APPROXIMATE SOLUTION OF PLANE THERMAL STRESS PROBLEMS IN DOUBLY-CONNECTED REGION BASED ON AN INTEGRAL EQUATION

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SUMMARY

The advent of nuclear reactors has confronted the researchers or designers with new problems of thermal stresses. In accordance with this rapid development in engineering, a great number of papers have been published about thermoelasticity and also a lot of solving methods have been used in these papers. Unfortunately, if the body has irregular boundaries, it is almost impossible for us to obtain exact solution. For such problems, we had to use the finite difference method or the finite element method as one of approximation. Both numerical techniques, however, are accompanied with an unavoidable error or are time-consuming.

The purpose of this paper is to explain a new method calculating thermal stresses in two dimensions. In this method the fundamental partial differential equation has to be replaced by an integral equation and then, this new fundamental equivalent equation can be solved by an adaption of a numerical integral technique or graphical integration method.

In this new approach, a foundation for finding $\Delta\Delta\chi$ is given by next integral form after Taylor's expansion about location $x+\xi$ and $y+\eta$

$$
\begin{aligned}
204\times 10^4 s^6\, \Delta\Delta\chi(x,y) = 9\Big\{5^8 \int_{-2s}^{2s}\int_{-2s}^{2s} \chi(x+\xi, y+\eta)\, d\xi d\eta - 4^8 \int_{-5s/2}^{5s/2}\int_{-5s/2}^{5s/2} \times \\
\times\, \chi(x+\xi, y+\eta)\, d\xi d\eta\Big\} + 120 s\Big[5^7\Big\{\int_{-2s}^{2s} \chi(x+\xi, y)\, d\xi + \\
+ \int_{-2s}^{2s} \chi(x, y+\eta)\, d\eta\Big\} - 4^7\Big\{\int_{-5s/2}^{5s/2} \chi(x+\xi, y)\, d\xi + \\
+ \int_{-5s/2}^{5s/2} \chi(x, y+\eta)\, d\eta\Big\}\Big] - 205\times 10^3 s^2 \times \\
\times (116X_1 + X_2 + 40X_3 - 2X_4) + 28616\times 10^2 s^2 \times \\
\times\, \chi(x,y) + O(s^{10})
\end{aligned}
$$

where

$$X_1 = \chi(x+s, y) + \chi(x-s, y) + \chi(x, y+s) + \chi(x, y-s)$$

Other X_i have similar forms as X_1.

As one of practical procedure, we can show the following patterns of weight of the nodal points for the conversion of plane thermal stress problems.

For example, the pattern of $\Delta\Delta\chi$ for steady state thermoelasticy is

$$
\begin{bmatrix}
0, & -1, & 8, & -1, & 0 \\
-1, & 20, & -62, & 20, & -1 \\
8, & -62, & 144, & -62, & 8 \\
-1, & 20, & -62, & 20, & -1 \\
0, & -1, & 8, & -1, & 0
\end{bmatrix} \chi(x_i, y_i) = 0
$$

A numerical example is carried out for the problem of rectangular plate with a similar rectangular hole under non-uniform temperature distribution.