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LYAPUNOV EXPONENTS FROM EXPERIMENTAL TIME SERIES OF BISTABLE FLOWS ON TRIANGULAR TUBE BANK ARRAYS

Alexandre Vagtinski de Paula[1] and Sergio Viçosa Möller[2]

[1]PhD Student, Federal University of Rio Grande do Sul, Porto Alegre - RS, Brasil (vagtinski@mecanica.ufrgs.br).
[2]Professor, Federal University of Rio Grande do Sul, Porto Alegre - RS, Brasil (svmoller@ufrgs.br).

ABSTRACT

Banks of tubes or rods are found in the nuclear and process industries, being the most common geometry used in heat exchangers. Evidences from experimental measurements shown the presence of instabilities in tube banks with square arrangements, generated after the second row of the tube bank (Olinto et al., 2005, 2007). This behavior was also observed after the second row of a tube bank with triangular arrangement by De Paula et al. (2009), and it is associated to the phenomenon of bistability known in the flow on two cylinders side-by-side.

Chaotic time series are observed routinely in experiments on physical systems (Abarbanel et al., 1993). Although unknown, the study of the dynamic process of bistability can reveal new features about the chaotic behavior of these time series.

This work presents a study about the determination of the Lyapunov exponents from experimental time series of bistable flows bistable flows on triangular tube bank arrays. A discrete wavelet transform is used to make a multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level. The Rosenstein's method is applied to calculate the largest Lyapunov exponent of the time series.

INTRODUCTION

Banks of tubes or rods are found in the nuclear and process industries, being the usual simplification for fluid flow and heat transfer in the study of shell-and-tube heat exchangers. Geometric characterization of a tube bank is made by the p/d-ratio, being d the tube diameter and p, the pitch, which is the distance between the centerlines of adjacent tubes.

The turbulent flow impinging on circular cylinders placed side-by-side presents a floppy and random phenomenon that changes the flow mode. This behavior is called bistable flow (Bearman and Wadcock, 1973; Fitzhugh, 1973) and is characterized by a wide near-wake behind one of the cylinders and a narrow near-wake behind the other, which generates two dominant Strouhal numbers, each one associated with one of the two wakes formed, as observed in Fig. 1. Since bistability was found recently in tube banks with square and triangular arrangements (Olinto et al., 2009; De Paula et al., 2012), the study of the behavior of the bistability phenomenon in simplified geometries, as in the case of two tubes placed side-by-side, helps in understanding the parameters and variables that influence more complex geometries as in a complete tube bank. As flow induced vibration and structure-fluid interaction are very dependent of the arrangement or configuration of the cylinders, new studies are necessary to improve its understanding, since bistability can be an additional excitation mechanism on the tubes.

Measurements of non-stationary phenomena, like the bistable flow, produce time varying series, where the Fourier analysis cannot be used. Instead, modern literature presents the wavelet technique as a valuable tool to analyze non-stationary time series and their possible singularities (Farge et al., 1999) and especially for the switching phenomenon in two side-by-side cylinders (Alam et al., 2003; Alam and Sakamoto, 2005).

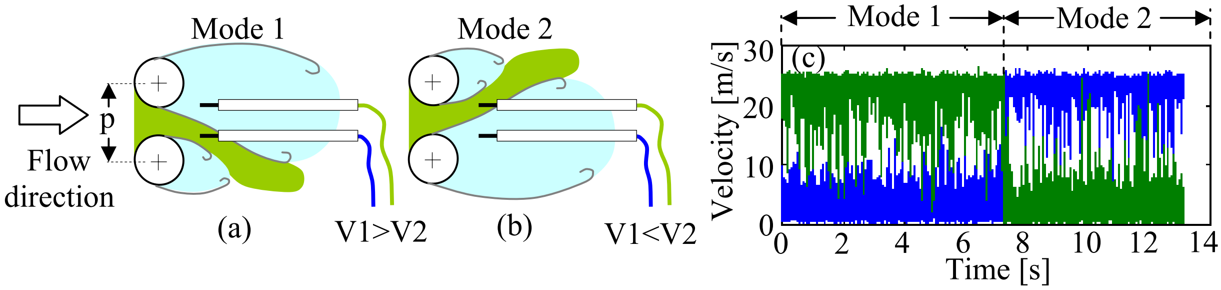

Figure 1. Bistability scheme for (a) mode 1 and (b) mode 2, and their respective characteristic hot wire anemometry signals (c).

