IMPACT LOAD TIME HISTORIES FOR VISCOELASTIC MISSILES

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SUMMARY

Missiles in nuclear power plants can be of various types depending on their origin. Missiles may be propelled by winds or by steam resulting from pipe breaks. Containment shell structure may be subjected to the aircraft crash. Tornado borne missiles may include steel rods and pipes, wooden poles, and automobiles, to name just a few. Internal accident generated missiles may be turbine blades, steel pipes and perhaps pieces of radiation shielding materials resulting from pipe breaks as is the case in the event of the loss of coolant accident. The targets, that may be subjected to these missiles, are concrete containment walls, internal concrete barrier walls, and other nuclear power plant components such as piping systems. Missiles can be considered to be either rigid or soft depending upon whether the deformation of the missile is small or large with respect to the target deformation. This paper deals with the effects of soft viscoelastic missile impact.

Generation of the impact load time history at the contact point between a viscoelastic missile and its targets is presented. In the past, in the case of aircraft striking containment shell structure, the impact load time history was determined on the basis of actual measurements by subjecting a rigid wall to aircraft crash. The effects of elastic deformation of the target upon the impact load time history is formulated in this paper. The missile is idealized by a linear mass-spring-dashpot combination using viscoelastic models. These models can readily be processed taking into account the elastic as well as inelastic deformations of the missiles. The target is assumed to be either linearly elastic or rigid. In the case of the linearly elastic target, the normal mode theory is used to express the time-dependent displacements of the target which is simulated by lumped masses, elastic properties and dashpots in discrete parts. In the case of Maxwell viscoelastic model, the time-dependent displacements of the missile and the target are given in terms of the unknown impact load time history. This leads to an integral equation which may be solved by Laplace transformation.

The normal mode theory is provided. The target structure may be composed of different materials with different components. Concrete and steel structural components have inherently different viscous friction damping properties. Hence, the equivalent modal damping depends on the degree of participation of these components in the modal response. An approximate rule for determining damping in any vibration mode by weighting the damping of each component according to the modal energy stored in each component is considered.

Examples are given for bricks with viscoelastic materials as missiles against a rigid target. Impact load time histories are plotted for various damping coefficients and stiffness constants of the missiles. These bricks were under consideration for radiation shielding purposes at the reactor coolant loop penetrations permitting them to be jettisoned, in the case of the loop break, and becoming missiles. The impact load time histories for viscoelastic missile models should be substantiated with experimental results.

1. Introduction

Nuclear power plant structures and systems are often postulated to be struck by flying objects and missiles. In the past, very little attention has been given to the development of impact loading, which plays an important role in the design and analysis of structures and systems subjected to missile impact. Impact load time histories may be determined analytically and experimentally.

Missiles may be simulated by viscoelastic models, such as those of Voigt or Kelvin, Maxwell, and standard linear solid model [1,2] in the analytical approach to the problem. These models are comprised of masses, springs, and dashpots in various combinations as simple models and may be built up to more complicated models by combining masses, springs, and dashpots in a more complex manner. Test results may indicate which model is more appropriate for a particular case.

Only the Voigt and Maxwell viscoelastic models to simulate missiles, as illustrated in Figure 1(a) and (b), are considered in this paper.

The target is considered to be either elastic or rigid. The elastic target is represented by a mathematical model consisting of lumped masses, elastic properties, and dashpots in discrete parts for the use of normal mode theory. The elastic target may be any structure or piping system, such as the pipe run in a pressurized water reactor (PWR) plant shown in Figure. 2.

2. Viscoelastic Models

Two simple viscoelastic models, as mentioned in the preceeding section, are considered to represent missiles for the development of impact load time histories, namely the Voigt model and the Maxwell model, all of which are composed of combinations of linear springs with spring constant k and dashpots with coefficient of viscosity c, as indicated in Figure 1(a) and (b). The Voigt model, which is comprised of spring and dashpot in parallel, is shown in Figure 1(a). The Maxwell model, consisting of spring and dashpot in series, is presented in Figure 1(b).

The missile is considered as a lumped mass, and the contact between the missile mass point and the target lumped mass point is established by a linear spring-dashpot combination, as illustrated in Figure 1(a) and (b) for the two simple viscoelastic models. If the two models can represent the missile, a linear relation can be formulated between u(t), the missile deformation, and F(t), the unknown impact load between the missile and the target. Elastic as well as inelastic deformations of the missile can be considered.

