THE USE OF SIMPLEX DIRECT SEARCH AND FINITE ELEMENT METHODS FOR DETERMINATION OF HEAT TRANSFER COEFFICIENTS

J. CHLÁDEK

ČKD Praha Research Institute, 19002 Praha 9, Czechoslovakia

J. JINOCH

National Institute for Machine Design, 25097 Praha 9 — Běchovice, Czechoslovakia

SUMMARY

The difficulty in solving heat transfer tasks in machine structures is often involved in determination of boundary conditions, especially of heat transfer coefficient on the surface of the given part. Determination of the values would then be based on experimental results. The experiment, however, could not always be arranged to allow simple calculation of local heat transfer coefficient values. The method considered enables this calculation when based only on values of temperature, measured at several arbitrary points within the part. The points may be placed even outside the exposed region.

Let us consider a body of general shape with heat transfer on its surface $\underline{S}$. Boundary conditions on the part $\underline{S}_A$ of the surface $\underline{S}$ may be known, and on the other part $\underline{S}_B$ of the surface $\underline{S}$ are given either in terms of surrounding temperature, which is supposed to be known, and of heat transfer coefficient. The spatially variable distribution of the latter can be expressed by Langrange's polynomial, determined by unknown values of the heat transfer coefficient in several points on the surface $\underline{S}_B$. These values form the vector $\overline{V}$, that describes the heat transfer coefficient distribution with accuracy, proportional to the chosen dimension of the vector. In this way the vector defines also a temperature field of the given body.

The task is now to find a vector determining such temperature field, that proves the best agreement with experimental results. This is performed by Nelder and Meads direct search optimizing method. The method requires the evaluation of temperature fields, corresponding to the initial set of vectors $\overline{V}$. The vector of this set that gives worse agreement with experimental results is then replaced by a new one. Generation of the new vector is controlled so as to result sequentially in reduction of the penalty function value. The value depends exponentially on the modulus of the difference between calculated and measured temperatures. The comparison is done in several points, which are identical with measuring points and the greater is their number, the better is the accuracy of the results obtained. The temperature field is computed by the finite element method using triangular elements with linear approximation of temperatures. Hitherto only plane and axisymmetric steady state cases were considered. In each step of optimizing procedure described above the preceeding temperature field is used with advantage for iterative solving the finite element method equations.

In accordance with the foregoing outlines a FORTRAN program for the ICL 1905 computer was written. It was verified by solving several simple problems and then used for evaluation of the heat transfer coefficient on the inner surface of diesel engine cylinder liner and on the inner surface of a steam generator storage container.

1. Introduction.

Many important components of machines and plants are subjected to a considerable thermal loading. Determination of operating temperatures and thermal stress distributions within the part is the point of vital importance for satisfying performance of the given system. A lot of helpful methods have been developed to solve this problem but all of them require an initial information about heat transfer conditions on the boundaries of the component. Boundary conditions may be calculated under quite simple assumptions only and in many cases there is no theoretical approach available. Determination of their values would then be based on experimental results. The experiment, however, could not always be arranged to allow simple calculation of heat transfer coefficient values in sufficient number of points. The method considered is intented to give the most detailed information possible about heat transfer conditions, when based only on values of operating temperatures, measured at several arbitrary chosen points within the part. The points may be placed even outside the exposed regions. The method is based on utilising of all available information about known or pre-assumed heat transfer boundary conditions and on the knowledge of heat transfer and conduction laws. This information is automatically transformed and integrated for experimentally obtained results.

2. Description of the method.

2.1 Initial assumptions.

Let us consider a body of general shape with heat transfer on its surface $\underline{S}$. Boundary conditions on the part $\underline{S_1}$ of the surface $\underline{S}$ may be known, and on the other part $\underline{S_2}$ of the surface $\underline{S}$ are given either in terms of surrounding temperatures, which are supposed to be known, and of the unknown spatially variable distribution of heat transfer coefficient values. At several controlling points within the body there are given local values of temperatures, having been obtained from experiment. The unknown distribution of heat transfer coefficient may be advantageously characterized by its values at just a certain number of points on the surface of the body.