EXPERIMENTAL TECHNIQUE

The velocity of the flow and its fluctuations are measured by means of a DANTEC *StreamLine* constant hot-wire anemometry system. One single hot wire probe (type DANTEC 55P11) was used in the experiments, which has a wire perpendicular to the main flow. The aerodynamic channel used in the experiments is made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The air is impelled by a centrifugal blower of 0.64 kW, and passes through two honeycombs and two screens, which reduce the turbulence intensity to about 1 % in the test section. Upstream the test section, placed in one of the side walls, a Pitot tube measures the reference velocity of the non-perturbed flow. Data acquisition is performed by a 16-bit A/D-board (NATIONAL INSTRUMENTS 9215-A) with USB interface. The acquisition frequency of time series was of 1 kHz, and a low-pass filter of 300 Hz was used to avoid aliasing. The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), are rigidly attached to the top wall of test section and their extremities are closed. The probe support is positioned with 3D transverse system placed 200 mm downstream the outlets (Fig. 2b). The measurements were performed aligning the probes along the tangent to the external generatrices of the tubes (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3 %. The Reynolds number of the experiment is 21,000, computed with the tube diameter and the reference velocity of 12.9 m/s.

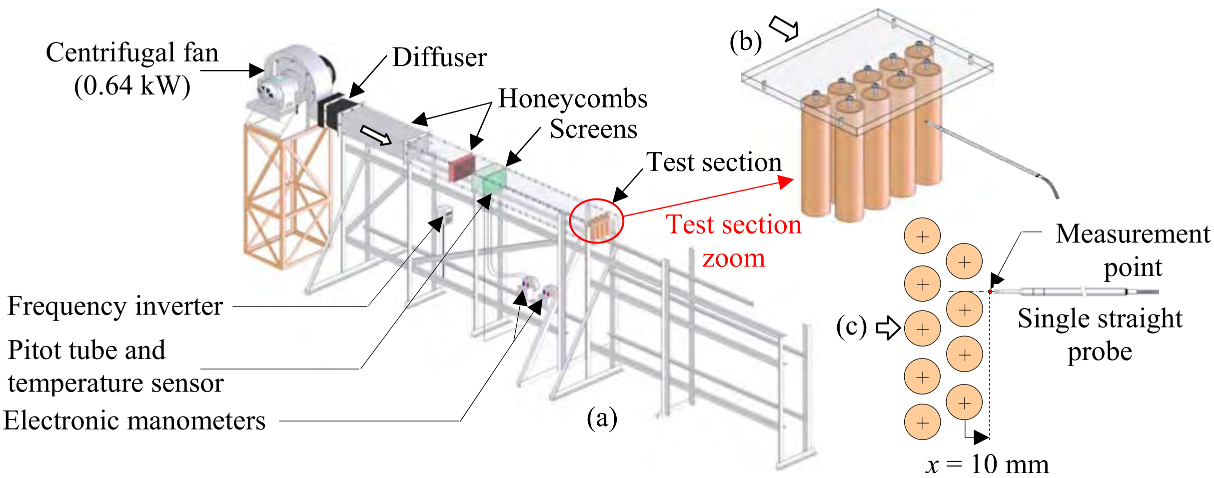

Figure 2. Schematic views: (a) aerodynamic channel, (b) test section and (c) probe position.

METHODOLOGY

The Rosenstein's method (Rosenstein et al., 1993) allows estimating the largest Lyapunov exponent from an experimental time series and takes into account all available data. Easy to implement, fast and robust with respect to changes in the embedding dimension, sample size of data, reconstruction step and noise level, the Rosenstein algorithm also allows simultaneously to calculate the correlation dimension. Thus, the following calculation generates estimates both as the level of complexity of the chaos system. The first step to estimating the largest Lyapunov exponent is to reconstruct the dynamics of the attractor from a time series. In this task, the method of time delays (Takens, 1981) was applied. The reconstructed trajectory, $\mathbf{X}$, can be expressed as a matrix, in which each column is a vector of phase space, in the form

$$\mathbf{X} = \left(X_1, X_2, \ldots, X_M\right)^T \tag{1}$$

where X_i is the state of the system in the discrete time i. For a time series with N points, $\{x_1, x_2, \ldots, x_N\}$, each X_i is given by

