ABSTRACT

HICKLIN, KAREN THEODORA. Decision Models for Mode of Delivery Combining Patient and Clinician Risk Perceptions and Preferences. (Under the direction of Dr. Julie S. Ivy.)

Of the nearly 4 million births that occur each year in the United States, almost one in every three is a cesarean delivery. Although the C-section rate has risen in recent years, there is no evidence that this increasing rate has caused a decrease in neonatal or maternal mortality or morbidity. Cesarean delivery is associated with increased risk of neonatal respiratory morbidity and can cause major complications in subsequent pregnancies such as placenta previa, placenta accreta, and uterine rupture. Overall, since C-sections have more maternal and neonatal complications, typically require more hospital time than vaginal delivery, may interfere with early maternal-infant bonding, and increase the overall costs of care, there is concern that current C-section rates are too high. Efforts to reduce cesarean rates have had limited success because C-sections are performed for various indications. In order to address the C-section rate, there is a need for a better understanding of patient and clinician risk perceptions and their preferences for the process of labor and delivery and their impact on decision making.

This dissertation research focuses on the development of analytical and simulation models to inform decision making related to birth with the goal of helping to facilitate communication between patients and their health care providers regarding birth. These decision models can be used to analyze and support efforts to address the increasing C-section rate due to indications which have variable clinical evidence such as a "failure-to-progress." Failure-to-progress (or labor arrest) refers to a lack of progressive cervical dilation or a lack of fetal descent, and it occurs when effective labor contractions do not lead to delivery of the baby. When a woman enters labor, she will deliver in one of two ways: vaginal

delivery or cesarean delivery. Due to the increased risk of infection and complications, allowing patients who require a C-section to continue labor is not recommended. However, performing a C-section on patients believed to be able to have a successful vaginal delivery can have a large impact on their future health. To evaluate this trade-off, stochastic decision models were developed to analyze the mode of delivery decision through the evaluation of trade-offs between prolonging labor and performing a C-section. Discrete event simulation (DES), Bayesian decision analysis, Markov decision processes (MDP) and a Bayesian MDP are used to evaluate this decision. This approach includes evaluating the perspectives and perceptions of risk pertaining to the patient.

Using DES, the natural progression of labor is modeled to understand total time in labor in absence of any interventions or C-sections. The DES is also used to derive parameters that are used as inputs into the Bayesian decision model, MDP, and Bayesian MDP. Using Bayesian decision analysis, the conditions are decided under which it is appropriate to prolong labor in order to learn more about the patient before deciding the appropriate mode of delivery. This model determines the conditional value of information and determines the conditions under which information has positive value. The finite horizon MDP is used to evaluate the appropriate mode of delivery in a 24-hour time period in order to maximize the expected utility of health outcomes for both patients and the valuation of the delivery process. The Bayesian MDP is used to predict the mode of delivery for each time period as a function of cervical dilation progression and patient type (i.e., failure-to-progress or not). The results of this work can be used in developing a decision aid tool to be used by women and their health care providers to explore various options and the risks associated with them.

Decision Models for Mode of Delivery Combining Patient and Clinician Risk Perceptions and Preferences

by
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DEDICATION

This dissertation is dedicated to my mother, Gail Jennings, my aunt, Navada Warren, and my best friend, Rosemary Enobakhare, for their continuous encouragement, love, support, and for seeing more in me than I often see in myself. For those things, I am truly grateful.

BIOGRAPHY

Karen T. Hicklin is currently a PhD candidate in the Edward P. Fitts Department of Industrial and Systems Engineering at North Carolina State University. Originally from Washington, DC, Karen received her BS degree in Mathematics from Spelman College in Atlanta, GA. Upon graduating from Spelman College, she returned to Washington, DC to work with the University of Southern California Office of Federal Relations and the United States Census Bureau while completing her MS degree in Mathematics and Statistics from Georgetown University. Her research interests are mathematical modeling of stochastic systems with emphasis on statistical and decision analysis as applied to health care and service environments. Karen is active with the National Society of Black Engineers (NSBE) and the Minority Issues Forum (MIF) within the Institute for Operations Research and the Management Sciences (INFORMS).

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CHAPTER

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BIRTH IN THE UNITED STATES

The decision of when a cesarean section (C-section) is needed for uncomplicated singleton births where there is no prior indication is often complicated. Indications such as fetal distress, uterine rupture, hypertensive disorders, along with the notion of perceived failure-to-progress often lead to a C-section. This decision process is illustrated in the following birth story (Ruby's birth story):

> *"At approximately 10:00 pm, contractions began for Ruby and around 2:00 am it was decided that she would go to the hospital. There she was told that she*

was 1-2 centimeters dilated with persistent contractions. At 7:00 am, no further dilation changes prompted the doctor to break her water and start her on Pitocin (medication used to speed labor). She received a epidural shortly after and she experienced no pain until afternoon. Around 1:00 pm, she experienced more intense and continuous contractions and was measured to be between 3 and 4 centimeters dilated. At this time, she was told that her slow progress meant she may need a C-section. Although the doctor would allow her to labor an additional 10 hours, Ruby was informed that there may not be much progress which means she would need a C-section anyway. At 1:30 pm, although she was afraid and unprepared, it was decided that Ruby would undergo a cesarean delivery."

This story provokes questions about whether that decision was the best decision and was the timing of the decision appropriate.

Of the nearly 4 million births that occur each year in the United States, almost one in every three is a cesarean delivery. With a rate of 32.2% in 2014 [64], the proportion of babies delivered by C-section has steadily increased over the years from an initial recording of 4.5% in 1965 [13]. The current C-section rate is more than three times the guidelines established by the World Health Organization (WHO), which suggest that a C-section rate of 5-10% correlates with the best outcomes for women and babies [17]. Although the C-section rate has risen in recent years, there is no evidence that this increasing rate has caused a decrease in neonatal or maternal mortality or morbidity [40].

Cesarean delivery is the most commonly performed operating room procedure in the United States and many parts of the world. In addition to being a major abdominal

surgery, cesarean delivery is associated with increased risk of neonatal respiratory morbidity and can cause major complications in subsequent pregnancies such as placenta previa, placenta accreta, and uterine rupture [5, 38, 66, 69, 87, 101]. There is also an increased risk of neonatal intensive care unit admission and perinatal death associated with C-sections as compared to vaginal delivery [44, 63, 85]. Overall, since C-sections have more maternal and neonatal complications, typically require more hospital time than vaginal delivery, may interfere with early maternal-infant bonding, and increase the overall costs of care, there is general consensus that current C-section rates are too high [5]. Efforts to reduce cesarean rates have had limited success because C-sections are performed for various indications. A study in 2011 discovered the leading physician-documented indications for C-sections, in order of frequency, were labor dystocia (i.e., difficult or slow labor progression typically characterized by a lack of continuous cervical dilation change or instances in which the baby's head or body is too large to fit through the mother's pelvis), abnormal or indeterminate fetal heart rate tracing, fetal malpresentation, multiple gestation, and suspected fetal macrosomia (i.e., fetal growth beyond a specified weight of 8 pounds and 13 ounces) [7]. In addition to the role that maternal characteristics (e.g., age, weight, ethnicity, etc.) play in the growing C-section rate, there is reasonable concern regarding how patient preferences and practice variation among hospitals and health care providers contribute to cesarean delivery rates [5]. In order to address the C-section rate, there is a need for a better understanding of patient and clinician risk perceptions and their preferences for the process of labor and delivery and their impact on decision making.

In 1916, Edwin Cragin coined the phrase, "once a Cesarean, always a Cesarean." Cragin urged physicians to avoid performing "unnecessary" C-sections due to the belief that future

C-sections would follow for subsequent pregnancies [22, 71]. In recent years this ideology has changed; partially due to the availability of vaginal birth after cesarean (VBAC) which is a viable option for many eligible women [17, 28, 42, 43, 52]. Although the increasing C-section rate can be attributed to many factors, the decreased use of VBAC has had a significant impact. Figure 1.1 provides the C-section rates from 2003 to 2014. During that time period the highest C-section rate was in 2009 with a rate of 32.8%. Figure 1.2 shows the C-section rates from 1989-2011 [5] taken from National Vital Statistics data [64]. The figure displays how the rate of C-section has increased as the rate of VBAC has decreased. This is an indication that the growing C-section rate is largely impacted by the number of repeat cesarean deliveries. Therefore, focus has been placed on studying factors that can safely prevent the first cesarean delivery [5, 13, 52]. In early 2014 new recommendations, targeted

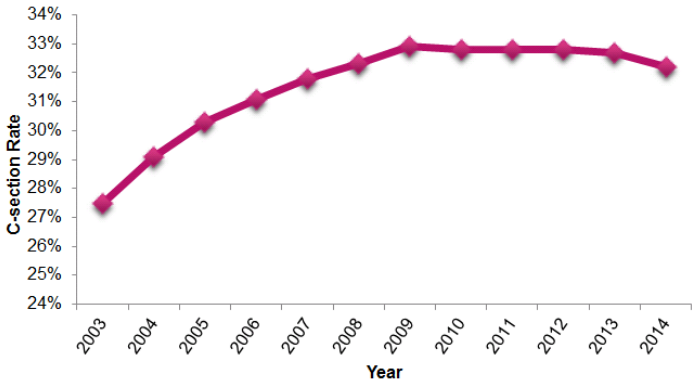


Figure 1.1 C-section rates from 2003 to 2014 according to the National Vital Statistics Reports reported by the Centers Disease Control and Prevention.

at reducing the cesarean rate by taking measures to prevent the first cesarean delivery, were issued jointly by the American College of Obstetricians and Gynecologists (ACOG) and the

Society for Maternal-Fetal Medicine (SMFM) [5]. The report is a compilation of findings regarding the various reasons that a woman may undergo a C-section and the suggested interventions that will lead to fewer C-sections. By avoiding the first C-section, it is believed that the C-section rate will decrease due to the reduction in repeat cesareans, which make up a large proportion of the C-section rate.

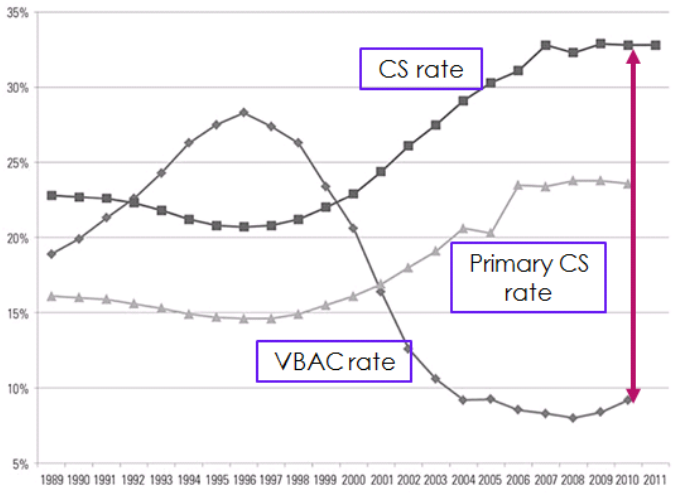


Figure 1.2 United States delivery rates from 1989-2011 and display of increasing gap between C-section (CS) rate and vaginal birth after cesarean (VBAC) rate in the using data from Martin et al. (2013) [64]. (Please see [5] for original image.)

1.1 Labor Progression Trends

Although a C-section may be performed for various reasons and, in some cases, may be the safest delivery mode, the increasing C-section rate over the years has been a cause of major concern for many health care practitioners. In particular, there is concern regarding

the number of "unnecessary" C-sections which are usually performed for soft indications (i.e., conditions experienced during labor that are considered suggestive of a C-section), such as a labor dystocia (i.e., a "failure-to-progress" diagnosis). A "failure-to-progress" diagnosis occurs when it is believed that a laboring woman will not successfully progress to full cervical dilation to ensure safe delivery of the fetus within a time period deemed appropriate for safe delivery. For many years the guidelines surrounding the length of time a woman labors were governed by the Friedman Curve which was established in 1954 by Emanuel Friedman [34, 72]. Friedman was the first to divide labor into stages and phases. Each stage can be described as follows: (1) early and active labor, (2) delivery of baby, and (3) delivery of the placenta. Active labor begins at the onset of labor until the cervix is fully dilated. Once full dilation is achieved, the second stage starts, which is also termed the pushing stage, resulting in the delivery of the fetus.

At any point in early or active labor a C-section may be performed to ensure the safe delivery of the fetus. There are various reasons someone may need a C-section. According to [102], failure-to-progress accounted for 44.7% of all intrapartum C-sections. Figure 1.3 shows the percentage of the various complications that lead to a cesarean delivery. The Friedman Curve provides a guide to understand abnormal labor progression, which was defined as cervical dilation of less than 1.2 cm per hour for nulliparous women (i.e., women giving birth for the first time) and cervical dilation less than 1.5 cm per hour for multiparous women (i.e., women who have previously given birth) [34]. Friedman further defined no change in dilation for more than two hours as labor arrest. Friedman studied the rate of cervical dilation for 500 laboring nulliparous women who received routine labor care. He determined the rate of cervical dilation per hour by recording the cervical dilation status

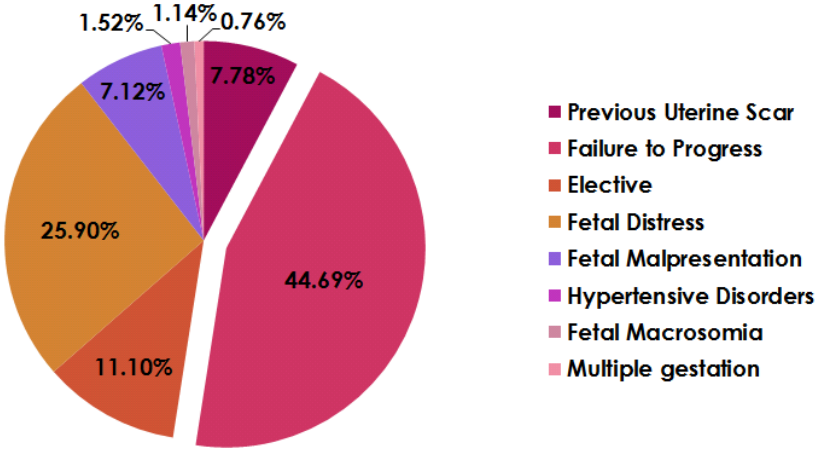


Figure 1.3 Various indications that may lead to a C-section for uncomplicated singleton births with no prior complications [102].

(x-axis) and time in hours (y-axis) at different periods throughout the labor process. After connecting the points on the graphs, he was then able to calculate the slope of each line. Friedman (1954) described the procedure for finding cervical dilation per hour as follows: "if the cervix dilates 2.0 to 3.5 cm in 2 hours, the slope is $(3.5-2.0)\div 2=0.75$. [34]" Therefore the amount of cervical dilation change would be 0.75 per hour in this example.

Friedman ultimately used the labor curves of 200 women who he considered to have "adequate pelvis, vertex presentation, occiput-anterior position, and whose labor progressed normally with average-sized infants spontaneously" to establish an "ideal" labor curve for nulliparous women. He defined normal labor progression to be within two standard deviations greater than the mean which resulted in a maximum first stage of labor of 19.8 hours. He assumed any rates of cervical dilation resulting in a first stage labor duration greater than 19.8 hours to be abnormal [35]. In years prior to Friedman Curve utilization, fetal assessment techniques used to determine the fetal oxygenation and general well-

being of the fetus during labor were nonexistent. During this time period labors were considered to be long and often ended in negative outcomes related to hypoxic injury and birth trauma [13]. The Friedman Curve was established to set a standard for performing a cesarean delivery in order to avoid some of the harmful outcomes of prolonging labor. The goal of the Friedman Curve was to provide guidance regarding an appropriate delivery time frame that would result in the best outcomes. A C-section was performed if the woman was not able to deliver the baby vaginally within the established time frame.

Due to changes in the delivery process and changes in the demographics of laboring women, a re-evaluation of the use of the Friedman Curve guidelines is needed. The Friedman Curve provides a statistical estimate of labor progression regardless of changes in the rate of dilation for different dilation states. The report developed by ACOG and SMFM mention the length of time a woman should be allowed to labor as one of the contributing factors to the growing cesarean rate for nulliparous women. Dr. Aaron B. Caughey who helped develop the guidelines said in a statement, "Evidence now shows that labor actually progresses slower than we thought in the past, so many women might just need a little more time to labor and deliver vaginally instead of moving to a cesarean delivery [5]." In a retrospective study using labor and delivery information from electronic medical records in 19 hospitals across the United States known as the Consortium on Safe Labor, Zhang et al. (2010) examined current labor patterns. They discovered that labor progression for a cervical dilation of 4 to 5 cm could take as long as 6 hours and more than 3 hours to progress from 5 to 6 cm, which differs greatly from expectations established by the Friedman Curve. They summarized their results by noting that allowing labor to continue for an extended period of time for cervical dilations 6 cm and under, would aid in reducing the rate of

intrapartum cesarean deliveries [102, 104]. An evaluation of the most appropriate time to perform a C-section is a combination of many elements. In particular, the status of the fetus and mother is consistently checked to ensure the health of both patients.

The mode of delivery is typically decided by the provider and the extent to which mode of delivery accounts for patient preferences is unclear [46]. For a given indication, several factors affect women and their provider's decisions regarding mode of delivery, including prior information or education about the likelihood of specific outcomes, preferences for those outcomes, and preferences for other attributes of the birth experience such as prior pregnancy/birth experiences. These factors can also enhance communication between women and their providers during pregnancy and birth experiences thus creating an improved experience for mothers and clinicians. In our research, we have developed decision models that can be used to analyze and support the concerns regarding efforts to reduce the increasing C-section rate due to a "failure-to-progress" with the goal of helping to facilitate communication between patients and their health care providers regarding birth. Our long-term research objective is to use systems science to identify and evaluate policies that incorporate patient preferences in decisions related to birth, improve a woman's birth experience, and achieve optimal maternal and neonatal outcomes to help women, health care providers, researchers, and health systems. Our approach includes evaluating the perspectives and perceptions of risk pertaining to the patient to develop single- and shared-decision models that maximize the likelihood of healthy outcomes for the mother and child as well as maximize the utility of the labor and delivery process. The results of this work can be used in developing a decision aid tool to be used by women and their health care providers to explore various options and the risks associated with them.

1.2 Decision Aid Tools

Dugas et al. (2012) conducted a systematic review and meta-analysis to evaluate different decision aids used during pregnancy and birth to foster communication between women and their health care providers regarding the risks, benefits, and preferences for different delivery options [27]. They considered improvement of knowledge [36, 67, 70, 83, 88], effect of interventions on anxiety [67, 70], decisional conflict [36, 67, 70, 83], and impact on final choice [70, 83] and final outcome [67]. Their analysis showed that computer-based information tools, decision analysis tools, individual and group counseling interventions were beneficial in reducing anxiety levels, decision analysis tools and computer-based information tools resulted in reduced levels of decisional conflict, and decision analysis tools had an impact on the final choice and outcome [27].

Montgomery et al. (2007) compared the effectiveness of two computer-based decision aids on decisional conflict and determination of mode of delivery among women with a previous C-section [67]. The women in the study were divided into three groups of care and instruction: (1) usual care, (2) informational program, and (3) decision analysis. Women in the usual care group were provided basic standard care given by obstetric and midwifery staff. Women in the information program were provided descriptions and probabilities of clinical outcomes for the mother and baby associated with planned vaginal birth, elective C-section, and emergency C-section. Lastly, women in the decision analysis group were provided with outcomes for the mother and baby associated with each delivery mode and were required to place a value on each outcome which served as utility approximations. This information was ultimately used in a decision tree to determine each woman's pre-

ferred delivery option in order to maximize utility. The results showed that women in the information program had reduced decisional conflict compared with women in the usual care group. In addition, the rate of vaginal delivery was higher for the decision group than the usual care group. This study shows the significance of decision tools in the mode of delivery decision process and how these tools help women to be more comfortable and informed decision makers.

To evaluate the effectiveness of a decision aid for women with a breech presentation, Nassar et al. (2007) conducted a randomized trial to test the difference in decisional conflict, knowledge, anxiety, satisfaction, and experience [70]. In the trial, 188 women were randomly assigned to one of two groups: usual care group and decision aid group. Women in the decision aid group were given a booklet, CD, and worksheet to be reviewed along with usual care. Women in the usual care group received usual care only, which comprised of standard counseling from their care provider. The statistical analysis showed women in the decision aid group had a higher degree of knowledge, and a lower degree of decisional conflict as compared to women in the usual care group. Overall they were able to show that the decision aid helped women feel more comfortable and informed about their birth plan. A similar study was conducted by Shorten et al. (2005) for women considering a trial of labor after cesarean delivery [83]. The results in their study also showed significant benefits for incorporating decision aids in the mode of delivery decision process.

1.3 Delivery Decision Models

Many mode of delivery decision models focus on risk prediction of cesarean delivery as governed by various epidemiological factors that may affect the health outcomes for the

mother and child [24, 47, 60, 86]. These models are for a single decision maker and most use logistic regression, statistical analysis, decision trees, and machine learning to model the decision. We classify these mode of delivery models into three categories: (1) descriptive models (Section 1.3.1), (2) predictive models (Section 1.3.2), and (3) optimization models (Section 1.3.3). Descriptive models are ones in which the state of knowledge regarding labor or conditions surrounding labor are explored to summarize policies or guidelines for labor or delivery. We define predictive models as those that use various input parameters to determine which mode of delivery is most appropriate. These factors vary from demographic characteristics of the mother to health conditions of either the mother or fetus. Optimization models are defined as those that determine the best decision in order to optimize a particular outcome or health state.

1.3.1 Descriptive Models

Using characteristics of the mother and pregnancy [7], labor progression [102, 103], and clinician practices [12, 39], researchers have been able to glean informative properties that influence one delivery mode over another. Barber et al. (2011) analyzed the physician-documented causes of a cesarean delivery for all births which occurred at Yale-New Haven Hospital during January 2003 and December 2009. Some of the categories used to classify cesarean delivery include repeat cesarean, arrest of labor dilation or descent (including failed vacuum and failed forceps), nonreassuring fetal heart tracing, multiple gestation, suspected macrosomia, elective per maternal request, preeclampsia or eclampsia, maternal or fetal indications, and obstetric indications. The maternal indications include conditions prior to pregnancy such as maternal cardiac disease and maternal HIV. Fetal indications in-

clude prenatal complications that preceded labor such as fetal anomalies and intra-uterine growth restriction. Obstetric indications were classified as complications brought about during pregnancy. Examples of these are placental abruption, placental accreta, placental previa, and cord prolapse. The authors were able to use a logistic regression model to estimate the risk of primary cesarean delivery for each indication, which was then used in the calculation of the primary cesarean delivery trend. The results from their analysis showed that the major causes of primary cesarean delivery were labor arrest disorders, nonreassuring fetal heart rate tracings and malpresentation. For all cesarean deliveries, nonreassuring fetal heart tracing, arrest of dilation, multiple gestation, preeclampsia, suspected macrosomia, and C-section per maternal request were the most significant indications of cesarean delivery [7].

Using data from the National Institute of Child Health and Human Development Consortium on Safe Labor, extensive studies have been developed to understand current labor trends and the need for a cesarean versus vaginal delivery. Data from the Consortium on Safe Labor was gathered through a retrospective study containing detailed labor and delivery electronic medical records for 19 hospitals across the United States from 2002 to 2008 [31, 102, 103]. Zhang, Landy et al. (2010) used labor data pertaining to singleton term gestations who experienced spontaneous onset of labor and delivered virginally with normal perinatal outcomes to examine trends in contemporary labor progression. Using interval-censored regression, they were able to estimate the duration of labor for each cervical dilation state, starting from the initial cervical dilation measurement observed at admission. The outcome of this study showed that labor progression from 4 cm to 5 cm could take as long as 6.4 hours for nulliparous women (i.e., women giving birth for the first

time) and 7.3 hours for multiparous women (i.e., women who have previously given birth). The labor trends showed that labor progression is quicker for multiparous women than for nulliparous women after 6 cm which is the point when labor progression increases. Zhang, Landy et al. (2010) were able to provide support for efforts to allow women to labor longer for cervical dilation prior to 6 cm, which they believe will help in reducing the rate of cesarean delivery due to labor arrest [103].

Zhang, Troendle et al. (2010) also used data from the Consortium on Safe Labor to describe current cesarean delivery practice in the United States. The results from their study showed that many of the C-sections were attributed to a repeat cesarean due to a uterine scar. In this study almost half of the women attempting vaginal delivery had induced labor and within that group the C-section rate was twice as high as for women who had a spontaneous onset of labor. They also discovered that more than half of the C-sections associated with labor dystocia were performed before 6 cm in induced labors and in women who had a previous uterine scar [102].

Ghaffarzadegan et al. (2013) used a learning-based simulation to model the decision process from the perspective of clinician experience. They found that physicians are more likely to schedule C-sections the longer they have practiced because of experiences from their past as well as lessons they have learned [39]. Caruana et al. (2003) also studied clinician behavior and its impact on the C-section rate [12]. They developed a model to determine whether the variations in the observed cesarean rates for 17 physician groups were attributed to the natural risk of the patient sub-populations or to differences in physicians practices. Using machine learning (i.e., minimum message length (MML) decision trees with Bayesian smoothing), they were able to show that although there is variation in

intrinsic risk among the groups, there is also reason to believe there is much variation in physician practice. In particular, they found 3 of the 17 physician practices to have higher C-section rates that may not be attributed to patient characteristics.

1.3.2 Predictive Models

Feghali et al. (2015), using data from the Consortium on Safe Labor, developed a study to evaluate labor curves for women who underwent a preterm induction of labor with the goal of characterizing predictors of vaginal delivery amongst this group. Conditions that affect the risk of preterm induction of labor include hypertensive disorders, diabetes mellitus, and fetal conditions such as fetal anomalies and stillbirth. Feghali et al. (2015) used multiple logistic regression analysis to determine which characteristics among maternal age, parity, body mass index, cervical effacement, cervical dilation, and fetal station were significantly correlated with vaginal delivery. Results showed that the number of vaginal births increased as gestational age increased and showed that parity was the strongest predictor of vaginal delivery for women who undergo preterm induction of labor [31].

In order to understand which variables (maternal age, weight, height, completed weeks of gestation, and weight of newborn) can be used as predictors of labor arrest, Dahan et al. (2005) analyzed retrospective data for 228 patients [24]. They used analysis of variance and stepwise discriminant analysis to evaluate the difference between vaginal delivery and C-section due to labor arrest. The results of their model showed that although birth weight, maternal age and maternal height were statistically significant risk factors, they are not good predictors of C-section due to labor arrest specifically. In another study, Smith et al. (2004) used the results of a multivariate logistic regression model as inputs into a converted

adjusted livelihood ratio which was used to estimate the risk of emergency C-section among patients who underwent prostaglandin induction of labor. They found that maternal age, height, gestational age, and fetal sex were the strongest predictors of emergency C-section risk [86]. Yang et al. (2013) used multivariate logistic regression analysis in a study that considered three different decision models which contain risk factors for cesarean delivery associated with each of the three stages of childbirth, more specifically, prenatal care, admission and labor as predictors of risk factors for cesarean delivery. They found that the models corresponding to later stages of labor produced more accurate risk factors of cesarean delivery [99].

Although this document focuses on the initial cesarean delivery, there are many cesarean predictive models that analyze the decision between vaginal birth after cesarean (VBAC) and trial of labor after cesarean (TOLAC) [44]. In particular, Grobman et al. (2007) used multivariable logistic regression to develop a model that can predict the chance of a successful VBAC for trial of labor patients using factors available at the first prenatal visit such as maternal age, ethnicity, pre-pregnancy body mass index (BMI), variables related to prior cesarean delivery, variables related to obstetric history, and pre-existing medical conditions [41]. The prediction model was developed into a graphic nomogram that would be used to assess the probability of a successful VBAC. The results showed that women who were younger, had lower pre-pregnancy BMI, and who were of white race had a better chance of successful VBAC. Other significant factors were history of vaginal delivery and cases in which the prior C-section was performed for reasons other than labor arrest.

1.3.3 Optimization Models

The use of decision trees is a common method to optimize the mode of delivery decision. Sims et al. (1997) validated the use of decision tree-based methods in the prediction of cesarean delivery through a comparison of the results achieved through logistic regression models [84]. Using decision tree-based methods that use recursive partitioning, they determined these methods can be used to predict cesarean delivery and suggested decision trees are small enough to be intelligible to physicians and handle missing values more easily than logistic regression methods. Mankuta et al. (2003) developed a decision tree for VBAC patients in order to analyze the trade-off between a trial of labor or a repeat cesarean section [62]. The decision tree optimizes the delivery by evaluating the risk of perinatal death associated with labor and the increased risk of maternal morbidity and mortality from placental previa or accreta from repeated cesarean sections. The results from their model show that trial of labor was favored if the chance of success was predicted to be 50% or greater. Xu et al. (2010) used a decision tree to determine mode of delivery by evaluating the cost-effectiveness of different health interventions in regards to pelvic floor consequences [98]. Decision trees were also used by Emmett et al. (2007) to predict mode of delivery for VBAC patients through computer-based decision aided utility assessments [29, 67]. Hill et al. (2008) used decision trees and recursive partitioning to identify biomarker (i.e., white blood cell count, cortisol, corticotrophin-releasing hormone, cervical dilatation of 2-4 cm, parity, smoking status, and maternal age) thresholds for determining the probability of preterm birth for women exhibiting early symptoms of labor. Using the algorithm of Atkinson and Therneau [93], they produced a classification tree and optimum threshold values for predictors in order of their relative discrimination power relative. The results

identified maternal white blood cell count and corticotrophin-releasing hormone as the most accurate biomarkers for predicting preterm birth [50].