Applying Newton's law, one obtains the following equation:

$$m\,\ddot{u}_m(t) = -\,F(t) \qquad (1)$$

in which m is the mass of the missile, $\ddot{u}_m(t)$ is the second derivative of the missile mass point displacement $u_m(t)$ with respect to time t, and F(t) is the unknown impact load time history. The weight of the missile is omitted in eq. (1). The initial conditions at time t=0, when the missile model just touches the target at the beginning of impact with initial velocity v_0, are:

$$u_m(0) = 0, \ \dot{u}_m(0) = v_0 \tag{2}$$

Then the displacement $u_m(t)$, as a function of the impact load $F(t)$, is obtained by integrating eq. (1) and utilizing the initial conditions of eq. (2), as follows:

$$u_m(t) = v_0 t - \frac{1}{m} \int_0^t (t-\tau) F(\tau) d\tau \tag{3}$$

2.1 Voigt Model

In this section, the missile is simulated by the Voigt viscoelastic model. The missile is represented with a lumped mass m, and a linear elastic spring with spring constant k and a linear dashpot with damping coefficient c in parallel, as shown in Figure 1(a). Hence, the load-deflection relationship for this model is

$$c\dot{u}(t) + ku(t) = F(t) \tag{4}$$

where $u(t)$ is the missile deformation as a function of time and $\dot{u}(t)$ is the time-dependent velocity.

The general integral of the nonhomogeneous linear differential eq. (4) for the initial condition $u(0) = 0$ is

$$u(t) = \frac{1}{c} e^{-\frac{k}{c}t} \int_0^t e^{\frac{k}{c}\tau} F(\tau) d\tau \tag{5}$$

2.2 Maxwell Model

If the missile can be represented by the Maxwell model, the missile consists of a lumped mass m, a linear elastic spring with rigidity constant k, and a linear dashpot with coefficient of viscosity c. The linear spring and the dashpot are arranged in series as indicated in Figure 1(b). The load-deflection relationship for this model is

$$\dot{u}(t) = \frac{\dot{F}(t)}{k} + \frac{F(t)}{c} \tag{6}$$

where $\dot{F}(t)$ is the first derivative with respect to time t of the unknown impact load. When eq. (6) is to be integrated, the initial conditions at t=0 must be prescribed as follows:

$$u(0) = \frac{F(0)}{k}, \ \ u(0) = 0 \tag{7}$$

By integrating eq. (6) and utilizing initial conditions of eq. (7), the following missile deformation $u(t)$ is obtained:

$$u(t) = \frac{F(t)}{k} + \frac{1}{c} \int_0^t F(\tau) d\tau \tag{8}$$

3. Flexible Target

When the target subjected to missile impact can be considered to be linear elastic, the normal mode theory or the modal superposition method can be used in the analysis of the linear elastic system, such as the one shown in Figure 2. For this purpose, the mathematical model simulating the target is comprised of lumped masses, elastic properties, and dashpots in discrete parts.

The primary advantage of the modal superposition method is that the differential equations of motion are decoupled when the target displacements are expressed in terms of normal modes.

The coupled equations of motion in matrix form for a linear elastic lumped parameter system with viscous damping subjected to a time-dependent load, F(t), can be written as follows:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = F(t)\{D\} \tag{9}$$

where [m] denotes the mass matrix, [c] the matrix of viscous damping coefficients, [k] the stiffness matrix, $\{\ddot{u}\}$, $\{\dot{u}\}$ and {u} column matrices of accelerations, velocities, and displacements at the coordinates of the system, respectively, and {D} the column matrix governing the direction of the load.

3.1 Target Displacement

Having obtained free vibration characteristics of a linear elastic system, namely natural frequency matrix $[\omega]$ and the corresponding mode shape matrix $[\phi]$, the following transformations in terms of normal coordinates are applicable [3,4,5]:

$$\{u\} = [\Phi]\{q\} \tag{10}$$

$$\{\dot{u}\} = [\Phi]\{\dot{q}\} \tag{11}$$

$$\{\ddot{u}\} = [\Phi]\{\ddot{q}\} \tag{12}$$

Substitution of these equations into eq. (9) and premultiplication of eq. (9) by the transpose of the mode shape matrix, $[\phi]^T$, yields

$$[\phi]^T[m][\phi]\{\ddot{q}\} + [\phi]^T[c][\phi]\{\dot{q}\} + [\phi]^T[k][\phi]\{q\} = F(t)[\phi]^T\{D\} \tag{13}$$