$$X_i = \left(x_i,\, x_{i+p}, \ldots, x_{i+(m-1)p}\right) \tag{2}$$

where p is the reconstructing step. Thus, $\mathbf{X}$ is a $M \times m$ matrix, and the constants M, m, p and N are related by

$$M = N - (m-1)p \tag{3}$$

The embedding dimension is generally estimated according to the Takens' theorem, i.e., to $m \geq 2D_0 + 1$ (where D_0 is the fractal or Hausdorff dimension), although the algorithm often presents good results when m is below this criterion. With respect of the choice of reconstruction step, the algorithm appears quite robust. After the reconstruction of the state space, the algorithm finds the "nearest neighbors" of each point of the trajectory. The near neighbor $X_{\hat{j}}$ is found by finding the point that minimizes the distance from a particular reference point X_j expressed by

$$d_j(0) = \min_{X_j} \left\| X_j - X_{\hat{j}} \right\| \tag{4}$$

where $d_j(0)$ is the initial distance of the j^{th} point to the next neighbor and || || denotes the Euclidean norm. An additional constraint imposed on nearby neighbors is that their temporal separation should be greater than the average length of the time series, given by

$$| i - \hat{j} | > T, \tag{5}$$

where T is the average period of the time series. This restriction allows us to consider each pair of neighboring as close initial conditions to different trajectories. The average period is estimated reciprocate the average frequency of the power spectrum, typically using a Fast Fourier Transform (FFT). However, in this work a power spectral density function (PSD) will be used instead. The largest Lyapunov exponent is estimated as the average separation of nearest neighbors

$$\lambda_1(i) = \frac{1}{i\,\Delta t}\;\frac{1}{(M-i)}\sum_{j=1}^{M-i}\;\ln\frac{d_j(i)}{d_j(0)}, \tag{6}$$

where Δt is the sampling period of time series, $d_j(i)$ is the distance between the j^{th} pair of nearest neighbors after step discrete time i or $i\Delta t$ seconds, and M is the number of reconstructed points. According to Rosenstein et al. (1993), the mathematical definition of the largest Lyapunov exponent can be expressed by

$$d(t) = C\,e^{\lambda_1(t)}, \tag{7}$$

where $d(t)$ is the average divergence in the time t and C is the constant that minimizes the initial separation. From the determination of λ_1 (equation 6), it is assumed that the j^{th} pair of nearest neighbors diverge about the rate given by the largest Lyapunov exponent

$$d_j(i) \approx C_j\,e^{\lambda_1(i\,\Delta t)}, \tag{8}$$

where C_j is the initial separation. Applying the logarithm on both sides of the equation (8), we obtain

$$\ln d_j(i) \approx \ln C_j + \lambda_1(i\,\Delta t). \tag{9}$$

For $j = 1, 2, \ldots, M$, the above equation represents a set of approximately parallel lines, each with a slope proportional to λ_1. Normalization in the separation of neighboring, given by C_j in equation (8) it is unnecessary to estimate λ_1, as shown in equation (9), reducing the computational cost. Thus, the largest Lyapunov exponent can be calculated using a linear fit by least squares method, applied to a mean line defined by

$$y(i) = \frac{1}{\Delta t}\langle \ln d_j(i)\rangle, \tag{10}$$

wherein $\langle\;\rangle$ denotes averaging over all values of j. According to Rosenstein et al. (1993), this process of obtaining the average value of the differences is the key to obtaining accurate values of λ_1 using series with few data points and in the presence of noise. A graph of $\langle \ln d_j(i)\rangle$ or the average value of ln(divergence) versus $i\Delta t$ or time (s) is constructed for various values of m (Fig. 3).

After a short transition, there is a long linear region which is used to extract the value of the largest Lyapunov exponent. The curves saturate for long values of $i\Delta t$, because the system is bounded in phase space and the divergence cannot exceed the average "length" of the attractor.

It is important to assess the value of λ_1 for various values of the embedding dimension m, because usually there is no prior knowledge of this value for experimental series. With the Rosenstein's algorithm is possible to obtain satisfactory results for $m \geq n$ (well below the Takens' criterion), where n is the topological dimension of the system. This is due to the fact that chaotic systems are effectively stochastic when embedded in a phase space that is too small to accommodate the true dynamic. The linear region for determining λ_1 can also be derived from a graph of slope vs. time, where a plateau, i.e., a zone with constant values (zero slope) indicates the region of interest.