A cost-effectiveness analysis, using a decision tree, was completed by Culligan et al. (2005) to analyze a policy for elective C-sections for macrosomic infants in order to prevent maternal anal incontinence, urinary incontinence, and newborn brachial plexus injuries [23]. The likelihood of an ultrasound detecting macrosomia was estimated using Bayes theorem. The sub-branches of the tree represented possible clinical consequences of choosing between elective C-section and trial of labor using probabilities of each chance event derived from medical literature. Other clinical consequences considered include emergency hysterectomy, hemorrhage requiring blood transfusion and maternal mortality. The outcome measures associated with the policies included: (1) number of brachial plexus injuries or cases of incontinence averted, (2) incremental monetary cost per 100,000 deliveries, (3) expected quality of life for the mother and child, and (4) a combined quality-adjusted life years (QALYs) measure for the mother and child. For every 100,000 deliveries, they showed the elective C-section policy resulted in fewer incidents of permanent brachial plexus injuries and anal incontinence, lower cost and marginally higher QALYs. They interpreted the results as a prevention of one case of anal incontinence for every 539 elective C-sections performed [23].

The models presented above are for single decision makers and do not evaluate the complex nature of having two decision makers. These papers model the decision from a societal perspective and do not consider a woman's role as an active decision maker. The proposed research will explicitly model how women and their providers interact in determining the mode of delivery incorporating those attributes associated with labor

progression and other factors that influence a cesarean delivery that are valued by each decision maker. This work fills this void by developing a shared-decision modeling framework to be used jointly by the patient and health care provider considering preferences and risk perceptions for both decision makers. In addition, to the best of our knowledge there is no mode of delivery decision model that can be used to identify the appropriate time to end a trial of labor based on the time currently spent in labor and current dilation state.

1.4 Review of Methods

The methods we propose to analyze the mode of delivery decision differ from those traditionally used. Methods described above use logistic regression, simulation, machine learning, decision and classification trees to analyze characteristics that influence delivery mode. Figure 1.4 is a depiction of the different modeling methods and the relevant decision models. The methods we propose in Chapters 2, 3, and 4 use discrete event simulation, Bayesian decision analysis, and Markov decision processes, respectively. Computer simulation is an effective tool because it allows for the ability to develop and test various strategies for treatment and prevention in a controlled environment. Simulation has many uses in healthcare. In particular, discrete event simulation (DES) is used to model and improve clinic and patient flow [45], disease progression [2, 37, 56, 59, 91, 92], and the effects of differing treatment policies [53, 65, 81, 97].

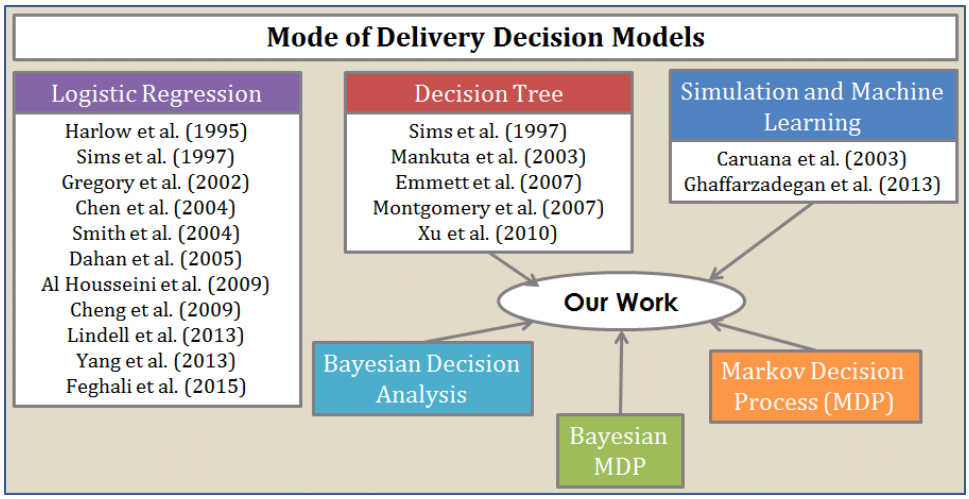


Figure 1.4 Display of current relevant mode of delivery decision models characterized by modeling methods.

1.4.1 Discrete Event Simulation

Kreke et al. (2004) provided a review of critical care decision models which achieve optimal decision for complex diseases through the evaluation of available interventions and various patient characteristics. They describe models that use Markov modeling to model the clinical course of sepsis [8] and Monte Carlo simulation in health issues such as heart disease [97], HIV infection [10, 20, 33, 53], and stroke [65]. They also provide insight into the use of DES in critical care [57]. Costa et al. (2003) developed a DES model to simulate the flow of patients through a critical care unit [21]. Kreke et al. (2004) describes how DES can incorporate the biology of diseases progression to evaluate policy making and treatment decisions. Shechter et al. (2005) developed a biologically-based DES to test policies for liver allocation. Through the simulation of end-stage liver disease and the allocation process, they were able to predict measures relevant to the organ allocation process within 1%

or 2% of the actual measure [81]. Alagoz et al. (2005) also developed a DES to model the natural history of end stage liver disease and the organ allocation system with an objective of predicting the effects of policy changes for allocation. Kreke et al. (2002) developed a simulation of liver allocation that estimates patient survival, financial cost and the number of wasted organs [56]. Similar to these studies, we present a DES of a biological system used to analyze the natural history of labor progression and to identify stopping rules for determining the appropriate time to end a trial of labor.

1.4.2 Bayesian Decision Analysis

Wald (1950) introduced methods for making a decision between null and alternate hypotheses regarding the value of an unknown parameter considering the effects of type I and type II errors. In this method the ratio of type I and type II errors are used to make a decision regarding the hypothesis [68, 80, 96]. We use this technique to decide mode of delivery, between trial of labor and C-section, for two types of laboring patients whose "failure-to-progress" status is unknown. Since C-sections are suggested due to a belief that a patient may "fail-to-progress," our decision model determines the optimal delivery decision as a function of cervical dilation progress. A similar approach was presented in Schechter (1988) where he developed a Bayesian sequential sampling method for diagnosing hypertension based on the measurement of diastolic blood pressure. Since blood pressure measurements change from one time period to the next, the diagnosis of hypertension is made from an accumulation of data. In his model, Schechter was able to apply methods developed by Wald (1950) to determine upper and lower limits to diagnose and exclude hypertension [78–80].

Using similar methods, Forsyth (1997) investigated the rehabilitation of contaminated

groundwater areas [32]. Research by Taskin and Lodree (2010) used Bayesian decision analysis to model the preparation for a demand surge in response to tropical disturbance [90] and emergency supplies inventory management using hurricane prediction measures [89]. Choi et al. (2004) used Bayesian information updating to model inventory ordering policies for seasonal products [18] and to model a retailer's optimal single ordering policy with different delivery options [19].

1.4.3 Markov Decision Processes

A Markov decision processes (MDP) is a dynamic program that determines the best decision among a set of alternatives which are associated with outcomes that optimize the objective over a defined time horizon. MDPs, used for sequential decision making, have been implemented in various realms such as ecology, economics, communications engineering, defense, and health care. Similar to decision trees, MDPs identify the best decision in order to optimize a system that is uncertain. However, the benefit of a MDP is the ability to model a system that is believed to change over time [75]. Beck and Paulkner (1983) are along the first to apply a Markov model to analyze medical prognosis. They described how a Markov mathematical model tool could be used to provide life expectancy and health status assessments [9]. There have been several MDP models to follow that analyze the decision process for other health care issues. Alagoz et al. (2010) developed a tutorial about MDPs use in medical decision making. They include the benefits of using MDPs and note that MDPs are best in handling decision problems that contain a large number of embedded decisions or decisions that are repeated over time [3].

MDPs have been successfully used to model various medical decisions. Alagoz et al.

(2004) developed an MDP to model to the liver acceptance problem [1]. Sandikci el al. (2008) also looked into the liver allocation problem by developing a MDP to model when to accept or reject the organ through the evaluation of the patient's health, the quality of the liver, and the rank of the patient in the waiting list [77]. To analyze treatment protocols for diabetes, Denton et al. (2009) used a MDP to evaluate the optimal start time of statin treatment based on patient attributes and cardiovascular risk models [25] and developed an MDP to analyze the use of statin treatment for cholesterol reduction [58]. Faissol et al. (2007) used an MDP to determine the best timing for testing and treating diseases while providing examples of MDPs to analyze the treatment plans for Hepatitis C and Chlamydia [30]. Chhatwal et al. (2010) developed a finite horizon MDP to evaluates when a women should be sent for a biopsy based on her mammography results and demographic characteristics [16]. Similarly, Alagoz et al. (2013) developed a MDP to help radiologist make decisions for patient management by focusing on a patient's total expected quality adjusted life years [4]. Maillart et al. (2008) developed a partially-observable Markov chain model to analyze the mammography screening decision for premenopausal and postmenopausal women in order to reduce breast cancer mortality [61]. An MDP comparing the benefits and risks associated with delaying treatment of HIV by optimizing the expected lifetime of patients [82]. Kreke et al. (2008) present a model of when a patient with pneumonia-related sepsis should be discharged [55]. We present a finite-horizon MDP that provides the total number of hours a woman should be allowed to labor at each state of cervical dilation within a 24 hour laboring period. The results correspond to the maximization of a utility function that incorporates the health outcomes for the mother and child along with a valuation of the delivery process.

Using DES, Bayesian decision analysis and MDPs, we have been able to develop single-decision maker models that lend to understanding how long a woman can labor and under what conditions labor should be prolonged. There are many studies that apply similar mechanisms to model and evaluate medical decisions and many mode of delivery decision models that evaluate the best mode of delivery. However, to the best of our knowledge, there are no models that consider the mode of delivery decision as a function of cervical dilation using the methods we present here.

The remainder of this dissertation is structured as follows. In Chapter 2, a simulation model of labor progression is introduced along with analysis of labor progression rules. In Chapter 3, we present a Bayesian decision model for a single decision maker that determines the conditions under which it is most appropriate to gather more information about the state of the patient before making a decision about mode of delivery with the goals of: (i) minimizing the loss/penalty associated with making an incorrect decision, and (ii) determining the value of information associated with gathering more information and identifying key attributes that are most influential to the decision. A Markov decision process is presented in Chapter 4 which provides policies regarding the length of time a woman can be allowed to safely labor before deciding a C-section is needed with the goal of maximizing the utility of health outcomes and the delivery process. In Chapter 5, we conclude with a Bayesian MDP which provides the optimal mode of delivery for each dilation state and time period when the patient type is unknown.

CHAPTER

2

SIMULATION OF LABOR

The discrete event simulation model presented in this chapter provides additional insight regarding mode of delivery decision making as a function of labor progression. The goals of the model are to: (1) model the natural progression of labor in absence of cesarean deliveries, (2) determine the underlying rules responsible for the current rate of cesarean deliveries due to a "failure-to-progress" diagnosis, and (3) develop ways to reduce "unnecessary" cesarean deliveries while also reducing the rate of complications. The results from the simulation provide estimates of the maximum time in labor to reduce the rate of cesarean

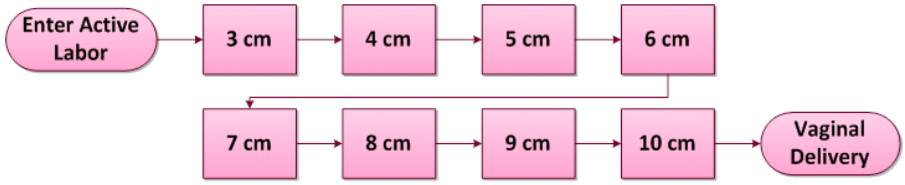


Figure 2.1 Process flow for simulation Model 1 of the natural progression of labor.

deliveries and increase the likelihood of health outcomes for the both mother and child. We develop two simulation models that model labor progression. The first model, Model 1, described in Section 2.1.1, characterizes the natural progression of labor. Figure 2.1 shows the process flow for Model 1. This model seeks to understand the length of time a woman may labor if no interventions or C-sections are introduced. The second model, Model 2, described in Section 2.6.1 with process flow shown in Figure 2.2, is similar to the first model; however, upon completion of each dilation state, the total time in that dilation state is evaluated to determine whether the patient should undergo a C-section based on a defined stopping criterion. That is, if a stopping rule of two hours (i.e., result of Friedman Curve) is implemented and a patient requires 3.6 hours to dilate from 4 cm to 5 cm, then she would be given a C-section at 4 cm. If she had progressed from 4 cm within two hours, she would continue to labor and proceed to 5 cm.

2.1 Formulation of Models

To evaluate the current cesarean delivery rate, we simulate the delivery process for women undergoing a trial of labor. In Model 2 we evaluate the Friedman Curve and other labor progression rules to identify circumstances in which the cesarean rate can be decreased by

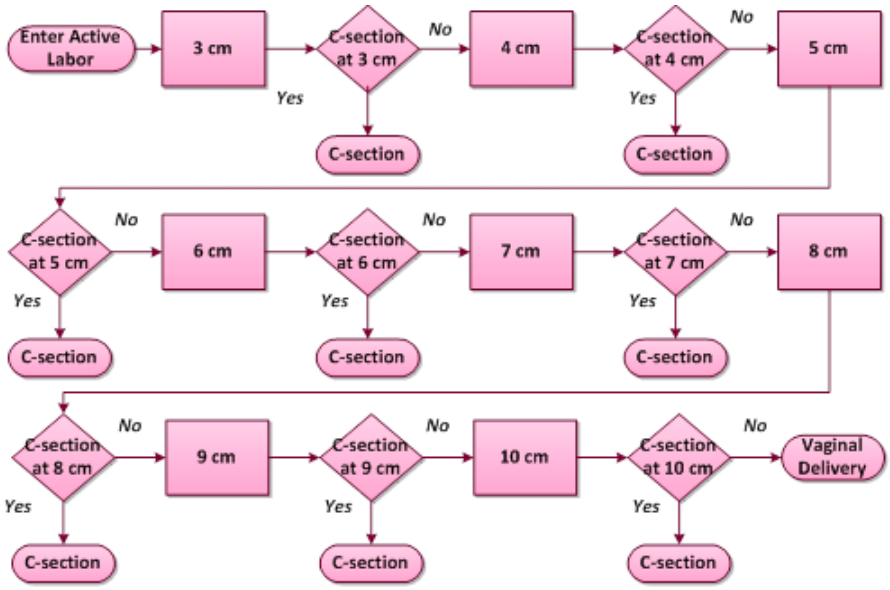


Figure 2.2 Process flow of simulation Model 2 of the duration of labor when C-sections due to a "failure-to-progress" diagnosis are considered.

analyzing the length of time a woman spends in labor and more specifically the duration of time a woman remains at one particular cervical dilation. We only seek to identify cesarean deliveries that are performed after a trial of labor has started and only consider "failure-to-progress" as the indication for a cesarean delivery for singleton nulliparous deliveries with no prior indications for cesarean delivery. Other indications for cesarean delivery such as fetal distress, uterine rupture, and cephalopelvic disproportion (CPD) are not considered in this study.

2.1.1 Model 1: Natural Progression of Labor

Labor is often described in three stages: (1) early and active labor, (2) delivery of baby, and (3) delivery of the placenta. In the first stage (the primary focus of our study) there are

three phases (early labor phase, active labor phase, and transition phase) that begin at the onset of labor until the cervix is fully dilated to 10 cm [6]. A Simio® model is developed to simulate labor to understand the duration of labor in absence of any stopping rules or interventions. Patients (i.e., entities) enter "active" labor starting at cervical dilation of 3 cm and we model the progression of cervical dilation to 10 cm. Once the cervix is dilated to 10 cm, the patient then moves into the second stage of labor (known as the pushing phase).

We divide the cervical dilation of active labor, starting at 3 cm, into states. Figure 3.5a shows the Simio® model of the natural progression of labor. Patients enter the labor server (SrcTOL) according to an exponential probability distribution with a rate parameter of 1 patient per hour and proceed through each dilation server (Srv $j$cm) until they reach the vaginal delivery sink (SnkVD). We assumed a percentage (90%) of women receives epidural analgesia prior to SnkVD [103]. In this model we do not allow C-sections because we want to understand how long active labor can last given the probability distributions derived from the data sources described in Section 2.2.

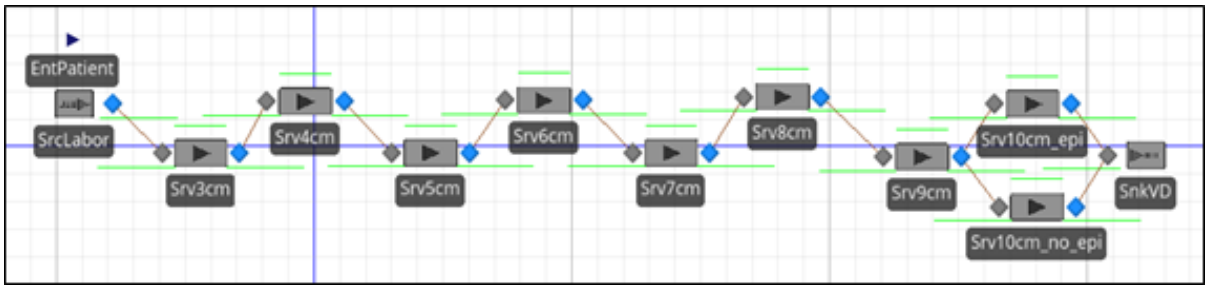


Figure 2.3 Screenshot of Simio® simulation Model 1 of labor progression from 3 cm to 10 cm in absence of cesarean deliveries.

2.1.2 Model 2: Various Labor Progression Rules

Similar to Model 1, women progress through labor according to a log normal distribution derived from publicly available literature sources in Model 2. The 5th, 50th, and 95th percentile for each laboring type and cervical dilation state is presented in Table 2.1. After the patient has finished laboring through a dilation state, it is decided whether the patient should continue to the next dilation state or be transferred to a cesarean delivery. This decision is based on the processing time (i.e., how long the patient remains in a particular dilation). If this time is greater than the established cutoff time, then the patient is given a C-section due to a "failure-to-progress" diagnosis. The stopping rules represents the 95th percentile rounded to the nearest integer as presented in Zhang et al. (2010) [103]. The stopping rules for each dilation state are as follows: 8 hours for 3 cm, 6 hours for 4 cm, 3 hours for 5 cm, 2 hours for dilation states 6-9 cm, and 4 hours for 10 cm. We use this model (Figure 3.5b) to simulate the current cesarean delivery rate and to experiment with various stopping rules.

2.1.2.1 Current C-section Rate for Failure-to-progress

Zhang et al. 2010, found that 2% of pre-labor cesarean deliveries and 47.1% of intrapartum cesarean deliveries are attributed to failure-to-progress or CPD (occurs when a baby's head or body is too large to fit through the mother's pelvis). They estimated that the total cesarean rate was 30.5%. In order to identify the decision rule used to achieve a cesarean rate of 14.98% (i.e., $30.5\% * 49.1\% = 14.98\%$) for a failure-to-progress/CPD diagnosis, we simulated 200 scenarios with differing stopping rules. We assume the minimum stopping rule to be 1 hour. The maximum stopping rule value we identify to be the stopping rule defined for

Table 2.1 The 5th, 50th, and 95th percentile for induced, augmented, and spontaneous labor types.

	INDUCED			AUGMENTED			SPONTANEOUS		
Cervical Dilation	x_{5th}	x_{50th}	x_{95th}	x_{5th}	x_{50th}	x_{95th}	x_{5th}	x_{50th}	x_{95th}
3 cm	0.2	1.4	8.1	0.2	1.2	6.8	0.1	0.4	2.3
4 cm	0.2	1.3	6.8	0.3	1.4	7.6	0.1	0.5	2.7
5 cm	0.1	0.6	4.3	0.1	0.7	4.9	0.06	0.4	2.7
6 cm	0.05	0.4	2.8	0.06	0.5	3.9	0.03	0.3	2.1
7 cm	0.03	0.2	1.5	0.05	0.3	2.2	0.04	0.3	1.7
8 cm	0.03	0.2	1.3	0.05	0.3	2	0.03	0.2	1.3
9 cm	0.04	0.3	1.9	0.05	0.3	2.4	0.04	0.3	1.8
10 cm (epi)	-	1.1	5	-	1.1	5	-	1.1	5
10 cm (no epi)	-	0.6	5	-	0.6	5	-	0.6	5

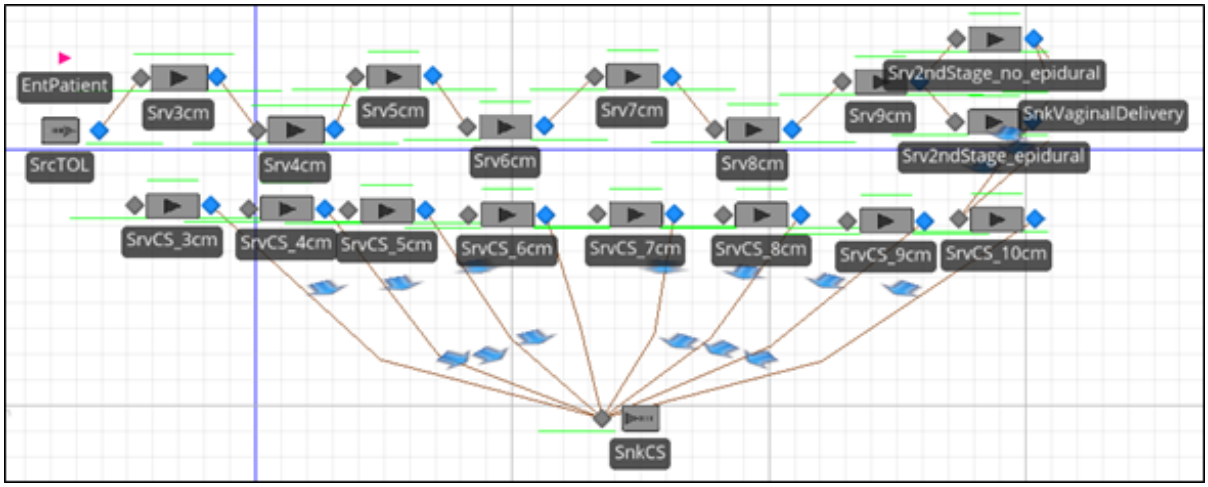


Figure 2.4 Screenshot of Simio simulation Model 2 where patients (EntPatient) enter the system (SrcTOL) according to an exponential probability distribution and progress through each server which is represented by Srv$j$cm for each cervical dilation j. Once the patient completes each cervical dilation state (Srv$j$cm), a decision (represented by the diamond in Figure 3.4) is made of regarding whether to perform a C-section given the length of time she was in that particular dilation. If the length of time exceeds the stated cutoff time, she delivers through cesarean and the entity enters into SrvCS_$j$cm to undergo a C-section at cervical dilation j. The simulation ends with each patient delivering either through cesarean (SnkCS) or vaginally (SnkVaginalDelivery).

Model 2 plus an additional two hours. We allowed the stopping rule to vary from 1 to 10 hours for 3 cm, 1 to 8 hours for 4 cm, 1 to 5 hours for 5 cm, 1 to 4 hours for states 6 cm through 9 cm, and 1 to 6 hours for 10 cm by an increment of 0.5 hours and calculated the cesarean delivery rate as well as the absolute value of the difference between the calculated C-section rate and the target rate of 0.1498. Scenarios closest to the target rate are identified as potential stopping rules used to produce a C-section rate of 14.98%.

2.1.2.2 Experiments with Various Stopping Rules

To gain insight regarding the effects of allowing women to labor longer before deciding to deliver by cesarean, we experiment with different stopping rules to estimate the resulting cesarean section rate and the corresponding number of complications. We use probability estimates from Cheng et al. (2009) who estimated the rate of complications for labors lasting 0-12 hours, 12-18 hours, 18-24 hours, and greater than 24 hours to calculate the expected number of complications. The complications included birth trauma, NICU admissions, and a neonatal composite variable of 5-min Apgar <7, shoulder dystocia, and birth trauma. By adding the probability of the complications mentioned for each time interval, we calculate the expected number of complications that occurred from labor durations of 0-12 hours ($\text{Complications}_{0-12}$), 12-18 hours ($\text{Complications}_{12-18}$), 18-24 hours ($\text{Complications}_{18-24}$), and greater than 24 hours ($\text{Complications}_{24}$) which corresponded to a probability of 0.059, 0.061, 0.066, and 0.101, respectively. The expected number of

complications for each simulation run is computed using the following formula:

$$\begin{aligned}\mathbb{E}(\text{Number of Complications}) = 0.059 * \text{Complications}_{0-12} + 0.061 * \text{Complications}_{12-18} \\ + 0.066 * \text{Complications}_{18-24} + 0.101 * \text{Complications}_{24}\end{aligned}$$

and the rate of complication is calculated by taking

$$\frac{\mathbb{E}(\text{Number of Complications})}{\text{Total Number of Births}}.$$

We estimate the C-section rate and number of complications using a stopping criterion of 2 hours, 3 hours, 4 hours, ..., 9 hours for each dilation. We then experiment with various two-dimensional stopping rules to identify the effects these rules have on the estimated C-section rate and rate of complications. We define the two-dimensional stopping rule to be (a, b) where a represents the stopping rule for dilation states 3 cm, 4 cm, and 5 cm and b represents the stopping rule for dilation states 6 cm - 10 cm. The value of a and b vary from 2 hours to 9 hours. The two-dimensional stopping rule $(6,3)$ refers a stopping rule of 6 hours for dilation states 3-5 cm and a stopping rule of 3 for dilation states 6-10 cm.

2.2 Data Description

Data for the model were derived from three sources: (1) *Contemporary Patterns of Spontaneous Labor with Normal Neonatal Outcomes* [103] and (2) *Normal Progress of Induced Labor* [48], and (3) *The Association between the Length of First Stage of Labor, Mode of Delivery, and Perinatal Outcomes in Women Undergoing Induction of Labor* [15]. The Consortium

on Safe Labor conducted retrospective studies using labor and delivery data from electronic medical records in 19 hospitals across the United States [103] and data from Harper et al. (2012) was the result of a four-year retrospective cohort study of deliveries at Washington University Medical Center in St. Louis, Missouri [48]. Each data source evaluated current labor progression patterns. In particular, these studies used an interval-censored regression [54], under the assumption that the labor duration is lognormally distributed to estimate the median time to progress 1 cm in cervical dilation. Interval-censored regression is used when only the information available regarding an event's occurrence is the interval of time in which the event occurred [54]. The lognormal distribution has been stated as a natural fit for the duration of labor because the distribution is skewed leaning to the right [95]. To account for the complications that may arise based on time in labor, we used information from Cheng et al. (2009) who conducted a retrospective cohort study of pregnancies with labor induction who delivered during the second stage of labor to identify the rate of maternal and neonatal complications as a function of the duration of labor [15].

We model three different types of patients: (1) patients who had an induction of labor, (2) patients who had an augmented labor and (3) patients who had a spontaneous labor as described in Harper et al. (2012). Harper et al. (2012) sought to compare normal labor progress for women whose labor was induced versus labor progression for those women who labored spontaneously. Those patients in the spontaneous labor category are women who received no augmentation with oxytocin nor underwent artificial rupture of the membranes. Augmented labors were defined for women who were diagnosed as having a spontaneous labor but subsequently received oxytocin augmentation. There were 5,388 women included in the Harper et al. (2012) study, 2,021 of which were spontaneous

labors, 1,720 were augmented labors, and 1,647 were induced labors accounting for 37.5%, 31.9%, and 30.6% of the study population, respectively. We used these percentages to assign identities to the entities (patients) in the simulation. All women included in the Harper et al. (2012) study delivered in the second stage of labor. Therefore if a C-section was given it was done after a cervical dilation of 10 cm [48].

Harper et al. (2012) provided the median, 5th percentile and 95th percentile of the duration of labor for dilation states 3-10 cm. The median and 95th percentile of the duration of labor from 10 cm to delivery was presented in Zhang et al. (2010) [103]. Under the assumption that labor duration times are lognormally distributed we developed a percentile matching procedure that can be used to model the duration of labor progression for each patient type and each cervical dilation state.

