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TEMPERATURE EFFECT ON THE DYNAMIC RESPONSE OF SPHERICAL SHELLS

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ABSTRACT

Modified Berger's method has been extended to the case thermal loading for investigating the large amplitude free vibration of shallow spherical shells subjected to a thermal gradient.

1 INTRODUCTION

In high-speed space vehicles and in nuclear reactors certain parts have to be operated under elevated temperature and as a result modulus of elasticity of materials becomes functions of space variables [1], and as such vibrational characteristics of continuous elastic media must then be based on non-homogeneous elastic theory.

Compared to linear theory, nonlinear ana-lysis of shallow spherical shells appears to be rare. The usual method for investigating the behaviour of plates and shells exhibiting large amplitude is to solve Karman field equations [2] extended to a dynamic case. However, such equations pose problem even in finding approximate solutions. To simplify the problem Berger [3] proposed an alternative method which appeared to be useful in finding approximate solutions but later some authors became critical and inferred that Berger's approximation may lead to absurd results if freely used [4], though in certain cases the method appears to be useful and yields results quite satisfactory for practical purposes [5]. Sinha Roy and Banerjee [6] tried to modify Berger's approximation, the limit of accuracy of which is yet to be tested, for, like Berger's hypothesis, it lacks in providing with a rigorous physical significance in support of the present modification. Utilising this modification, the present paper aims at investigating the behaviour of shallow spherical shells subjected to a thermal gradient and vibrating at large amplitude.

2 DEDUCTION OF GOVERNING DIFFERENTIAL EQUATIONS

Considered here a spherical shell with clamped edges and subjected to a steady thermal gradient. The coordinate system employed in the present analysis has been shown in Fig. 1. The vertical component of displacement of the middle surface of the shell is denoted by 'w', considered to be positive in the direction shown. The radial displacement of a point in the middle surface is denoted by 'u' measured horizontally. The elevation of the middle surface of the shell above the base plane is denoted by 'z' and is given by

$$z = (R^2/2R_o)(1 - r^2/R^2) \qquad (1)$$

where the quantities R, R_o and R have been depicted in Fig. 1.

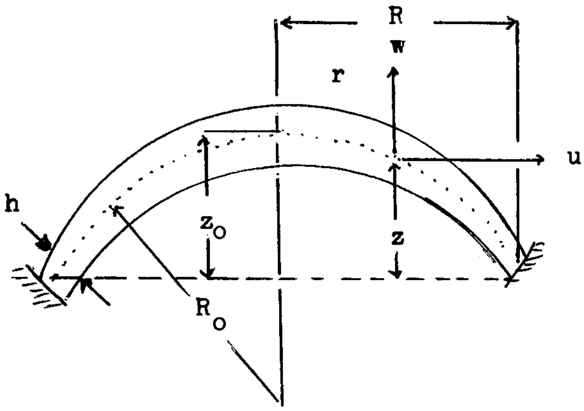

Fig. 1. Shell geometry

The temperature distribution 'T' is assumed to be linear function of the radial distance 'r' and the modulus of elasticity 'E' is assumed to be a linear function of 'T'. As a result the stiffness parameter 'D' becomes a linear function of 'r' so that

$$T = T_o \ (1 - r/R) \tag{2}$$

$$E = E_o \ (1 - m \ T) = E_o \left[1 - \beta (1 - r/R) \right] \tag{3}$$

where $\beta = m \ T_o$ and T_o is the reference temperature, E_o, the reference modulus of elasticity, 'm' is the slope of variation of 'E' with 'T'. The flexural rigidity 'D' is then given by

$$D(r) = D_o \ \left[1 - \beta (1 - r/R) \right] \tag{4}$$

D_o being the reference flexural rigidity.

The potential energy due to bending and strtching may be written in the following form, using the modified form [6]

$$V = \tfrac{1}{2} \iint D(r) \Big[(\nabla^2 w)^2 - 2 (1 - \nu) \ w_{,r} \ w_{,rr}/r + (12/h^2) \ e_1{}^2 + (12/h^2) \lambda \left(\tfrac{1}{2} w_{,r}^2 + w_{,r} \ z_{,r} \right)^2 \Big] r \ dr \ d\theta \tag{5}$$

where λ is unknown depending on Poisson's ratio of the shell material, and

$$e_1 = du/dr + \nu u/r + \tfrac{1}{2} w_{,r}^2 + dw/dr \ . \ dz/dr \tag{6}$$

The kinetic energy of the shell is given by,

$$E_k = (\rho h/2) \iint (u_{,t}^2 + w_{,t}^2) \ r \ dr \ d\theta \tag{7}$$

ρ being the density of the shell material.