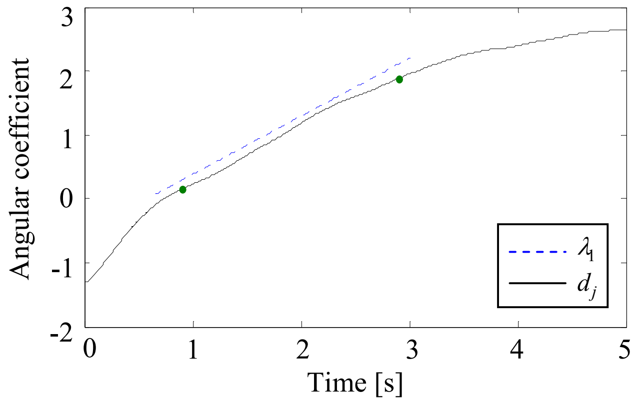

Figure 3. Typical graph of the average of ln(divergence) versus time for the Lorenz attractor.

RESULTS

Measurements performed after two rows of tubes in a triangular arrangement with a pitch-to-diameter ratio of P/D = 1.26 with a single hot wire probe placed in the gap between and downstream the tubes are shown in Fig. 4a, according to the position presented by Fig. 2c. The time series of axial velocity of the flow , which has a total of 2^{15} = 32768 data points (N), presents five changes between two distinct velocity levels, concerning to 3.0 m/s (wide near-wake - mode 1) and 18.6 m/s (narrow near-wake - mode 2). A reconstruction of the signal, filtered by discrete wavelet transfor (Fig. 4b) is performed accordingly with a selected decomposition level (n = 8, with frequency content from 0 to 1.9531 Hz), where the bistable phenomenon can be dissociated from turbulence.

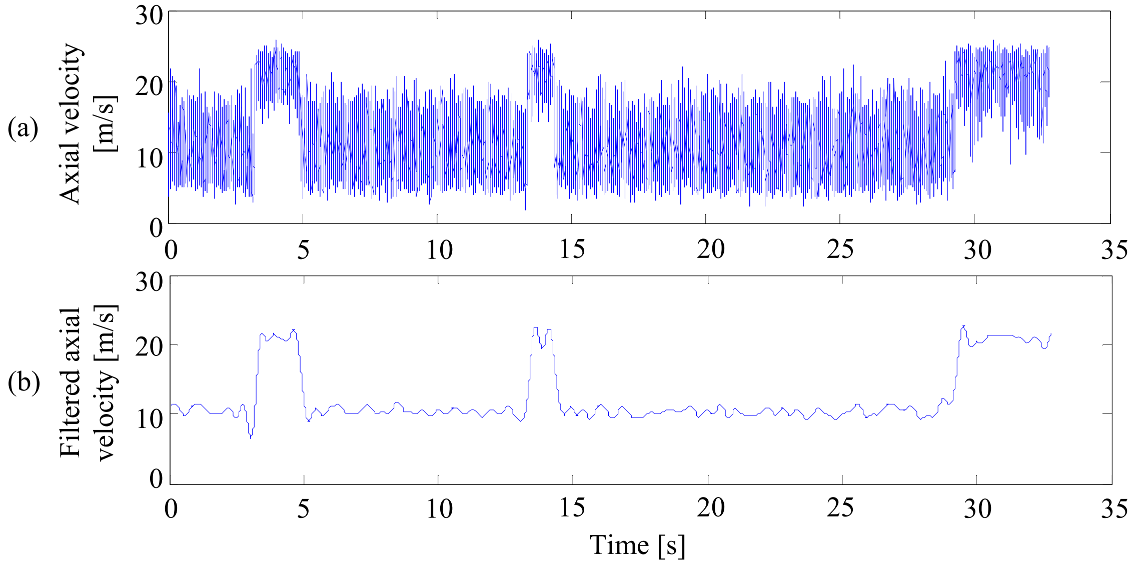

Figure 4. (a) Time series measured after two rows of tubes in a triangular arrangement with a pitch-to-diameter ratio of P/D = 1.26 with a single hot wire probe. (b) Filtered time series by means of discrete wavelet transform with decomposition level n = 8 (frequencies from 0 to 1.9531 Hz).

