

STUDY ON BEHAVIOURS OF MULTI-PLY BELLOWS SUBJECTED TO PRESSURE AND DISPLACEMENT LOADS

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ABSTRACT

Multi-ply bellows is widely used because its flexibility and pressure tightness. However, the analytical treatment of its behaviours is difficult compared with that of single-ply bellows, since the uncertainty of friction between plies exists. In this paper the strength evaluation equations of multi-ply bellows are examined and the friction effect between plies is investigated through simplified models and detailed FEM analyses of two-ply bellows for example including the effect of material plasticity and internal pressure.

Mainly following results are obtained.

- The behaviours of two-ply bellows without friction can be analysed approximately by using single-ply bellows model even in the elastic-plastic region. The effort of numerical analysis can be reduced if two-ply bellows analyses can be replaced by the analyses of single-ply bellows model.
- The effect of friction is relatively small up to about 0.4 by friction coefficient. Therefore, the behaviours of two-ply bellows can be evaluated practically ignoring friction effect.
- The strength of multi-ply bellows against pressure, i.e. buckling, can be evaluated conservatively by assuming no friction, even if considered the internal pressure with friction. Therefore, the evaluation method without friction is recommended for design.

INTRODUCTION

Multi-ply bellows is widely used because its flexibility and pressure tightness. However, the analytical treatment of its behaviours is difficult compared with that of single-ply bellows, since the uncertainty of friction between plies exists. EJMA Standard (1980) provides strength evaluation equations for multi-ply bellows, however, no friction effect between plies is considered. Preliminary examination on the effect of friction of multi-ply bellows was conducted by Tsukimori (2014). However, Detailed behaviours of multi-ply bellows have not investigated, especially the effect of internal pressure on the friction force. In this paper the strength evaluation equations of multi-ply bellows are examined based on EJMA Standard (1980) and the results by Tsukimori (2014) and the friction effect between plies is investigated through simplified models and detailed FEM analyses of two-ply bellows for example including the effect of material plasticity and internal pressure.

The aim of this study is to clarify the effect of friction between plies on the behaviours of multi-ply bellows and to develop evaluation methods of the stress components due to internal pressure and/ or displacement load and buckling strength due to internal pressure, i.e., in-plane squirm and column squirm for design. Another point of this study is the modelling of multi-ply bellows for numerical analysis. Detailed three dimensional analyses of multi-ply bellows including plasticity, geometrical nonlinearity and friction effect will be very complicated and still difficult to obtain stable results. If it can be replaced by some equivalent single ply bellows without friction, that will be very effective to reduce numerical efforts and to obtain practical results for engineering. In this paper the behaviours of multi-ply bellows,

especially two-ply bellows for example, are investigated by simple analytical ways and also FEA with friction and the results are discussed from the above points of view.

EVALUATION OF STRESS COMPONENTS

EJMA Standard (1980) provides simple evaluation equations of stress components for design. The effect of friction on stress components is investigated by using axisymmetric FE models of two-ply bellows for example with considering some friction coefficients and the results are compared with the extreme cases, friction coefficient $\mu = 0$ (by EJMA Standard (1980)) and ∞ (by Tsukimori (2014)).

In Case of $\mu = 0$ (by EJMA)

The equations of stress components of multi-ply bellows with no friction by EJMA Standard (1980) are summarized as follows.

(1) *stress components due to displacement*

- meridional membrane stress;

$$\sigma_{mm}^d = \frac{Et_p^2}{2H^3 Cf} e, \quad t_p = t \cdot \sqrt{\frac{d}{d_p}}, \quad d_p = d + H \quad (1)$$

-meridional bending stress;

$$\sigma_{mb}^d = \frac{5Et_p}{3H^2 Cd} e \quad (2)$$

where, E : Young's modulus, H : convolution height, e : equivalent axial displacement per convolution, Cf and Cd : coefficient by EJMA, t_p : modified thickness, t : nominal thickness, d_p : pitch diameter, d : diameter at convolution root.

As can be seen from these equations, the stress components due to displacement of multi-ply bellows are as the same as single ply bellows with thickness of one ply.

