ANALYSES OF THE DYNAMIC BEHAVIOR OF NUCLEAR POWER REACTOR COMPONENTS CONTAINING FLUID

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SUMMARY

This paper presents the application of a finite element method for the analysis of the dynamic behaviour of nuclear power reactor components containing fluids.

If the acoustic assumption is made for the fluid a numerical solution of fluid solid interaction problem is easily obtainable. In the formulation it is assumed that the fluid velocity is negligible and Eulerian coordinate are used, the solid structure is described by a classical Lagrangian Formulation. In the case of incompressible fluid the pressure equations can be eliminated and the structure mass matrix is augmented by an added mass matrix.

At the free surface the pressure can be stated as zero and in this case the surface wave is not described. To introduce the wave the vertical displacement of the horizontal free surface is simulated by a solid Lagrangian line and a boundary condition on pressure must be introduced in the fluid Eulerian mesh. Neigleting the inertia non linear terms, the pressure is proportional to the wave height.The same result is achieved using solid boundary element of vertical stiffness.

This formulation has been implemented in the finite element program NOVAX specialised for the calculation of the response of axisymetric structures by the technique of Fourier series decomposition.

To check the asumption made a comparison has been made with three test problems.

Problems 1 and 2 are the free vibration in the vertical and horizontal direction of a rigid cylindrical tank filled with liquid.

Problem 3 is the sloshing mode of a fluid in a partially filled rigid spherical tank. Comparison is made with experiments involving several fluids.

Application to a typical fast reactor of the pool type is shown, effect of the free surface boundary condition is discussed.

1. INTRODUCTION

In the dynamic analysis of a LMFBR reactor vessel, the fluid structure interaction effect plays an important role in the load distribution.

Generally the fluid behaviour is difficult to describe mathematically due to the highly non linear nature of the flow equations. However if the acoustic assumption is made, that is, if the flow velocity is small and if viscosity effects are neglected the fluid equations are linear and the fluid solid interaction problem can be solved.

For earthquake analysis the additional assumption of fluid incompressibility can be used and the size of the problem is greatly reduced.

This paper :

- describes the basic assumption related to the acoustic hypothesis and free surface formulation,
- validates the solution by comparison with known analytical solution and experimental results,
- gives an example of the application to the seismic analysis of a LMFBR of the pool type.

2. ACOUSTIC EQUATIONS FOR THE FLUID

The fluid behaviour is defined by the three following equations :

- continuity $\frac{d\rho}{dt} + \rho \, \mathrm{div} \; V = 0$
- Euler $\rho \frac{dV}{dt} + \mathrm{grad} \; (P + \rho g z) = 0$
- Compressibility $\delta P = C^2 \delta \rho$

P, ρ, V, Z, g, C^2 and t are respectively :

Pressure, fluid mass density, velocity, elevation, acceleration of gravity, sound speed and time.

In the acoustic approximation the non linear term of the Eulerian derivatives are neglected then :

$$\nabla^2 P - \frac{1}{C^2} \ddot{P} = 0$$

The boundary condition at the fluid boundary is :

$$\frac{\partial P}{\partial n} = - \rho \ddot{U}_n - \rho \, g n$$

where $\frac{\partial P}{\partial n}$ and U_n are pressure gradient and velocity normal to the boundary.

At the free surface the pressure is set equal to zero, and on the Eulerian mesh :

$$P \simeq \rho \ddot{U}_n \cdot U_n + \rho g_n \cdot U_n$$

The first term of this equation can be dropped leaving :

$$P = \rho g_n U_n$$

This relation can be simulated by mechanical boundary elements of vertical stiffness ρg_n.

If the free surface motion is neglected the pressure is set to zero.

3. FINITE ELEMENT FORMULATION

Using a weighted residual method the following variational formulation is derived :

$$W = \int_V \nabla F . \nabla P \; dV + \int_V F . \frac{\ddot{P}}{C^2} \; dV + \int_S F \rho (\ddot{U}_n + g_n) \; dS = 0 \qquad (3)$$

Finite element discretization using the same shape function for F and P yields the following matrix equation :

$$Q\ddot{P} + HP = -\rho L\ddot{U} - \rho G$$

P is the modal pressure of the Eulerian mesh. Q and H are matrices due to the volume integral of equation (3). L and G are matrices due to the surface integral introduced by the boundary conditions (2).

For an incompressible fluid Q = 0 and in dynamic problems G can be neglected.