For an infinite number of points and without the presence of noise, the Takens' theorem shows that the choice of reconstruction step (p) is in most cases arbitrary. However, due to the finite size of the experimental time series and its contamination by external noise, reconstruction depends much in the correct choice of step reconstruction. Two limiting cases can occur when the incorrect choice of the parameter p: if it is chosen a value too small for p, $x(t)$ and $x(t+p)$ will have similar values, with the reconstructed attractor presenting a linear dependence, i.e., they may be compressed around the diagonal $y = x$ (in the case of an embedding dimension $m = 2$) which does not occur in real components x and y. On the other hand, if the parameter p is choosing very large, the reconstructed vectors will be completely uncorrelated, covering the whole state space. A simple and widely used criterion for choosing the parameter p is to choose a reconstruction step the order of time autocorrelation of the signal, τ. Then, the step p is chosen to be approximately equal to τ. Thus, it is guaranteed that x_i and x_{i+p} are linearly independent, but not totally uncorrelated.

The autocorrelation coefficient function (Fig. 5a) shows no local minimum, and the power spectral density function (PSD) of the signal, presented in Fig. 5b, shows the behavior of a broadband process. The mean period of the time series cannot be established from this graph.

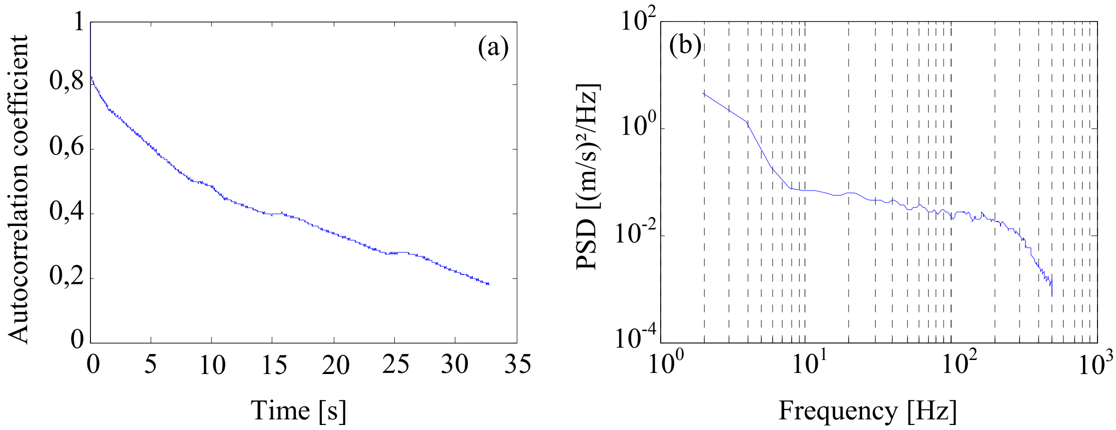

Figure 5. (a) Autocorrelation coefficient function. (b) Power spectral density function (PSD) of the signal.

An alternative way to reconstruct the attractor is choosing several values of the time delay. When the attractor is not compressed around the diagonal nor covering the whole state space, it will be sufficiently unfolded. The time step chosen in this process is p = 150 data points.

The reconstruction of the attractor from the original time series (with all the frequency content) is presented in Fig. 6, where the points are spread in over a large region in the state space. To overcome this situation, the attractor is reconstructed from the filtered signal (Fig. 4b), when only the frequency content between 0 to 1.9531 Hz is present. The result of this reconstruction is shown in Fig. 7, where the complex dynamic system can be evidenced.

The percent of false neighbors is calculated from embedding dimension m = 1 to 10 and with a coverage radius L_c = 15 data points, where the result is shown in Fig. 8a. Above m = 7 there is no false neighbors. However, for $m \geq 5$, the number of false neighbors is small (below 1.15 %), being m = 5 an adequate embedding dimension.

The mean period of the time series is considered as T = 0.015 s or 15 data points. The highest Lyapunov exponent are estimated with the Rosenstein's method for m = 3, 5, 7 and 10 with 6000 iterations, where the results are presented in Fig. 8b. The final value of the highest Lyapunov exponent is an average over the calculus for different embedding dimensions, where the standard deviation of the solution is also presented. The highest Lyapunov exponent was calculated as λ_1 = 0.5318 ± 0.0441, indicating the presence of chaos.