2.3 Probability Distributions

In order to provide each patient type with a random labor progression (i.e., estimation of the probability distribution of labor progression) for each dilation state, we used a percentile matching procedure assuming a lognormal distribution. Vahratian et al. (2006) explained how the lognormal distribution is a natural fit for the duration of labor times due to its shape which is skewed to the right as demonstrated by evidence that some labors may produce a long right tail [95]. The goal of the percentile matching procedure is to find the exact (or a close approximation) of the lognormal distribution that provides the same median and 5th and 95th percentile values provided in the literature [48, 103]. Due to the limitation that it is not known exactly when a cervical dilation change occurs, the authors each used interval-censored regression to estimate the percentiles for the duration of labor.

Interval-censoring is used when the time of an event is not known; however, it is known that the event occurred within a certain interval of time. A minimum and maximum possible time range was computed from each labor time record for each interval of cervical dilation (i.e., from 4 cm to 5 cm, 5 cm to 6 cm, etc.). By fitting a model with a lognormal distribution along with interval-censored time-to-event data, the authors were able to determine the median and 5th or 95th percentiles of labor duration time [95].

The procedure described below was developed to match the 5th, 50th (median) and 95th percentiles of the labor duration data presented in [48, 103]. By matching percentiles, we are then able to fit lognormal distributions for each dilation state and each patient type.

Percentile Matching Procedure

Given a nonnegative random variable X with probability distribution function (pdf), $f(x)$, cumulative distribution function (cdf), $F(x) = \int_{-\infty}^{x} f(u)du$ for $-\infty < x < \infty$, and three selected quantiles of the distribution of X we have

$$x_{p_i} = F^{-1}(p_i) \text{ for } i = 1,2,3,$$

where p_i represents a given percentile satisfying the following relationship $0 < p_1 < p_2 < p_3 < 1$. Given the quantile estimates $\hat{x}_{p_i} : i = 1,2,3$, we seek to fit the shifted lognormal distribution with the pdf

$$\frac{1}{(x-a)\beta\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\frac{ln(x-a)-\lambda}{\beta}\right]^2\right\} \text{ for } x \geq a$$

so that we have the cdf

$$F(x;\lambda,\beta,a) = \int_{-\infty}^{x} f(u;\lambda,\beta,a)\,du \tag{2.1}$$

$$= \begin{cases} 0 & \text{if } x < a \\ \Phi\left[\frac{ln(x-a)-\lambda}{\beta}\right] & \text{if } x \geq a \end{cases} \tag{2.2}$$

where

$$\Phi(z) \equiv \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}u^2\right\} du$$

is the standard normal cdf. We seek estimates $\hat{a}$, $\hat{\lambda}$, $\hat{\beta}$ of the lognormal parameters a, λ, β based on the quantile estimates $\left\{\hat{x}_{p_i} : i = 1,2,3\right\}$ such that

$$F(\hat{x}_{p_i};\hat{a},\hat{\lambda},\hat{\beta}) \approx p_i \text{ for } i = 1,2,3. \tag{2.3}$$

Combining Equations (2.2) and (2.3), we seek to solve the following system of equations:

$$\frac{\ln(\hat{x}_{p_i}-\hat{a})-\hat{\lambda}}{\hat{\beta}} = \Phi^{-1}(p_i) \equiv z_{p_i} \text{ for } i = 1,2,3. \tag{2.4}$$

Since we know the 5th, 50th, and 95th percentile, we have $p_1 = 0.05$, $p_2 = 0.50$, and $p_3 = 0.95$. So Equation (2.4) has the following form

$$\frac{\ln(\hat{x}_{p_1}-\hat{a})-\hat{\lambda}}{\beta} = -1.6449 = z_{p_1} \tag{2.5}$$

$$\frac{\ln(\hat{x}_{p_2}-\hat{a})-\hat{\lambda}}{\beta} = 0 = z_{p_2} \tag{2.6}$$

$$\frac{\ln(\hat{x}_{p_3}-\hat{a})-\hat{\lambda}}{\beta} = 1.6449 = z_{p_3} \tag{2.7}$$

From Equation (2.6), we obtain

$$\hat{a} = \hat{x}_{p_2} - e^{\hat{\lambda}}. \tag{2.8}$$

Dividing Equations (2.5) and (2.7), we obtain

$$\frac{\ln(\hat{x}_{p_1} - \hat{a}) - \hat{\lambda}}{\ln(\hat{x}_{p_3} - \hat{a}) - \hat{\lambda}} = -1,$$

which simplifies to

$$\ln[(\hat{x}_{p_1} - \hat{a})(\hat{x}_{p_3} - \hat{a})] = 2\hat{\lambda},$$

which we rewrite as

$$(\hat{x}_{p_1} - \hat{a})(\hat{x}_{p_3} - \hat{a}) = e^{2\hat{\lambda}}. \tag{2.9}$$

Plugging the right-hand side of Equation (2.8) into the left-hand side of Equation (2.9) as an "approximation" for $\hat{a}$, then we have the following form

$$[e^{\hat{\lambda}} - (\hat{x}_{p_2} - \hat{x}_{p_1})][e^{\hat{\lambda}} + (\hat{x}_{p_3} - \hat{x}_{p_2})] = e^{2\hat{\lambda}}. \tag{2.10}$$

If we let $\delta_{21} \equiv \hat{x}_{p_2} - \hat{x}_{p_1}$ and $\delta_{32} \equiv \hat{x}_{p_3} - \hat{x}_{p_2}$ where δ_{ij} presents the difference between quantile estimates for p_i and p_j. Then Equation (2.10) simplifies to

$$e^{2\hat{\lambda}} + (\delta_{32} - \delta_{21})e^{\hat{\lambda}} - \delta_{21}\delta_{32} = e^{2\hat{\lambda}},$$

which has the solution

$$\begin{aligned}\hat{\lambda} &= \ln\left(\frac{\delta_{32}\delta_{21}}{\delta_{32}-\delta_{21}}\right) \\ &= \ln\left[\frac{(\hat{x}_{p_3}-\hat{x}_{p_2})(\hat{x}_{p_2}-\hat{x}_{p_1})}{\hat{x}_{p_3}-2\hat{x}_{p_2}+\hat{x}_{p_1}}\right].\end{aligned}$$

Using this value of $\hat{\lambda}$, we evaluate $\hat{a}$ as follows:

$$\hat{a} = \max\left\{\hat{x}_{p_2} - e^{\hat{\lambda}}, 0\right\}.$$

This modification of Equation (2.8) is needed because we must have $\hat{a} \geq 0$ for a nonnegative shifted lognormal random variable X. Finally, we evaluate $\hat{\beta}$ by plugging the values of $\hat{\lambda}$ and $\hat{a}$ into Equation (2.7), yielding

$$\hat{\beta} = \frac{\ln(\hat{x}_{p_3}-\hat{a})-\hat{\lambda}}{z_{p_3}};$$

then the associated lognormal distribution has mean

$$\hat{\mu} = \exp\left\{\hat{\lambda}+\frac{1}{2}\hat{\beta}^2\right\}$$

and standard deviation

$$\hat{\sigma} = \hat{\mu}\sqrt{\exp(\hat{\beta})-1}.$$

We can check the validity of the distribution by calculating the the cdf for each percentile (Table 2.2). The goal would be to have

$$F(\hat{x}_{0.05}) = 0.05,$$
$$F(\hat{x}_{0.5}) = 0.5,$$
$$F(\hat{x}_{0.95}) = 0.95.$$

Using this information we were able to derive the distributions for nulliparous women in the induction, augmented, and spontaneous labor groups as presented in Table 2.2. A graphical depiction of the distributions developed for dilation states 4 cm, 7 cm, and 10 cm for induced spontaneous, and augmented labors is presented in Figure 2.5.

Overall, the derived distributions provide reasonable estimates for the duration of labor for each dilation state. The cdf for the 50th and 95th percentiles are matched exactly and the cdf for the 5th percentile is matched for particular dilation states. Further analysis regarding the derived lognormal distributions is provided in Section 2.4.1.

2.4 Results

2.4.1 Natural Progression of Labor

Simulation Model 1 was run for 52 weeks to emulate a hospital setting for one year. Using 10 replications in which patients enter labor according to an exponential probability distribution with a mean of 1 hour, there were approximately 8,741 ± 89.94 total births where 2,672

Table 2.2 Parameters λ, β, and a corresponding to the lognormal distributions along with the cdf for each percentile for each dilation state and three patient laboring types derived from the percentile matching procedure.

	Cervical Dilation	x^a_{5th}	x^a_{50th}	x^a_{95th}	*Parameters*			$F(x_{5th})$	$F(x_{50th})$	$F(x_{95th})$
					λ	β	a			
INDUCED	*3 cm*	0.2	1.4	8.1	0.3365	1.0672	0	0.0341	0.5	0.95
	4 cm	0.2	1.3	6.8	0.2624	1.0059	0	0.0314	0.5	0.95
	5 cm	0.1	0.6	4.3	-0.5480	1.2168	0.0219	0.05	0.5	0.95
	6 cm	0.05	0.4	2.8	-0.9163	1.1830	0	0.0394	0.5	0.95
	7 cm	0.03	0.2	1.5	-1.6318	1.2368	0.0044	0.0500	0.5	0.95
	8 cm	0.03	0.2	1.3	-1.6094	1.1380	0	0.0477	0.5	0.95
	9 cm	0.04	0.3	1.9	-1.2040	1.1222	0	0.0363	0.5	0.95
	10 cm (epi)	-	1.1[b]	5[b]	0.0953	0.9205	0	-	0.5	0.95
	10 cm (no epi)	-	0.6[b]	5[b]	-0.5108	1.2890	0	-	0.5	0.95
AUGMENTED	*3 cm*	0.2	1.2	6.8	0.1823	1.0546	0	0.0447	0.5	0.95
	4 cm	0.3	1.4	7.6	0.2906	1.0513	0.0627	0.05	0.5	0.95
	5 cm	0.1	0.7	4.9	-0.3567	1.1830	0	0.05	0.5	0.95
	6 cm	0.06	0.5	3.9	-0.6931	1.2488	0	0.0448	0.5	0.95
	7 cm	0.05	0.3	2.2	-1.2452	1.2330	0.0121	0.05	0.5	0.95
	8 cm	0.05	0.3	2	-1.2272	1.1654	0.0069	0.05	0.5	0.95
	9 cm	0.05	0.3	2.4	-1.2595	1.2939	0.0162	0.05	0.5	0.95
	10 cm (epi)	-	1.1[b]	5[b]	0.0953	0.9205	0	-	0.5	0.95
	10 cm (no epi)	-	0.6[b]	5[b]	-0.5108	1.2890	0	-	0.5	0.95
SPONTANEOUS	*3 cm*	0.1	0.4	2.3	-1.0321	1.1222	0.0438	0.05	0.5	0.95
	4 cm	0.1	0.5	2.7	-0.7156	1.0364	0.0111	0.05	0.5	0.95
	5 cm	0.06	0.4	2.7	-0.9188	1.1622	0.0010	0.05	0.5	0.95
	6 cm	0.03	0.3	2.1	-1.2040	1.1830	0	0.0258	0.5	0.95
	7 cm	0.04	0.3	1.7	-1.2040	1.0546	0	0.0280	0.5	0.95
	8 cm	0.03	0.2	1.3	-1.6094	1.1380	0	0.0477	0.5	0.95
	9 cm	0.04	0.3	1.8	-1.2040	1.0893	0	0.0322	0.5	0.95
	10 cm (epi)	-	1.1[b]	5[b]	0.0953	0.9205	0	-	0.5	0.95
	10 cm (no epi)	-	0.6[b]	5[b]	-0.5108	1.2890	0	-	0.5	0.95

Sources:
a - Harper et al. (2012)∼[48]; b - Zhang et al. (2010)∼[103]

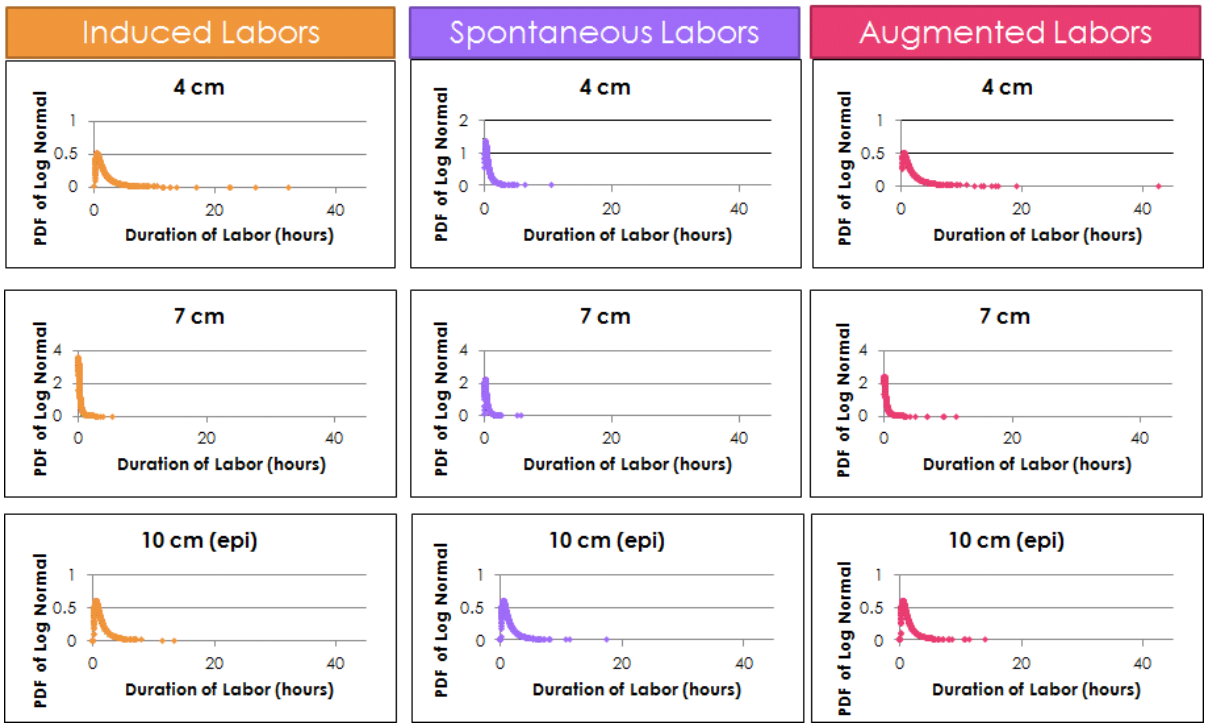


Figure 2.5 Lognormal curves for dilation states 4 cm, 7 cm, ..., 10 cm for induced, spontaneous, and augmented labors.

(30.57%) were induced labor births, 2,801 (32.04%) were augmented labor births and 3,267 (37.38%) were spontaneous labor births. The average time in system, which we define as the length of active labor from 3 cm to vaginal delivery, was 8.61 ± 0.05 hours (Figure 2.6). The minimum and maximum times were 1.11 ± 0.13 hours (Figure 2.7) and 92.29 ± 21.99 hours (Figure 2.8), respectively. In addition to the average, minimum, and maximum values for duration of labor, the 5th, 50th, and 95th percentile estimates along with the corresponding 95% confidence interval are provided in Table 3.1 for all three patient types. The results provide reasonable estimates of the total length of active labor assuming labor duration is lognormally distributed. Having such an estimate can inform the definition of policies

for determining appropriate times to end a trial of labor for those patients considered "failure-to-progress."

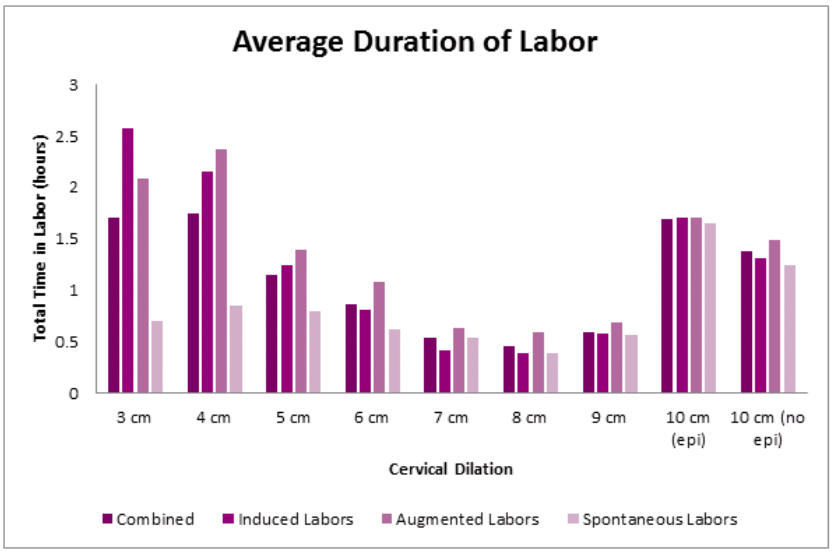


Figure 2.6 Average labor times for Model 1 measured in hours.

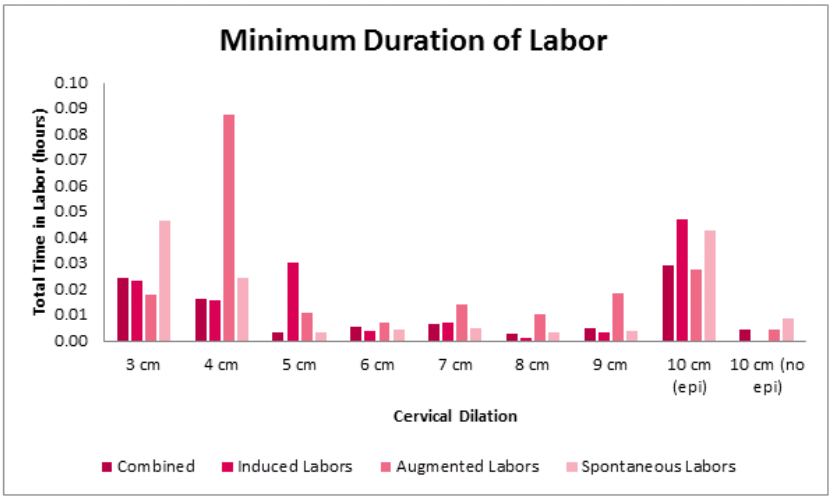


Figure 2.7 Minimum labor times for Model 1 measured in hours.

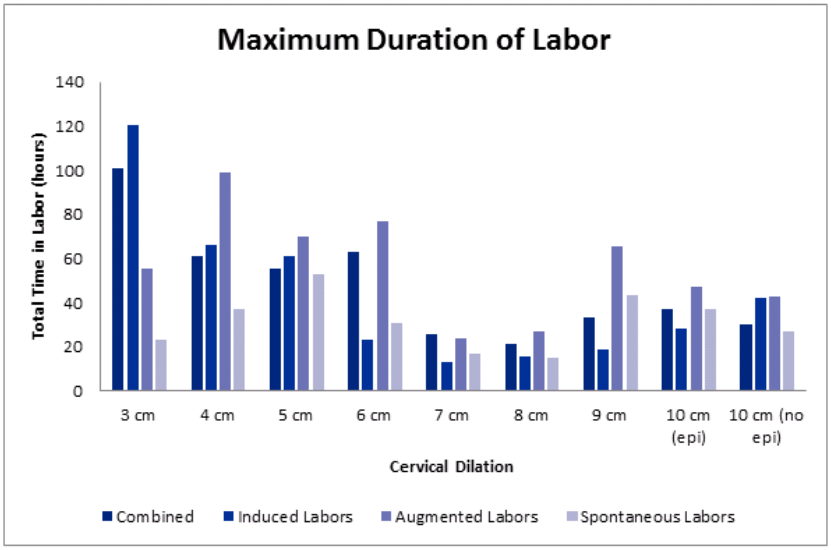


Figure 2.8 Maximum labor times for Model 1 measured in hours.

Table 2.3 The 5th, 50th, and 95th percentile, minimum, average, and maximum duration of labor for each dilation state and labor type.

	3 cm (hours)	4 cm (hours)	5 cm (hours)	6 cm (hours)	7 cm (hours)	8 cm (hours)	9 cm (hours)	10 cm (epi) (hours)	10 cm (no epi) (hours)
5th Percentile	0.24 (0.011)*	0.25 (0.011)*	0.10 (0.006)	0.06 (0.003)*	0.03 (0.002)	0.03 (0.002)	0.05 (0.002)*	0.25 (0.010)	0.07 (0.004)
50th Percentile	1.42 (0.040)	1.28 (0.030)	0.60 (0.020)	0.40 (0.010)	0.20 (0.010)	0.20 (0.010)	0.31 (0.010)	1.10 (0.030)	0.58 (0.02)*
95th Percentile	8.22 (0.398)	6.89 (0.314)	4.35 (0.238)	2.90 (0.160)	1.48 (0.080)	1.34 (0.071)	1.87 (0.091)	5.12 (0.218)	4.46 (0.220)*
Min	0.024	0.016	0.030	0.004	0.007	0.001	0.003	0.047	0.011
Average	2.575	2.147	1.242	0.805	0.411	0.391	0.572	1.701	1.308
Max	120.324	66.081	61.285	23.034	12.909	15.605	19.018	27.915	42.267

(a) Induced Labors

	3 cm (hours)	4 cm (hours)	5 cm (hours)	6 cm (hours)	7 cm (hours)	8 cm (hours)	9 cm (hours)	10 cm (epi) (hours)	10 cm (no epi) (hours)
5th Percentile	0.22 (0.010)*	0.31 (0.014)	0.10 (0.005)	0.06 (0.004)*	0.05 (0.003)	0.05 (0.003)	0.05 (0.003)	0.24 (0.010)	0.07 (0.004)
50th Percentile	1.17 (0.032)	1.39 (0.038)	0.68 (0.021)	0.49 (0.016)	0.30 (0.010)	0.30 (0.009)	0.31 (0.010)	1.13 (0.027)	0.65 (0.022)*
95th Percentile	6.75 (0.311)	7.39 (0.329)	4.64 (0.225)*	3.91 (0.217)	2.26 (0.128)	1.92 (0.094)	2.39 (0.136)	5.03 (0.207)	5.93 (0.425)*
Min	0.018	0.088	0.011	0.007	0.014	0.010	0.019	0.028	0.004
Average	2.078	2.358	1.385	1.081	0.637	0.590	0.680	1.706	1.479
Max	55.260	99.007	69.664	76.837	23.943	26.785	65.287	47.217	42.905

(b) Augmented Labors

	3 cm (hours)	4 cm (hours)	5 cm (hours)	6 cm (hours)	7 cm (hours)	8 cm (hours)	9 cm (hours)	10 cm (epi) (hours)	10 cm (no epi) (hours)
5th Percentile	0.10 (0.005)	0.10 (0.005)	0.06 (0.003)	0.04 (0.002)*	0.05 (0.003)*	0.03 (0.002)	0.05 (0.002)*	0.24 (0.010)	0.07 (0.004)
50th Percentile	0.40 (0.012)	0.49 (0.013)	0.40 (0.012)	0.30 (0.009)	0.30 (0.008)	0.20 (0.006)	0.30 (0.009)	1.10 (0.027)	0.62 (0.021)
95th Percentile	2.26 (0.109)	2.68 (0.122)	2.75 (0.146)	2.15 (0.116)	1.73 (0.083)	1.30 (0.066)	1.80 (0.087)	4.83 (0.185)	4.45 (0.220)
Min	0.046	0.024	0.003	0.004	0.005	0.003	0.004	0.043	0.009
Average	0.701	0.848	0.797	0.614	0.533	0.384	0.557	1.651	1.243
Max	23.233	37.097	52.774	30.457	16.914	15.224	43.524	37.329	26.985

(c) Spontaneous Labors

Data are point estimates (95% confidence interval half-width)
* represent confidence intervals that do not contain the literature source value

Using the median times of labor duration, we were able to plot the labor curves for each labor type, the combined labors, and the Friedman Curve estimates starting at 3 cm. The results are presented in Figure 2.9. The chart shows a comparison of labor duration for differing labor types and suggests that actions pertaining to birth decisions are a function of the labor type. The results also show that labors in the combined, augmented, and induced cases last longer than those estimated by the Friedman Curve.

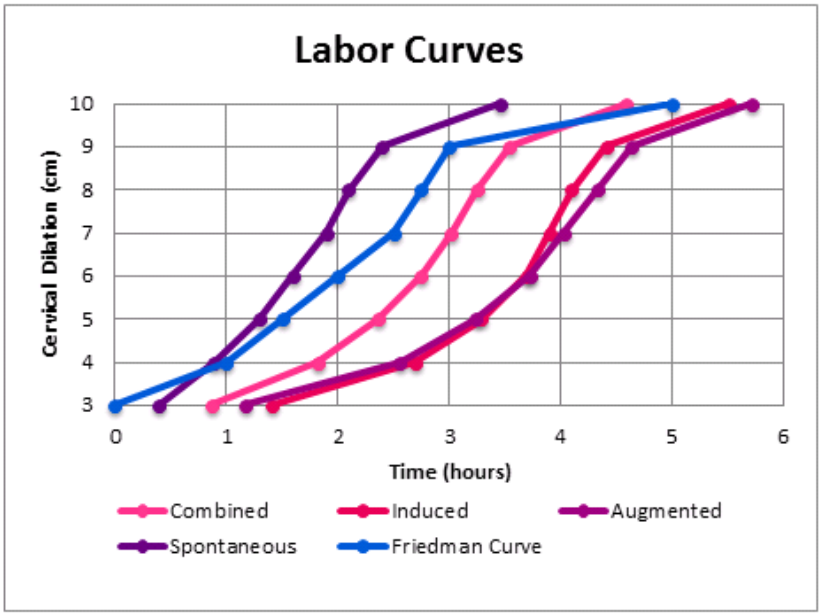


Figure 2.9 Median labor curves for induced, augmented, and spontaneous labors along with a Friedman Curve estimate [35].

2.4.2 Estimate of Current C-section Rate Due to Failure-to-progress

We were able to identify three scenarios that resulted in a cesarean section percentage close to 0.1498, presented in Table 2.4. The stopping criterion for most dilation states is longer than two hours, which differs from the labor progression rules suggested by the Friedman Curve. The scenarios identified agree with Zhang et al. (2010) suggestion that women should be allowed to labor longer in the first stage of labor for cervical dilation at and below 6 cm. Overall the results provide evidence that allowing women to have a longer first stage of labor can aid in reducing the rate of cesarean deliveries. With a difference of 0.0008 from a C-section rate of 14.98%, Scenario 1 has a stopping criterion closest to the rate observed by the Consortium on Safe Labor. Since the 95% confidence intervals for the three scenarios overlap, we assume that the scenarios are not statistically different and that any of the scenarios identified (or slight variations) were used in practice for the 19 hospitals observed by Zhang et al. (2010).

2.4.3 One-dimensional Stopping Rules

To understand the effect of waiting before deciding a patient has failed to progress and a C-section is necessary, we estimate the C-section rate and rate of complications using a stopping criterion of 2 hours, 3 hours, 4 hours, etc., up to 9 hours for all dilation states. We assume the stopping rule is the same for all dilation states. The comparison of rates and rate of complications are displayed in Figures 2.10a and 2.10b and Table 2.5. The results confirm the inverse relationship between the number of cesarean deliveries and the number of complications. The largest benefit for waiting an additional hour is for extending the 2

Table 2.4 Display of duration of labor (in hours) to achieve current rate of 14.98%.

	Scenario 1	Scenario 2	Scenario 3
Duration of Labor 3 cm	8.2	8.3	8.5
Duration of Labor 4 cm	6.9	7	7
Duration of Labor 5 cm	5	4.8	4.9
Duration of Labor 6 cm	5	4.8	4.9
Duration of Labor 7 cm	4.6	4.8	5
Duration of Labor 8 cm	4.6	4.8	5
Duration of Labor 9 cm	5.5	5.5	5.3
Duration of Labor 10 cm	6.6	6.5	6.5
Difference from 0.1498	0.0008	0.0010	0.0023
Cesarean Section Percentage	0.1506 ± 0.0024	0.1508 ± 0.0043	0.1521 ± 0.0048
Number Complications	517.07 ± 11.45	513.90 ± 4.41	515.73 ± 7.97
Percentage of Complications	0.059 ± 0.000	0.059 ± 0.000	0.059 ± 0.000

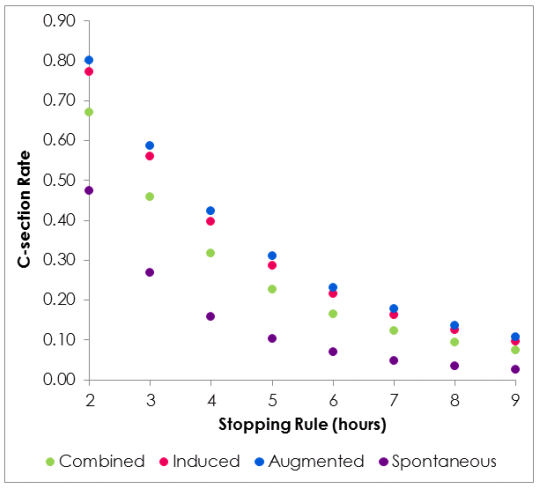
hour and 3 hour rules. That is, allowing women to labor longer than two or three hours before deciding the patient is failure-to-progress provides the most benefit in terms of the reduction in the number of cesarean deliveries without significantly affecting the risk of complications. The C-section plot shows that as the stopping rule increases in length, the marginal benefit is minimal. That is, the percentage decrease decreases as the stopping rule length increases. The rate of complications plot shows a differing effect. Although the increase in the risk of complications is increasing by a small amount, the marginal increase grows as the stopping rule length increases. The change in risk of complication is greatest for augmented labors and lowest for spontaneous labors.