Due to the orthogonality conditions of the normal modes, the following relations can be written:

$$[\phi]^T[m][\phi] = [M] \tag{14}$$

$$[\phi]^T[k][\phi] = [K] \tag{15}$$

$$[\phi]^T[c][\phi] = [C] \tag{16}$$

in which [M] is the generalized mass matrix. The generalized stiffness matrix $[K] = [\omega^2][M]$ and $[C] = 2[\beta][\omega][M]$ where $[\beta]$ denotes the matrix of damping ratios expressed as fractions of critical damping [4,5,6,7,8,9,10,11,12,13]. Using these equations in substitution of eqs. (14),(15), and (16) into eq. (13) and premultiplication of this equation by $[M]^{-1}$, one obtains the following uncoupled equations:

$$\{\ddot{q}\} + 2[\beta]\ [\omega]\ \{\dot{q}\} + [\omega^2]\ \{q\} = F(t)\ \{\gamma\} \tag{17}$$

where column matrix $\{\gamma\} = [M]^{-1}\ [\phi]^T\{D\}$.

According to matrix eq. (17) a single equation associated with the nth mode can be written as

$$\ddot{q}_n + 2\beta_n\omega_n\dot{q}_n + \omega_n^2 q_n = \gamma_n\ F(t) \tag{18}$$

The solution of this equation in terms of a convolution integral is

$$q_n(t) = \frac{\gamma_n}{\omega_n} \int_0^t \frac{1}{\sqrt{1-\beta_n^2}} F(\tau) e^{-\beta_n\omega_n(t-\tau)} \sin[\omega_n\sqrt{1-\beta_n^2}(t-\tau)]d\tau \tag{19}$$

Then the displacement of the target at the lumped mass point of impact, according to eq. (10), is

$$u_s(t) = \sum_{n=1}^{N} \frac{\phi_n\ \gamma_n}{\omega_n} \int_0^t \frac{1}{\sqrt{1-\beta_n^2}} F(\tau) e^{-\beta_n\omega_n(t-\tau)} \sin[\omega_n \sqrt{1-\beta_n^2}\ (t-\tau)]d\tau \tag{20}$$

3.2 Determination of Equivalent Modal Damping

The target may consist of different components with various materials having inherently different damping properties, such as concrete structures, steel structures and systems, and foundation media [5,8,9,11,12,13]. Hence, the effective damping in any vibration mode of the total system depends upon the degree of participation of these components in the modal response. This can be accomplished using the concept of weighted modal damping according to Biggs [13]. This is an approximate rule for determining damping for a mode by weighting the damping associated with the individual components according to the modal energy stored in each component.

The equivalent modal damping ratio of the nth mode, β_n, may be obtained in some cases by

$$\beta_n = \frac{\{\phi_n\}^T[\bar{k}]\{\phi_n\}}{\{\phi_n\}^T[k]\{\phi_n\}} \tag{21}$$

where $\{\phi_n\}$ is the nth mode eigenvector, $[k]$ is the overall system stiffness matrix, and $[\bar{k}]$ is a modified system stiffness matrix formed from component matrices obtained as the product of the component damping ratio and its stiffness matrix.

It should be noted that there are other ways of arriving at the equivalent modal damping for a particular system or structure, such as the one presented in reference [12].

3.3 Impact Load Time Histories

The deformation u(t) for both the Voigt and Maxwell viscoelastic models, presented by eqs. (5) and (8), can be expressed in terms of the target deflection $u_s(t)$ at the lumped mass impact point, eq. (20), and the displacement $u_m(t)$ of eq. (3), as follows:

$$u(t) = u_m(t) - u_s(t) \tag{22}$$

Substituting eqs. (5), (3), and (20) into eq. (22) yields the following integral equation for the Voigt missile model:

$$\frac{1}{c}\int_0^t e^{-\frac{k}{c}(t-\tau)} F(\tau)d\tau = v_0 t - \frac{1}{m}\int_0^t (t-\tau)F(\tau)d\tau$$

$$- \sum_{n=1}^{N} \frac{\phi_n \gamma_n}{\omega_n} \int_0^t \frac{1}{\sqrt{1-\beta_n^2}} F(\tau) e^{-\beta_n \omega_n (t-\tau)} \sin[\omega_n \sqrt{1-\beta_n^2}(t-\tau)]d\tau \tag{23}$$

where N corresponds to the number of modes to be considered.