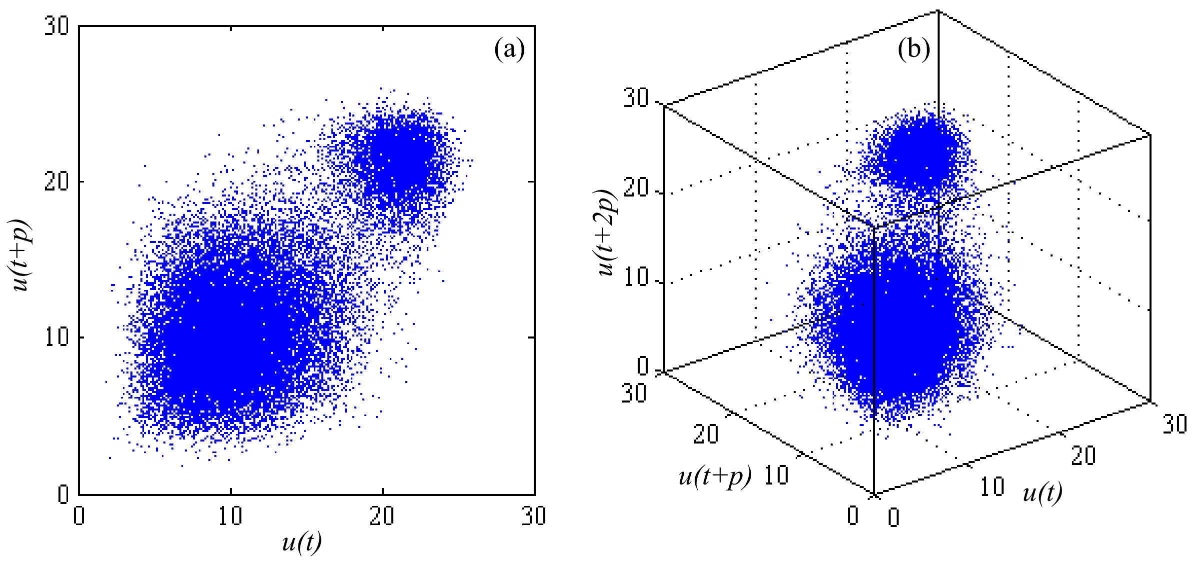

Figure 6. Reconstruction of the attractor from the original time series (with all the frequency content). (a) 2D reconstruction. (b) 3D reconstruction.

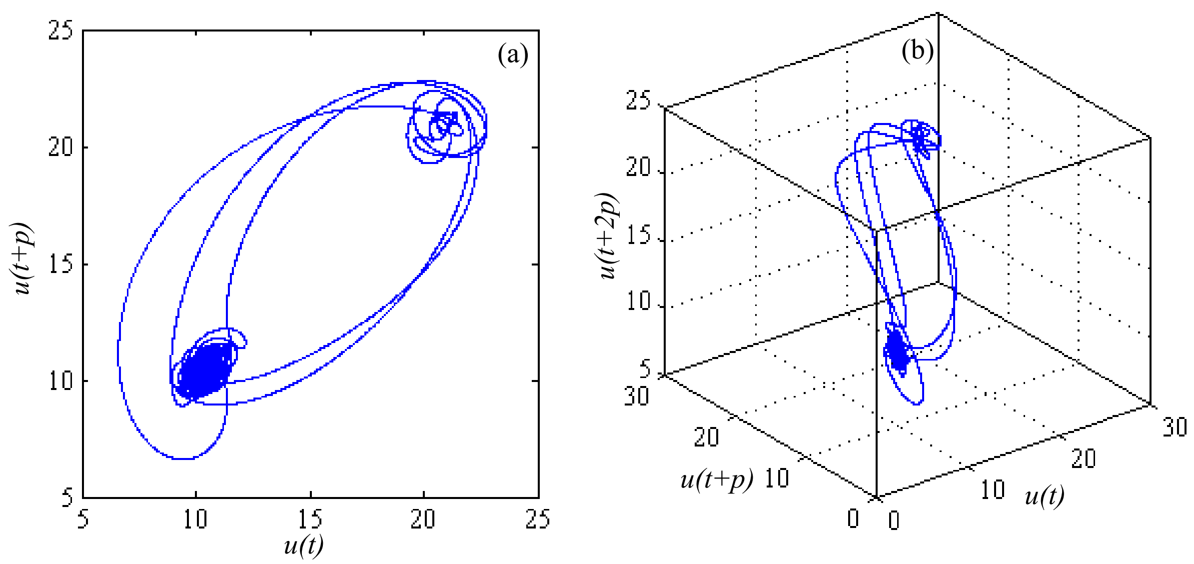

Figure 7. Reconstruction of the attractor from the filtered signal (with a frequency content between 0 to 1.9531 Hz) (a) 2D reconstruction. (b) 3D reconstruction.