(2) *stress components due to pressure*

- circumferential membrane stress;

$$\sigma_{cm}^p = \frac{pd_p}{2nt_p} \cdot \frac{1}{0.571 + 2H/q} \quad (3)$$

- meridional membrane stress;

$$\sigma_{mm}^p = \frac{pH}{2nt_p} \quad (4)$$

- meridional bending stress;

$$\sigma_{mb}^p = \frac{p}{2n} \left(\frac{H}{t_p} \right)^2 Cp \quad (5)$$

where, q : bellows pitch, p : pressure, n : number of plies, Cp : coefficient by EJMA.

As can be seen from these equations, the stress components due to pressure of multi-ply bellows are as the same as single ply bellows with thickness of one ply subjected the pressure of p/n .

In Case of $\mu = \infty$ (by Tsukimori)

The equations of stress components of multi-ply bellows with friction coefficient $\mu = \infty$ (by Tsukimori (2014)) are summarized as follows. Single ply bellows with thickness of total plies is regarded here as multi-ply bellows with $\mu = \infty$.

(1) stress components due to displacement

- meridional membrane stress;

$$\sigma_{mm}^{d\infty} = \frac{Et_p^2 n^2}{2H^3 Cf} e + \frac{n-1}{n} \sigma_{mb}^{d(nt)} = \left(\frac{Et_p^2 n^2}{2H^3 Cf} + \frac{5(n-1)Et_p}{3H^2 Cd} \right) e \quad (6)$$

-meridional bending stress;

$$\sigma_{mb}^{d\infty} = \frac{1}{n} \sigma_{mb}^{d(nt)} = \frac{5Et_p}{3H^2 Cd} e \quad (7)$$

Equation (7) is as same as equation (2).

(2) stress components due to pressure

- circumferential membrane stress;

$$\sigma_{cm}^p = \frac{pd_p}{2nt_p} \cdot \frac{1}{0.571 + 2H/q} \quad (8)$$

- meridional membrane stress;

$$\sigma_{mm}^{p\infty} = \frac{pH}{2nt_p} + \frac{n-1}{n} \sigma_{mb}^{p(nt)} = \left(\frac{H}{2nt_p} + \frac{(n-1)H^2}{2n^3 t_p^2} Cp \right) p \quad (9)$$

- meridional bending stress;

$$\sigma_{mb}^{p\infty} = \frac{1}{n} \sigma_{mb}^{p(nt)} = \frac{p}{2n^3} \left(\frac{H}{t_p} \right)^2 Cp \quad (10)$$

Equation (8) is as same as equation (3).

Evaluation by Numerical Analysis

In order to understand the effect of friction between plies of multi-ply bellows, FEA has been conducted for two-ply bellows for example.

(1) specification of numerical analysis

- software : FINAS ver. 21.0 (2013)

- analysis model and dimension : a half convolution with root radius 721.5mm, convolution height 61.5mm, pitch 50mm and thickness 1.5mm × 2 plies. And two single ply bellows with thickness 1.5mm and 3.0mm are added for comparison.

- finite elements : 4 node quadrilateral axisymmetric element QAX4, 2 node line contact element LCONT2 (for two-ply bellows), spring-damper element LCOMB2 (for displacement loading case)

- material characteristic : Young's modulus E=160000MPa and Poisson's ratio $\nu=0.302$

- boundary conditions : axial movement restrained at crown side end and some axial movement at root side end (for displacement loading case) / axial movement restrained at both ends (for pressure loading case)

- loading conditions : 0.4mm axial displacement at root side end (for displacement loading case) / 0.3MPa for two-ply bellows and single ply bellows with thickness 3.0mm and 0.15MPa for single ply bellows with thickness 1.5mm on the inside surface (for pressure loading case)

- friction coefficient (for two-ply bellows) : $\mu = 0, 0.2, 0.3, 0.4, 1000$

Analysis models are shown in Figure 1 for two-ply bellows and single ply bellows with thickness 1.5mm. And boundary conditions and loading conditions are shown in Figure 2 for displacement load case and pressure load case.