2.4.4 Two-dimensional Stopping Rules

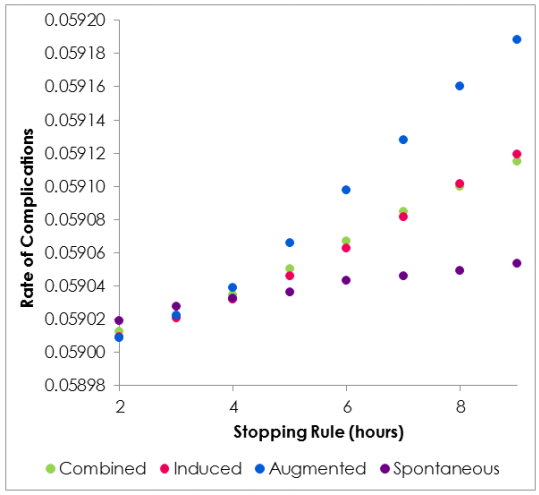
We extend the uniform stopping rules to consider a single change in stopping rule. That is, allow for one stopping rule for early dilation states (less than 6 cm) and a different

Table 2.5 The marginal percentage of decrease for C-section rates and marginal percentage increase for expected rate of complications as a function of changes in the stopping rule.

	Stopping Rule (hours)	C-sections Percentage	Rate of Decrease	Rate of Complications	Rate of Increase
Combined	2	0.67	-	5.9013%	-
	3	0.46	31.61%	5.9022%	0.02%
	4	0.32	30.87%	5.9035%	0.02%
	5	0.23	28.66%	5.9050%	0.03%
	6	0.16	27.54%	5.9067%	0.03%
	7	0.12	25.57%	5.9085%	0.03%
	8	0.09	22.22%	5.9100%	0.03%
	9	0.07	21.15%	5.9115%	0.03%
Induced	2	0.77	-	5.9009%	-
	3	0.56	27.58%	5.9020%	0.02%
	4	0.40	29.19%	5.9032%	0.02%
	5	0.29	27.43%	5.9046%	0.02%
	6	0.21	25.28%	5.9063%	0.03%
	7	0.16	24.61%	5.9082%	0.03%
	8	0.12	23.23%	5.9101%	0.03%
	9	0.10	22.13%	5.9119%	0.03%
Augmented	2	0.80	-	5.9009%	-
	3	0.59	26.53%	5.9022%	0.02%
	4	0.42	28.06%	5.9039%	0.03%
	5	0.31	26.67%	5.9066%	0.05%
	6	0.23	25.10%	5.9098%	0.05%
	7	0.18	23.58%	5.9128%	0.05%
	8	0.14	22.85%	5.9160%	0.06%
	9	0.11	22.17%	5.9188%	0.05%
Spontaneous	2	0.48	-	5.9019%	-
	3	0.27	43.57%	5.9028%	0.01%
	4	0.16	40.69%	5.9032%	0.01%
	5	0.10	35.75%	5.9036%	0.01%
	6	0.07	31.95%	5.9043%	0.01%
	7	0.05	32.12%	5.9046%	0.00%
	8	0.03	26.27%	5.9049%	0.01%
	9	0.03	25.26%	5.9054%	0.01%



(a) The C-section rate for varying stopping rules.



(b) The rate of complications for varying stopping rules.

Figure 2.10 The C-section rate (a) and rate of complications (b) for varying one-dimensional stopping rules.

stopping rule for later dilation states (6 through 10 cm). By analyzing the collection of two-dimensional and one-dimensional stopping rules, we have developed an efficient frontier which allows us to explore the trade-off between C-section rate and complications. The efficient frontier enables us to identify the rules that minimize both the C-section rate and rate of complications. The C-section rate and the corresponding rate of complications for each stopping rule is presented in Figure 2.11. Table 2.6 provides the list of the efficient stopping rules that lie on the efficient frontier for each labor type. Longer stopping rules correspond to a lower C-section rate which is also supported by our analysis of the one-dimensional rules. A comparison of the C-section rate against the rate of complications is presented in Figure 3.4. Overall, augmented labors have a higher risk of complications and spontaneous labors have the lowest risk of complications as well as C-section rates.

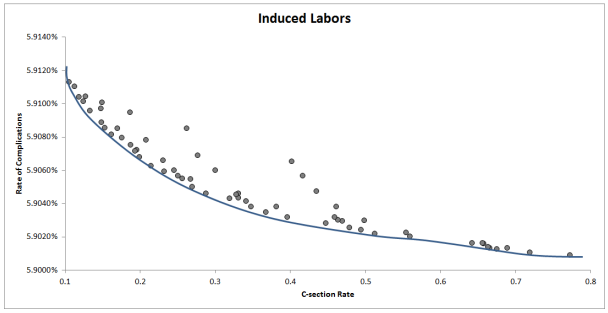
The C-section rate for induced labors and augmented labors are similar. Although the rate of complications increases as the stopping rule increases, the rate of complications is relatively small.

Table 2.6 Two-dimensional stopping rules corresponding to the efficient frontiers for each laboring types.

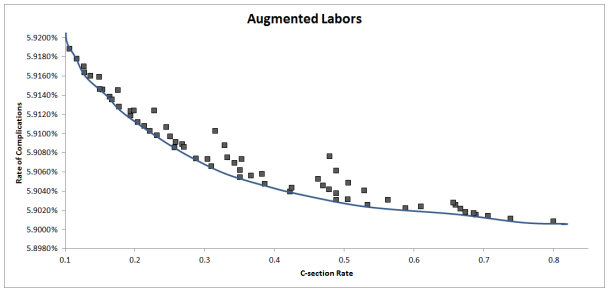
Stopping Rules	
Combined	(2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (4,3), (4,4), (4,5), (4,6), (4,7), (5,5), (5,6), (5,7), (6,5), (5,8), (6,6), (7,5), (6,7), (6,8), (7,6), (6,9), (7,8), (7,9), (8,7), (8,8), (8,9), (9,9), (9,8)
Induced	(2,2), (2,3), (2,4), (2,5), (2,7), (3,2), (3,3), (3,4), (3,5), (3,6), (4,3), (4,4), (4,5), (4,6), (4,7), (5,4), (5,5), (5,6), (5,7), (5,8), (6,5), (6,6), (6,7), (6,8), (6,9), (7,6), (7,7), (7,8), (7,9), (8,7), (8,8), (8,9), (9,7), (9,8),(9,9)
Augmented	(2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (3,3), (3,4), (4,3), (4,4), (4,5), (5,4), (5,5), (5,6), (6,5), (6,6), (6,7), (6,8), (7,5), (7,6), (7,7), (7,8), (8,6), (8,7), (8,8), (8,9), (9,8), (9,9)
Spontaneous	(2,2), (2,3), (2,4), (2,6), (2,8), (3,2), (3,4), (3,5), (4,5), (4,6), (4,8), (5,5), (5,6), (5,7), (5,8), (5,9), (6,8), (7,7), (7,8), (7,9), (8,8), (8,9), (9,9)

2.5 Discussion

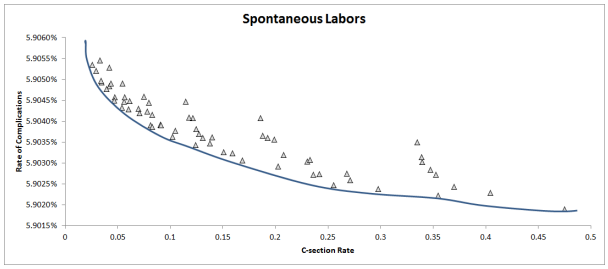
Model 1 provides interesting insight into the total length of active labor for nulliparous women. Since various complications correspond to the duration of active labor, it is important to understand exactly how long active labor can last. The model suggests that the average length of active labor was approximately 9 hours and the longest labor length was 165 hours. Although it may be common practice to observe a labor length closer to 9



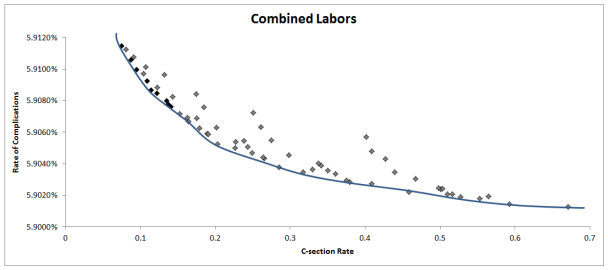
(a) Efficient frontier for induced labors.



(b) Efficient frontier for augmented labors.



(c) Efficient frontier for spontaneous labors.



(d) Efficient frontier for the general population.

Figure 2.11 The C-section rate against the rate of complications for varying two-dimensional stopping rules separated by induced, augmented, spontaneous, and combined laboring types.

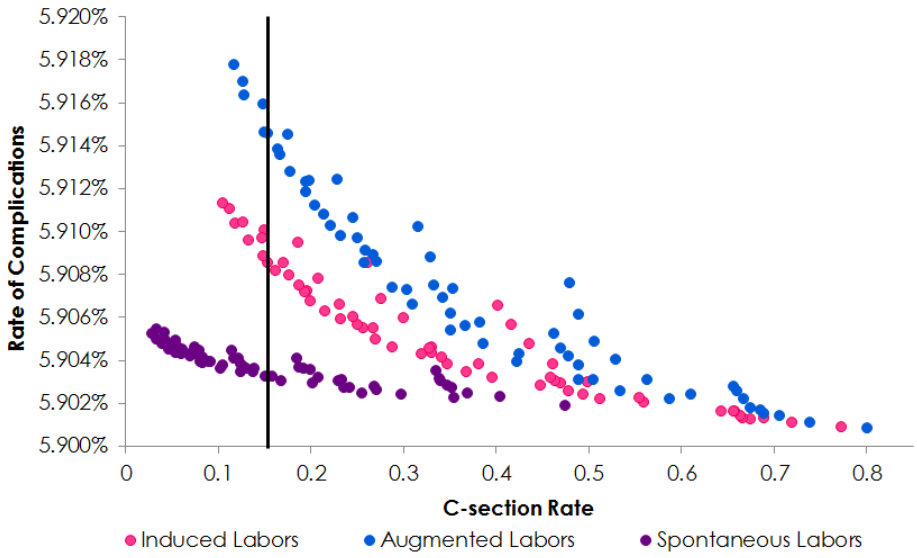


Figure 2.12 The C-section rate against the rate of complications for varying two-dimensional stopping rules for each laboring type. The solid line corresponds to the current C-section rate attributable to failure-to-progress diagnosis.

hours, this work shows that labor can be much longer without the use of interventions and interruptions. By evaluating the minimum, average, and maximum duration of labor for each dilation state it is possible to shape how labor duration is viewed and the potential for allowing longer labors while also monitoring the resulting health effects. Similar to Zhang et al. (2010) [103] and Harper et al. (2012) [48], the longest labor duration is for the progression from 3 to 4 cm and labor duration times decrease as cervical dilation increases until reaching 10 cm, which is similar to other studies.

The results of Model 2 show the importance of dilation-state-specific decision rules for determining when a patient has failed to progress which confirms the results established by Zhang et al. (2010). They suggested that thresholds of appropriate labor length should be based on the level of cervical dilation due to the inconsistency of the speed of cervical

dilation. The authors believe dilation-state-specific thresholds are more appropriate to defining labor arrest than a "one-size-fits-all" approach [105]. The nature of the Friedman Curve provided a blanket stopping rule which determined that a failure-to-progress diagnosis should be given after a patient has not progressed from one centimeter to the next after two hours. We also see from Figures 2.10a and 2.10b that the lower C-section rates correspond to a higher number of complications and vice-versa. Since it is not possible to lower the cesarean delivery rate without increasing the number of complications, an optimal policy for ending a trial of labor is one which does not exceed an established number of complications while also minimizing the C-section rate. The C-section rate and number of complications can be best minimized by employing a varying time stopping rule dependent on the dilation state.

Another interesting aspect of this problem, not explicitly considered, is the effect that epidural analgesia may have on labor progression and health outcomes. Vahratian et al. (2004) [94] and Thorp et al. (1991) [51] conducted studies comparing the use and placement of epidural analgesia in labor and its effect on labor progression for singleton nulliparous deliveries with a spontaneous labor. The results from Vahratian et al. (2004) showed that the use of an epidural analgesia did not have much effect on labor progression and the results from Thorp, et al. (1991) determined that administering an epidural analgesia resulted in a longer first and second stage of labor. Although these two studies differ in their results, this suggests there is reason to consider the effect that something like epidural analgesia may have on labor progression.

2.6 Conclusion

The labor and delivery process has changed a great deal over time [5, 102, 103]. Although it can be argued that the Friedman Curve is no longer effective, its use may still be prevalent. Since there are different delivery guidelines practiced by various obstetricians, midwives, and other labor and delivery specialists, we developed a simulation model of labor progression and evaluated stopping conditions for a trial of labor to mimic the rate of cesarean deliveries given a failure-to-progress diagnosis with the goal of identifying guidelines to assist in reducing the rate of C-sections and rate of complications. This research provides insight into how to model the labor process and lays the foundation for further research in this area. The decision to administer a C-section is a combination of the health of the mother and child, the length of time of time a woman labors, and her discomfort level. We have determined that an appropriate labor progression plan is one in which nulliparous singleton births of women with no prior complications should be allowed to labor longer than two hours which was established by the Friedman Curve; instead the time should be a function of dilation stage. This is in line with the current guidance regarding labor arrest released by ACOG and SMFM who developed guidelines for diagnosing failure-to-progress in terms of cervical dilation. Their recommendations suggest cesarean delivery due to prolonged first stage of labor should not be considered until 6 cm has been achieved for women with ruptured membranes. They also suggest that a prolonged latent phase (which they describe as cervical dilation less than 6 cm) of more than 20 hours for nulliparous women and more than 14 hours for multiparous women should not be reason for a C-section [5]. The work presented in this chapter uses a non-traditional mechanism of obstetrics to offer

insight into a decision that affects many women and children. The results provided can be used to shape how labor length is evaluated and governed.

2.6.1 Simulation Purposes

In addition to providing estimates of appropriate labor length, this simulation model also serves as a basis for parameter estimation in our different decision models used to optimize the mode of delivery decision. The simulation model is used to estimate probability transition matrices for the Markov decision process (MDP) that is used to develop policies that govern how long a patient should remain in labor before deciding a C-section is needed with the goal of maximizing utility (discussed in Chapter 4). In addition, a variation of the simulation model is used in the Bayesian decision model which approximates the appropriate times in which the decision maker has enough information about the patient to decide whether to continue trial of labor or deliver through C-section and when to prolong labor before making such a decision (discussed in Chapter 3). Both models use a time period of one hour.

2.6.1.1 Probability Transition Matrices for MDP

Since there are two actions in the MDP (trial of labor or C-section), two matrices containing the possible state transitions are needed. The simulation described above is used to develop the matrix for the trial of labor action. The states reflect cervical dilation starting from 3 cm to 10 cm and three modes of delivery, (1) vaginal delivery, (2) C-section, and (3) emergency C-section. In this problem, emergency C-section refers to a C-section performed due to a "failure-to-progress" diagnosis. We capture the probability that labor will end in emergency

C-section by modeling the natural progression of labor and after the processing time has expired in each dilation state, the total time in labor is calculated. If the total time in labor exceeds a pre-specified maximum allowable time for labor, then the labor ends and a C-section is incorporated. However, if the total time is less than the maximum allowable time, then labor continues and the patient moves to the next dilation state. To capture the probability of moving from the starting dilation to another within an hour, the simulation is run for one hour. After the simulation has finished, the probability of moving to another dilation state, vaginal delivery, or remaining in the current dilation state is calculated.

2.6.1.2 Probability Transition Matrices for Bayesian Model

Similar to the method described in Section 2.6.1.1, the Bayesian model considers two actions and each transition probability matrix contains states that reflect cervical dilation measurement from 3 to 10 cm. This model does not contain mode of delivery states since the Bayesian model is primarily concerned with identifying times in which a trial of labor is appropriate versus C-section through labor progression analysis. The Bayesian decision model utilizes two matrices similar to that described in Section 2.6.1.1. The first matrix is used to model the transitions from one dilation state to another for "not failure-to-progress patients" defined as patients who undergo a successful trial of labor. For this matrix labor progression is assumed to be lognormally distributed. The second matrix models labor progression times for "failure-to-progress patients" defined as patients who experience labor arrest and ultimately receive a C-section. The "failure-to-progress" matrix uses labor progression times that assume an exponential distribution. The percentile matching procedure was not used due to lack of information for at least three percentile measurements for

C-section patients. Similar to the matrix for the MDP, both simulations ran for one hour and the probability of moving to a different dilation state was calculated.

CHAPTER

3

BAYESIAN DECISION ANALYSIS

The study considered in this chapter uses a Bayesian sequential sampling framework to develop threshold values for when it is appropriate to gather additional information prior to making a decision about a woman's birth plan during a trial of labor. In this model, we assume all women enter a trial of labor which is defined as the time period in which a pregnant woman enters active labor (i.e., stage of labor where a woman experiences strong contractions and the cervix dilates to 10 cm) for safe delivery of a child. During a trial of labor, cervical dilation is used as a benchmark of labor progression. Women who experience

spontaneous onset of labor are typically admitted to the hospital at what is considered to be 4 cm. Once the cervix has disappeared (often termed as fully dilated to 10 cm), the woman then moves into the pushing stage of labor. At any point during the trial of labor, a C-section may be needed because the (presumed) risks of vaginal delivery complications are too great. Since a laboring woman will deliver in only one of two ways, vaginal delivery or cesarean delivery, we develop a method that can determine when to gather additional information in order to determine which type of delivery is most appropriate as a function of the belief that the woman will have a safe vaginal delivery as a function of the cervical dilation progression. We develop a decision model that determines whether more information about the woman's cervical dilation progression should be gathered before deciding about the mode of delivery. The model provides threshold values when more information is needed before making a decision. Capturing these threshold values is important because if the duration of labor is too short, the patient may undergo an unnecessary cesarean delivery. However, if the time period is extended too long and the patient does indeed need a C-section, then the chance of complications for the mother and fetus is increased.

During the time the decision maker waits before choosing the mode of delivery, it is possible to learn more about the patient. It is possible to better understand her medical state and by assessing the amount of time it takes her to progress from one dilation state to the next, we can more clearly define the likelihood of a successful vaginal delivery. The model can also provide the best time to end a trial of labor in case a C-section is safest due to the increasing risk of complications for prolonging labor when a C-section is needed.

A description of relevant Bayesian decision models is introduced in Section 3.1. In Sections 3.2, 3.3, and 3.4, we describe the methods and each model considered. We then

evaluate the effects of changing parameter values and explore the value of information and willingness-to-pay in Section 3.5. In Section 3.6, we conclude with a discussion of our findings.

3.1 Bayesian Decision Models

Our Bayesian model, based on techniques developed by Wald (1950) [96] and presented in [68], is used to estimate a parameter of interest by weighing the option of estimating the parameter using current knowledge of the system against waiting and taking an observation to learn more about the state of the system before making an estimate. Schechter (1990) developed a study that used this Bayesian framework to analyze sequential decision making problems when estimates of continuous disease states, such as diastolic blood pressure or serum cholesterol, were needed. The focus of the study was to determine whether to perform a test to assess health status before administering a treatment plan or to develop a treatment plan using the information already gathered about the patient. This model was used to find a stepwise stopping rule which compared the expected utility of the decision to "treat"' or "not treat now" [78, 79]. Studies using similar methods have been applied to on-site investigations for the rehabilitation of contaminated groundwater areas [32], preparation for a demand surge in response to tropical disturbance [89], and hurricane prediction in regards to emergency supplies inventory management [89]. Bayesian information updating has also been used in inventory ordering policies for seasonal products [18] and in a retailer's optimal single ordering policy with different delivery modes [19].

3.2 Methods

We develop a Bayesian decision model to aid in the prediction of "patient type" in order to minimize total expected risk associated with birth. Each patient who enters a ToL will either deliver vaginally or through C-section. For those patients considered "ftp," a C-section is needed and for patients who are not ftp ($\overline{ftp}$), it is possible to have to a successful ToL. This research focuses on C-sections due to a ftp diagnosis and does not explicitly consider C-section performed for reasons other than ftp or presumed "ftp" (although the prior belief may be derived based on various factors including patient demographics, risk profile, provider's prior experience, etc.). We next introduce the terminology and notation used in this study.

Let $W = \{w_1, w_2\}$ be the actual patient type, where w_1 is a $\overline{ftp}$ patient and w_2 is an ftp patient. Let $D = \{d_1, d_2\}$ be the decision about patient type made by the provider or clinician where d_1 is to decide the patient is $\overline{ftp}$ and d_2 is to decide the patient is ftp. For each dilation state, there are two actions: (i) to make a decision about the type of patient (ftp or $\overline{ftp}$), or (ii) decide to gather more information before making the decision. In this study, we assume that patients are checked each hour to monitor the progression of labor and "gathering additional information" is defined as observing the patient's cervical dilation within a one hour time frame. Each state of dilation provides insight into patient type and is associated with a probability of a successful ToL.

The prior distribution of W is specified by $\xi_i = \Pr(W = w_1|i) = 1 - \Pr(W = w_2|i)$, where i represents the cervical dilation state ranging from 3 cm to 10 cm. The value of ξ_i represents the DM's belief regarding the chance a patient will have a successful ToL dependent on cer-

vical dilation. To identify the best decision, the expected loss when no observation is taken ($\rho_0(\xi_i)$), is compared to the expected loss associated with waiting at least one additional hour before selecting a delivery option plus the additional loss or penalty, C, incurred for waiting: $\mathbb{E}[\rho_0(\xi_i(X))] + C$, where X represents the outcome of an observation and $\xi_i(X)$ is the updated belief that the patient is $\overline{ftp}$ (posterior probability) when observation, X, is observed.

The optimal action depends largely on the consequences or penalty associated with choosing an incorrect option (making a mistake). The goal of the decision model is to identify the action that minimizes this expected penalty. We define $L_{k,l}$ as the penalty for choosing patient type k when the patient is of type l. The losses for each decision are defined as:

$$L_{k,l} = \begin{cases} 0 & \text{if } k = d_1 \text{ and } l = w_1 \\ L_{1,2} & \text{if } k = d_1 \text{ and } l = w_2 \\ L_{2,1} & \text{if } k = d_2 \text{ and } l = w_1 \\ 0 & \text{if } k = d_2 \text{ and } l = w_2. \end{cases}$$

where $L_{2,1}$ and $L_{1,2}$ represent the loss (penalty) for making the wrong decision, i.e., $L_{2,1}$ represents the penalty for choosing CS when ToL is appropriate and $L_{1,2}$ represents the penalty for choosing ToL when CS is needed. It can be argued that either of the following options is worse: (i) deciding CS when ToL is best can be detrimental to a woman's and baby's health with consequences for future births, or (ii) continuing the ToL when a CS is needed, which increases the risk of short- and long-term maternal and neonatal complications [15]. Therefore, three cases are considered: $L_{1,2} > L_{2,1}$, $L_{1,2} < L_{2,1}$, and $L_{1,2} = L_{2,1}$. These

relationships capture the perspective of different stakeholders. For example, the $L_{1,2} > L_{2,1}$ relationship may represent the perspective of a clinician (or health system) who believes it is worse to allow labor to continue for an $f\,t\,p$ patient. Whereas the relationship, $L_{1,2} < L_{2,1}$, may represent a patient, payer, or characterize the perspectives of a midwife compared to an obstetrician. In this relationship, prolonging labor for a CS patient is valued.

We explore and compare two models to identify belief thresholds for when it is appropriate to take an observation. Model 1 is primarily concerned with whether the patient has progressed from the current state of dilation. Model 2 is an extension of Model 1 that takes into account how much the patient has dilated incorporating the probability of moving to particular dilation states. In Model 1, the duration of labor is assumed to be exponentially distributed and Model 2 simulates labor progression assuming a lognormal distribution. Both models assume all patients enter hospitalized "active" labor at a cervical dilation of 3 cm and progress through labor at various rates until reaching the second stage of labor.

3.3 Model 1: Exponential Time

Within an hour several cervical dilation changes may occur. A woman observed at i cm may remain at i cm or progress to $i+1$ cm, $i+2$ cm, $i+3$ cm, etc., with some probability. The model determines the type of patient ($f\,t\,p$ or $\overline{f\,t\,p}$) either based on the current state of knowledge (i.e., taking no observations), or after an observation (i.e., the DM attempts to learn more about the patient by waiting an hour before making a decision about the patient type). The likelihood function, $f_W(x|i)$, is defined as the probability that a change

in dilation ($X = 1$) is observed for each patient type, W, and dilation state, i, where

$$f_{w_1}(1|i) = \Pr\left(X = 1 | W = \overline{f\,t\,p}, i\right) = 1 - f_{w_1}(0|i)$$

and

$$f_{w_2}(1|i) = \Pr\left(X = 1 | W = f\,t\,p, i\right) = 1 - f_{w_2}(0|i).$$

To model the time it takes for a woman to progress from one dilation state to the next, we use the duration of labor data from Zhang, Troendle et al. (2010) and Zhang, Landy et al. (2010) assuming the duration of labor is exponentially distributed. Using the median labor duration for successful vaginal deliveries [103] and the median labor arrest duration before CS [102], we find the rate parameter, $\lambda_{W\in(w_1,w_2)}$, with the median formula for the exponential distribution:

$$\text{median}(i)_W = \frac{\ln(2)}{\lambda_{i,W}} \text{ for } W \in \{w_1, w_2\}. \tag{3.1}$$

We calculate the probability that labor will progress within one hour assuming $T_{i,W}$ is the duration of labor for dilation state i and patient type W. Thus for $W \in \{w_1, w_2\}$,

$$\begin{aligned}\Pr(T_{i,W} \leq 1 \text{ hour}) &= 1 - e^{\left(-\lambda_{i,W}\right)} \\ &= f_W(1|i) \\ &= \Pr(X = 1|W, i).\end{aligned}$$

The probability of labor progression for dilation states 3 cm to 10 cm for women who delivered vaginally and for women who had a C-section are shown in Table 3.1a.

Table 3.1 Distribution of the duration of labor progression by cervical dilation.

Cervical Dilation (cm)	**Vaginal Delivery**			**Cesarean Delivery**		
	Median (hours) [103]	λ_{i,w_1}	$\Pr(T_{i,w_1} > 1)$	Median (hours) [102]	λ_{i,w_2}	$\Pr(T_{i,w_2} > 1)$
3-4	1.8	0.3852	0.6804	4.0	0.1733	0.8409
4-5	1.3	0.5332	0.5867	4.0	0.1733	0.8409
5-6	0.8	0.8664	0.4204	3.5	0.1980	0.8203
6-7	0.6	1.1552	0.3150	2.9	0.2390	0.7874
7-8	0.5	1.3863	0.2500	2.8	0.2476	0.7807
8-9	0.5	1.3863	0.2500	3.0	0.2310	0.7937
9-10	0.5	1.3863	0.2500	2.2	0.3151	0.7297

(a) Model 1: Exponentially distributed duration of labor progression. The median duration of labor progression, the corresponding $\lambda_{i,W}$ parameter, and the probability that labor will progress within one hour for vaginal delivery [103] and cesarean delivery [102].