By the same token, substituting eqs. (8), (3), and (20) into eq. (22) yields the following integral equation for the Maxwell missile model:

$$\frac{F(t)}{k} + \frac{1}{c}\int_0^t F(\tau)d\tau = v_0 t - \frac{1}{m}\int_0^t (t-\tau)F(\tau)d\tau$$

$$- \sum_{n=1}^{N} \frac{\phi_n \gamma_n}{\omega_n} \int_0^t \frac{1}{\sqrt{1-\beta_n^2}} F(\tau) e^{-\beta_n \omega_n (t-\tau)} \sin[\omega_n \sqrt{1-\beta_n^2}(t-\tau)]d\tau \tag{24}$$

The linear integral eqs. (23) and (24), given in terms of the unknown impact load F(t), may be solved by Laplace transformation. Eq. (23), associated with the Voigt model can be written, using Laplace transforms, as

$$\frac{1}{c}\frac{1}{s+\frac{k}{c}} f(s) = \frac{v_0}{s^2} - \frac{f(s)}{ms^2} - \sum_{n=1}^{N} \phi_n \gamma_n \frac{f(s)}{(s+\beta_n\omega_n)^2 + \omega_n^2(1-\beta_n^2)} \tag{25}$$

From this equation, the following expression for f(s), which is the Laplace transform of F(t), is obtained:

$$f(s) = \frac{mv_0}{\frac{m}{c}\frac{s^2}{s+\frac{k}{c}} + 1 + m\sum_{n=1}^{N}\phi_n\gamma_n\frac{s^2}{s^2+2\beta_n\omega_n s+\omega_n^2}} \tag{26}$$

Similarly, using Laplace transforms in eq. (24), corresponding to the Maxwell model, the expression for f(s), as the Laplace transform of F(t), is as follows:

$$f(s) = \frac{mv_0}{\frac{m}{k}s^2+\frac{m}{c}s + 1 + m\sum_{n=1}^{N}\phi_n\gamma_n\frac{s^2}{s^2+2\beta_n\omega_n s+\omega_n^2}} \tag{27}$$

The functions f(s) of eqs. (26) and (27) can be written in terms of a quotient of two polynomials as

$$f(s) = mv_0\frac{A(s)}{B(s)} \tag{28}$$

By means of partial fractions, this equation becomes

$$f(s) = mv_0\sum_{i=1}^{r}\frac{C_i}{s-s_i} \tag{29}$$

where the constants C_i are given by

$$C_i = \left.\frac{(s-s_i)\;A(s)}{B(s)}\right]_{s=s_i} \tag{30}$$

in which s_i are the roots of the denominator polynomial B(s).

The inverse transformation gives the impact load time history F(t) in terms of a sum of exponential functions:

$$F(t) = mv_0\sum_{i=1}^{r}C_i e^{s_i t} \tag{31}$$

4. <u>Rigid Target</u>

In the case of a rigid target, the target displacement $u_s(t)$ becomes zero in eq. (22). Then the impact load time history for the Voigt missile model and the rigid target becomes

$$F(t) = \frac{mv_0}{\lambda}\, e^{-\frac{c}{m}t}\left[\left(\frac{k}{m} - \frac{c^2}{2m^2}\right)\sin\lambda t + \frac{c}{m}\lambda\cos\lambda t\right] \tag{32}$$

where $\lambda = \sqrt{\frac{k}{m} - (\frac{c}{2m})^2}$.

The impact load time history for the Maxwell missile model and the rigid target is

$$F(t) = \frac{v_0 k}{\sqrt{(\frac{k}{2c})^2 - \frac{k}{m}}} e^{-\frac{k}{2c}t} \sinh \sqrt{(\frac{k}{2c})^2 - \frac{k}{m}}\, t, \tag{33}$$

for $(\frac{k}{2c})^2 > \frac{k}{m}$

and

$$F(t) = \frac{v_0 k}{\sqrt{\frac{k}{m} - (\frac{k}{2c})^2}} e^{-\frac{k}{2c}t} \sin \sqrt{\frac{k}{m} - (\frac{k}{2c})^2}\, t, \tag{34}$$

for $\frac{k}{m} > (\frac{k}{2c})^2$.

5. Examples

Impact load time histories for bricks with viscoelastic materials as missiles against a rigid target are plotted for various damping coefficients c and stiffness constants k of the missiles in Figures 3 through 6. The missile mass m, for the assumed brick shape of 2" x 4" x 8", is .956 x 10^{-2} lb·sec^2/in. The initial velocity v_0 is assumed to be 200 ft/sec.