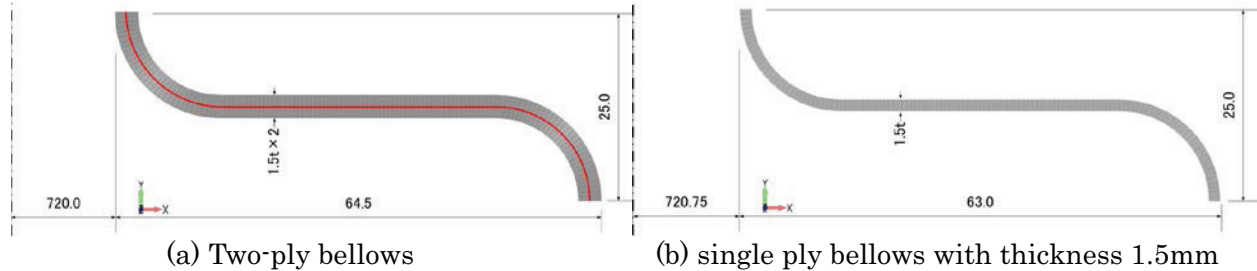


Figure 1. Axisymmetric analysis models of a half convolution of bellows

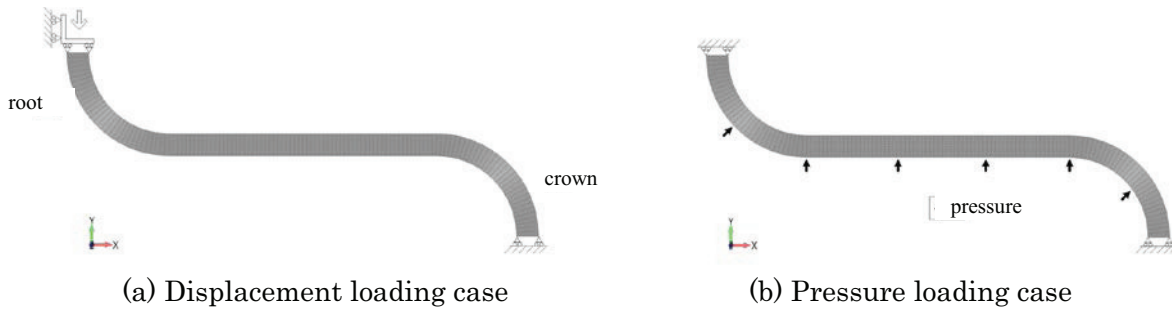


Figure 2. Boundary conditions and loading conditions

(2) analysis results

- displacement loading case;

Figure 3 shows meridional membrane stress and meridional bending stress with different friction coefficients subjected to displacement load obtained by FEA compared with the simple evaluation by EJMA Standard (1980) and Tsukimori (2014). It is found that stress values in the realistic region of friction coefficient μ from 0.2 to 0.4 are almost same as those of $\mu = 0$. The stress values by FEA of single ply with one ply thickness and two ply thickness agree well with the calculations by EJMA Standard (1980) and by Tsukimori (2014), respectively, however, they are a little smaller than those by FEA of two-ply bellows model. The maximum bending stress occurs at the root of outside ply where its meridional curvature becomes slightly larger than that of inside ply. It is guessed that the reason comes from the difference of curvature between inside and outside ply, since the average bending stress of inside and outside ply yields very close value as the bending stress of single ply bellows model with one ply thickness.

- pressure loading case;

Figure 4 shows circumferential membrane stress, meridional membrane stress and meridional bending stress with different friction coefficients subjected to pressure load obtained by FEA compared with the simple evaluation by EJMA Standard (1980) and Tsukimori (2014). It is also found that stress values in the realistic region of friction coefficient μ from 0.2 to 0.4 are close to those of $\mu = 0$. The stress values by FEA of single ply with one ply thickness and two ply thickness agree well with the calculations by EJMA Standard (1980) and by Tsukimori (2014), respectively, however, they are a little smaller than those by FEA of two-ply bellows model. It is guessed that the reason comes from the difference of curvature between inside and outside ply as mentioned above.