Depending of the shape function used the matrix H may be singular and to remove the "rigid" mode of matrix H pressure are defined by the relation (Ref. 1) :

$$P = (J,R) \begin{vmatrix} \lambda \\ \tilde{P} \end{vmatrix}$$

J is a column vector of one R contains only zero and one, after some algebraic manipulation the equation is reduced to :

$$\left| \begin{array}{l} \tilde{H}\tilde{P} = -\rho \tilde{L} \ddot{U} \\ J^T L \ddot{U} = 0 \end{array} \right. \qquad \text{with} \qquad \left| \begin{array}{l} \tilde{H} = R^T H R \\ \tilde{L} = R^T L \end{array} \right.$$

The last equation is equivalent to :

$$J^T LU = 0$$

The structure equation is :

$$M \ddot{U} + K U = L^T P$$

Definig an added mass matrix :

$$\tilde{M} = \rho \tilde{L}^T \tilde{H}^{-1} \tilde{L}^T$$

The following set of equation is derived

$$\begin{vmatrix} M + M & 0 \\ 0 & 0 \end{vmatrix} \times \begin{vmatrix} \ddot{U} \\ \ddot{\lambda} \end{vmatrix} + \begin{vmatrix} K & -L^T J \\ -J^T L & 0 \end{vmatrix} \times \begin{vmatrix} U \\ \lambda \end{vmatrix} = 0$$

The above formulation has been implemented in an axisymetric code NOVAX using Fourier series decomposition.

4. COMPARISON WITH THEORICAL AND EXPERIMENTAL RESULTS

Problem 1.

Free vibration in the vertical direction of a rigid cylindrical tank filled with liquid.

The following data have been used :

cylinder radius R = 72 m (6 and 16 elements)

" " " height H = 28,8 m (6 and 12 elements)

acceleration of gravity g = 9,81 m/s2

circular frequencies w are correlated by the dimensionless variable

$$\lambda = w\sqrt{\frac{R}{g}}$$

The theorical value of λ is :

$$\lambda = \xi \left[\tanh \left(\xi \frac{H}{R} \right) \right]^{\frac{1}{2}} \qquad \text{with} \quad J'_0(\xi) = 0$$

or ξ = 3.832, 7.016, 10.173, 13.324.

J_0 Bessel function of the first kind.

The following results have been obtained for the first three modes.

	Frequencies		
	1	2	3
NOVAX - mesh 6 x 6	1,86	2,68	3,32
mesh 16 x 12	1,87	2,65	3,22
Theorical result	1,87	2,64	3,19

Problem 2.

Free vibration in the horizontal direction of a rigid cylindrical tank filled with liquid.

The following data have been used :

cylinder radius R = 1 m (10 elements)

cylinder height H = 1 m (10 elements)

acceleration of gravity g = 9,81 m/s2

circular frequencies w are correlated by the dimensionless variable

$$\lambda = w\sqrt{\frac{R}{g}}$$

The theorical value of λ is :

$$\lambda = \left[\tanh\left(\xi \frac{H}{R}\right)\right]^{\frac{1}{2}} \quad \text{with} \quad J'_1(\xi) = 0$$

or ξ = 1.841, 5.331, 8.536, 11.706, ...

The following results have been obtained for the first four modes.

	Frequencies			
	1	2	3	4
NOVAX	1.324	2.331	3.000	3.598
Theory	1.323	2.308	2.921	3.421

Free surface deformation is :

$$\delta = A\, J_1\left(\xi \frac{r}{R}\right)$$

The pressure disturbance is :

$$\frac{P}{\rho g} = \delta \frac{\cosh\left[\xi (Z + H)/R\right]}{\cosh\left[\xi H/R\right]}$$

Figure 1 gives the pressure distribution.

Problem 3.

Sloshing mode of a fluid in a partially filled rigid sphere.

The non axisymetric modes for horizontal vibration (harmonic 1) have been extracted for three levels. $\frac{H}{R}$ = 0.5, 1., 1.6,

Circular frequencies w are correlated by the dimensionless variable

$$\lambda = w\sqrt{\frac{R}{g}}$$

Comparison made with other numerical formulations and experimental results involving several fluids (Ref. 2) shows good agreement (Figure 2).

5. SEISMIC ANALYSIS OF A SODIUM COOLED REACTOR

Computational model

The simplified model of a fast breeder reactor of the pool type is given by figure 3.

The structures represented are :

The main vessel, internal vessel, core, core support, control rod support structure, and the upper deck with the rotating plugs.

Pumps and intermediate heat exchangers are represented by their masses lumped on the upper deck and the core support structure. The sodium mass is 40 % of the mass supported by the main vessel.