In our case the body was considered to be plane or axisymmetrical. That enables to express the distribution by the Lagrange's polynomial expansion, depending on one variable only. This variable was chosen to be the distance along the perimeter or meridian of the body. The polynomial expansion is performed on an interval of the body periphery section which is divided into equal parts by a number of points A_k. At each of the points we define a parameter W_k, that expresses the local value of heat transfer coefficient and is used as the parameter of the polynomial expansion. By means of the expansion a value of heat transfer coefficient is assigned to any point within the interval. There may be a number of such intervals of the heat transfer distribution even in the case of its discontinuity.

The set of values of parameters W_k forms the components of a vector $\mathbf{W}$ that describes the heat transfer coefficient distribution with particulars according to its dimension. In this way the vector also defines a temperature field of the body.

The task now is to find a vector, determining such a temperature field, which proves the best agreement with experimental results.

2.2 The method of search.

Determination of the required vector $\mathbf{W}$ was originally performed by the simplex direct search optimizing method developed by Nelder and Mead [1]. The method is designed for optimizing a function of n variables without constraints. In the case of minimizing we have to chose $(n+1)$ vectors $\mathbf{R}_i$, $i = 1,2, \ldots (n+1)$ in n dimensional space, defining the current "simplex". The vectors must contain the base of the n - dimensional space. Calculating the optimized function values y_i at all points end points of vectors $\mathbf{R}_i$ we shall find the respective minimum and maximum values.

Let us write y_H for maximum and y_L for minimum of y_i values and $\mathbf{R}_H$ and $\mathbf{R}_L$ for the corresponding vectors. Further we define $\mathbf{R}_S$ as the vector end point of which is the centroid of end points of all the vectors with exception of $\mathbf{R}_H$.

At each stage of optimizing the vector $\mathbf{R}_H$ is replaced by a new one, which is obtained by following operations:

a, reflection:

$$\mathbf{R}_A = (1 + a)\cdot\mathbf{R}_S - a.\mathbf{R}_H \quad , \quad a>0 \quad , \qquad (1)$$

When reflection is succesfull, i.e. $y_a \quad y_L$, then it is followed by

b, expansion:

$$\mathbf{R}_B = (1 - b)\cdot\mathbf{R}_S + b.\mathbf{R}_A \quad , \quad b>1 \quad , \qquad (2)$$

In the case of failed reflection, i.e. $y_a \quad y_L$, following step is

c, contraction:

$$\mathbf{R}_C = (1 - c)\cdot\mathbf{R}_S + c.\mathbf{R}_M \, , \quad 0<c<1 \, , \qquad (3)$$

$\mathbf{R}_M$ is that of $\mathbf{R}_A, \mathbf{R}_H$, which yields the lower function value y.

When the lower of the values y_A , y_B is less than y_H , then $\mathbf{R}_H$ is replaced by $\mathbf{R}_A$ or $\mathbf{R}_B$ respectively. If not, a new simplex defined by replacing of all $\mathbf{R}_i$'s by $(\mathbf{R}_i+\mathbf{R}_L)/2$.

In this way the optimizing process goes on, generating automatically new vectors $\mathbf{R}$, end points of which approach consecutively the coordinates of minimum of the optimized function.

In the course of our investigations we developed a new variant of this optimizing method in order to speed up the rate of its convergence. In our method the search is performed in the direction $\mathbf{R}_H - \mathbf{R}_L$ in contradiction to the original method, where the search was carried out in the direction $\mathbf{R}_H - \mathbf{R}_B$. In addition to the original two steps of the search in an optimizing stage a third step was introduced, direction and size of which depends on results of the preceding two steps. Analogical to the original method, substituing for $a = 1$, $b = 2$ and $c = 1/4$ we have:

$$\mathbf{R}_A = (1 + a)\cdot\mathbf{R}_L - a\cdot\mathbf{R}_H = 2\cdot\mathbf{R}_L - \mathbf{R}_H \qquad \text{/reflection/} \qquad (4)$$