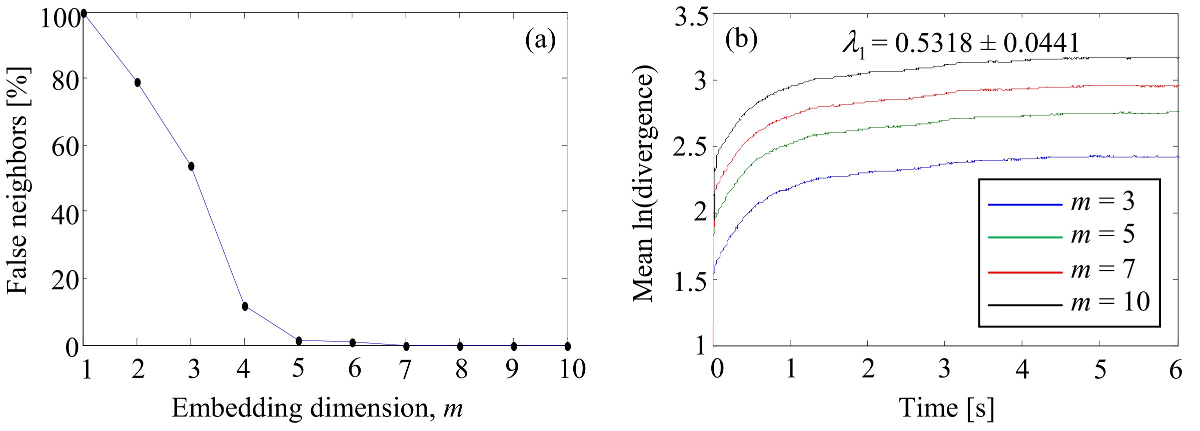

Figure 8. (a) Percent of false neighbors according to the embedding dimension. (b) Largest Lyapunov exponent according to the embedding dimension.

CONCLUSIONS

In the study of the bistable phenomenon can be applied techniques from dynamic systems based on time series analysis. Techniques of state space reconstruction have been extensively used when only time series are available. The Lyapunov exponents provide both qualitative and quantitative characterization of the chaotic behavior of nonlinear systems, where one positive Lyapunov exponent is indicative of chaos. In this respect, this paper presents the reconstruction of the state space of both original time series, as the series filtered by discrete wavelet transform. To obtain the delay used in the reconstruction of the state space, the autocorrelation function of the signal seems to be not representative. An alternative way to reconstruct the attractor is choosing a delay where the attractor is not compressed around the diagonal nor covering the whole state space. The embedding dimension of the attractor was determined by analyzing the percentage of false neighbors, giving an indication of the number of equations needed to describe the system. The largest Lyapunov exponent is calculated to identify the chaotic behavior of the experimental time series. Rosenstein's method was used in this step, which is fast and takes into account all available data. Easy to implement, the method is robust with respect to changes in most embedding parameters. The time series analyzed present largest Lyapunov exponents with positive sign, which is indicative of chaotic behavior. After filtering the series by discrete wavelet transform, the details of the reconstructed attractor can be observed, showing that its trajectory in state space is complex and its pattern is rich in details.

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NOMENCLATURE

D	Diameter of the tubes
D_0	Fractal or Hausdorff dimension
$d(t)$	Average divergence in the time t
$d_j(0)$	Initial distance
Δt	Sampling period of time series
C_j	Constant that minimizes the initial separation
$i\Delta t$	Discrete time
i	Discrete time
λ_1	Largest Lyapunov exponent
M	Number of reconstructed points
m	Embedding dimension
N	Number of points of the time series
P	Pitch or distance between the centers of two adjacent tubes
P/D	Pitch-to-diameter ratio
p	Reconstructing step
T	Average period of the time series
t	Time
τ	Time autocorrelation of the signal
x	Distance of the probe downstream the center of the tubes
$\mathbf{X}$	Reconstructed trajectory
X_i	State of the system
X_j	Particular reference point
$X_{\hat{j}}$	Near neighbor
$\langle \ln d_j(i) \rangle$	Average value of the logarithm of the divergence

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