These bricks were under consideration for radiation shielding purposes at the reactor coolant loop penetrations in a PWR plant. In the event of a loop break, they would be jettisoned and become missiles.

The plot of Figure 3 results from eq. (32). By the use of eq. (33), plots of Figures 4 and 5 are obtained. Figure 6 is the result of eq. (34).

6. Conclusions

Further studies are needed for other more complex viscoelastic missile models than the ones considered in this paper. Whether a given missile performs according to one or another of these models is a question to be resolved by testing. If it does, then the theory is applicable.

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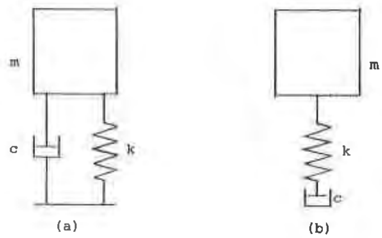

Figure 1. Viscoelastic models:
(a) Voigt, (b) Maxwell.

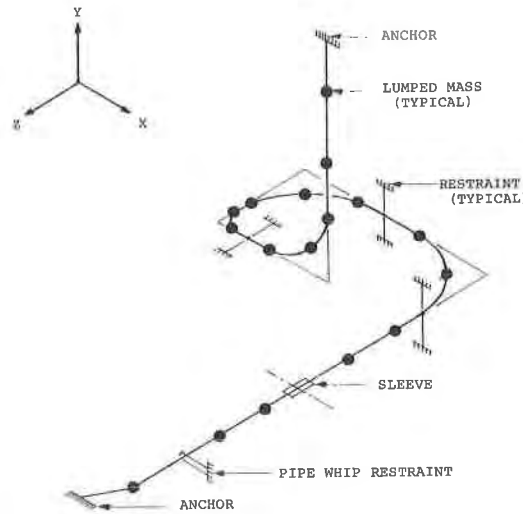

Figure 2. Lumped parameter model of a pipe line in a PWR plant.

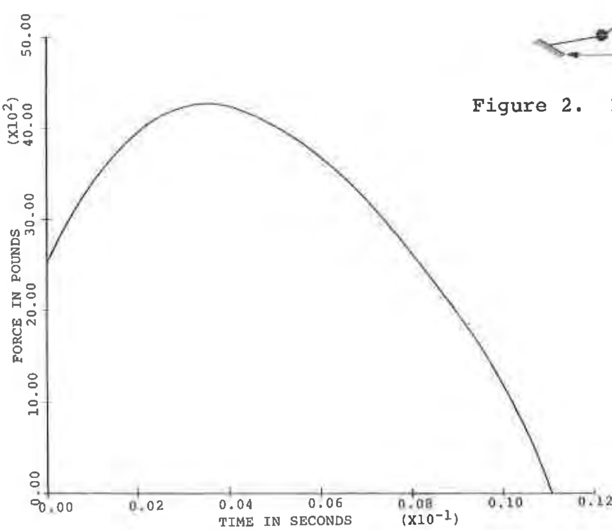

Figure 3. Impact load time history for Voigt viscoelastic missile model against rigid target with c=1.09 lb·sec/in and k=500 lb/in.

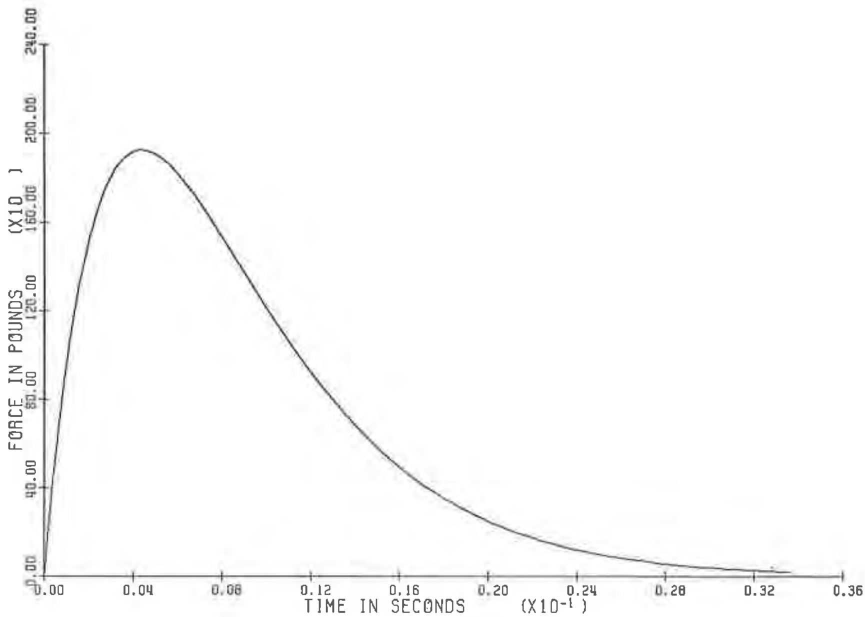

Figure 4. Impact load time history for Maxwell viscoelastic missile model against rigid target with c=1.09 lb·sec/in and k=500 lb/in.