if $y_A < y_L$ then

$$\mathbf{R}_B = (1 - b)\cdot\mathbf{R}_L + b\cdot\mathbf{R}_A = 3\cdot\mathbf{R}_L - 2\cdot\mathbf{R}_H \qquad \text{/expansion/} \qquad (5)$$

if $y_B < y_L$ then

$$\mathbf{R}_B = 1/2\cdot(\mathbf{R}_A + \mathbf{R}_B) \qquad \text{/mean value/} \qquad (6)$$

if not, then

$$\mathbf{R}_C = 1/2\cdot(\mathbf{R}_A + \mathbf{R}_L) \qquad \text{/mean value/} \qquad (7)$$

if $y_A \geqslant y_L$ then

$$\mathbf{R}_B = (1 - c)\cdot\mathbf{R}_L + c\cdot\mathbf{R}_M = 1/4\cdot(3\cdot\mathbf{R}_L + \mathbf{R}_M) \qquad \text{/contraction/} \qquad (8)$$

$\mathbf{R}_M$ is that of $\mathbf{R}_A, \mathbf{R}_B$, which yields thelower function value $\underline{y}$.

If $y_B > y_M$ we must form a new simplex in the same way as in the original method. If $y_B \leqslant y_L$

$$\mathbf{R}_C = 1/2\cdot(\mathbf{R}_L + \mathbf{R}_M) \qquad \text{/mean value/} \qquad (9)$$

if not, then

$$\mathbf{R}_C = (1 + c)\cdot\mathbf{R}_L - c\cdot\mathbf{R}_M = 1/4\cdot(5\cdot\mathbf{R}_L - \mathbf{R}_M) \qquad \text{/semireflection/} \qquad (10)$$

The vector $\mathbf{R}_H$ is then replaced by that of $\mathbf{R}_A, \mathbf{R}_B$ and $\mathbf{R}_C$, which yields the lowest value $\underline{y}$.

This modified method was used with advantage, giving substantially quicker convergence than the original one.

2.3 Determination of temperatures.

In our case a temperature field of the body is determined by heat transfer coefficients and ambient temperatures on the surface S_1 , which are known, and by unknown conditions on the surface S_2, which are defined by a vector $\mathbf{W}$ .The set of such vectors $\mathbf{W}_i$, $i = 1,2, \ldots (n+1)$ stands for the set of vectors $\mathbf{R}_i$, forming the simplex in the optimizing method. By this method such new vectors are generated which determine temperature fields giving consecutively better agreement with the given experimental results.The optimizing is controlled by minimizing the value of special penalty function P .The value of the function depends exponentially on the modulus of the difference between calculated temperatures T_i ,respective to the vector $\mathbf{W}_i$,and given temperatures $\overline{T}_i$, obtained from experiment.The comparison is done at the above mentioned controlling points, number of them be NC.We can write for the value of the penalty function

$$P_i = \sum_{j=1}^{NC} \left\{ \exp \left[C_j \left| T_{i,j} - \overline{T}_j \right| \right] - 1 \right\} \tag{11}$$

C_j is a positive constant,expressing the importance of the j - th controlling point.

The lower is the value of the penalty function,the better is the agreement between measured and calculated temperatures.The value may be therefore used as a convergence criterion.The optimizing process goes on until the value of penalty function falls bellow a pre-set limit.

The dimension of the simplex of vectors depends on the number of controlling points,holding $i = 1,2, \ldots (NC+1)$. Dimension of each of the vectors should be equal to NC.Being greater it would cause a multiplicity of the result,the form of which would then depend on the form of the initial simplex of arbitrary chosen vectors $\mathbf{W}_{i0}$

The above described optimizing method requires multiple computations of temperature fields within the machine part considered. The economy of the solution is therefore primarily influenced by the efficiency of temperature field calculations. The well known finite element method [2], chosen for the purpose,has proved to be a very versatile and efficient tool for solving various field problems.The authors already used its linear triangular elements for plane and axisymmetric problems for calculations of temperature fields in machine parts [3] ,as well as for the analysis of grain boundary diffusion in metals [4],[5] .Similar modification of finite element method was therefore applied for calculations of steady state axisymmetrical temperature distribution in our optimizing problem.

