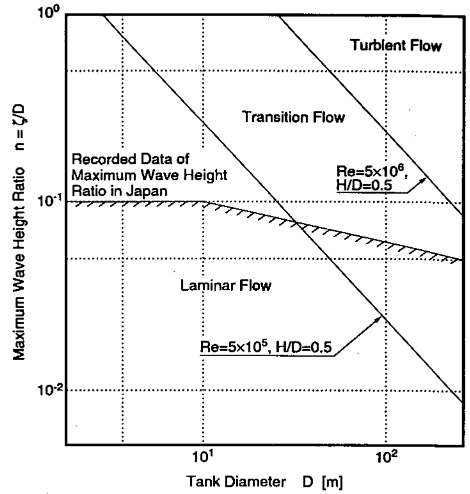

Fig. 7 Application Limit of Theoretical Solution for Damping Ratio (ν = 1.0 cst)

7. Conclusion

The vibration tests of a cylindrical tank were carried out to investigate the viscous damping in sloshing mode. The effects of kinematic viscosity, liquid level and tank dimensions on the viscous damping were investigated.

The experimental results of the viscous damping in small wave height have good agreement with calculated values due to the theoretical solution in an assumed laminar boundary layer by K.M. Case et al.. The viscous damping in sloshing mode depends on liquid viscosity, liquid level and tank dimensions. Especially tank diameter effects on the viscous damping, and the viscous damping decreases with the increase of tank diameter.

The theoretical solution is a good method to estimate the viscous damping of small cylindrical tanks under 25m in diameter. However, it is necessary to take notice of the estimation of viscous damping of the tanks over 25m in diameter by the theoretical solution because of the occurrence of transition and turbulent flow.

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