	Induced Labor						**Augmented Labor**						**Spontaneous Labor**					
	5th[a]	*50th*[a]	*95th*[a]	*Parameters*			*5th*[a]	*50th*[a]	*95th*[a]	*Parameters*			*5th*[a]	*50th*[a]	*95th*[a]	*Parameters*		
Dilation				λ	β	α				λ	β	α				λ	β	α
3	0.2	1.4	8.1	0.37	1.04	0	0.2	1.5	8.1	0.48	0.98	0	0.1	0.4	2.3	-1.03	1.12	0.04
4	0.2	1.3	6.8	0.32	0.97	0	0.3	1.35	7.6	0.23	1.08	0.09	0.1	0.5	2.7	-0.72	1.03	0.01
5	0.2	0.6	4.3	-0.80	1.35	0.15	0.1	0.75	4.9	-0.26	1.12	0	0.06	0.4	2.7	-0.92	1.16	0.001
6	0.05	0.4	2.8	-0.89	1.17	0	0.06	0.55	3.9	-0.56	1.16	0	0.03	0.3	2.1	-1.15	1.15	0
7	0.02	0.2	1.5	-1.57	1.20	0	0.05	0.4	2.8	-0.89	1.17	0	0.04	0.3	1.7	-1.14	1.02	0
8	0.03	0.2	1.3	-1.60	1.13	0	0.05	0.4	3	-0.91	1.22	0	0.03	0.2	1.3	-1.60	1.13	0
9	0.04	0.3	1.9	-1.17	1.10	0	0.05	0.4	2.2	-0.83	0.99	0	0.04	0.3	1.8	-1.16	1.06	0
10 (epidural)	0.2[b,c]	1.1[b,c]	5[b,c]	0.38	0.75	0	0.4[b,c]	1.1[b,c]	5[b,c]	0.40	0.73	0	0.4[b,c]	1.1[b,c]	5[b,c]	0.40	0.73	0
10 (no epidural)	0.2[b,c]	0.6[b,c]	5[b,c]	-0.44	1.25	0	0.4[b,c]	0.6[b,c]	5[b,c]	-0.40	1.22	0	0.4[b,c]	0.6[b,c]	5[b,c]	-0.40	1.22	0

Sources:
a - Harper et al. (2012) [48]; b - Zhang, Landy et al. (2010) [103]; c - Zhang, Troendle et al. (2010) [102]

(b) Model 2: Lognormally distributed duration of labor progression. Parameters λ, β, and a to the corresponding lognormal distributions for each dilation state and three patient types derived from the percentile matching procedure.

For ξ_i (the prior probability that the patient is $\overline{ftp}$) given dilation state, i, the best decision (i.e., Bayes decision) minimizes the expected loss when no observation is taken which we define as

$$\rho_0(\xi_i) = \min\left\{L_{d_2,w_1}\Pr(w_1|i), L_{d_1,w_2}\Pr(w_2|i)\right\} \text{ for } \xi_i \in [0,1]$$

and

$$\Pr(W|i) = \begin{cases} \xi_i & \text{when } W = w_1 \\ 1-\xi_i & \text{when } W = w_2. \end{cases}$$

For dilation state i, the Bayes decision is d_2 when

$$\begin{aligned} & L_{2,1}[\Pr(W = w_1|i)] \leq L_{1,2}[\Pr(W = w_2|i)] \\ & \implies L_{2,1}\xi_i \leq L_{1,2}(1-\xi_i) \\ & \implies \xi_i \leq \frac{L_{1,2}}{L_{1,2}+L_{2,1}} = a. \end{aligned} \tag{3.2}$$

So we have

$$\rho_0(\xi_i) = \begin{cases} L_{2,1}\xi_i & \text{for } 0 \leq \xi_i \leq a, \\ L_{1,2}(1-\xi_i) & \text{for } a \leq \xi_i \leq 1. \end{cases} \tag{3.3}$$

That is, when no observations are taken, the best decision is CS (d_2) when ξ_i is less than a and ToL (d_1) when ξ_i is greater than a. In order to identify the Bayes decision when at least one observation is considered, we must define $\rho_0(\xi_i(X))$ for $X = 0, 1$. Using Bayes theorem,

the posterior probability that the patient is $\overline{f\,t\,p}$ conditional on dilation i and observation, X, is $\xi_i(X) = \Pr(W = w_1 | X = x, i)$. Therefore,

$$\xi_i(x) = \frac{f_{w_1}(x|i)\xi_i}{f_{w_1}(x|i)\xi_i + f_{w_2}(x|i)(1-\xi_i)}, \text{ for } x \in \{0,1\}.$$

According to Equation (3.3), for d_2 to be optimal

$$\xi_i(x) = \frac{f_{w_1}(x|i)\xi_i}{f_{w_1}(x|i)\xi_i + f_{w_2}(x|i)(1-\xi_i)} \leq a. \tag{3.4}$$

Hence,

$$\xi_i \leq \frac{a f_{w_2}(x|i)}{f_{w_1}(x|i) + a(f_{w_2}(x|i) - f_{w_1}(x|i))} = b_x(i), \text{ where } x \in \{0,1\}.$$

Therefore,

$$\rho_0(\xi_i(0)) = \begin{cases} L_{2,1}\xi_i(x) & \text{for } 0 \leq \xi_i \leq b_x(i), \\ L_{1,2}(1-\xi_i(x)) & \text{for } b_x(i) \leq \xi_i \leq 1. \end{cases}$$

for $x \in \{0,1\}$. That is,

$$\rho_0(\xi_i(0)) = \begin{cases} L_{2,1}\xi_i(0) & \text{for } 0 \leq \xi_i \leq b_0(i), \\ L_{1,2}(1-\xi_i(0)) & \text{for } b_0(i) \leq \xi_i \leq 1. \end{cases}$$

and

$$\rho_0(\xi_i(1)) = \begin{cases} L_{2,1}\xi_i(1) & \text{for } 0 \leq \xi_i \leq b_1(i), \\ L_{1,2}(1-\xi_i(1)) & \text{for } b_1(i) \leq \xi_i \leq 1. \end{cases}$$

where $0 \leq b_1(i) \leq b_0(i) \leq 1$. The marginal probability is defined as

$$\Pr\{X = x|i\} = f_{w_1}(x|i)\xi_i + f_{w_2}(x|i)(1-\xi_i).$$

Finally,

$$\begin{aligned}\mathbb{E}[\rho_0(\xi_i(X))] &= \sum_{x=0}^{1} \rho_0(\xi_i(x)) \Pr\{X = x|i\} \\ &= \begin{cases} L_{2,1}\xi_i(0)\Pr\{0|i\} + L_{2,1}\xi_i(1)\Pr\{1|i\} & \text{for } 0 \leq \xi_i \leq b_1(i), \\ L_{2,1}\xi_i(0)\Pr\{0|i\} + L_{1,2}(1-\xi_i(1))\Pr\{1|i\} & \text{for } b_1(i) \leq \xi_i \leq b_0(i), \\ L_{1,2}(1-\xi_i(0))\Pr\{0|i\} + L_{1,2}(1-\xi_i(1))\Pr\{1|i\} & \text{for } b_0(i) \leq \xi_i \leq 1. \end{cases}\end{aligned}$$

and

$$\rho_1(\xi_i) = \min\left\{\rho_0(\xi_i),\ \mathbb{E}[\rho_0(\xi_i(X))] + C\right\}. \tag{3.5}$$

The Bayes decision, considering the option for at least one observation, corresponds to the action that minimizes the expected loss including the cost of waiting and the expected loss with no observations.

3.4 Model 2: Lognormal Labor

Model 1 examines the appropriate time to take an observation considering whether the patient's cervical dilation has progressed within an hour. Model 2 incorporates a more detailed characterization of cervical dilation progression. The underlying problem is to determine the classification of patient type, which is static, but the observation space in Model 2 is dynamic. The likelihood function, $f_W(X = i+n|i)$, represents the probability that

there is $n \in \{0, 1, ..., 11-i\}$ additional centimeter change in dilation from i cm dilation given the patient is type W. The states $3, 4, ..., 10$ represent cervical dilation and "11" represents the vaginal delivery (VD) state. A probability transition matrix is used as an input for the likelihood functions.

In Chapter 2, a discrete event simulation model of CS decision making was used as a function of labor progression with the goal of (1) modeling the natural progression of labor in absence of cesarean deliveries, (2) determining the underlying rules responsible for the current rate of cesarean deliveries due to an ftp diagnosis, and (3) developing ways to reduce unnecessary cesarean deliveries while also reducing the rate of complications [49]. The probability transition matrix is described in Section 3.4.1.

3.4.1 Probability Transition Matrix

To determine the probability of moving from one level of cervical dilation to another in one hour for both types of patients, we used the discrete event simulation model [49] parameterized with data from Zhang, Troendle et al. (2010), Zhang, Landy et al. (2010), and Harper, et al. (2012). Assuming labor duration is lognormally distributed and given the 5th, 50th, and 95th percentiles of this distribution, we were able to use a percentile matching procedure to determine labor time distributions for each cervical dilation state and three labor categories: induced labors, augmented labors, and spontaneous labors (see Table 3.1b). These labor time distributions were used in the simulation to calculate the probability that labor dilation would progress to another dilation state. To approximate the matrix for the ftp patient, the median duration (in hours) of labor arrest before cesarean delivery was used to estimate the parameter for the exponential distribution using Equation (3.1). The

simulation model ran for one hour for each dilation state and at the end of each run, the percentage of women who progress to a subsequent state of cervical dilation or remain in the current state is calculated. The probability transition matrices for vaginal delivery ($\overline{f\,t\,p}$) and cesarean delivery ($f\,t\,p$) are displayed below.

Probability Transition Matrix for Vaginal Delivery

	3 cm	4 cm	5 cm	6 cm	7 cm	8 cm	9 cm	10 cm	VD
3 cm	0.7098	0.2101	0.0565	0.0170	0.0053	0.0009	0.0003	0.0001	0
4 cm	0	0.7257	0.1854	0.0592	0.0198	0.0064	0.0025	0.0010	0
5 cm	0	0	0.5662	0.2596	0.1033	0.0416	0.0203	0.0083	0.0007
6 cm	0	0	0	0.4970	0.2545	0.1302	0.0816	0.0341	0.0026
7 cm	0	0	0	0	0.4190	0.2607	0.1919	0.1145	0.0139
8 cm	0	0	0	0	0	0.3743	0.3239	0.2607	0.0411
9 cm	0	0	0	0	0	0	0.4473	0.4549	0.0978
10 cm	0	0	0	0	0	0	0	0.7630	0.2370
VD	0	0	0	0	0	0	0	0	1

Probability Transition Matrix for Cesarean Delivery

	3 cm	4 cm	5 cm	6 cm	7 cm	8 cm	9 cm	10 cm	VD
3 cm	0.908	0.0886	0.0034	0	0	0	0	0	0
4 cm	0	0.9154	0.0815	0.0031	0	0	0	0	0
5 cm	0	0	0.8965	0.0918	0.0117	0	0	0	0
6 cm	0	0	0	0.8765	0.1066	0.0152	0.0017	0	0
7 cm	0	0	0	0	0.8838	0.1044	0.0101	0.0017	0
8 cm	0	0	0	0	0	0.8638	0.1202	0.016	0
9 cm	0	0	0	0	0	0	0.844	0.156	0
10 cm	0	0	0	0	0	0	0	1	0
VD	0	0	0	0	0	0	0	0	1

3.4.2 Bayes Decision Formulation for Model 2

To solve for the Bayes decision when at least one observation is taken, we define

$$\rho_0(\xi_i(X = i+n)) \text{ for } n \in \{0, 1..., 11-i\},$$

to be the expected loss given the observation, X, that dilation has progressed to state $i+n$ starting from cervical dilation state i. The Bayes decision to determine if at least one observation is taken is determined by Equation (3.5). The Bayes decision for the no observation case is the same as in Model 1, as shown in Equation (3.3).

To represent the structure of the expected loss for the Bayes decision, we define the order statistics, $b_{(x)}(i)$, for the set $b_x(i)$. Given the set $b_x(i)$ for $x \in \{0, 1, ..., 11-i\}$, reordering the set such that $\min\{b_x(i)\} = b_{(1)}(i) < b_{(2)}(i) < ... < b_{(12-i)}(i) = \max b_x(i)$. The solution for each $i \in \{3, 4, 5, ..., 10\}$ has the structure presented in Equation (3.6), which translates to deciding CS for ξ_i less than the minimum $b_x(i)$ (or $b_{(1)}(i)$), deciding ToL for ξ_i greater than the maximum $b_x(i)$ (or $b_{(12-i)}(i)$), and deciding to take an observation for all ξ_i values

between $b_{(1)}(i)$ and $b_{(12-i)}(i)$.

$$\mathbb{E}[\rho_0(\xi_i(X))] = \begin{cases} \sum_{x=i}^{11} L_{2,1}\xi_i(x)\Pr\{X=x|i\} & \text{for } 0 \le \xi_i \le b_{(1)}(i), \\ \sum_{x=i}^{10} L_{2,1}\xi_i(x)\Pr\{X=x|i\} & \\ \quad + L_{1,2}(1-\xi_i(11))\Pr\{11|i\} & \text{for } b_{(1)}(i) \le \xi_i \le b_{(2)}(i), \\ \vdots & \\ \sum_{x=i}^{12-(k+1)} L_{2,1}\xi_i(x)\Pr\{X=x|i\} & \\ \quad + \sum_{x=12-k}^{11} L_{1,2}(1-\xi_i(x))\Pr\{X=x|i\} & \text{for } b_{(k)}(i) \le \xi_i \le b_{(k+1)}(i), \\ \vdots & \\ \sum_{x=i}^{11} L_{1,2}(1-\xi_i(x))\Pr\{X=x|i\} & \text{for } b_{(k)}(i) \le \xi_i \le b_{(k+1)}(i), \end{cases} \tag{3.6}$$

Illustration of Model Formulation

To further clarify the solution for Model 2, we provide an example for the 8 cm state. From 8 cm, there are four possible transitions that can occur as shown in Figure 3.1. The values

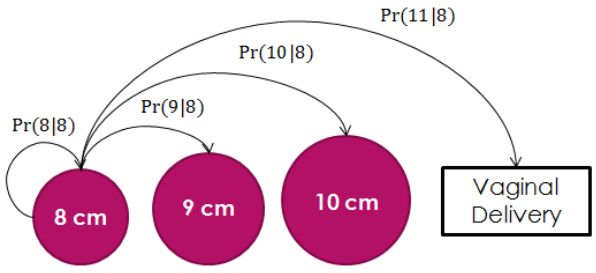


Figure 3.1 Possible transitions from 8 cm within a one hour time period.

for $b_{x=i+n}(i)$ where $i = 8$ and $n \in \{0,1,2,3\}$ are $b_8(8) = 0.82$, $b_9(8) = 0.43$, $b_{10}(8) = 0.11$, and $b_{11}(8) = 0$. Since $b_{11}(8) < b_{10}(8) < b_9(8) < b_8(8)$, then $b_{(1)}(8) = b_{11}(8)$, $b_{(2)}(8) = b_{10}(8)$, $b_{(3)}(8) = b_9(8)$, and $b_{(4)}(8) = b_8(8)$. Therefore the solution is

$$\mathbb{E}[\rho_0(\xi_8(X))] = \begin{cases} L_{2,1}\sum_{x=8}^{11}\xi_8(x)f_{w_1}(x|8) & \text{for } \xi_8 = b_{(1)}(8) = 0, \\ L_{2,1}\sum_{x=8}^{10}\xi_8(x)f_{w_1}(x|8) + L_{1,2}(1-\xi_8(11))f_{w_2}(x|8) & \text{for } 0 < \xi_8 \le b_{(2)}(8), \\ L_{2,1}\sum_{x=8}^{9}\xi_8(x)f_{w_1}(x|8) & \\ +L_{1,2}\sum_{x=10}^{11}(1-\xi_8(x))f_{w_2}(x|8) & \text{for } b_{(2)}(8) \le \xi_8 \le b_{(3)}(8), \\ L_{2,1}\xi_8(8)f_{w_1}(x|8) & \\ +L_{1,2}\sum_{x=9}^{11}(1-\xi_8(x))f_{w_2}(x|8) & \text{for } b_{(3)}(8) \le \xi_8 \le b_{(4)}(8), \\ L_{1,2}\sum_{x=8}^{11}(1-\xi_8(x))f_{w_2}(x|8) & \text{for } b_{(4)}(8) \le \xi_8 \le 1. \end{cases} \tag{3.7}$$

This represents the solution for 8 cm when at least one observation is allowed. The minimum of the values in Equation 3.3 and Equation 3.7 for each interval is then used to find the Bayes decision.

3.5 Model Exploration

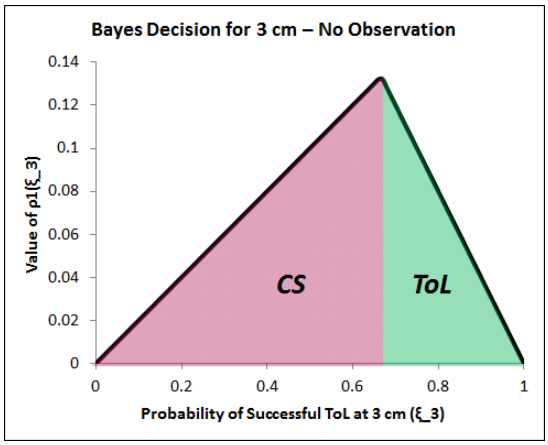
The results identify values of ξ_i (i.e., the DM's prior belief that the patient is $\overline{f\,t\,p}$) when an observation should be taken, when the decision should be d_2 (an indication for CS), and when the decision should be $d_1 = \overline{f\,t\,p}$, an indication for ToL. As a base case, the penalty for continuing a ToL when CS is most appropriate ($L_{1,2}$) is defined to be a multiple of the

penalty for deciding a CS is needed when a ToL is best ($L_{2,1}$), i.e., $L_{1,2} = m * L_{2,1}$ where $L_{2,1} = 0.2$ and $m = 2$. The results for dilation states 3 cm to 10 cm for Models 1 and 2 are summarized in Table 3.2. Figure 3.3 shows a pictorial representation of the loss functions and corresponding Bayes decision for each dilation state.

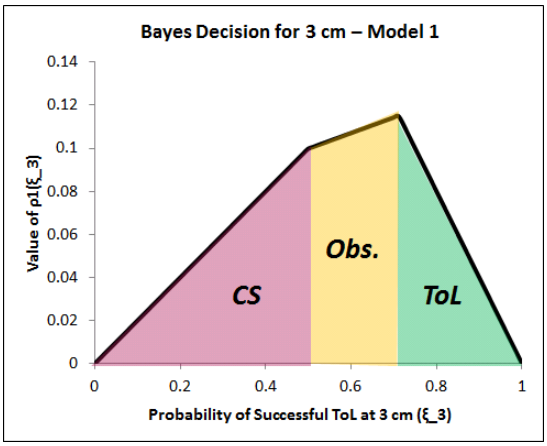
Table 3.2 The Bayes decision threshold values for ξ_i (the belief that a patient is $\overline{ftp}$) for both models and all cervical dilation states.

Dilation	No Observation		One Observation–Model 1		One Observation–Model 2	
3 cm			*CS*	$0 \le \xi_3 \le 0.499$	*CS*	$\xi_3 = 0$
			Take Obs.	$0.499 \le \xi_3 \le 0.712$	*Take Obs.*	$0 \le \xi_3 \le 0.719$
			ToL	$0.712 \le \xi_3 \le 1$	*ToL*	$0.719 \le \xi_3 \le 1$
4 cm			*CS*	$0 \le \xi_4 \le 0.435$	*CS*	$\xi_4 = 0$
			Take Obs.	$0.435 \le \xi_4 \le 0.741$	*Take Obs.*	$0 \le \xi_4 \le 0.716$
			ToL	$0.741 \le \xi_4 \le 1$	*ToL*	$0.716 \le \xi_4 \le 1$
5 cm			*CS*	$0 \le \xi_5 \le 0.383$	*CS*	$\xi_5 = 0$
			Take Obs.	$0.383 \le \xi_5 \le 0.796$	*Take Obs.*	$0 \le \xi_5 \le 0.760$
			ToL	$0.796 \le \xi_5 \le 1$	*ToL*	$0.760 \le \xi_5 \le 1$
6 cm			*CS*	$0 \le \xi_6 \le 0.383$	*CS*	$\xi_6 = 0$
	CS	$0 \le \xi_i \le 0.667$	*Take Obs.*	$0.383 \le \xi_6 \le 0.833$	*Take Obs.*	$0 \le \xi_6 \le 0.779$
	ToL	$0.667 \le \xi_i \le 1$	*ToL*	$0.833 \le \xi_6 \le 1$	*ToL*	$0.779 \le \xi_6 \le 1$
7 cm			*CS*	$0 \le \xi_7 \le 0.369$	*CS*	$\xi_7 = 0$
			Take Obs.	$0.369 \le \xi_7 \le 0.862$	*Take Obs.*	$0 \le \xi_7 \le 0.808$
			ToL	$0.862 \le \xi_7 \le 1$	*ToL*	$0.808 \le \xi_7 \le 1$
8 cm			*CS*	$0 \le \xi_8 \le 0.355$	*CS*	$\xi_8 = 0$
			Take Obs.	$0.355 \le \xi_8 \le 0.864$	*Take Obs.*	$0 \le \xi_8 \le 0.822$
			ToL	$0.864 \le \xi_8 \le 1$	*ToL*	$0.822 \le \xi_8 \le 1$
9 cm			*CS*	$0 \le \xi_9 \le 0.419$	*CS*	$\xi_9 = 0$
			Take Obs.	$0.419 \le \xi_9 \le 0.854$	*Take Obs.*	$0 \le \xi_9 \le 0.791$
			ToL	$0.854 \le \xi_9 \le 1$	*ToL*	$0.791 \le \xi_9 \le 1$
10 cm			*CS*	$0 \le \xi_{10} \le 0.416$	*CS*	$\xi_{10} = 0$
			Take Obs.	$0.416 \le \xi_{10} \le 0.758$	*Take Obs.*	$0 \le \xi_{10} \le 0.724$
			ToL	$0.758 \le \xi_{10} \le 1$	*ToL*	$0.724 \le \xi_{10} \le 1$

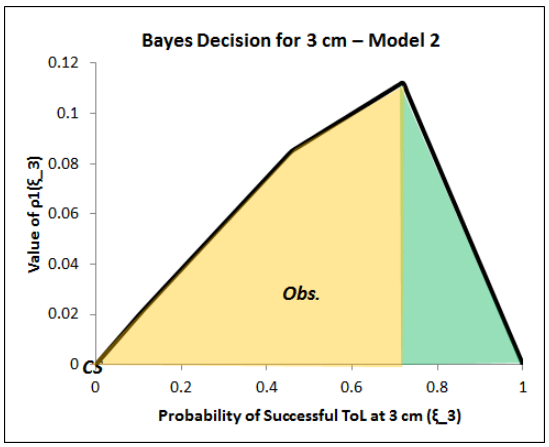
The results for 3 cm in Model 1 show it is best to decide ftp when ξ_3 is between $[0, 0.499]$,



(a) Bayes decision when no observation are considered.



(b) Bayes decision when at least one observation is considered for Model 1.



(c) Bayes decision when at least one observation is considered for Model 2.

Figure 3.2 Expected loss function for the no observation case, Model 1, and Model 2 for dilation state 3 cm.

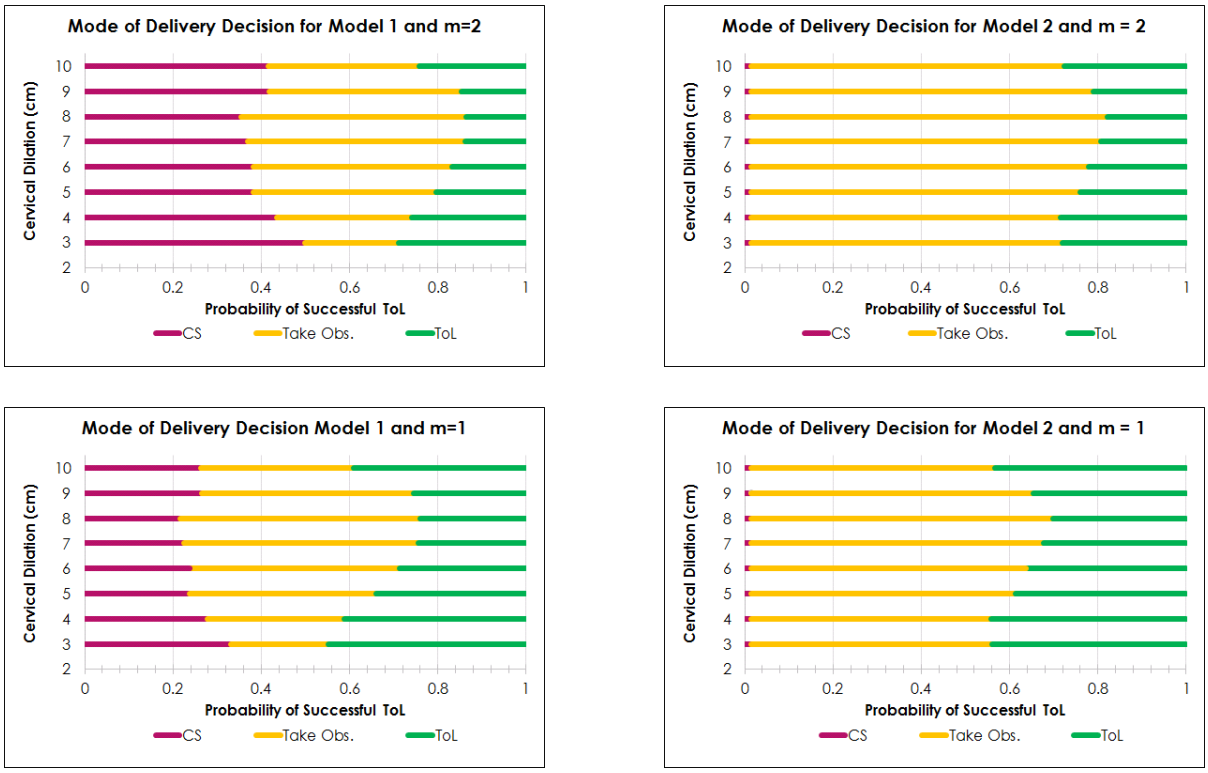


Figure 3.3 Expected loss function for Model 1 and Model 2 for each cervical dilation states 3 cm through 10 cm. The right side of the chart represents the decision to continue ToL and the left side represents the CS decision. The area in the middle represents the decision to wait and take an observation before making a decision regarding patient type.

decide $\overline{f\,t\,p}$ when ξ_3 is between $(0.712, 1]$, and to take an observation when ξ_3 lies in the interval $(0.499, 0.712]$. This implies that a CS is most appropriate if the belief that the patient is $\overline{f\,t\,p}$ is less than 0.499. For the 3 cm case in Model 2, the results show that it is best to declare the patient is $f\,t\,p$ when $\xi_3 = 0$, decide $\overline{f\,t\,p}$ when ξ_3 is in the interval $(0.719, 1]$ and to take an observation when ξ_3 lies in the interval $(0, 0.719]$. Comparing the two models for 3cm, we note that the width of the interval for taking an observation is smaller for Model 1 (which is 0.213 compared to 0.719 for Model 2). Model 2 requires additional information for a wider range of ξ_3 than Model 1 before making a decision about the type of patient and mode of delivery. Not only is the observation interval smaller for Model 1 but the threshold for deciding when a CS is needed is higher than Model 2. The policy for Model 2 requires more evidence prior to deciding CS, and is more likely to take observations; in general, information is more valuable. This is due to the long right tail associated with the lognormal distribution which represents the potential for longer labors.

Assuming $T_{i,W}$ is the duration of labor for dilation state i, the probability that labor will last longer than t hours is calculated in order to further examine the structure of both models. Table 3.3 presents the value for $\Pr(T_{i,W} > t)$ assuming an exponential distribution and lognormal distribution for $t = 0.5, 1, 1.5, 2$. The results show that for all states where $t = 0.5$ (except the 3 cm and 5 cm states), the probability of longer labors is greater for the lognormal distribution. This trend continues for all other values of t in every state. Thus indicating that the lognormal distribution favors longer labor durations by assigning a higher probability.

Table 3.3 Comparison of the value for the probability that labor will last longer than t hours for exponential distribution and lognormal distribution.

Dilation	***t***	***Model 1***	***Model 2***	***Dilation***	***t***	***Model 1***	***Model 2***
3 cm	0.5	0.8249	0.8192	7 cm	0.5	0.5612	0.9282
	1	0.6804	0.7627		1	0.3150	0.7118
	1.5	0.5612	0.7257		1.5	0.1768	0.5119
	2	0.4629	0.6977		2	0.0992	0.3650
4 cm	0.5	0.7660	0.8528	8 cm	0.5	0.5000	0.9586
	1	0.5867	0.7875		1	0.2500	0.7315
	1.5	0.4494	0.7425		1.5	0.1250	0.4854
	2	0.3443	0.7078		2	0.0625	0.3083
5 cm	0.5	0.7660	0.7606	9 cm	0.5	0.5000	0.9254
	1	0.5867	0.6691		1	0.2500	0.7423
	1.5	0.4494	0.6098		1.5	0.1250	0.5743
	2	0.3443	0.5661		2	0.0625	0.4438
6 cm	0.5	0.6484	0.8613	10 cm	0.5	0.5000	0.9411
	1	0.4204	0.7209		1	0.2500	0.8750
	1.5	0.2726	0.6152		1.5	0.1250	0.8182
	2	0.1768	0.5339		2	0.0625	0.7694

3.5.1 The Cost of Errors

The multiplier, m, is varied in the equation $L_{1,2} = m * L_{2,1}$ in order to explore the relationship between the penalty associated with the two types of errors and its impact on the decision structure. For the base case, it was initially assumed that $m = 2$. In Figure 3.3, the intervals for when an observation should be taken are shown for $m = 1$ (the penalty for making either incorrect decision is equivalent) and $m = 2$. The interval for recommending CS in Model 1 and the interval for taking an observation are narrower for $m = 1$. The interval corresponding to continuing a ToL is wider for all dilation states than when $m = 2$. For Model 2, the interval for deciding CS is unchanged, the interval for taking an observation is decreased and the interval for deciding ToL is increased when $m = 1$. The results suggest that when the penalty values are equal, more consideration should be given to continuing a ToL.