Applying Hamilton's principle and then using Euler's variational equations to the Lagrangian function $L = (E_k - V)$, in-plane inertia being neglected, one can obtain the following equations

$$12 \ D \ e_1 / h^2 = \alpha^2 \ f(t) \ r^{-1 + \nu} \tag{8}$$

where α is a constant and f(t) is an unknown function of time, and

$$D(r)\left[\nabla^4 w - (12\lambda/h^2)\left\{\tfrac{1}{2} w_{,r}^3/r - (3/R_o)\, w_{,r}^2 + (3/2)\, w_{,r}^2 w_{,rr} - (3r/R_o)\, w_{,r}\, w_{,rr} + (r^2/R_o^2)\, w_{,rr} + (3r/R_o^2)\, w_{,r}\right\}\right]$$
$$+ dD(r)/dr\left[2 w_{,rrr} + (3/8)\, w_{,rr} - w_{,r}/r^2 - (12\lambda/h^2)\left\{\tfrac{1}{2} w_{,r}^3 - (3/2)(r/R_o)\, w_{,r}^2 + (r^2/R_o^2)\, w_{,r}\right\}\right] + d^2D(r)/dr^2\left[w_{,rr} + (\nu/r)\, w_{,r}\right]$$
$$+ \rho h\, w_{,tt} - \alpha^2 f(t) \quad r^{\nu-1} w_{,rr} + r^{\nu-2} w_{,r} - r^{\nu-1}(1+\nu)/R_o \qquad (9)$$

3 METHOD OF SOLUTION

For a spherical shell with clamped immovable edges one may assume

$$w = A\, w_o(t)\, (1 - r^2/R^2)^2 \qquad (10)$$

where 'A' stands for the maximum deflection in the positive direction.

Inserting the expression for w in equation (8) and integrating over the surface area of the shell one obtains the value of the unknown constant in the form

$$\frac{\alpha^2 h^2}{12} = D_o\,(1-\beta)\left[\frac{16\, A\, w_o(t)\, R^{1-\nu}}{(5-\nu)(7-\nu)R_o} + \frac{128\, A^2\, w_o^2(t)\, R^{-1-\nu}}{(5-\nu)(7-\nu)(9-\nu)}\right]$$
$$+ 128\, R^{-1-\nu}/(6-\nu)(8-\nu)(10-\nu)\quad A^2\, w_o^2(t)$$
$$+ 16\, A\, w_o(t)\,(R^{1-\nu}/R_o)\quad /\,(6-\nu)(8-\nu) \qquad (11)$$

With the above value of α given by (11) and that of 'w' given by (10) a Galerkin procedure is applied to equation (9) to obtain the time differential equation in the form

$$w_{o,tt} + C_1 w_o + C_2 w_o^2 + C_3 w_o^3 = 0 \qquad (12)$$

where, C_1, C_2 and C_3 are known constants depending on the known parameters, and the parameter λ which occurs in the foregoing equations can be determined from the minimum potential energy and is given by

$\lambda = 2\nu^2$ (for clamped edge) and $\lambda = \nu^2$ (for simply-supported edge) [6].

4 SOLUTION OF THE TIME DIFFERENTIAL EQUATION

The solution of the time differential equation has been given in Ref. [7] subject to the initial conditions $W = w_o(0) = 1$ and $dw_o/dt = 0$ for $t = 0$. The ratio of the frequencies for nonlinear and linear vibrations may be written in the form

$$\omega^*/\omega = 1 + (A/h)^2\left\{\tfrac{3}{4}(C_3/C_1) - (5/6)(C_2/C_1)^2\right\}^{\frac{1}{2}} \qquad (12)$$

