

Seismic loadings and measured parameters combined by power spectral density methods on a microcomputer

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1 INTRODUCTION

The qualification of equipment to resist the design seismic loading may require determining what motions will be felt at the points of support for the equipment, and many other calculations of seismic and vibrational effects require that dynamic loadings be established at points other than those at which they have been prescribed or calculated in the course of structural analysis. These calculations have always presented a number of experimental and theoretical difficulties. In particular the engineer needs a procedure for expressing the seismic excitation in a form that is convenient for calculation, a technique for determining the amplification functions for the structures and equipment through which the seismic loadings are transmitted, and a computational implementation that is convenient and efficient. This paper describes the use of previously published algorithms to develop the power spectral density function for a seismic excitation previously described in terms of its response spectrum and to compute the amplification function for a system whose modal shapes, but not its mass distribution, are known. The resulting computer program, which runs on an IBM PC/XT microcomputer, includes graphical screen output that permits the engineer to determine the current state of the calculations and to modify the course of the calculations.

2 COMPUTING THE POWER SPECTRAL DENSITY

The algorithm for finding the power spectral density function, given the response spectrum, follows the previous work of Kaul (1978) and Unruh and Kana (1981). In the latter's notation, the basic equation for the relative motion of a single-degree-of-freedom system is

$$\ddot{z} + 2\beta\omega_0\dot{z} + \omega_0^2 z = -\ddot{x} \quad (1)$$

where z is the relative displacement, \ddot{x} is the base input time history of acceleration, β is the critical damping ratio, ω_0 is the resonant frequency of the undamped oscillator, and superposed dots indicate differentiation with respect to time. It is well known that, for any specified time history \ddot{x} , a response spectrum $R(\omega_0)$ can be calculated. In what follows the response spectrum will be expressed in terms of

acceleration, unless otherwise noted.

If the seismic input is assumed to be a stationary, random, and Gaussian process with a zero mean, it can also be described in terms of its power spectral density $\Phi(\omega)$, where ω is now understood to be any value of frequency and is not related to the oscillator frequency ω_0 . Vanmarcke (1976) provides a summary of the application of the power spectral density in earthquake engineering and also points out some of the idiosyncracies that affect numerical integration. The power spectral density of the oscillator response is expressed by $\Phi(\omega, \omega_0)$, and a basic result of linear dynamics is that

$$\Phi(\omega, \omega_0) = \left[\frac{\omega_0^4 + 4\omega_0^2 \beta^2 \omega^2}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 + 4\omega_0^2 \beta^2 \frac{\omega^2}{\omega_0^2} \right]} \right] \Phi(\omega) \quad (2)$$

The variance of the oscillator response is

$$\sigma^2(\omega_0) = \int_{-\infty}^{\infty} \Phi(\omega, \omega_0) d\omega \quad (3)$$

and the variance of the time derivative of the response is

$$\dot{\sigma}^2(\omega_0) = \int_{-\infty}^{\infty} \omega^2 \Phi(\omega, \omega_0) d\omega \quad (4)$$

Once these functions have been found, they can be combined with other parameters to obtain the amplitude factor:

$$F_0(\omega_0) = \left[-2 \ln \left\{ \frac{\pi}{T} \frac{\sigma}{\dot{\sigma}} \ln(1-r) \right\} \right] \quad (5)$$

where T is the duration of the seismic motion and r is the probability that the computed motion will exceed the estimates calculated. The response spectrum can then be expressed approximately but very accurately by

$$R(\omega_0) = F_0(\omega_0) \alpha(\omega_0) \quad (6)$$

Kaul (1978) states, in effect, that this expression deviates from the exact value for very short durations of motion or for very large values of damping, and neither situation is of interest here.

Unruh and Kana (1981) continue the above development to propose a method of finding $\Phi(\omega)$ given $R(\omega_0)$. They start with Kaul's (1978) approximate expression for the power spectral density evaluated at the same points at which the response spectrum is defined:

$$\hat{\Phi}(\omega_0) = \frac{2\beta}{\pi\omega_0} R^2(\omega_0) \left\{ 2 \ln \left[\left[\frac{\pi}{\omega_0 T} \right] \ln(1-r) \right] \right\}^{-1} \quad (7)$$

First the approximate value of $\Phi(\omega)$ is computed at all desired frequencies. Then the response spectrum $R(\omega)$ is evaluated at the same

frequencies from equation (6). The estimate of $\Phi(\omega)$ is then corrected in proportion to the square of the ratios between the desired and the back-computed response spectra. The procedure is repeated for the new estimate of $\Phi(\omega)$ until the iterations converge satisfactorily. Unruh and Kana (1981) report satisfactory convergence after five cycles.

The authors have programmed the method proposed by Unruh and Kana (1981) for an IBM PC/XT. The response spectrum can be described by a number of values of frequency and acceleration, and the program will interpolate between the input values to obtain a description at up to 400 points distributed either logarithmically or linearly over a desired range of frequencies. The input and interpolated response spectra as well as the results of any number of iterations can be plotted on the screen to allow the user to control the number of iterations and the convergence criterion. The power spectral density can also be plotted on the screen, and all plots can also be made on peripheral plotter or printers to provide hard copies of the work.