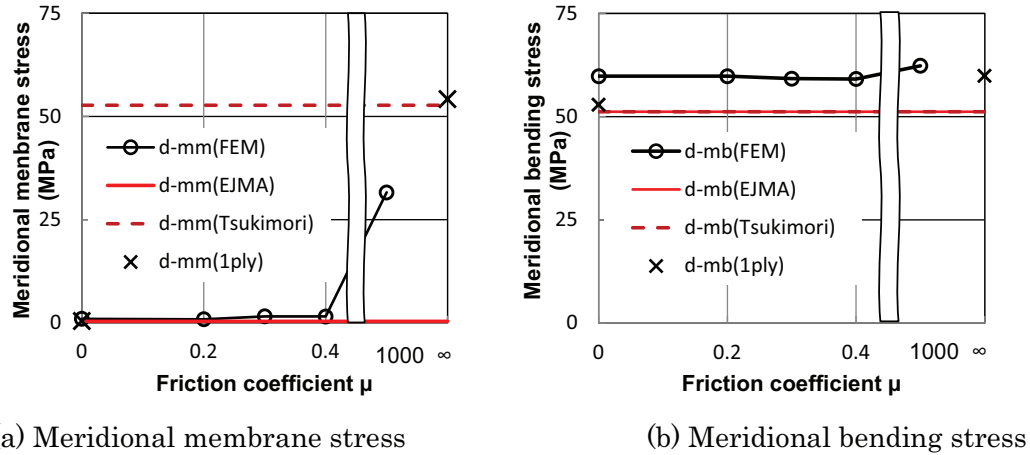


Figure 3. Stress components vs friction coefficient (Displacement loading case)

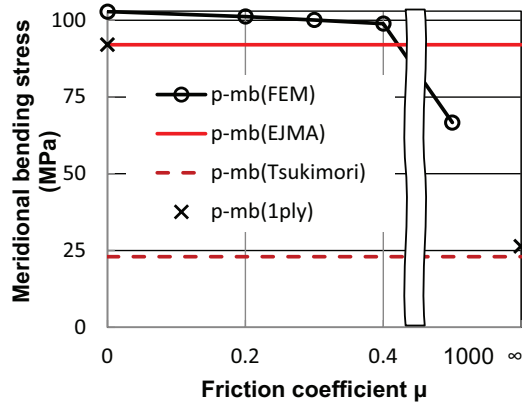
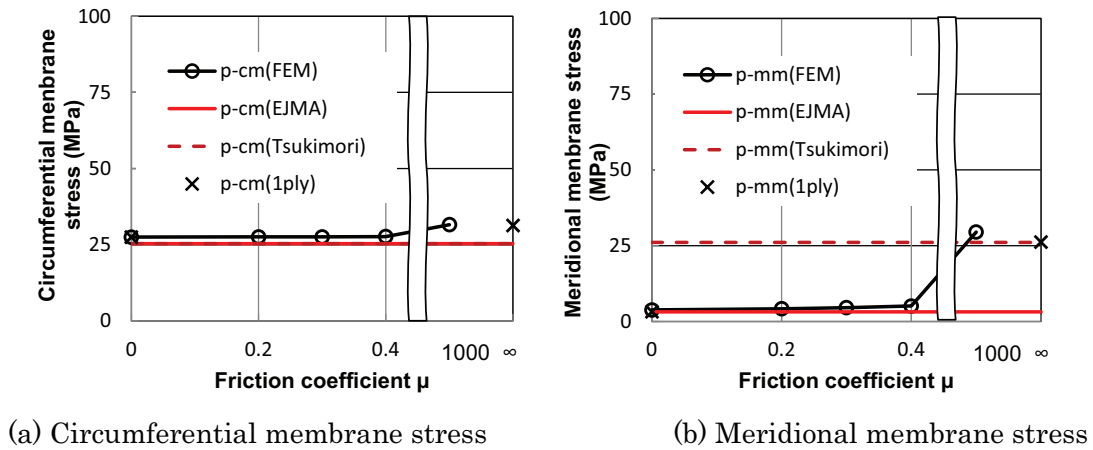


Figure 4. Stress components vs friction coefficient (Pressure loading case)

BUCKLING STRENGTH AGAINST INTERNAL PRESSURE

It is known that bellows yields two types of unique buckling modes, i.e. in-plane squirm and column squirm due to internal pressure load. EJMA Standard (1980) provides simple evaluation equations for

these buckling strengths of multi-ply bellows, in which no friction effect is considered. In this study not only the effect of friction coefficient but also the effect of pressure on the large deformation of bellows are investigated by using axisymmetric FE models of two-ply bellows for example and the results are compared with the extreme cases, friction coefficient $\mu = 0$ (by EJMA Standard (1980)) and ∞ (by Tsukimori (2014)).

In Case of $\mu = 0$ (by EJMA)

The equations of buckling of multi-ply bellows with no friction by EJMA Standard (1980) are summarized as follows.