When it is assumed that $L_{1,2} > L_{2,1}$ (i.e., $m < 1$), increasing values of m shows it is more favorable to decide a CS since the penalty for waiting is high. Alternatively, when $L_{1,2} < L_{2,1}$, the region for deciding a CS decreases and the region for ToL increases as m decreases since the loss for incorrectly deciding ToL is not as detrimental as performing a CS for a ToL patient. Figure 3.4 provides a pictorial depiction of the optimal policy as a function of m for the 6 cm cervical dilation state. If m is allowed to continuously increase, the Bayes decision for the no observation scenario will be CS unless the belief that the patient is a $\overline{ftp}$ patient is definite. Alternatively, as m decreases the reverse relationship is exemplified.

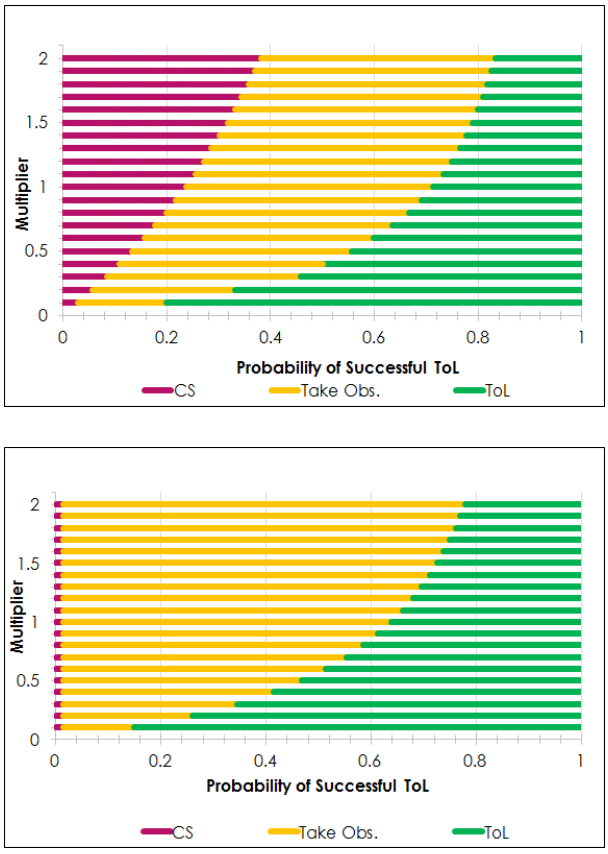


Figure 3.4 The optimal decision for different multiplier values, m, in the relation $L_{1,2} = m * L_{2,1}$ for the 6 cm cervical dilation state for Model 1 and Model 2.

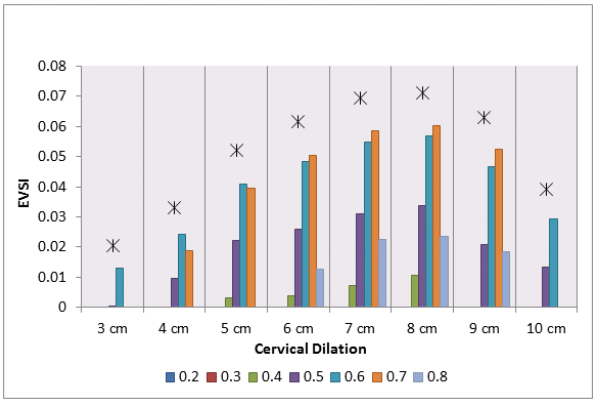
3.5.2 Value of Information

The DM is more certain about the patient type for belief values closest to the endpoints. Values closer to zero result in a stronger preference for CS whereas values closer to one favor ToL. For each dilation state there is an interval of beliefs for which the DM is uncertain. Analysis of these regions provides additional insight about the value of information (VOI) associated with learning more about the patient type for each dilation state. We examine

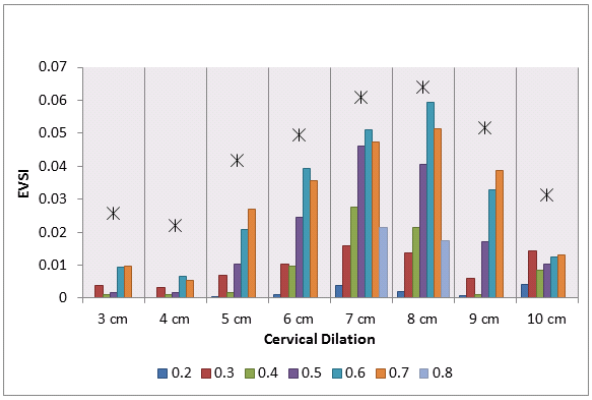
the expected value of sample information (EVSI) but in the context of delivery decision making from a Bayesian perspective. We determine the conditional value of information (conditional on the DM's prior belief) and determine the conditions (in terms of the DM's prior belief) under which information has positive value. We have developed a model to determine which states of cervical dilation are the most uncertain through the analysis of particular ξ_i values. For the values $\xi_i = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,\ 0.8\ \forall i$, the EVSI is calculated where EVSI= $\rho_0(\xi_i) - \rho_1(\xi_i)$. Plotting the cervical dilation against the EVSI, the willingness-to-pay (WTP) for each dilation in each model is identified. Figure 3.5 shows the VOI for each model by cervical dilation.

A comparison of the models shows a similar trend for each value of ξ_i. Figure 3.5 shows the VOI is highest for ξ_i between 0.6 and 0.7. For both models, the VOI is highest for cervical dilation of 7 cm and 8 cm for most values of ξ_i. As expected, values of ξ_i closer to the endpoints have a lower VOI since there is more confidence (less uncertainty) in those regions. Interestingly, the VOI for $\xi_i = 0.8$ is positive for 6 cm, 7 cm, 8 cm, and 9 cm in Model 1 and 7 cm and 8 cm in Model 2. That is, there is a degree of uncertainty for $\xi_i = 0.8$ even though it is fairly close to 1.

To further explore the VOI and WTP, the penalty for taking an observation is varied to understand how it affects when an observation is taken. The initial model assumed the additional penalty cost to be zero, i.e., there is no additional penalty associated with prolonging labor. Using the loss function relationship for the base model ($L_{1,2} = 2 \times L_{2,1}$) for each dilation state, the maximum penalty value for which it is optimal not to take an observation was identified. As shown by the stars in Figure 3.5, the WTP is a function of cervical dilation and is influenced by the distribution assumptions for the duration of labor.



(a) Model 1



(b) Model 2

Figure 3.5 Value of information for Model 1 and Model 2 for each cervical dilation state. The willingness-to-pay value for each model is denoted by a star.

This suggests that the observation penalty should also be a function of the dilation state as well as the presumed risk (i.e., the prior belief) for waiting before making a decision.

Since the WTP is higher for Model 1 in all states except 7 cm, it suggests that there is more uncertainty in the observation interval under the assumption of exponential labor duration (i.e., for Model 1 than for Model 2). This analysis also indicates that for dilation states 7 cm and 8 cm, taking an observation to assess labor progression is most important since the

WTP is highest for those dilation states. Interestingly, this analysis demonstrates that the VOI and WTP has a nonlinear and non-monotonic relationship with the dilation state and the DM's prior belief. Overall, the VOI provides insight into which dilation states are more uncertain and for which states the newly acquired information impacts the decision.

3.6 Discussion

This paper presents models to inform decision making regarding mode of delivery; in particular, when it is worthwhile to continue a ToL. A Bayesian framework is developed for making a birth decision when there is uncertainty regarding patient type (either ftp or $\overline{ftp}$) for laboring women. Model 1 uses information regarding whether a patient has progressed from the current dilation state to another within an hour and does not take into account the destination of labor progression. Model 2 is an extension where the destination of labor progression is considered. Here the patient type is static; however, the observation space is dynamic as represented by the probability transition matrices. The Bayesian models are used to identify threshold values for when an observation is needed based on the belief that the patient is a $\overline{ftp}$ patient. Exploration of the model shows that the mode of delivery decision is a function of the cervical dilation.

A few themes that influence the decision process are revealed through this analysis. The first theme focuses on the exploration of the VOI. VOI and WTP are conditional on both the cervical dilation and the DM's prior belief and the relationships are nonlinear. The VOI and WTP identified the dilation states with the most uncertainty. Cervical dilation states 7 cm and 8 cm seem to be the most uncertain because they had the highest EVSI and WTP value for all values of ξ_i. The results suggest that more attention should be given to women who

are in this stage of labor. It can be assumed that in these dilation states, it is beneficial to acquire more information before making a decision about delivery mode.

Second, the labor progression distributions strongly influence the duration of labor for each dilation state, which is to be expected. The exponential distribution was chosen to model labor progression for both delivery types in Model 1 primarily because the objective of the model was to capture whether the patient has progressed based on the median times for labor progression taken from literature. In Model 2, we use the lognormal distribution for vaginal delivery and exponential distribution for cesarean delivery in the simulation. The results show the use of the lognormal may be more appropriate given Model 2 identifies more conditions for which an observation should be taken and does not favor the CS decision for lower values of ξ_i as Model 1 does. This analysis shows that using an exponential distribution to model labor progression may not be ideal because the number of cesarean deliveries actually needed could be overestimated.

The third theme is the importance of the loss function. It was explained earlier that the loss function is a combination of the relative penalty for choosing the wrong decision. The loss function should be based on expert opinion/experience and should incorporate the consequences for choosing incorrectly. There may be various forms of the loss function. For example, a regression function could be developed that incorporates risk factors, gestation period, as well as maternal and neonatal characteristics to represent the consequences for making an incorrect decision. The consequence for choosing incorrectly in later dilation states may be more detrimental than for earlier dilation states.

Current practice suggests delivery decisions are based on clinicians' professional opinions. We have provided a framework for using information regarding labor progression to

diagnose ftp patients. A feature of this decision model is that the parameters concerning the loss functions and ξ_i values are patient-provider pair specific and can be modified on a case by case basis. This model can be used to facilitate shared decision making enabling providers to communicate with patients about potential risks and what to expect during labor.

CHAPTER

4

MARKOV DECISION PROCESS

A Markov decision process (MDP) is a dynamic program used to model a stochastic system which determines the best decision among a set of alternatives that are associated with outcomes with the goal of optimizing an objective over a time horizon [74]. The MDP presented in this chapter provides insight into the length of time a woman should remain in labor before deciding a C-section is needed in order to (i) maximize the utility of health outcomes and the delivery process and (ii) identify utility attributes that drive the decision. The MDP is an extension of a decision tree where the decision is made once incorporating

the probability of health outcomes and the valuation of those outcomes. The MDP incorporates the dynamic nature of the decision process since a decision is made every hour from the onset of labor until delivery occurs or until the end of the horizon.

The active stage of labor, the focus of this research, is defined to be the period of time that begins at the onset of labor until the cervix is fully dilated. In order to model decision making for laboring women, we divide cervical dilation of active labor into states starting at 3 cm and ending at 10 cm. In addition to each cervical dilation state, we include three modes of delivery as states: (1) vaginal delivery (VD), (2) C-section (CS), and (3) emergency cesarean (ECS). In this model "CS" refers to the choice to perform a C-section based on the health of the mother or child and "ECS" refers to undergoing a C-section due to a "failure-to-progress" diagnosis based on the Friedman Curve and other labor progression distinctions of normal labor progression [5, 103]. In the discussion that follows, we describe the decision process considered with the MDP in the context of a decision tree.

4.1 Decision Trees

Decision trees are a common decision support tool used in medical decision making. Podgorelec et. al. (2002) described decision trees as a reliable and effective representation of gathered knowledge that provides a great degree of accuracy in decision making [73]. Decision trees combine the probability of different outcomes and the valuation of those outcomes for each decision and state of nature to make a one time treatment decision in order to optimize the stated objective. These models have had success in predicting the best mode of delivery for VBAC patients considering a collection of health outcomes [62, 67], predicting delivery mode based on physician practices [11], predicting preterm birth [50],

and the choice between delivery modes as to prevent anal incontinence [23] and to evaluate pelvic floor consequences [98]. A simple decision tree consists of a decision node, the outcomes of those decisions and the probability and utility associated with those outcomes. We represent the mode of delivery decision with a decision tree by comparing the best delivery mode between trial of labor (ToL) and CS based on the estimated utility of health outcomes for the mother and child in combination with a disutility value associated with the delivery process.

We model the decision process that begins during a trial of labor after the start of active labor and assume the decision to stop a trial of labor can occur at any point during active labor prior to delivery. Figure 4.1 shows the simple decision tree considered. It models the trade-off between mother and child outcomes as a function of various patient characteristics and risk factors, with the long-term goal of identifying potential strategies for optimizing health outcomes. The relationship between the outcomes of the mother and child is complex. Although not an active decision maker, the child is directly affected by the decision. Different delivery modes have different risks for the mother and child. This collection of outcomes, represented by the terminal nodes of the decision tree, are collapsed into the following categories: healthy mother and healthy child (HMHC), healthy mother and unhealthy child (HMUC), unhealthy mother and healthy child (UMHC), and unhealthy mother and unhealthy child (UMUC). The characterization “unhealthy” is a broad collection of ailments that may affect the mother or child. In this analysis, “unhealthy mother” encompasses uterine rupture, hysterectomy, or death after delivery and “unhealthy child” refers to those neonates who have experienced birth trauma or death after delivery. We assumes health outcomes for the mother and child are independent.

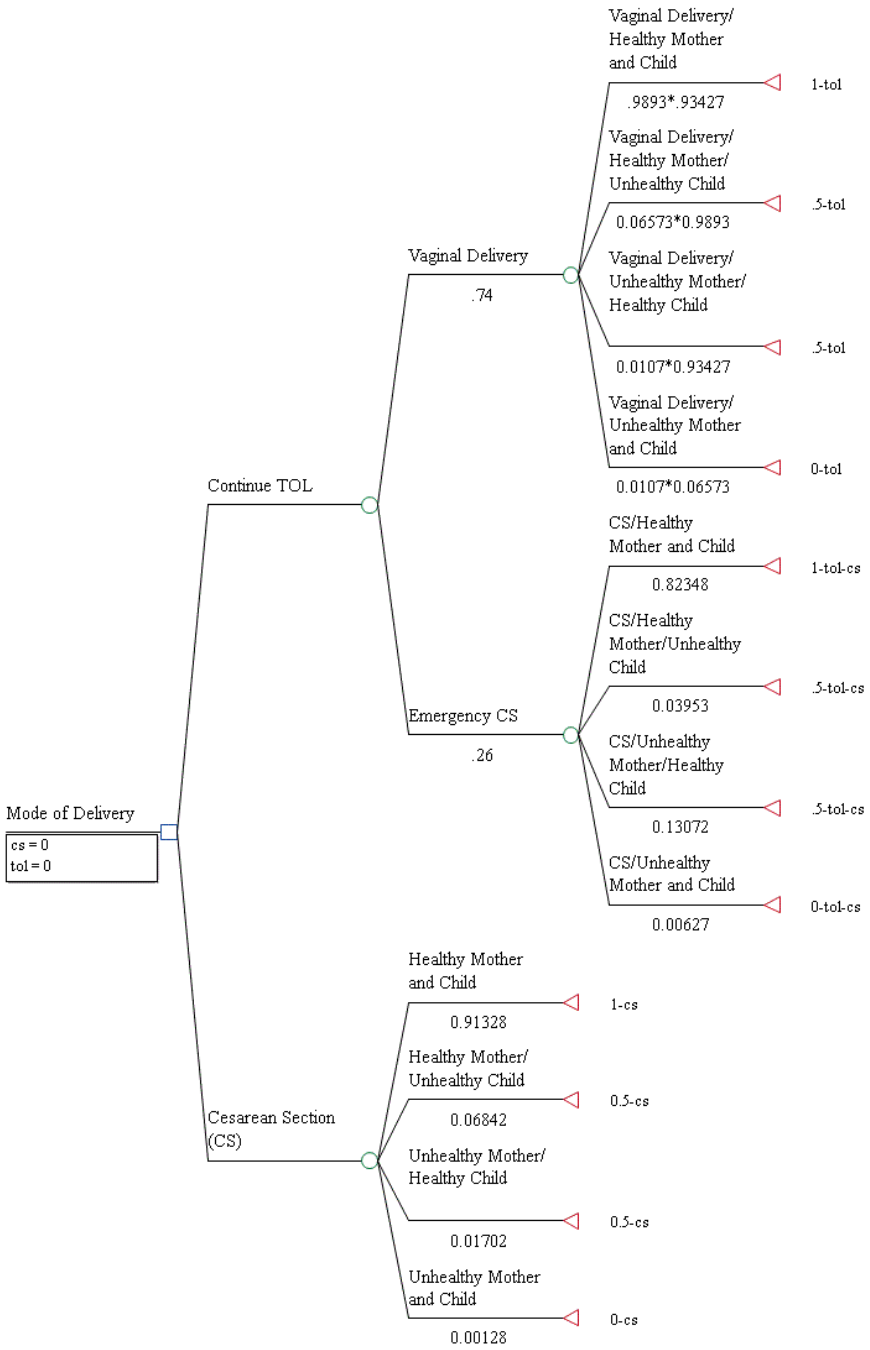


Figure 4.1 Decision tree to analyze the trade off between trial of labor and C-section dependent on utility values corresponding to the mother's and child's health.

The probabilities for each of the four scenarios were derived from the 2010 Evidence Report/ Technology Assessment, Vaginal Birth After Cesarean: New Insights [44]. However, there is little research regarding the valuation patients (or providers) place on these outcomes, or how best to combine the utility associated with the outcomes of these intertwined decision makers. It is not clear that the outcomes of the mother and child are independent or how to quantify their relationship. In this study, we assume the outcomes for the mother and child to be independent for simplicity and calculate the probability of maternal and child outcomes using inputs shown in Table 4.1. The independence of the outcomes for

Table 4.1 Probability values for healthy maternal and child outcomes derived from [44].

ToL		*CS*
$\Pr(VD) = 0.74$	$\Pr(ECS) = 0.26$	
$\Pr(HM\|VD) = 0.9893$	$\Pr(HM\|ECS) = 0.9543$	$\Pr(HM\|CS) = 0.9817$
$\Pr(HC\|VD) = 0.9343$	$\Pr(HC\|ECS) = 0.8630$	$\Pr(HC\|CS) = 0.9303$

the mother and the child implies that for every heath outcome and delivery mode we have

$$\Pr(ij|ToL,k) = \Pr(i|ToL,k) * \Pr(j|ToL,k), \;\; \Pr(ij|CS) = \Pr(i|CS) * \Pr(j|CS)$$

for

$i = \{HM, UM\}$, $j = \{HC, UC\}$, and $k = \{VD, ECS\}$.

In order to examine the relationship between the combined utility associated with maternal and child outcome values and the mode of delivery decision, we consider four scenarios as presented in Table 4.2. The differing values of utility for the HMUC and UMHC cases represent the valuation placed on the health of the mother versus that placed on the

child. The second scenario, which shows U(HMUC) = 0.9 and U(UMHC) = 0.1, means more weight is placed on the health of the mother than that of the child. The reverse is true for the third scenario.

Table 4.2 Scenarios to represent the varying values of utility for the health of the mother and child.

	U(HMHC)	*U(HMUC)*	*U(UMHC)*	*U(UMUC)*
Scenario 1 (blue)	1	0.5	0.5	0
Scenario 2 (red)	1	0.9	0.1	0
Scenario 3 (green)	1	0.1	0.9	0

We capture the effect of the mode of delivery or birth process in terms of disutility, which is subtracted from the utility, associated with the health of the mother and child. The final reward for each path of the tree is derived from a combination of the mother/child utilities and actions, ToL or CS. By varying the disutility associated with trial of labor, $d(ToL)$, and C-section, $d(CS)$, from 0 to 1 by an increment of 0.01, we derive a trade-off curve representing the threshold for choosing a ToL versus a CS, presented in Figure 4.2. The trade-off curve shows the optimal decision as a function of the trial of labor and C-section disutilities. The results show that trial of labor is more favorable as the disutility of C-section increases. However, when the disutility for trial of labor reaches 0.74, C-section is more favorable regardless of its disutility. These results suggest that the decision is more sensitive to the disutility associated with trial of labor than C-section. Figure 4.2 also shows how the curve shifts depending on the differing values of utility for the health of the mother and child (i.e., the second and third scenarios in Table 4.2). When a higher utility weight is placed on the mother's health, C-sections are favored. However, when higher value is placed on the

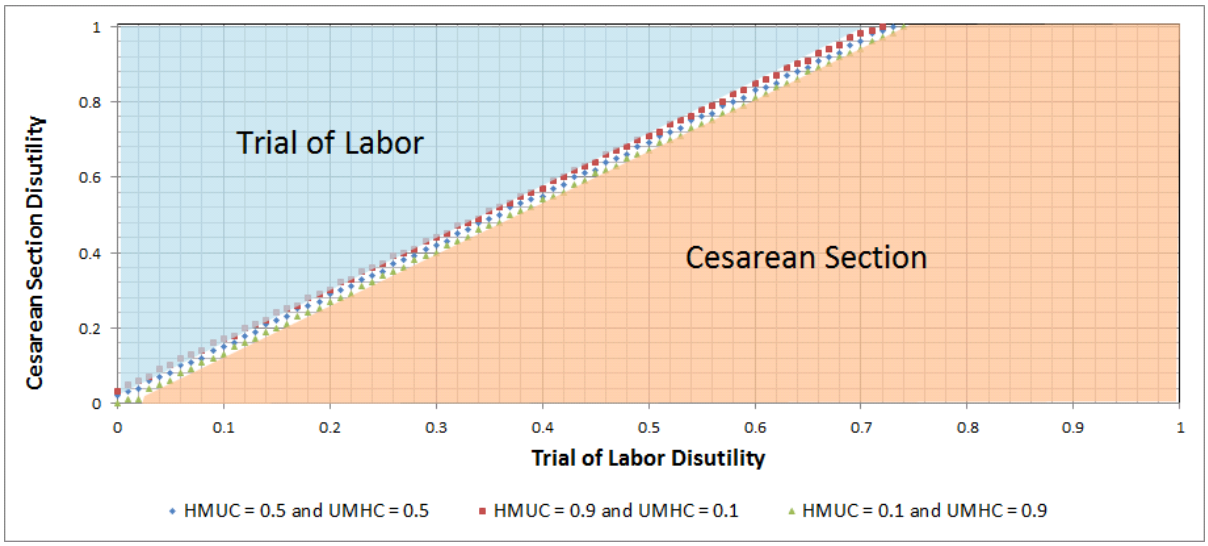


Figure 4.2 Trade-off curve between ToL and CS as a function of disutility values. The column headings are disutilities associated with ToL and the row headings are disutilities associated with CS.

child's health trial of labor is preferred. From this we see that when considering one patient's health over the other, the delivery mode can change. We extend this work by considering optimal policies for delivery mode for consecutive time periods.

4.2 MDP Model Formulation

The finite horizon MDP provides policies for when a C-section is needed due to a "failure-to-progress" diagnosis for each dilation state in order to maximize the expected utility of healthy outcomes for the mother and child. For every decision epoch, the decision is whether to perform a C-section or continue trial of labor where continuing trial of labor represents the decision to do nothing. In this case labor progresses naturally without intervention unless an emergency C-section is needed. The set of all possible states, S ,

include cervical dilation states and three modes of delivery: $S \in \{3,4,5,...,10,\ \text{VD, CS, ECS}\}$. Each component of the model, including the objective function, probability transition matrices, and reward structure are described in the next few sections.

4.2.1 Objective Function

For each time period, n, the goal is to identify the best decision to maximize the expected utility of healthy outcomes for the mother and child. Here n represents the number of time periods remaining in a 24-hour time period (which we assume is the maximum length of time for safe delivery of the fetus [14, 15]). The recursive optimality equation is

$$x_i(n) = \max_{a \in \{ToL, CS\}} \left\{ R_i^a(n) + \sum_{j=i}^{ECS} P_{ij}(a) x_j(n-1) \right\} \tag{4.1}$$

$$x_i(0) = \begin{cases} R_{VD} & \text{for } i = VD, \\ R_{CS} & \text{for } i = CS, \\ R_{ECS} & \text{otherwise.} \end{cases} \tag{4.2}$$

where

- $x_i(n)$ represents the maximum expected utility when n periods remain (in a 24 hour labor period), given the patient is in state $i \in S$
- $P_{ij}(a)$ represents the probability of moving from cervical dilation i to cervical dilation j in 1 hour for action $a \in \{ToL, CS\}$ and $i, j \in S$
- $R_i^a(n)$ represents the expected utility for action $a \in \{\ ToL, CS\}$ given the patient is in state i

- R_M is the reward for delivery mode $M \in \{VD, CS, ECS\}$

The initial condition reflects the expected utility associated with appropriate mode of delivery when no time periods remain. If a woman is allowed to labor for a maximum of 24 hours and she is still in labor at the end of the 24 hours, then a delivery must be made. Therefore, for states $i \in \{$ 3,4,5,...,10,ECS$\}$, the expected utility is ECS since an emergency C-section is most appropriate at the end of the horizon. The utility associated with VD is incurred when $i = VD$ and the utility associated with CS is incurred when $i = CS$.

4.2.2 Reward Structure

Similar to Section 4.1, the reward is a combination of the utility for health outcomes for the mother and child and the disutility for the mode of delivery. We assume the highest utility value of 1 represents when both patients are healthy, a utility value of 0.5 represents one of the patients being healthy and the other being unhealthy and a utility value of 0 represents when both patients are unhealthy. The disutility is subtracted from the health outcome utility and represents the effect that the delivery process has on utility. The lowest value of disutility 0 represents the most desirable delivery mode and alternatively a disutility value of 1 represents the least desirable delivery mode. The formulation of the expected utility is

$$R_i^a(n) = \begin{cases} 0 & \text{for } i = 3, ..., 10, \text{ CS and } a = \text{ToL} \\ R_{VD}(n) & \text{for } i = \text{VD and } a = \text{ToL} \\ R_{ECS}(n) & \text{for } i = \text{ECS and } a = \text{ToL} \\ R_{CS}(n) & \text{for } i \in S \text{ and } a = \text{CS} \end{cases} \tag{4.3}$$

where

$$R_{VD}(n) = \sum_o \Pr(o|VD,i,n) * (U(o|VD,i) - d(VD)), \tag{4.4}$$

$$R_{ECS}(n) = \sum_o \Pr(o|ECS,i,n) * (U(o|ECS,i) - d(ECS)), \tag{4.5}$$

and

$$R_{CS}(n) = \begin{cases} \sum_o \Pr(o|CS,i,n) * (U(o|CS,i) - d(CS)) & \text{for } i \in 3,...,10, \text{ CS} \\ 0 & \text{for } i = \text{VD, ECS} \end{cases} \tag{4.6}$$

for $o \in \{HMHC, MHUC, UMHC, UMUC\}$. Here $\Pr(o|M,i)$ is probability of health outcomes for delivery mode, M, and state, i, and time period n, $U(o|M,i)$ is utility of health outcomes for delivery mode M at state i, and $d(M)$ is the disutility associated with each delivery mode.

The result of ToL action will either be to continue labor, VD, or ECS. Since delivery for the trial of labor action does not occur until VD or ECS states, the immediate reward for states 3 cm, 4 cm,..., 10 cm is 0. The reward for VD and ECS states for trial of labor action reflects the expected utility corresponding to that mode of delivery. The reward for the CS state for the trial of labor action is 0 in this case, because one cannot have a CS when trial of labor is the decision. Similarly, the reward for VD and ECS states, given the C-section action, is 0 since it is not possible to have a C-section and VD or ECS. All other rewards for the C-section action, reflect the expected utility of health given a C-section at that particular dilation. The rewards are presented in Table 4.3. The initial utilities (boundary conditions) in this problem correspond with the end of horizon reward utility, which is defined as having no periods remaining (n = 0) equating to a time period of 24 hours. We determined

Table 4.3 Rewards for each state considering both actions, trial of labor and C-section.