NUMERICAL RESULTS AND DISCUSSION

Numerical results have been shown in the form of graphs Fig. 2. The results

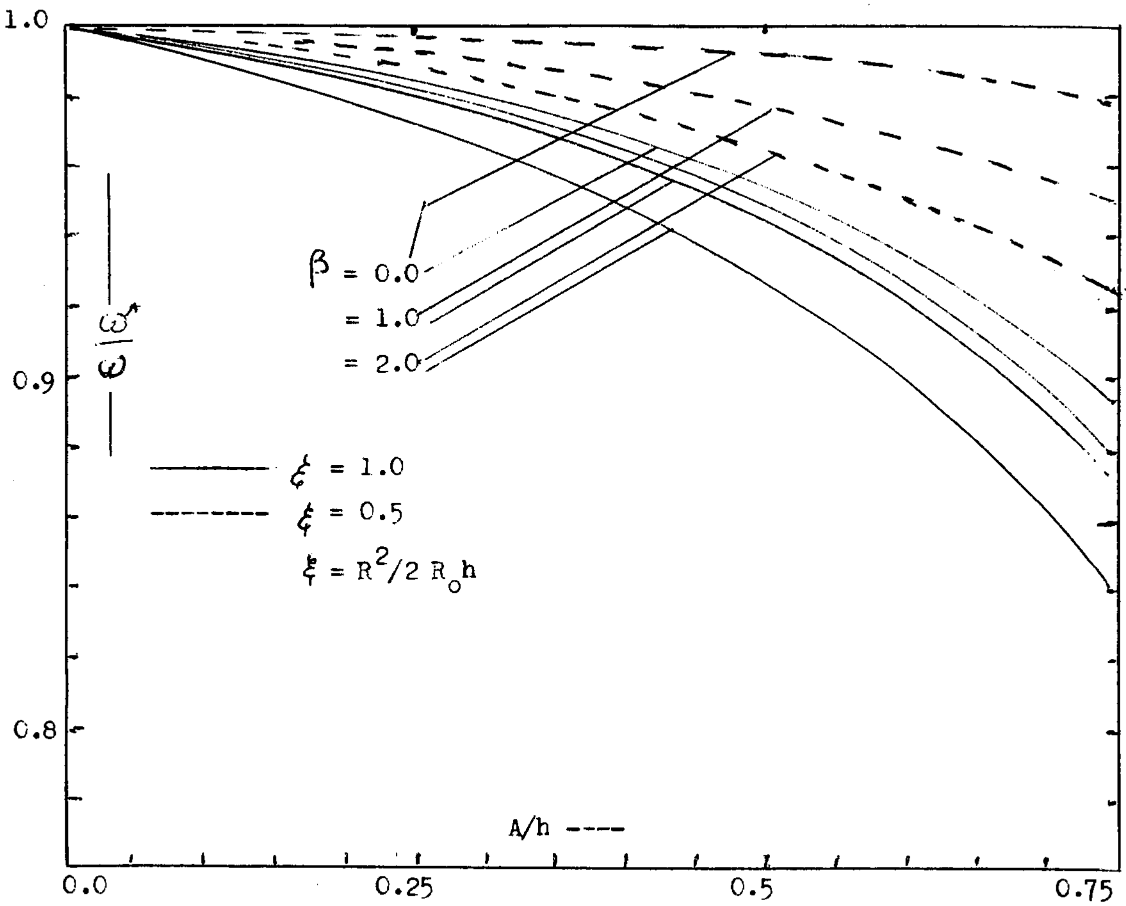

Fig. 2. Variation of non-dimensional frequency ratio for different values of non-dimensional amplitude and temperature parameter β .

have been computed for ξ = 1 and $\frac{1}{2}$, and for β = 0.0, 1.0 and 2.0 . Only available results for comparison are those of Ref. 6 in the limiting case, when $\beta \longrightarrow 0$. Deviation in the results could be observed because equation (7) of Ref. 6 appears to be falacious. The correct form should be that given by equation (11) of the present paper for $\beta = 0$. Equation (11) being a vital one, the time differential equation obtained in Ref. 6 thus becomes incorrect. Hence the accuracy of the method cannot be claimed with respect to the problem treated in Ref. 6. Thus the assumption of the authors of Ref. 6 that the results obtained in Ref. 7 may contain some errors does not stand. Moreover, Ramachandran [7] has treated the problem more rigorously. One more point may be added in this context that assuming values for $\xi < 1$ appears to be impractical when the radius of the base circle is large enough compared to the shell thickness.

The frequency ratio increases with the increase in the value of β . For better investigations and to find the effect of temperature parameter, both the values of ξ and β should be taken greater than unity, preferably 4. Present investigation is still in progress and would be published soon and the analysis will be based on more reliable basic governing equations derived from Karman field equations.

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