3. Practical realization.

3.1 Computational Programme.

A FORTRAN programme SAOP for ICL 1905 computer with the storage of 32 k words was written for solving the optimizing problem.The flow Chart /fig.1/ represents the progress of solution.

In the subroutine <u>SINPUT</u> all input data /mesh description, boundary conditions,material properties,specification of surface with unknown heat tra nsfer etc./ are read,printed,checked and stored.

Subroutine <u>SIUSIT</u> generates for every node of the finite elemebt mesh a list of numbers of neighbouring nodes.The auxiliary field of integer constants is used with advantage for assembling of total matrices and vectors of the finite element system and for iterative solution of linear equations.Checks of mesh descriptions are carried out here,too,the constant ICH being negative in case of any error.

In the subroutine <u>SLAGKO</u> the Lagrange's coefficients for polynomial expansion of the unknown heat transfer coefficients are calculated.

Subroutine <u>SESMAT</u> generates element heat conductivity matrices and right hand sides including known heat transfer terms.The modified total heat conductivity matrix $\overline{\mathbf{H}}$/including non-zero members only/ and total right hand side vector $\overline{\mathbf{F}}$ is assembled by organized adding of element matrices and vectors with respect to mesh topology. As only the known heat transfer members are considered here, $\overline{\mathbf{H}}$ and $\overline{\mathbf{F}}$ are constant throughout the whole process of solution. They are used as constant basis for assembling of total matrices and right hand side vectors by adding of trial heat transfer terms in later phases of calculation. Both $\overline{\mathbf{H}}$ and $\overline{\mathbf{F}}$ are therefore stored on a magnetic disc to be prepared for repeated reading into inner core.

In the subroutine <u>HTC (I)</u> the I-th set of "trial" values of heat transfer coefficients α is calculated with the use of Lagrange's coefficients prepared previously and of the I-th set of parameters $\underline{W}$.At the beginning of the calculation /first (NC + 1) steps/ values of $\underline{W}$ are the input values,in the next phases of solution they are computed in the above described way.

Subroutine <u>SOLVE(I)</u> performs:

a, calculation of $\Delta\mathbf{H}$ - heat transfer terms of element conductivity matrices and $\Delta\mathbf{F}$- corresponding of element right hand side members,both due to the I-th set of trial values $\underline{W}$ and α.

b, adding of $\Delta\mathbf{H}$ and $\Delta\mathbf{F}$ onto apprppriate positions in $\overline{\mathbf{H}}$ and $\overline{\mathbf{F}}$,thus assembling a complete total heat conductivity matrix $\mathbf{H}$ and right hand side vector $\mathbf{F}$ of a system of linear algebraic equations for nodal temperatures $\mathbf{T}$,corresponding to the I-th set of α :

$$\mathbf{H} \cdot \mathbf{T} = \mathbf{F} \tag{12}$$

c, iterative solution of system (12) by means of Gauss-Seidel method. The iterative solution process starts in all phases of calculation with a good approximation of solution vector $\mathbf{T}$.For the first solution the approximation is an input value, in next steps the solution of the preceeding step is used as a starting approximation for the following one. The sets of **W** trial values being arranged so that matrix **H** and vector **F** do not vary too much from step to step, only reasonable number of iterations is necessary for obtaining satisfactory results.

d, calculation of the value P(I) of the exponential penalty function which expresses the quality of the I-th set of trial values $\underline{W}$ and α.

Further information especially concerning the logics of the solution process is apparently seen from the flow chart itself. In addition to notation used in the text, following symbols are used:

P_{MA} ...maximum admissible value of penalty function,

REFLECTION , (4) .... reflection carried out by means of formula(4),

IL , IH .... numbers of sets with lowest and highest values of P ,

W(I,J) .... J-th component of the I-th vector **W**.