State(i)	Trial of Labor Reward	Cesarean Section Reward
3	0	$\mathbb{E}$(Utility for C-section at 3 cm)
4	0	$\mathbb{E}$(Utility for C-section at 4 cm)
5	0	$\mathbb{E}$(Utility for C-section at 5 cm)
6	0	$\mathbb{E}$(Utility for C-section at 6 cm)
7	0	$\mathbb{E}$(Utility for C-section at 7 cm)
8	0	$\mathbb{E}$(Utility for C-section at 8 cm)
9	0	$\mathbb{E}$(Utility for C-section at 9 cm)
10	0	$\mathbb{E}$(Utility for C-section at 10 cm)
VD	$\mathbb{E}$(Utility for Vaginal Delivery)	0
CS	0	$\mathbb{E}$(Utility for C-section)
ECS	$\mathbb{E}$(Utility for Emergency C-section)	0

the expected utility for this period by identifying the most appropriate delivery for each cervical dilation in the 24th hour of labor. We assumed all cervical dilation states receive the reward for ECS. For $n = 0$, states VD and CS receive the reward associated with those states.

4.2.3 Probability Transition Matrices

Simulation was used to derive the probability transition matrices for each action. The simulation used the probability distributions derived from literature regarding the duration of labor for each centimeter of dilation as described in Chapter 3. To determine the probability of moving from one level of cervical dilation to another in one hour for the "trial of labor" action, the simulation ran for one hour and the probabilities of moving to another cervical dilation state, remaining in the current dilation state, or moving to a delivery mode (i.e., VD or ECS) were calculated. For the second action, C-section, the probability transition matrix

represents the decision of evoking the cesarean delivery decision at any point during labor. In this case, all states except VD and ECS, go to CS with probability 1. Since a patient cannot experience C-section and vaginal delivery or C-section and emergency C-section, the state VD remains in VD and ECS remains in ECS with probability 1 (i.e., they are absorbing states) in the C-section matrix.

Probability Transition Matrix for Trial of Labor

$\mathbf{P_{ij}^{ToL}} =$

	3 cm	***4 cm***	***5 cm***	***6 cm***	***7 cm***	***8 cm***	***9 cm***	***10 cm***	***VD***	***CS***	***ECS***
3cm	0.4426	0.35282	0.1288	0.0472	0.0183	0.0053	0.0023	0	0	0	0.0019
4 cm	0	0.4785	0.2746	0.1372	0.0643	0.0242	0.0136	0.0045	0	0	0.0031
5 cm	0	0	0.3051	0.2741	0.1775	0.1126	0.0798	0.0453	0.0023	0	0.0033
6 cm	0	0	0	0.2229	0.2072	0.1954	0.1905	0.1653	0.0160	0	0.0028
7 cm	0	0	0	0	0.1308	0.1807	0.2674	0.3597	0.0603	0	0.0010
8 cm	0	0	0	0	0	0.1008	0.2413	0.5105	0.1465	0	0.0010
9 cm	0	0	0	0	0	0	0.1475	0.5809	0.2694	0	0.0022
10 cm	0	0	0	0	0	0	0	0.5092	0.4702	0	0.0206
VD	0	0	0	0	0	0	0	0	1	0	0
CS	0	0	0	0	0	0	0	0	0	1	0
ECS	0	0	0	0	0	0	0	0	0	0	1

Probability Transition Matrix for C-section

$\mathbf{P_{ij}^{CS}} =$

	3 cm	***4 cm***	***5 cm***	***6 cm***	***7 cm***	***8 cm***	***9 cm***	***10 cm***	***VD***	***CS***	***ECS***
3 cm	0	0	0	0	0	0	0	0	0	1	0
4 cm	0	0	0	0	0	0	0	0	0	1	0
5 cm	0	0	0	0	0	0	0	0	0	1	0
6 cm	0	0	0	0	0	0	0	0	0	1	0
7 cm	0	0	0	0	0	0	0	0	0	1	0
8 cm	0	0	0	0	0	0	0	0	0	1	0
9 cm	0	0	0	0	0	0	0	0	0	1	0
10 cm	0	0	0	0	0	0	0	0	0	1	0
VD	0	0	0	0	0	0	0	0	1	0	0
CS	0	0	0	0	0	0	0	0	0	1	0
ECS	0	0	0	0	0	0	0	0	0	0	1

4.2.4 Model Implementation

The horizon for the MDP is a 24 hour time period. Two methods are used in order to identify the best policy for labor.

Case 1: The decision is modeled assuming the expected utility for CS is unchanged over time for every cervical dilation state:

$$R_i^{CS}(n) = R_{CS} \text{ n} = 24, 23, ..., 0. \tag{4.7}$$

Case 2: The probabilities of unhealthy outcomes are increased over time thus changing the expected utility for CS. The formulation for Case 2 assumes the probability for UMUC increases by 10% and the HMUC and UMHC outcomes increase by 5% for each increasing period.

We define the probability of each C-section outcome as follows:

$$\Pr(HMUC|CS,i,n-1) = 1.05 * \Pr(HMUC|CS,i,n)$$

$$\Pr(UMHC|CS,i,n-1) = 1.05 * \Pr(UMHC|CS,i,n)$$

$$\Pr(UMUC|CS,i,n-1) = 1.1 * \Pr(UMUC|CS,i,n)$$

$$\Pr(HMHC|CS,i,n) = 1 - (\Pr(HMUC|CS,i,n) + \Pr(UMHC|CS,i,n)$$

$$+ \Pr(UMUC|CS,i,n))$$

from Equation (4.3). This reflects the notion that a cesarean delivery in an earlier period results in an increased risk of complications and infection. The longer labor is prolonged for a woman needing a cesarean delivery, the chance of a bad outcome increases [15].

In both cases the following utility values were assigned to each outcome: U(HMHC) = 1, U(HMUC) = 0.5, U(UMHC) = 0.5, and U(UMUC) = 0 and the probability of each outcome is the same values presented in Table 4.1. As a base case, the following disutility values were assumed: $d(VD) = 0$, $d(CS) = 0.8$, and $d(ECS) = 1$ which reflect a strong dislike for a cesarean delivery and strong preference for vaginal delivery. We also vary the $d(CS)$ values from 0.7 to 0.9 as initial sensitivity analysis. In our implementation, we assume the following ordering on the relationship between reward functions: $R_{VD} > R_{CS} > R_{ECS}$. This suggests that vaginal delivery is the preference and if a C-section is needed then a C-section is preferred to an emergency C-section. However, any ordering that captures the preference of the decision maker can be used.

4.3 Results

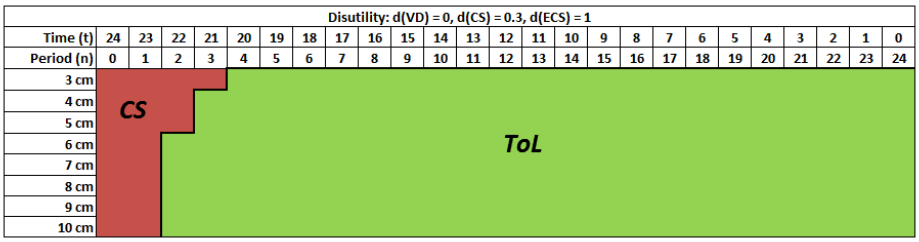
Using backwards induction to solve the MDP, the results determine the number of hours a woman should remain in labor conditional on the dilation before it is decided a C-section is needed based on the expected utility associated with each delivery mode presented in Figures 4.3a-4.3d for Case 1. For the base case, the results show that a patient observed at 3 cm, with a labor period not to exceed 24 hours, should be allowed to labor at 3 cm for at most 22 hours before deciding a C-section is needed. If she does progress to another dilation state, then the total time in that state minus the time already labored gives the amount of time she can remain at that cervical dilation. For example, if a woman starts labor at 3 cm and progresses to 5 cm after 5 hours, then the amount of time she can labor at 5 cm would be $23-5 = 18$ hours. If she progresses to 9 cm after 7 hours then the policy indicates she should be allowed to remain at 9 cm for $24-12 = 12$ hours before deciding

a C-section is needed. Using similar logic, we can design each possible path and identify the appropriate times to end a trial of labor given the total amount of time labored and the cervical dilation state. It is important to note that if a trial of labor is determined to be the best decision, the patient may deliver vaginally or need an emergency C-section. Overall, we see that as the dilation state increases so does the amount of time a patient should be allowed to remain in labor. Further, as the value of $d(CS)$ increases the patient should be allowed to labor longer. These results suggest that the amount of time to remain in labor is strongly dependent on the utility and disutility values of health outcomes and delivery process.

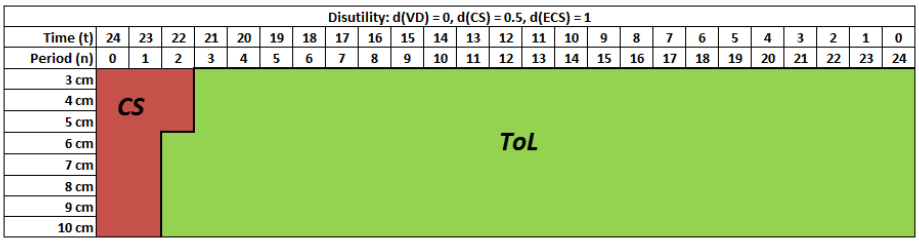
Figure 4.4 shows the policy for Case 2 in which the probability of unhealthy outcomes increases over time. The results show the increases (as described in Section 4.2.4) in the probability of unhealthy outcomes does not have a significant impact on laboring times. Although most of the labor duration times are the same between the two cases, we see that the length of time the patient is allowed to remain at certain dilation states is longer by at most one hour. As would be expected as $d(CS)$ increases, the amount of time a woman is allowed to labor increases for both cases 1 and 2 (refer to Figures 4.3a-4.3d and 4.4). The impact of the increase in $d(CS)$ decreases as dilation increases.

4.3.1 Sensitivity Analysis–Disutility Values

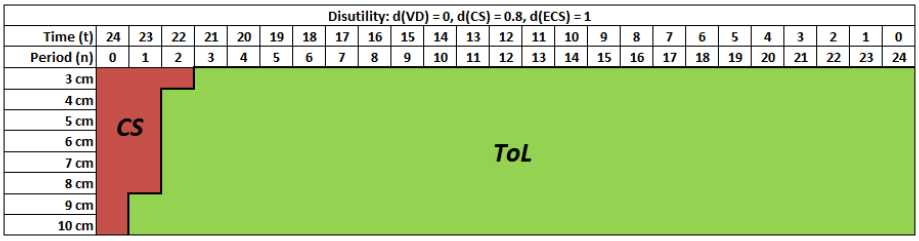
The delivery mode decision for each time period is based on the expected utility for health outcomes. Since the expected utility is a combination of utility of health outcomes and disutility of delivery process it is important to understand what changes to these values implies. The utility for health outcomes HMUC and UMHC and disutility values are patient-



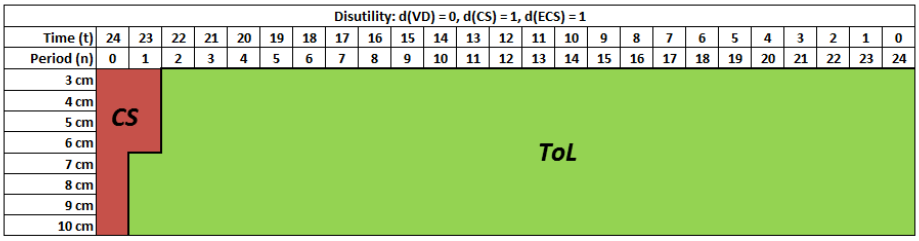
(a) Disutility values: $d(VD) = 0$, $d(CS) = 0.3$ and $d(ECS) = 1$.



(b) Disutility values: $d(VD) = 0$, $d(CS) = 0.5$ and $d(ECS) = 1$.



(c) (Basecase) Disutility values: $d(VD) = 0$, $d(CS) = 0.8$ and $d(ECS) = 1$.



(d) Disutility values: $d(VD) = 0$, $d(CS) = 1$ and $d(ECS) = 1$.

Figure 4.3 Optimal decision for each period and dilation state for varying CS disutility values when $d(VD) = 0$ and $d(ECS) = 1$.

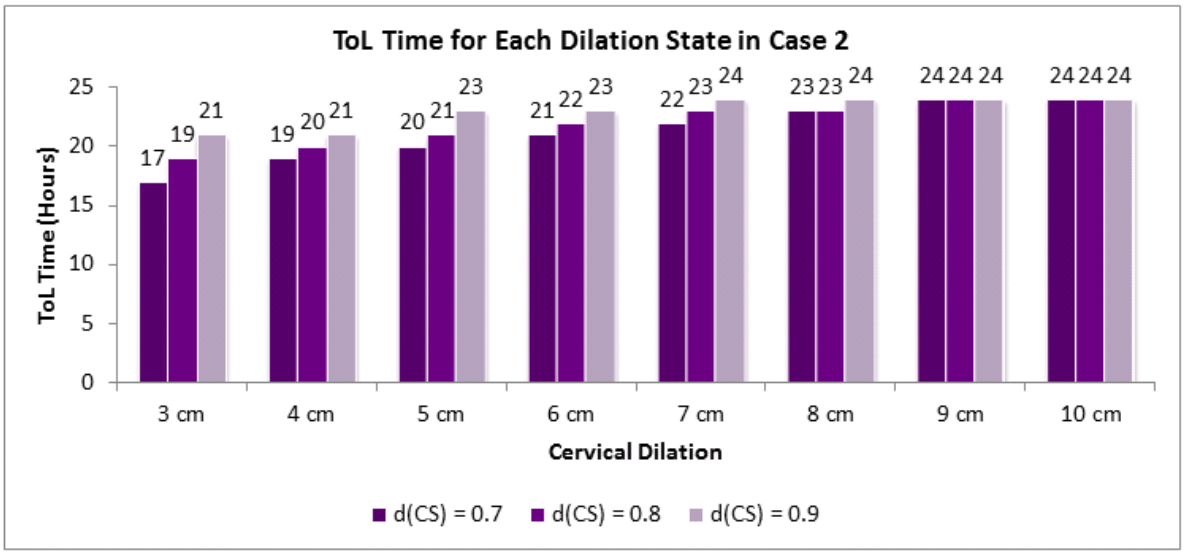
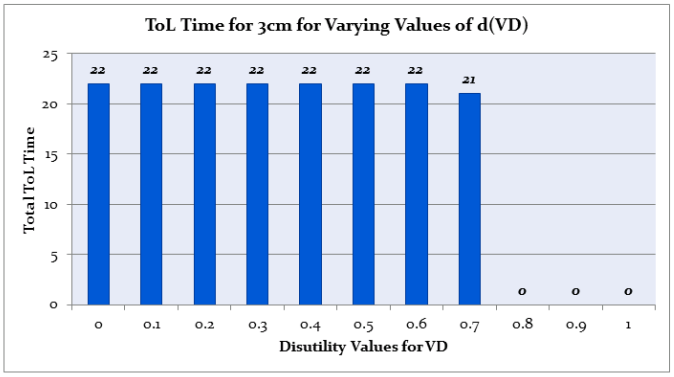


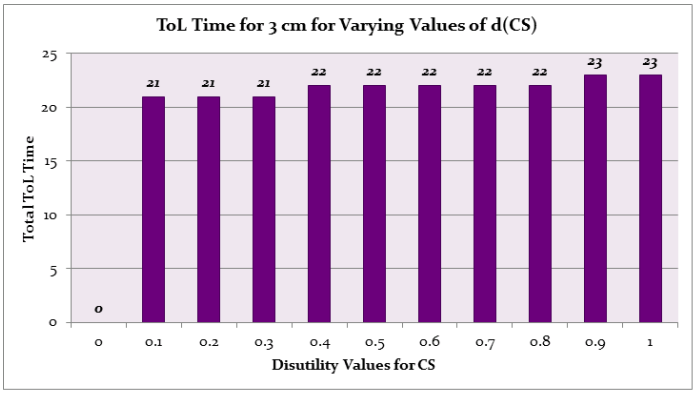
Figure 4.4 Amount of time a patient should remain in labor for case 2 with disutility values of $d(VD) = 0$, $d(CS) = 0.8$ and $d(ECS) = 1$.

centered and may be changed to reflect the desires of the patient and her valuation of the process. That is, a patient may increase the value of UMHC in comparison to HMUC to show that the child's health is more important than her own. It may also be decided that CS is the preference in which case, the disutility for CS would be lower than the disutility for VD and ECS. To analyze how the delivery mode decision changes for varying values of disutility, we conduct a sensitivity analysis on disutility values for the 3 cm state. In each case, we varied the disutility of each delivery mode from 0 to 1 by an increment of 0.1 and fixed the values of the other delivery modes. That is, for the disutility associated with VD, we varied $d(VD)$ between 0 and 1 to see how the decision changes for the 3 cm state when $d(CS) = 0.8$ and $d(ECS) = 1$ are fixed. We repeated this method for CS and ECS. The results for this analysis are presented in Figure 4.5.

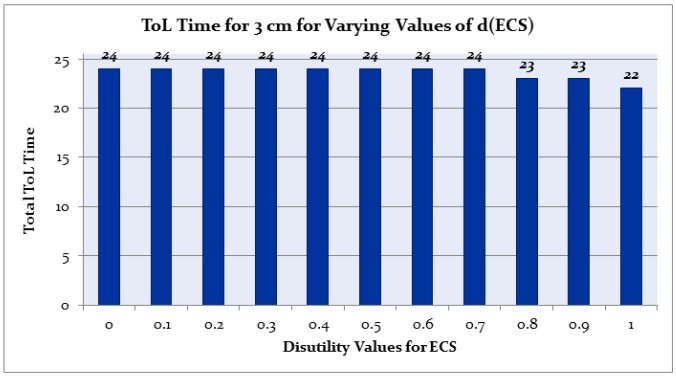
The results show that as the values of $d(VD)$ increase, the maximum allowable time in



(a) Disutility for VD, $d(VD)$, is varied from 0 to 1 by an increment of 0.1, $d(CS) = 0.8$, and $d(ECS) = 1$.



(b) Disutility for CS, $d(CS)$, is varied from 0 to 1 by an increment of 0.1, $d(VD) = 0$, and $d(ECS) = 1$.



(c) Disutility for ECS, $d(ECS)$, is varied from 0 to 1 by an increment of 0.1, $d(CS) = 0.8$, and $d(VD) = 0$.

Figure 4.5 Maximum allowable time a patient can remain in labor for cervical dilation state 3 cm when disutility values are varied.

labor decreases and for $d(VD) > 0.7$, CS is the best delivery mode for every time period (Figure 4.5b). This indicates that as the value of $d(VD)$ gets closer to the value of $d(CS)$, it is less desirable to allow a patient to labor for longer periods of time. That situation would reflect a patient who values both VD and CS almost equally. Figure 4.5b shows a reverse relationship for when the value, $d(CS)$, is varied. Here we see that for values of $d(CS) \geq 0.1$, the maximum allowable time in labor is increased. So for values of $d(CS)$ closer to 0 which is the fixed value of $d(VD)$, CS is the preferred mode. We see from both Figures 4.5b and 4.5c that when $d(VD)$ and $d(CS)$ are closer to each other, the best decision is CS. Lastly, we see there is not much of an effect associated with the varying values of $d(ECS)$. The maximum allowable time to remain in labor remains at 24 hours for values of $d(ECS) \geq 0$ and decreases by 2 hours to 22 hours for $d(ECS) = 1$. Since we do not see much change in the decision of when ToL is best, the optimal policy is robust with respect to the ECS disutility.

4.3.2 Sensitivity Analysis–Immediate Rewards

In our base case analysis (discussed in Section 4.2.4), we assumed there was no additional reward incurred for remaining in labor and reward was only received upon delivery. In order to understand how incorporating additional rewards for the labor process, i.e., continuing a trial of labor, affects the optimal policy, we evaluated the policies for reward values of 0, 0.05, 0.1, 0.15, 0.2 and 0.25. These results are presented in Figure 4.6 and show that as the reward is increased the allowable laboring time (up to 24 hours) increases for all states. Incorporating a reward of 0.25 ensures that the decision is ToL for every dilation state. This analysis assumes that the additional cost is the same for all states. However, the reward for

remaining in labor may differ by dilation state. For example, it could be assumed that the reward for lower dilation states (i.e., 3 cm, 4 cm, 5 cm, and 6 cm) may be higher than the reward for higher dilation states.

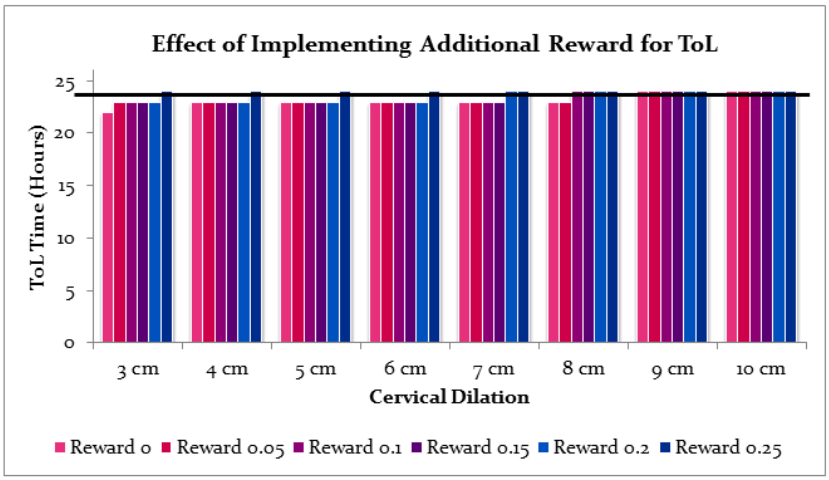


Figure 4.6 The effect of implementing additional rewards values of 0.05, 0.1, 0.15, 0.2 and 0.25 for remaining in labor. As the reward increases, the number of trial of labor hours also increases. The horizontal black line represents a total of 24 hours.

4.4 MDP Structural Property

The numerical examples explored in Section 4.2.4 suggests that there is a time period for each dilation state in which the decision changes. The numerical examples also suggests that for a period n, if ToL is the optimal decision for dilation state, i, then the decision must also be ToL for any dilation state greater than i. However, there are feasible numerical examples in which this structure does not hold, which are presented in Section 4.4.2. We

explore these properties further by examining the conditions under which the structure holds in Section 4.4.1.

4.4.1 Monotonic Optimal Policies in n

The structure of the MDP solution for the numerical examples explored above show that CS is optimal until a period, say $n = n_i$ and, for every period from n_i to $N = 24$, ToL is optimal. Here n_i is the number of time periods remaining in labor given patient is in state i. This relationship for the 3 cm state is displayed in Figure 4.7.The optimal decision, $d_n(i)$, for period, n, and state, i, has the following structure,

$$d_n(i) = \underset{a\in(ToL,CS)}{\operatorname{argmax}} \left\{x_i^a(n)\right\} = \begin{cases} CS & \text{when } 1 \le n < n_i \\ ToL & \text{when } n_i \le n \le N. \end{cases}$$

Let $\mathbb{E}[U(i)] = R_i$ for $i =$ CS, VD, ECS and let $R_i^{ToL}(n)$ be the immediate reward for being in state i for ToL decision and $R_i^{CS}(n)$ be the immediate reward for the CS decision, we define the maximum expected utility when the patient is in state i and there are n periods remaining as

$$x_i(n) = \max \begin{cases} R_i^{ToL}(n) + \sum_{j=i}^{ECS} P_{i,j}^{ToL} x_j(n-1) = x_i^{ToL}(n) \\ R_i^{CS}(n) + \sum_{j=i}^{ECS} P_{i,j}^{CS} x_j(n-1) = x_i^{CS}(n), \text{ for } n = 1,\ldots,N. \end{cases}$$

$$\text{where } x_i(0) = \begin{cases} R_{ECS} & \text{for } i = 3,\ldots,10 \\ R_{VD} & \text{for } i = VD \\ R_{CS} & \text{for } i = CS \\ R_{ECS} & \text{for } i = ECS. \end{cases} \tag{4.8}$$

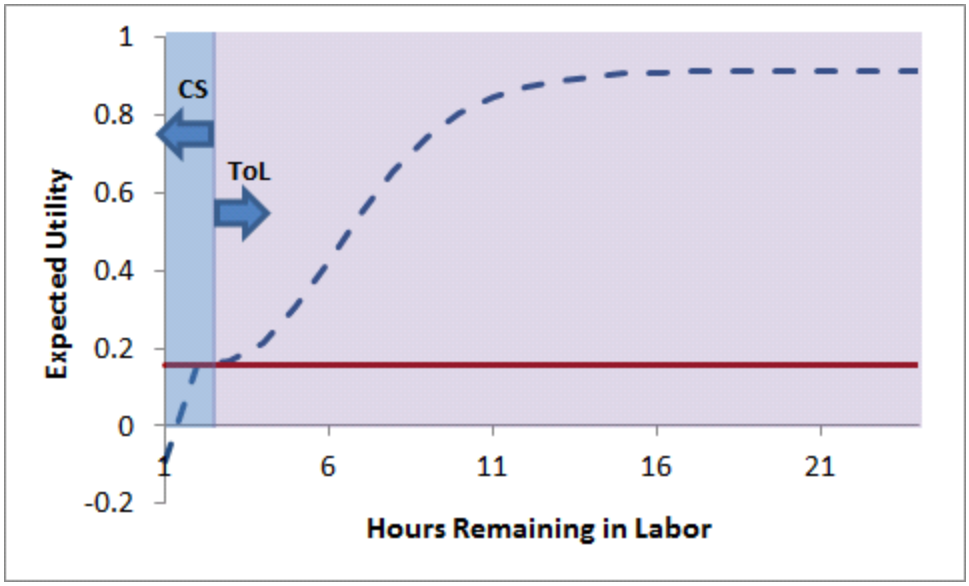


Figure 4.7 The trend of the value function for each time period for the 3 cm state. The figure shows that CS is the decision up to a value n_3 which represents time remaining in labor and the decision is ToL for every time period greater than n_3.

We can rewrite $x_i^{CS}(n)$ as follows

$$x_i^{CS}(n) = R_{CS} \text{ for } i = 3, ..., 10. \tag{4.9}$$

We assume the following relationship: $R_{VD} > R_{CS} > R_{ECS}$. We will show non-decreasing utility for each dilation state in three parts. The first component shows the utility function $x_i(n)$ is non-decreasing. We follow this by showing the two functions $x_i^{CS}(n)$ and $x_i^{ToL}(n)$ are also non-decreasing.

Proposition 1. *If $R_i^{ToL}(n)$ and $R_i^{CS}(n)$ are nondecreasing functions of n for $n = 0, 1, ..., 24$ and $i \in \{3, ..., 10, VD, CS, ECS\}$, then function $x_i(n) = \max\left\{x_i^{ToL}(n), x_i^{CS}(n)\right\}$ is also a nondecreasing function of n. That is, $x_i(n+1) - x_i(n) \geq 0$ for $n = 0, 1, ..., 23$ and $i \in \{3, ..., 10, VD, CS, ECS\}$.*

Proof. By induction on n.

1. Let $n = 1$, show $x_i(1) \geq x_i(0)$ for $i \in \{3, ..., 10, \text{VD}, \text{CS}, \text{ECS}\}$.

   First for $i = 3, \ldots, 10$, we want to show that

$$x_i(1) = \max\left\{x_i^{ToL}(1), x_i^{CS}(1)\right\} \geq R_{ECS} = x_i(0). \tag{4.10}$$

   According to Equation (4.8), we have $x_i(0) = R_{ECS}$ for $i \in \{3, ..., 10\}$. We know that for

$i \in \{3, ..., 10\}$,

$$\begin{aligned} x_i^{ToL}(1) &= R_i^{ToL}(1) + \sum_{j=i}^{ECS} P_{i,j}^{ToL} x_j(0) \\ &= \sum_{j=i}^{ECS} P_{i,j}^{ToL} x_j(0) \text{ since } R_i^{ToL}(1) = 0 \\ &= R_{ECS} \left[\sum_{j=i}^{10} P_{i,j}^{ToL} \right] + P_{i,VD}^{ToL} R_{VD} + P_{i,ECS}^{ToL} R_{ECS} + P_{i,CS}^{ToL} R_{CS} \\ &= R_{ECS} \left[\sum_{j=i}^{10} P_{i,j}^{ToL} + P_{i,ECS}^{ToL} \right] + P_{i,VD}^{ToL} R_{VD} + P_{i,CS}^{ToL} R_{CS}. \end{aligned}$$

According to Equation (4.9), we need to show that for $i = 3, ..., 10$,

$$\max \left\{ R_{CS}, R_{ECS} \left[\sum_{j=i}^{10} P_{ij}^{ToL} + P_{i,ECS}^{ToL} \right] P_{i,VD}^{ToL} R_{VD} + P_{i,CS}^{ToL} R_{CS} \right\} \geq R_{ECS} \tag{4.11}$$

because

$$x_i^{CS}(1) = R_{CS} \text{ for } i = 3, ..., 10 \tag{4.12}$$

by Equation (4.9). This is always true because

$$x_i(1) = \max\left\{x_i^{ToL}(1), x_i^{CS}(1)\right\} \text{ by definition} \tag{4.13}$$

$$= \max\left\{x_i^{ToL}(1), x_{CS}\right\} \text{ by Equation (4.12)}$$

$$\geq R_{CS} \text{ by definition of max operator}$$

$$> R_{ECS} \text{ by assumption}$$

$$= x_i(0) \text{ by Equation (4.8) for } i = 3, ..., 10. \tag{4.14}$$

Next we need to prove $x_i(1) \geq x_i(0)$ for $i \in \{\text{VD, CS, ECS}\}$. For state $i = \text{VD}$, we have

$$x_{VD}(1) = \max\left\{x_{VD}^{ToL}(1), x_{VD}^{CS}(1)\right\}. \tag{4.15}$$

Observe that for action $a = ToL$,

$$x_{VD}^{ToL}(1) = R_{VD}^{ToL}(1) + \sum_{j=VD}^{ECS} P_{VD,j}^{ToL} x_j(0) \tag{4.16}$$

$$= 0 + P_{VD,VD}^{ToL} x_{VD}(0) + P_{VD,CS}^{ToL} x_{CS}(0) + P_{VD,ECS}^{ToL} x_{ECS}(0) \tag{4.17}$$

$$= 0 + 1 \cdot R_{VD} + 0 \cdot R_{CS} + 0 \cdot R_{ECS}$$

$$= R_{VD} = x_{VD}(0). \tag{4.18}$$

In Equation (4.16), $R_{VD}^{ToL}(1) = 0$ because the patient is already in state VD at time $n = 1$, so the action $a = ToL$ is superfluous and earns no immediate reward at time $n = 1$. Moreover, in Equation (4.17), $P_{VD,VD}^{ToL} = 1$, $P_{VD,CS}^{ToL} = 0$, and $P_{VD,ECS}^{ToL} = 0$ because VD is

an absorbing state. Therefore Equation (4.18) follows from Equation (4.17).