The model uses 90 nodes, 64 shell elements and four fluid superelements. During the calculational process each fluid superelement is subdivided into smaller elements.

Seismic analysis involves only the two first terms of the Fourier series decomposition.

Effect of the boundary condition on the fluid

Two calculations have been made to investigate the effect of the fluid boundary condition :

$P = 0$ at the free surface

$P = \rho g Z$ at the free surface of superelement 2.

In the second calculation four sloshing modes appear, the other modes frequencies are unaffected.

Participation factors of the sloshing modes are important in the horizontal direction only and in that direction they are unaffected by the fluid boundary condition.

In the horizontal direction the mass affected to the first sloshing mode is close to the value given by the classical theory of cylindrical tank.

Stresses in the structure are practically not affected by the fluid boundary condition, less than 1 %.

The following table gives the first twelve eigenfrequencies and participation factors :

Vertical			horizontal		
Frequency	Participation factor		Frequency	Participation factor	
	P = 0	P = gZ		P = 0	P = gZ
0.321	-	2.04	0.172	-	546.
0.459	-	0.51	0.350	-	186.
0.669	-	1.81	0.466	-	59.6
0.670	66.9	- 62.6	0.678	-	37.4
1.65	- 94.2	-100.	0.799	80.4	70.
3.36	73.8	- 85.7	2.60	590.	589.
4.07	0.	0.	2.67	26.5	25.9
4.15	1040.	1060.	3.08	383.	386.
4.65	582.	559.	3.60	765.	763.
6.54	20.9	38.1	5.85	611.	612.
7.49	467.	-515.	6.22	218.	218.
8.68	44.1	98.8	7.93	171.	171.

The surface wave can only be calculated with the sloshing mode. Using NRC RG.1.60 response spectrum input at the upper deck level normalized to 0,1 g the height calculated is 0,27 m.

6. CONCLUSION

A formulation has been developped for the analysis of incompressible fluid and structure coupled problem. This method has been implemented in a finite element program for axisymetric structures and has been validated by comparison with analytical solutions and tests. The program has been used for the seismic analysis of a large pool type sodium cooled reactor. Surface wave has been shown to have a negligible effect on structure response.

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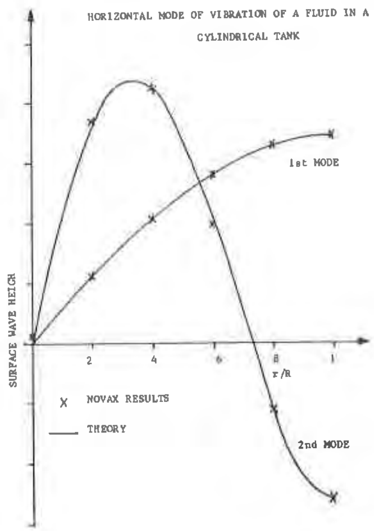

FIGURE 1 : Cylindrical tank sloshing mode

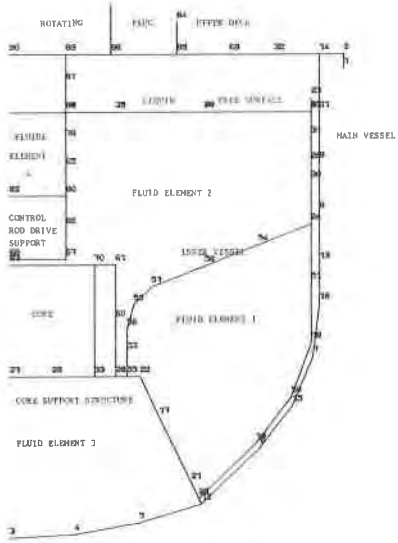

FIGURE 2 : Liquid natural frequency variation in a rigid spherical tank

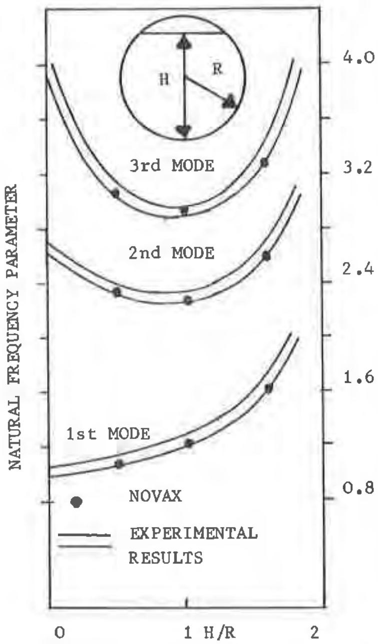

FIGURE 3 : Pool type breeder reactor seismic model