(1) in-plane squirm

$$p_{cr1} = \frac{3.8n \cdot t_p^2}{Cp \cdot H^2} Sy \quad \rightarrow \quad \frac{p_{cr1}}{n} = \frac{3.8 \cdot t_p^2}{Cp \cdot H^2} Sy \quad (11)$$

where, Sy : yield stress.

As can be seen from this equation, each ply takes the shared pressure, p_{cr1}/n . Therefore, the critical pressure of multi-ply bellows can be obtained by the critical pressure of single ply bellows multiplied by the number of plies.

(2) column squirm

$$p_{cr2} = \frac{0.3\pi \cdot fiu}{N^2 q} \quad , \quad fiu = \frac{1.7 \cdot d_p \cdot t_p^3 E n}{Cf \cdot H^3} \quad (12), (13)$$

From equation (12) and equation (13), the following equation can be derived.

$$p_{cr2} = \frac{0.51\pi \cdot d_p \cdot t_p^3 E \cdot n}{Cf \cdot H^3 N^2 q} \quad \rightarrow \quad \frac{p_{cr2}}{n} = \frac{0.51\pi \cdot d_p \cdot t_p^3 E}{Cf \cdot H^3 N^2 q} \quad (14)$$

where, N : number of convolutions of bellows, fiu : axial spring rate per convolution.

As can be seen from equation (14), each ply takes the shared pressure, p_{cr2}/n . Therefore, the critical pressure of multi-ply bellows can be obtained by the critical pressure of single ply bellows multiplied by the number of plies as same manner as equation (11). Equation (12) is based on elastic column buckling theory and the critical pressure is supposed to be reduced by some factor for plasticity and so on. Tsukimori et al. (1988) proposed an evaluation method of squirm in elastic-plastic region for single ply bellows.

In Case of $\mu = \infty$

The equations of buckling of multi-ply bellows with friction coefficient $\mu = \infty$ (Tsukimori (2014)) are summarized as follows. Single ply bellows with thickness of total plies is regarded here as multi-ply bellows with $\mu = \infty$.

(1) in-plane squirm

$$p_{cr1}^{\infty} = \frac{3.8n^2 \cdot t_p^2}{Cp \cdot H^2} Sy \quad \rightarrow \quad \frac{p_{cr1}^{\infty}}{n^2} = \frac{3.8 \cdot t_p^2}{Cp \cdot H^2} Sy \quad (15)$$

(2) column squirm

$$p_{cr2}^{\infty} = \frac{0.3\pi \cdot fiu^{\infty}}{N^2 q} \quad , \quad fiu^{\infty} = \frac{1.7 \cdot d_p \cdot t_p^3 E n^3}{Cf \cdot H^3} \quad (16), (17)$$

From equation (16) and equation (17), the following equation can be derived.

$$P_{cr2}^{\infty} = \frac{0.51\pi \cdot d_p \cdot t_p^3 E \cdot n^3}{Cf \cdot H^3 N^2 q} \rightarrow \frac{P_{cr2}^{\infty}}{n^3} = \frac{0.51\pi \cdot d_p \cdot t_p^3 E}{Cf \cdot H^3 N^2 q} \quad (18)$$

By comparing two couples of equations, equations (11) and (15), and equations (14) and (18), the following relations are obtained for in-plane squirm and column squirm.

$$\frac{P_{cr1}^{\infty}}{P_{cr1}} = n \text{ (for in-plane squirm)}, \quad \frac{P_{cr2}^{\infty}}{P_{cr2}} = n^2 \text{ (for column squirm)} \quad (19), (20)$$

Evaluation by Numerical Analysis

In order to understand the effect of friction between plies of multi-ply bellows, FEA has been conducted for two-ply bellows for example. In the displacement loading cases, the effect of pressure is studied, since the friction force might increase due to pressure even if the friction coefficient is same.