3.2 First results.

AS a first problem for testing of both the computational programme and the optimizing method a very simple example was chosen, as presented in fig.2. The body has the form of cylindrical shell, heated on the inner and cooled on the outer surface. The heat transfer conditions on the outer surface are known. The heat transfer coefficient distribution on the inner surface is to be found, conforming with given temperature values at the controlling points 1,2 and 3. By means of this example we verified various strategies of optimizing search, starting from various initial simplexes. The calculations proved that only a very small relative error of iteration in solving finite element method equations may be allowed. In accordance with its magnitude also the lower limit of penalty function, P_{MA} ,used as the criterion of the optimizing method convergence, must be chosen.

On the background of the experience with this simple problem more complex problems could be tackled, an example of which is shown in fig.3. It represents the solution of heat transfer coefficient distribution along the inner surface of water cooled diesel engine cylinder liner.

The method considered proved to be a very useful tool for solving complex heat transfer problems. It is also an attempt of a new use of optimizing method in machine design.

References

[1] NELDER, J.A., MEAD,R., "A simplex method for function minimization," Computer Journal 7 , 1965, p.308 - 313.

[2] ZIENKIEWICZ, O.C., "The Finite Element Method i Engineering Science", Mc Graw - Hill , London , 1971.

[3] JINOCH, J., "The solution of two-dimensional heat conduction problems by finite element method", /in Czech/, Strojírenství,24, 7, 1974, p. 397 - 403.

[4] JINOCH, J.,BÍLEK, Z., KNÉSL, Z., "Application of finite element method in metals diffusion", /in Czech/, Kovové materiály, 3/1974.12, p.305 - 317.

[5] JINOCH, J., BÍLEK,Z., KNĚSL, Z.,"The Influence of Grain Size on the Grain Boundary Diffusion", /in English/, Czechoslovak Journal of Physics, Vol. B 24, 1974, p.343 - 346.