The implementation of the algorithm required some attention to the details of the numerical integration of equations (3) and (4). The integrands have very sharp peaks near the undamped resonant frequency ω_0 , and for frequencies significantly less than ω_0 they are essentially equal to $\Phi(\omega)$ and $\omega^2 \Phi(\omega)$. After considerable experimentation, it was found that sufficient accuracy could be obtained by a scheme employing very small increments of ω at the peak near ω_0 and increasing the increments as the frequency moved away from that at the peak without permitting the interval of frequency to exceed the digitization interval.

3 AMPLIFICATION FUNCTION

Once the power spectral density is known for the input motion, the power spectral density can be computed elsewhere in the structure or equipment, provided an appropriate amplification or transfer function exists. If the complex transfer function is $H(\omega)$, then the magnitude of the power spectral density $\Phi(\omega)$ at the second point is

$$\Phi(\omega) = |H(\omega)|^2 \Phi(\omega) \tag{8}$$

Sadik, et al. (1984) describe several methods by which the transfer function can be found. The most direct approaches require that the distribution of mass in the amplifying structure be established, and it is clear that this poses many experimental difficulties. However, they do propose one technique for calculating the transfer function when the mass is not known explicitly but the shapes of the significant modes have been determined analytically or experimentally. The procedure starts with the matrix form of the basic equations of relative motion:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M]\{ \{I_x\}\ddot{x}_b + \{I_y\}\ddot{y}_b + \{I_z\}\ddot{z}_b \} \tag{9}$$

where the notation is the customary one with the addition that \ddot{x}_b is the input base acceleration in the x direction, $\{I_x\}$ is the vector of ones and zeros that selects and distributes the base acceleration \ddot{x}_b to the appropriate degrees of freedom, and the corresponding terms in y and z do the same for the other directions of motion. It then follows by the usual derivation that the modal responses will be governed by the

equivalent of equation (1):

$$\{\ddot{\alpha}\} + [2\beta_r \omega_r] \{\dot{\alpha}\} + [\omega_r^2] \{\alpha\} = -\{\Gamma_x\} \ddot{x}_b - \{\Gamma_y\} \ddot{y}_b - \{\Gamma_z\} \ddot{z}_b \quad (10)$$

where $\{\alpha\}$ is the modal displacement, r indicates the mode number, and the vectors of Γ are the modal participation factors for x , y , and z directions of excitation.

If $[\Phi]$ is the matrix of all the modal shapes expressed as columns, then the matrix of masses associated with each mode r is

$$[M_{rr}] = [\Phi]^T [M] [\Phi] \quad (11)$$

The modal participation factors are

$$\{\Gamma_x\} = [M_{rr}]^{-1} [\Phi]^T [M] \{I_x\} \quad (12)$$

and similar equations for the y and z directions of excitation. It also follows that, when all the modes are used,

$$\{I_x\} = [\Phi] \{\Gamma_x\} \quad (13)$$

Sadik et al. (1984) recognize that, when a reduced set of modes is used, $[\Phi]$ is replaced by a non-square matrix $[\Psi]$ and substituting it into the above equations and minimizing the error in the force vector leads to

$$\{\Gamma_x\} = ([\Psi]^T [\Psi])^{-1} [\Psi]^T \{I_x\} \quad (14)$$

They then show that the transfer function between a direction of input x and a degree of freedom s is

$$H_{sx}(\omega) = \sum_{r=1}^N \phi_{sr} \{\Gamma_r\}^T \{I_x\} h_r(\omega) \quad (15)$$

In this equation ϕ_{sr} is the component of mode r for the s degree of freedom and $h_r(\omega)$ is the complex modal transfer function:

$$h_r(\omega) = \frac{\omega_r^2 + i2\beta_r \omega_r \omega}{\omega_r^2 - \omega^2 + i2\beta_r \omega_r \omega} \quad (16)$$

Thus the complex transfer function can be computed without direct use of the mass matrix. The results will be exact if all the modes are used and approximate if a limited set is used. The accuracy of the approximation depends on the number of modes used to express $[\Psi]$, and that can be checked by determining how well equation (13) is satisfied.

The above methodology was also implemented on the IBM PC/XT. Graphic displays show the computed magnitude of the transfer function, and interactivity permits the user to determine whether the approximation to

equation (13) is satisfactory. The transfer function is then used in conjunction with the power spectral density to calculate the power spectral density at other points in the structure.

4 COMPUTER IMPLEMENTATION AND USE

The algorithms for generating power spectral densities and response spectra from each other and for developing amplification functions from the mode shapes were implemented in a computer program called SPEED (for Spectral Power Evaluation of Equipment for Dynamics), which operates on an IBM PC/XT. The code is written primarily in FORTRAN, but commercially available packages were also used to create interactive screens for input and output of alpha-numerical data and to develop graphical output on screens and peripheral plotting devices. Experience has shown that the program makes it possible for an engineer to find the power spectral density and to compute the response at another elevation within the time for one session on the computer. This has greatly improved productivity.

The principal use of the system to date has been in the qualification of equipment located in racks and other secondary supports. Field measurements provide data on the mode shapes, and the desired response spectra at the bases of the racks are the result of previous analyses. SPEED is then used to find the power spectral density and the amplification functions for the points of support of the equipment, from which the response spectra can be found. The system can also be used to calculate response spectra for different values of damping and at different points on the structure without having to generate a full artificial earthquake.

Developing SPEED involved the usual programming considerations and problems. In addition attention to interactive features and the use of commercially available productivity tools greatly enhanced the utility of the resulting system. It was also found that the details of the numerical integrations had to be treated carefully if undesirable errors were to be avoided. The program is now in productive use, and several improvements are planned for future versions.

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