(1) specification of numerical analysis

- software : FINAS ver. 21.0 (2013)
- analysis type : Large deformation analysis.
- analysis model and dimension : a half convolution with root radius 461.0mm, convolution height 41.0mm, pitch 33.0mm and thickness at root 1.0mm × 2 plies. And a single ply bellows with thickness 2.0mm at root is added for the case of $\mu = \infty$. Thickness distribution along meridional line is modelled, considering the press forming process.
- finite elements : 4 node quadrilateral axisymmetric element QAX4, 2 node line contact element LCONT2 (for two-ply bellows), spring-damper element LCOMB2 (for displacement loading case)
- material characteristic : Young's modulus $E=195000\text{MPa}$, Poisson's ratio $\nu=0.3$ and yield stress $\sigma_y = 237.6\text{ MPa}$. The constitutive relation is modelled by multi-linear approximation based on the following equation for plastic strain ε_p .

$$\varepsilon_p = \left(\frac{\sigma - \sigma_p}{K} \right)^{\frac{1}{m}}, \quad \sigma_p = \sigma_y - K(0.002)^m \quad (21), (22)$$

where $K=1058.1\text{ MPa}$, $m=0.6513$

- boundary conditions : axial movement restrained at both ends (for pressure loading case) / axial movement restrained at crown side end and some axial movement at root side end (for displacement loading case)
- loading conditions : Internal pressure on the inside surface is applied until contact between adjacent convolutions occurs. (for pressure loading case) / axial displacement at root side end is applied until contact between adjacent convolutions occurs by compression load at 4 pressure levels which are obtained by the pressure loading cases, i.e., non-pressure, initially yielding pressure, pressure at contact, and the average pressure of yielding pressure and contact pressure. (for displacement loading case).
- friction coefficient (for two-ply bellows) : $\mu = 0, 0.2, 0.3, 0.4, 1000$

Analysis models are similar and boundary conditions and loading conditions are as same as the previous analysis shown in Figure 1(a) and Figure 2.

(2) analysis results

- pressure loading case;

Figure 5 shows the deformation shapes of cases of $\mu=0$ and ∞ at contact pressure with Mises stress distribution. And Figure 6 shows the relation between internal pressure and bulging displacement. The horizontal line denotes the displacement at contact between adjacent convolutions. It is found that the results of the cases of $\mu = 0 \sim 0.4$ are almost same not only in elastic region but also in elastic plastic

region. Tsukimori (2014) obtained very similar results and he also obtained good agreement between the frictionless case and the case of single-ply bellows of 1 ply thickness, if the pressure multiplied by 2 in the latter. These results suggest that the effect of friction coefficient in the practical range ($\mu = 0.2 \sim 0.4$) can be ignored and the critical pressure of in-plane squirm can be estimated by single-ply bellows of 1 ply thickness subjected by half pressure of real pressure.