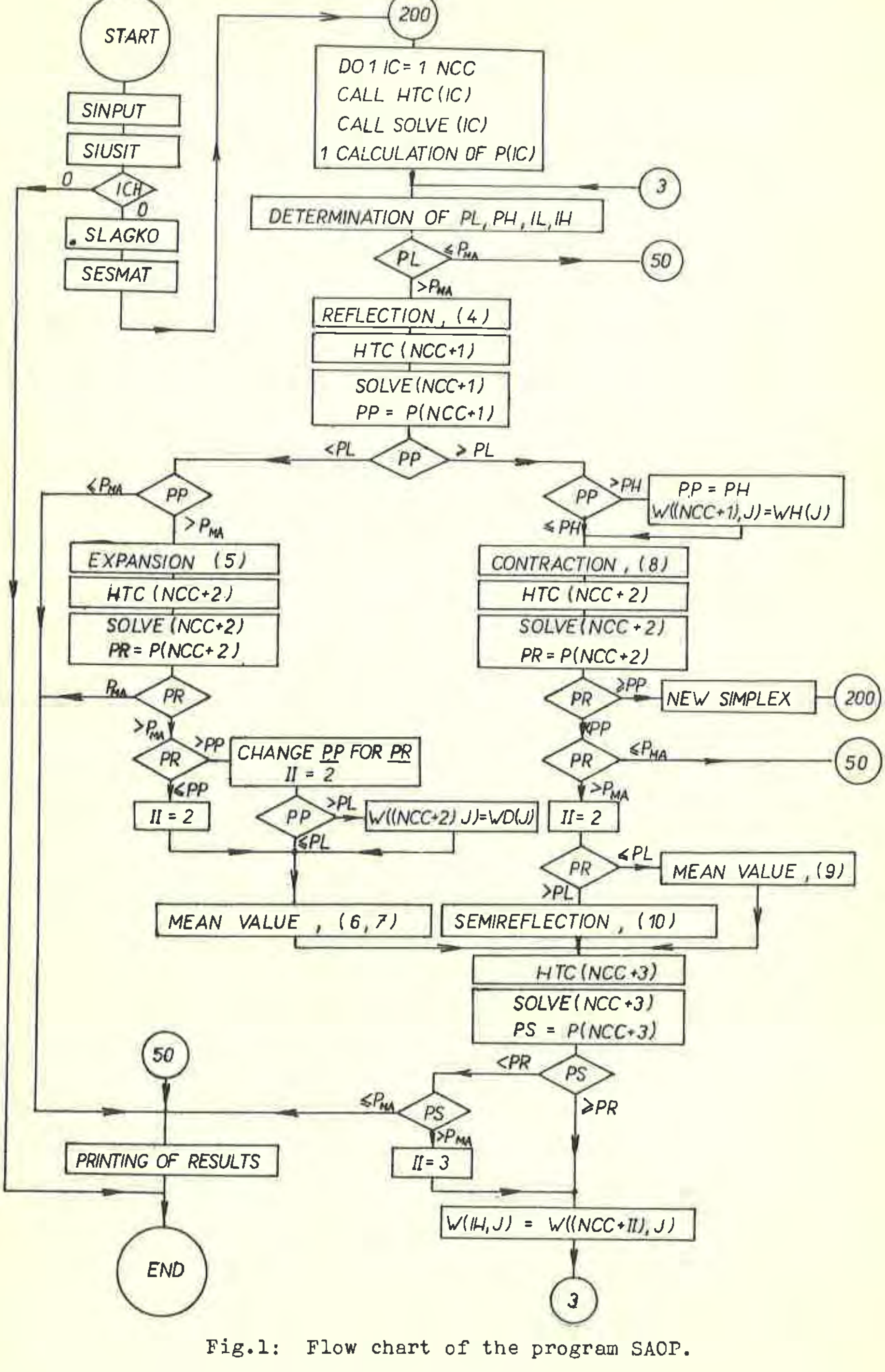

Fig.1: Flow chart of the program SAOP.

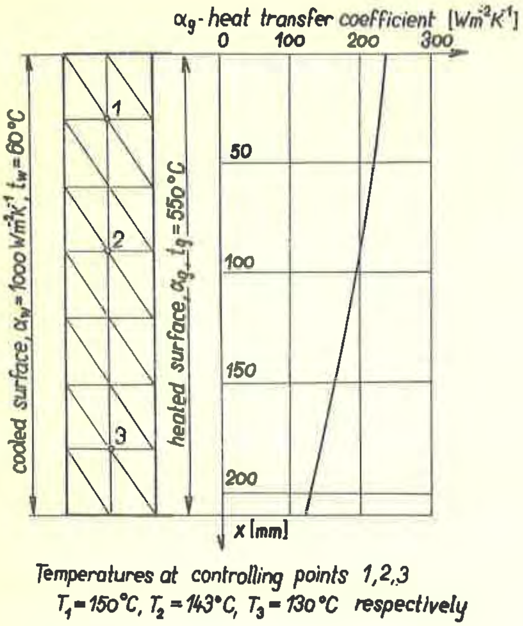

Fig.2 Simple test example and its results.

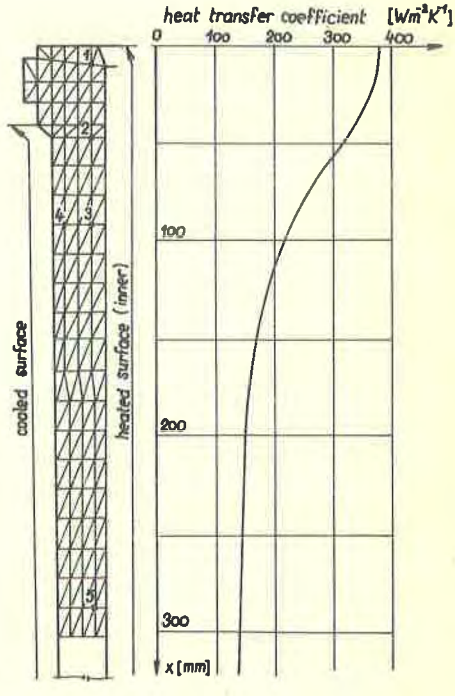

Fig.3 Finite element mesh, location of controlling points and obtained heat transfer coefficient distribution for diesel engine cylinder liner.