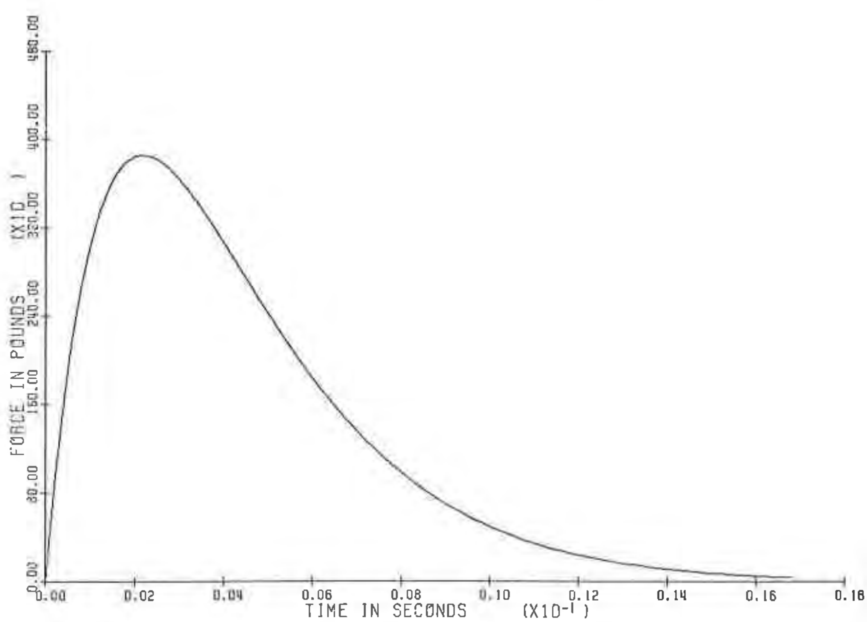

Figure 5. Impact load time history for Maxwell viscoelastic missile model against rigid target with c=2.185 lb·sec/in and k=2000 lb/in.

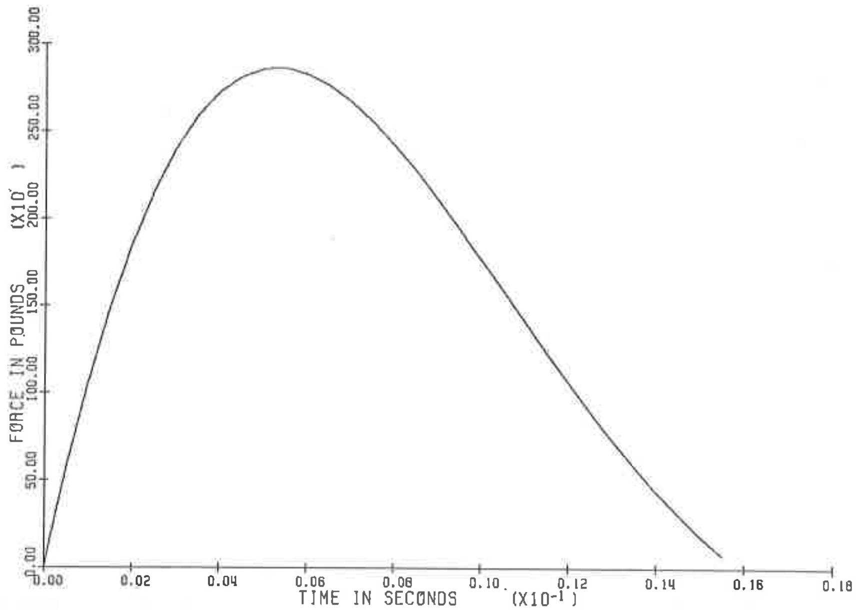

Figure 6. Impact load time history for Maxwell viscoelastic missile model against rigid target with c=2.186 lb·sec/in and k=500 lb/in.