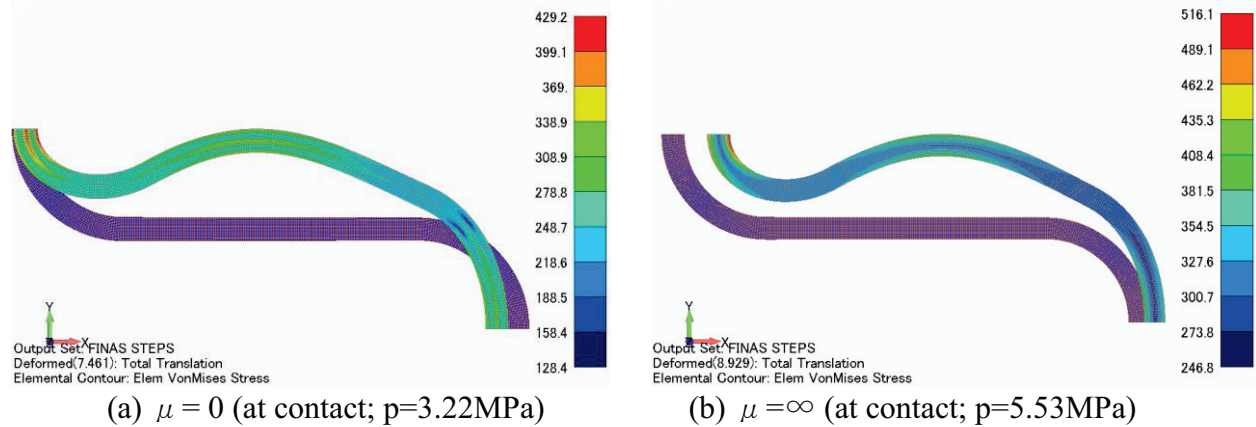


Figure 5 Deformation at contact pressure and stress distribution

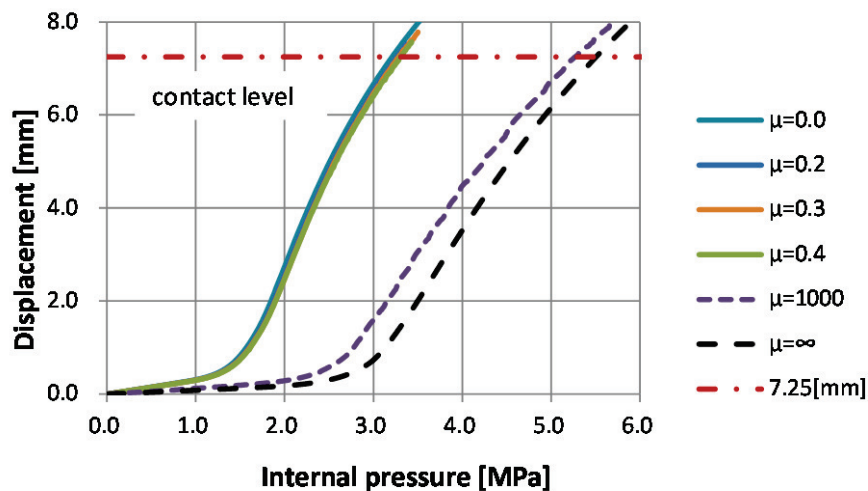


Figure 6 Internal pressure vs Bulging displacement

- displacement loading case;

Figure 7 shows the relation between axial displacement per convolution and reaction force at 4 pressure levels of the cases, $\mu = 0$ and 0.4 . Figure 8 shows the relation between pressure and bellows spring rate per convolution for each friction coefficient obtained from the relation between axial displacement per convolution by averaging the inclinations of compression side and tension side. Pressure levels are depicted by symbols and the critical pressures for in-plane squirm by equation (11) and (15) are also drawn. As can be seen from this figure, the spring rate of non-friction case at yielding start pressure is a little smaller than that at no pressure. After that it declines due to propagation of plastic region. And it increases again. The cases with some friction reveal similar tendency as non-friction case. But values are larger than that of non-friction case except the cases at no pressure, since pressure contributes to increase the friction force. The difference between non-friction case and the cases with practical friction coefficients, $\mu = 0.2 \sim 0.4$, is about 30% at the maximum in the pressure region less than p_{cr1} . Column squirm depends on spring rate and the spring rate is the function of pressure and friction coefficient for

multi-ply bellows as can be seen from Figure 8. But it can be said that the critical pressure estimated as non-friction case gives a conservative value from the point of design.

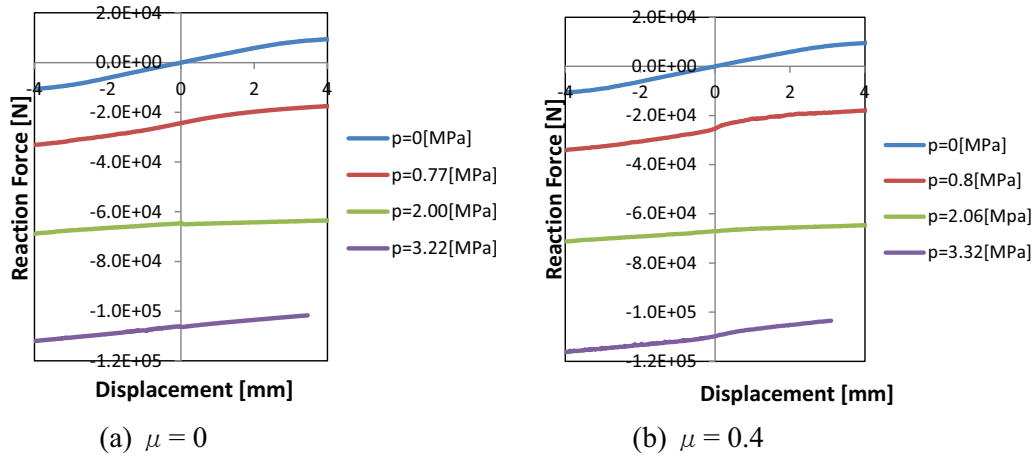


Figure 7 Axial displacement per convolution vs reaction force at 4 pressure levels

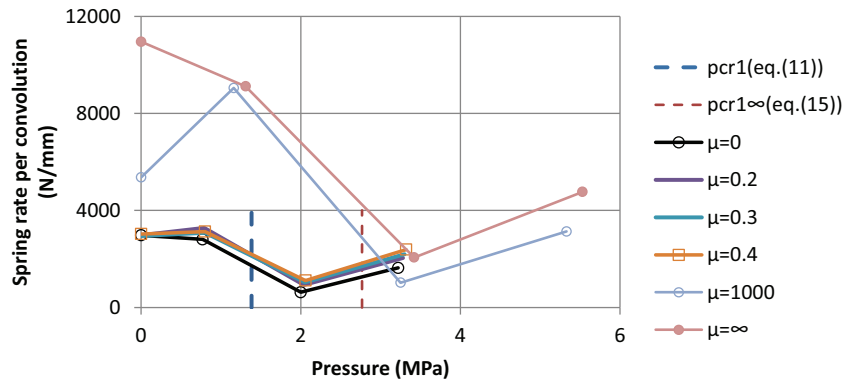


Figure 8 Pressure vs bellows spring rate per convolution

DISCUSSION

Evaluation of stress components; The effect of friction can be ignored for practical use. This means that stress components can be estimated by single-ply bellows of 1 ply thickness subjected by the pressure divided by number of plies. Simple evaluation by EJMA Standard (1980) is useful for approximation. But attention should be paid to that there is some possibility of increase of the stresses by displacements and of disadvantage for fatigue life, if stick between plies happens due to some reason.

Evaluation of internal pressure buckling; Buckling pressures can be also estimated by single-ply bellows model of 1 ply thickness subjected by the pressure divided by number of plies and the estimation will be a little conservative approximation.

Number of convolutions vs buckling pressure; Tsukimori and Iwata (1990) proposed an evaluation method to obtain the buckling pressure as the function of number of convolutions including the effect of plasticity for single-ply bellows. Similar relation can be drawn by using equations (11), (14), (15) and (18) for two-ply bellows of the previous example as shown in Figure 9. However, the trend lines by equations (14) and (18) are based on elastic buckling equation. As simple conservative approximation, equations (11) and (15) can be applied as discussed above. In order to introduce the effect of friction, if the ratios of the spring rates with friction to non-friction spring rate obtained from Figure 8 are applied to equation (14) as correction factors, an additional line can be drawn indicated by ‘pcr2*’ in Figure 9. It is pointed out that the possibility of change the buckling mode from column squirm to in-plane squirm due

to friction exists in the transient region between in-plane squirm and column squirm, however, the envelope line by equations ((11)and (14) gives conservative values.

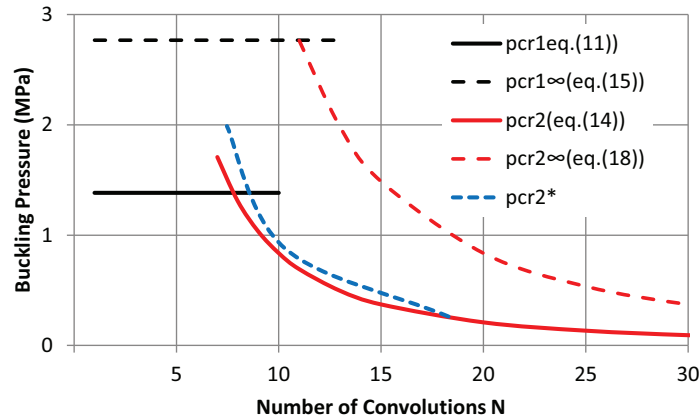


Figure 9. Number of convolutions vs buckling pressure

CONCLUSION

In this study the effect of friction on stresses and buckling pressures of multi-ply bellows was investigated by analytical way. Through this study the followings were clarified.

- (1) The effect of friction on the stress components is very small in the range of practical friction coefficient ($\mu = 0 \sim 0.4$).
- (2) The effect of friction coefficient in the practical range ($\mu = 0.2 \sim 0.4$) can be ignored and the critical pressure of in-plane squirm can be estimated by single-ply bellows of 1 ply thickness subjected by half pressure of real pressure.
- (3) Column squirm depends on spring rate and the spring rate is the function of pressure and friction coefficient. But the effect of friction seems to be small and the critical pressure estimated by non-friction assumption gives a little conservative value.

It is difficult to estimate the effect of friction in the structure or components by analytical ways because it involves many uncertainties. We have tried to evaluate the effect of friction on multi-ply bellows strength by analyses. It is desired to conduct experiments for validation of analytical methods in future.

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