Similarly for state $i = VD$ and action $a = CS$, we have

$$\begin{aligned} x_{VD}^{CS}(1) &= R_{VD}^{CS}(1) + \sum_{j=VD}^{ECS} P_{VD,j} x_j^{CS}(0) \\ &= 0 + P_{VD,VD}^{CS} x_{VD}(0) + P_{VD,CS}^{CS} x_{CS}(0) + P_{VD,ECS}^{CS} x_{ECS}(0) \\ &= 0 + 1 \cdot R_{VD} + 0 \cdot R_{CS} + 0 \cdot R_{ECS} \\ &= R_{VD}. \end{aligned} \tag{4.19}$$

Thus from (4.15), (4.18), and (4.19), we have

$$x_{VD}(1) = \max\left\{x_{VD}^{ToL}(1), x_{VD}^{CS}(1)\right\} \qquad = max\{R_{VD}, R_{VD}\} = R_{VD} \qquad = x_{VD}(0). \tag{4.20}$$

A similar argument for states $i/in\{CS, ECS\}$ and actions $a \in \{ToL, CS\}$ shows that

$$x_i^a(1) = R_i = x_i(0) \text{ for } i \in \{CS, ECS\} \text{ and } a \in \{ToL, CS\} \tag{4.21}$$

because CS and ECS are absorbing states.

Combining (4.14), (4.20), and (4.21), we have the first part pf the induction argument:

$$x_i(1) \geq x_i(0) \text{ for } i \in \{3, ..., 10, \text{ VD, CS, ECS}\}. \tag{4.22}$$

2. Assume $x_j(n) \geq x_j(n-1)$ for all $j \in \{3, ..., 10, VD, CS, ECS\}$ and for some $n \in \{1, 2, ..., 23\}$.

First we show $x_i(n+1) \geq x_i(n)$ for $i \in \{3,...,10\}$.

We know that for $i \in \{3,...,10\}$, we have

$$x_i(n+1) = \max\left\{x_i^{ToL}(n+1), x_i^{CS}(n+1)\right\}, \tag{4.23}$$

$$x_i(n) = \max\left\{x_i^{ToL}(n), x_i^{CS}(n)\right\} \tag{4.24}$$

Notice that

$$\begin{aligned} x_i^{ToL}(n+1) &= R_i^{ToL}(n+1) + \sum_{j=i}^{ECS} P_{ij}^{ToL} x_j(n) \\ &\geq R_i^{ToL}(n) + \sum_{j=i}^{ECS} P_{ij}^{ToL} x_j(n-1) \end{aligned} \tag{4.25}$$

because

$$R_i^{ToL}(n+1) \geq R_i^{ToL}(n) \text{ by assumption,}$$

and $x_j(n) \geq x_j(n-1)$ for all j by the induction hypothesis. Finally we observe that

$$r_i^{ToL}(n) + \sum_{j=i}^{ECS} P_{ij}^{ToL} x_j(n-1) = x_i^{ToL}(n). \tag{4.26}$$

Combining (4.25) and (4.26), we have

$$x_i^{ToL}(n+1) \geq x_i^{ToL}(n) \text{ for } i \in \{3,...,10\}. \tag{4.27}$$

Similarly we observe that for action $a = CS$,

$$x_i^{CS}(n+1) = R_i^{CS}(n+1) + \sum_{j=i}^{ECS} P_{ij}^{CS} x_j(n) \tag{4.28}$$

$$\geq R_i^{CS}(n) + \sum_{j=i}^{ECS} P_{ij}^{CS} x_j(n-1) \tag{4.29}$$

because

$$R_i^{CS}(n+1) \geq R_i^{CS}(n) \text{ by assumption} \tag{4.30}$$

and $x_j(n) \geq x_j(n-1)$ for all j by the induction hypothesis. Finally we observe that

$$R_i^{CS}(n) + \sum_{j=i}^{ECS} P_{ij}^{CS} x_j(n-1) = x_i^{CS}(n). \tag{4.31}$$

Combining (4.28), (4.29), and (4.31), we have

$$x_i^{CS}(n+1) \geq x_i^{CS}(n) \text{ for } i \in \{3, ..., 10\}. \tag{4.32}$$

Combining (4.27) and (4.32), we see that

$$\max\left\{x_i^{ToL}(n+1), x_i^{CS}(n+1)\right\} \geq \max\left\{x_i^{ToL}(n), x_i^{CS}(n)\right\} \text{ for } i \in \{3, ..., 10\}; \tag{4.33}$$

and from (4.23), (4.24), and (4.33) we finally obtain

$$x_i(n+1) \geq x_i(n) \text{ for } i \in \{3, ..., 10\}. \tag{4.34}$$

Next we need to show $x_i(n+1) \geq x_i(n)$ for $i \in \{VD, CS, ECS\}$. First we consider state

$i = VD$. For action $= ToL$, we have

$$\begin{aligned} x_{VD}^{ToL}(n+1) &= R_{VD}^{ToL}(n+1) + \sum_{j=VD}^{ECS} P_{VD,j}^{ToL} x_j(n) \\ &= R_{VD}^{ToL}(n+1) + x_{VD}(n) \\ &\geq R_{VD}^{ToL}(n) + x_{VD}(n-1) \\ &= x_{VD}^{ToL}(n) \end{aligned} \tag{4.35}$$

for the following reasons: (a) we assume $R_i^{ToL}(n+1) \geq R_i^{ToL}(n)$ for all i; (b) VD is an absorbing state; and (c) $x_j(n) \geq x_j(n-1)$ for all j by the induction hypothesis. A similar analysis for state $i = VD$ and action$a = CS$ yields

$$\begin{aligned} x_{VD}^{CS}(n+1) &= R_{VD}^{CS}(n+1) + \sum_{j=VD}^{ECS} P_{VD,j}^{CS} x_j(n) \\ &= R_{VD}^{CS}(n+1) + x_{VD}(n) \\ &\geq R_{VD}^{CS}(n) + x_{VD}(n-1) \\ &= x_{VD}^{CS}(n). \end{aligned} \tag{4.36}$$

It follows from (4.35) and (4.36) that

$$\begin{aligned} x_{VD}(n+1) &= \max\left\{x_{VD}^{ToL}(n+1), x_{VD}^{CS}(n+1)\right\} \\ &\geq \max\left\{x_{VD}^{ToL}(n), x_{VD}^{CS}(n)\right\} \qquad = x_{VD}(n). \end{aligned} \tag{4.37}$$

A similar argument shows that

$$x_i^{ToL}(n+1) \geq x_i^{ToL}(n) \text{ for } \{CS, ECS\}; \tag{4.38}$$

$$x_i^{CS}(n+1) \geq x_i^{CS}(n) \text{ for } \{CS, ECS\}; \tag{4.39}$$

and

$$\begin{aligned} x_i(n+1) &= \max\left\{x_i^{ToL}(n+1), x_i^{CS}(n+1)\right\} \\ &\geq \max\left\{x_i^{ToL}(n), x_i^{CS}(n)\right\} \\ &= x_i(n) \text{ for } i \in \{CS, ECS\}. \end{aligned} \tag{4.40}$$

Combining (4.34), (4.37), and (4.40), we have the desired conclusion of Proposition 1:

$$x_i(n+1) \geq x_i(n) \text{ for } i \in \{3, ..., 10, \text{ VD, CS, ECS}\} \text{ and } n = 0, 1, ..., 23. \tag{4.41}$$

□

We have shown that for every dilation state that the optimal decision is non-decreasing in n. Next we explore the properties which are needed to show the optimal decision is non-decreasing in i. Puterman (1994) provides conditions under which it can be shown that there exist an optimal decision $d_n(i)$ which is non-decreasing in i. In order to prove monotone optimal policies we must show the following conditions of Theorem 4.7.4 (Puterman, 1994) [75] for action a, state s, and time t.

Theorem 2. *(Theorem 4.7.4) Suppose for t = 1,..., N-1 that*

1. $r_t(s,a)$ *is nondecreasing in* s *for all* $a \in A'$,

2. $q_t(k|s,a)$ *is nondecreasing in* s *for all* $k \in S$ *and* $a \in A'$,

3. $r_t(s,a)$ *is a superadditive (subadditive) function on* $S \times A'$,

4. $q_t(k|s,a)$ *is a superadditive (subadditive) function on* $S \times A'$ *for all* $k \in S$, *and*

5. $r_N(s)$ *is nondecreasing in* s.

Then there exist optimal decision rules $d_t^*(s)$ *which are nondecreasing (nonincreasing) in* s *for* $t = 1, \ldots, N-1$.

Here S represents {3 cm, 4 cm, ..., 11 = VD, 12 = CS, 13 = ECS} and $A_s = A'$ for all $s \in S$ and

$$q_t(k|s,a) = \sum_{j=k}^{13} P_t(j|s,a). \tag{4.42}$$

In our problem formulation the above conditions do not hold. For our problem formulation,

$$r_t(s,a) = \begin{cases} 0 & \text{for } i \in \{3, \ldots, 10\} \\ R_{VD} = 0.962 & \text{for } i = VD \\ R_{CS} = 0.156 & \text{for } i = CS \\ R_{ECS} = -0.091 & \text{for } i = ECS \end{cases}$$

and

$$r_N(s) = \begin{cases} R_{ECS} & \text{for } s \in \{3, \ldots, 10\} \\ R_{VD} & \text{for } s = VD \\ R_{CS} & \text{for } s = CS \\ R_{ECS} & \text{for } s = ECS. \end{cases}$$

Since $R_{VD} > R_{CS} > 0 > R_{ECS}$, then we see that condition 1 and condition 5 do not hold of Theorem 2. By Equation (4.42), the summation of all future states must be nondecreasing in s. For $a = CS$, this condition is satisfied since $q_t(k|s, CS)$ is independent of s. The values for $q_t(k|s, ToL)$ are displayed below.

	3	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***VD***	***CS***	***ECS***
3 cm	1	0.5574	0.2046	0.0759	0.0286	0.0104	0.0051	0.0028	0.0019	0.0019	0.0019
4	1	1	0.5215	0.2468	0.1097	0.0454	0.0211	0.0076	0.0031	0.0031	0.0031
5	1	1	1	0.6949	0.4208	0.2433	0.1307	0.0508	0.0056	0.0033	0.0033
6	1	1	1	1	0.7771	0.5699	0.3746	0.1841	0.0188	***0.0028***	***0.0028***
7	1	1	1	1	1	0.8692	0.6885	0.4211	0.0613	***0.0010***	***0.0010***
8	1	1	1	1	1	1	0.8992	0.6579	0.1475	***0.0010***	***0.0010***
9	1	1	1	1	1	1	1	0.8525	0.2716	***0.0022***	***0.0022***
10	1	1	1	1	1	1	1	1	0.4908	***0.0206***	***0.0206***
VD	1	1	1	1	1	1	1	1	1	***0***	***0***
CS	1	1	1	1	1	1	1	1	1	***1***	***0***
ECS	1	1	1	1	1	1	1	1	1	1	1

For states 3 cm, 4 cm , 5 cm, …, 10 cm, VD, $q_t(k|s,a)$ is non-decreasing. However, for states CS and ECS, this property no longer holds. If we consider reordering the absorbing states such that ECS is before VD which is followed by CS, then $q_t(k|s, ToL)$ is non-decreasing (condition 2) and superadditive (condition 4) for all k and s combinations except for in the case where $k = VD$ and $s = VD$. In order to show a function is

superadditve, we must show that for a function $g(x, y)$,

$$g(x^+, y^+) - g(x^+, y^-) \geq g(x^-, y^+) - g(x^-, y^-)$$

for $x^+ \geq x^-$ in X and $y^+ \geq y^-$ in Y [75]. Therefore, we must show that

$$q_t(k|s^+, a^+) - q_t(k|s^+, a^-) \geq q_t(k|s^-, a^+) - q_t(k|s^-, a^-).$$

The $q_t(k|s, ToL)$ values for the reordered states are displayed below.

	3	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***ECS***	***VD***	***CS***
3	1	0.557	0.205	0.076	0.029	0.010	0.005	0.003	0.002	0	0
4	1	1	0.521	0.247	0.110	0.045	0.021	0.008	0.003	0	0
5	1	1	1	0.695	0.421	0.243	0.131	0.051	0.006	0.002	0
6	1	1	1	1	0.777	0.570	0.375	0.184	0.019	0.016	0
7	1	1	1	1	1	0.869	0.689	0.421	0.061	0.060	0
8	1	1	1	1	1	1	0.899	0.658	0.147	0.146	0
9	1	1	1	1	1	1	1	0.853	0.272	0.269	0
10	1	1	1	1	1	1	1	1	0.491	0.470	0
ECS	1	1	1	1	1	1	1	1	1	***0***	0
VD	1	1	1	1	1	1	1	1	1	1	0
CS	1	1	1	1	1	1	1	1	1	1	1

Although the we can show that $q_t(k|s, ToL)$ is non-decreasing except in the special case where $k = VD$ and $s = VD$, $r_t(s, a)$ is still non-increasing (violation of condition 1). Using the new state ordering, condition 3 holds for states 3 cm through 10 cm since

$$r_t(s+1, CS) - r_t(s+1, ToL) \geq r_t(s, CS) - r_t(s, ToL) \text{ for } s \in \{3 \text{ cm}, 4 \text{ cm}, \ldots, 10 \text{ cm}\}.$$

However, the condition does not hold for the absorbing states. For $s \in \{3 \text{ cm}, 4 \text{ cm}, \ldots, 10 \text{ cm}\}$

$$r_t(VD, CS) - r_t(VD, ToL) \geq r_t(CS, CS) - r_t(CS, ToL)$$

$$0 - 0.962 \geq 0.156 - 0$$

$$-0.962 \geq 0.156$$

which is not true. Another suggested ordering of the states is

$$\{0 = ECS, 3, 4, 5, 6, 7, 8, 9, 10, 11 = \text{ CS}, 12 = \text{ ECS}\}.$$

This ordering satisfies conditions 1 and 5 of Theorem 2 however conditions 2, 3 and 4 are violated. Since we can not determine an ordering on the states in which all conditions of Theorem 2 are satisfied, then we can not prove the optimal policy is monotone. The structure of our problem contains three absorbing states which adds a layer of complexity and typical ways of showing monotonicity may not apply. Next, we show optimal non-monotonic policies for particular disutility values.

4.4.2 Non-monotonic Optimal Policies

In Section 4.4, we provided conditions under which optimal monotone policies exist and and justification of why our solution does not have a traditional threshold policy. In addition to not satisfying the conditions of presented in Puterman (1994) (Theorem 2) [75], there exist particular combinations of disutility values of VD, CS, and ECS in which the policies do not follow a monotone optimal policy. In Figure 4.8, we assume $d(VD) = 0$, $d(CS) = 0.035$,

and $d(ECS) = 1$ and in Figure 4.9, $d(VD) = 0$, $d(CS) = 0.01$, and $d(ECS) = 0.5$. In fact, for $0 < d(CS) < 0.04$, the monotone optimal policy does not hold. Since these values are suitable parameters, we know there are practical conditions under which the threshold policy we once observed does not exist.

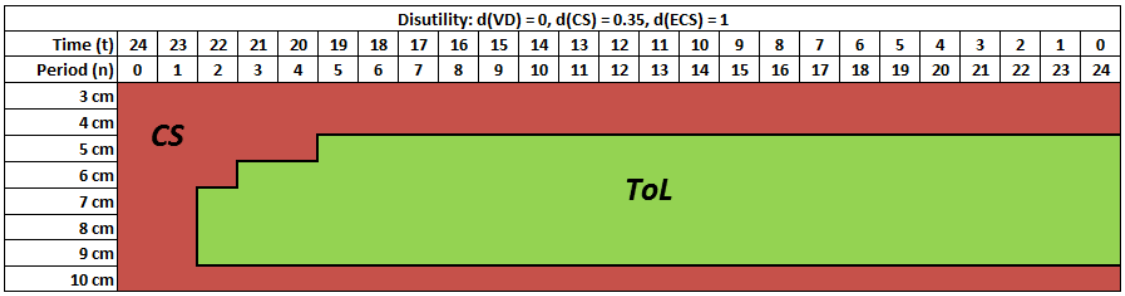


Figure 4.8 Example 1 of non-monotonic policy of when a CS is needed in labor.

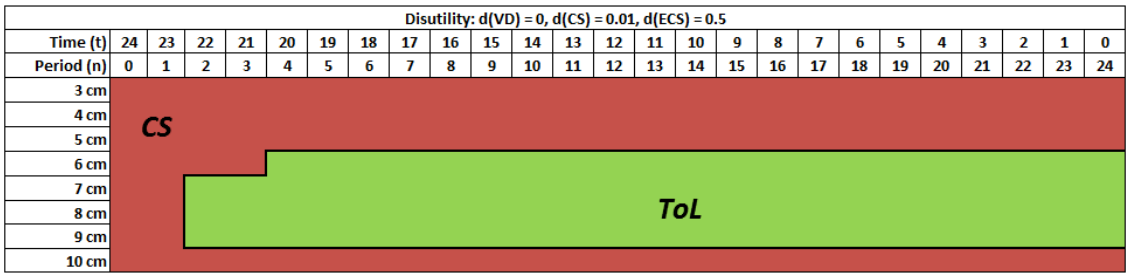


Figure 4.9 Second example of non-monotonic policy of when a CS is needed in labor for a particular set of disutility values.

4.5 Discussion

We have provided a method for modeling decision making during labor that can be used to develop policies regarding the length of time a woman can be allowed to labor as a function of cervical dilation and the utility of health outcomes and disutility of delivery process. This decision model can be used by multiple stakeholders as a decision tool. Clinicians can use the model to understand how patient preferences can influence delivery outcomes. They can also use the model as a resource to encourage shared decision making with their patients. The Consortium on Safe Labor study showed that it can take as long as 8.1 hours and 6.4 hours to progress from 3 to 4 cm and 4 to 5 cm, respectively [103]. The results from this model support this claim. This model suggests if labor progresses according to the data collected by the Consortium on Safe Labor, in absence of any other complications, it is optimal to allow women to labor longer before deciding a C-section is necessary. Further, this research suggests the optimal maximum allowable length of labor is a function of cervical dilation.

We consider two cases for the mode of delivery decision. In Case 2 the probability value of unhealthy outcomes is increased to represent the increasing rate of complications incurred from prolonging labor. However, this increasing rate does not affect the decision significantly. The process was designed to capture the effects of allowing the utility of cesarean delivery to decrease as labor progresses. This may also be captured by incorporating a probability of complications in the optimality equation. The probability of complications would be a function of the time in labor and the current dilation states. Further work to represent the decreasing utility of CS would improve our policy and capture the effect of

prolonging labor on health outcomes.

A significant component of this model is the assignment of utility and disutility values. We chose a particular set of utility and disutility values to represent a person who gives the highest regard to the health of both patients and strongly prefers a vaginal delivery. However, understanding the policy for varying utility values is important when trying to create a standard rule for duration of labor. Allowing the disutility values for VD, CS, and ECS to vary from 0 to 1 by an increment of 0.1, we see how the policy changes for the 3 cm state. The results suggest that understanding the disutility for CS and VD is important for classifying mode of delivery policies. In addition to the assignment of utility values, lack of understanding of how to best combine the utility of two dependent entities poses a limitation in this model. There is little research regarding how best to combine the utility associated with the outcomes of both patients. It is not clear that the outcomes of the mother and child are independent and how to quantify their relationship. Here we have assumed the utility for HMUC (healthy mother/unhealthy child) and UMHC (unhealthy mother/healthy child) to be equal at 0.5. However, changes in this estimate will change the results of the best policy. Further sensitivity analysis is needed to provide more insight into how the combination of utility values affects the policies.

An extension of this model would be to allow for interventions of labor such as assisted delivery. Here we do not explicitly explore these options. Including these interventions would provide more insight into what actually occurs in the delivery room. However, it is implicitly assumed that if labor is not interrupted by a C-section then other methods of labor intervention may have taken place in the course of the trial of labor.

Another extension of this model would be to explore other causes of emergency cesarean

delivery. We only use "failure-to-progress" as a basis for an emergency C-section. Zhang, et al. (2010) discovered that failure-to-progress (labor arrest) and cephalopelvic disproportion (occurs when a baby's head or body is too large to fit through the mother's pelvis) accounted for about 47.1% of intrapartum cesarean deliveries. Other indications for C-section include fetal distress, uterine rupture, and hypertensive disorders [103].

CHAPTER

5

BAYESIAN MARKOV DECISION PROCESS

In Chapter 3, we presented a Bayesian decision model that can be used to provide insight into when it is most appropriate to prolong labor with the intention of learning more about the patient and when the mode of delivery can be decided using current available information about the patient. In this model formulation, the objective is to minimize the losses incurred from making an incorrect decision regarding patient type (i.e., failure-to-progress or not). In Chapter 4, we introduced a Markov decision process (MDP) to explore how long a woman can remain in labor for each dilation state (i.e., 3 cm to 10 cm) depending

on the utility of health outcomes and the disutility associated with the birth process and delivery mode. We propose to combine these models to create a Bayesian MDP in which the duration of labor for each dilation state is determined based on the belief of patient type. Specifically, if we define two probability matrices for the trial of labor decision one detailing the probability transitions for failure-to-progress (ftp) patients and the other detailing the probability transitions for not-failure-to-progress ($\overline{ftp}$) patients. By varying the probability of belief in each patient type, we can define labor times as a function of cervical dilation state and patient type. Appropriate recommendations regarding delivery mode are a function of labor progression and ultimately whether the patient is in fact a failure to progress patient or not. This approach incorporates both factors to evaluate circumstances in which the decision should be based on observations by prolonging labor or based on a traditional MDP that does not consider patient type.

5.1 Bayesian MDP Literature

Ding et al. (2002) described a Bayesian MDP to be used in determining optimal inventory levels when demand is censored. Their formulation allows for the demand distributions to be updated in addition to policy updating as information is gathered [26]. Ryzhov et al. (2010) presented a newsvendor problem considering two agents where one agent learns from the actions of the first agent in an MDP with unknown transition probabilities to make the best decision. In the problem an MDP is used with transition rates based on the behavior of one agent. The other agent adapts and learns using a Bayesian model [76]. The proposed model we present differs from tradition partially-observable Markov decision processes where the transition probabilities are uncertain [100]. In this formulation, the

state is true and accurate for the decision maker however the patient type, which drives the transition rates, is unknown.

5.2 Methods

Similar to Chapter 4, we present a finite-horizon MDP which provides policies for when a C-section is needed due to a "failure-to-progress" diagnosis for each dilation state. The MDP incorporates two actions: 1) continue trial of labor (ToL) and 2) C-section (CS). The objective is to maximize the expected utility of healthy outcomes for both the mother and child. We represent patient type as $W \in \{\mathbf{w_1}, \mathbf{w_2}\}$ where w_1 is $\overline{f\,t\,p}$ and w_2 is $f\,t\,p$. We extend the MDP for the ToL action and incorporate two matrices to capture labor progression for $f\,t\,p$ and $\overline{f\,t\,p}$ patients.

The natural history of a trial of labor is described by the state transition diagram presented in Figures 5.1 and 5.2. For the $\overline{f\,t\,p}$ patient's state transition diagram, within an hour the dilation state may remain in the current dilation state, move to a higher dilation state, or deliver via vaginal delivery (VD) or emergency C-section (ECS). In the $f\,t\,p$ patient's state transition diagram, the dilation state can also remain in the current dilation, move to a higher dilation, or deliver through ECS only (i.e., vaginal delivery is not possible for $f\,t\,p$ patients). Although we include C-section (CS) as a state in the probability transition matrices, it is not possible to transition into state CS when the decision is $To\,L$.

In the Bayesian MDP, the utility for the trial of labor action is a weighted combination of the belief that the patient is a $\overline{f\,t\,p}$ patient and the belief that the patient is $f\,t\,p$. Similar to the models presented in Chapter 3, the belief that a patient is $\overline{f\,t\,p}$ at a particular dilation state i is ξ_i. We consider two probability transition matrices representing the trial of labor

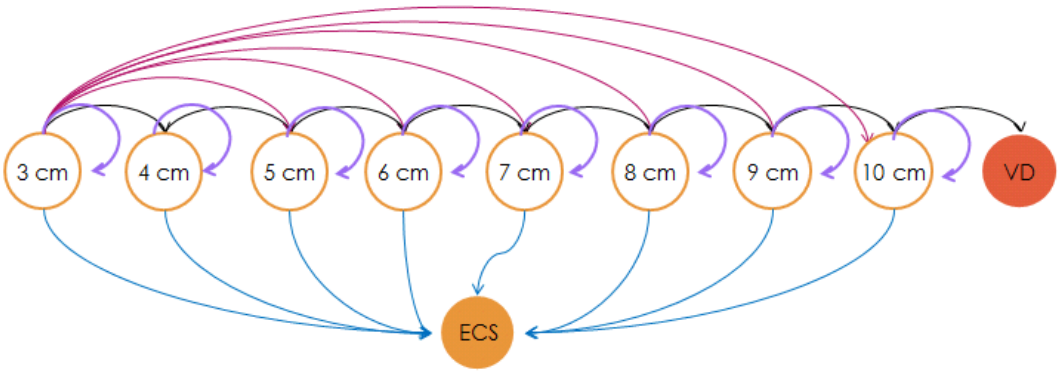


Figure 5.1 State transition diagram for $\overline{f\,t\,p}$ patients within one hour. Transitions are possible from any lower dilation state to a higher dilation state including VD and ECS.

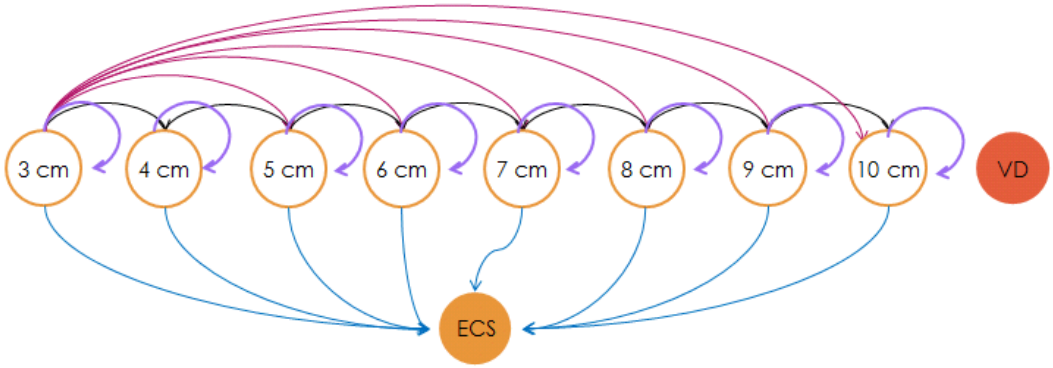


Figure 5.2 State transition diagram for $f\,t\,p$ patients within one hour. Transitions are possible from any lower dilation state to a higher dilation state and ECS. Transitions into VD are not possible.

action: $\mathbf{P}_{\overline{\mathbf{ftp}}}$ and $\mathbf{P}_{\mathbf{ftp}}$. The probability transition matrices were derived from the discrete event simulation presented in Chapter 2.

For the $\mathbf{P}_{\overline{\mathbf{ftp}}}$ matrix, the underlying probability distribution for the duration of labor in each dilation state is lognormal and the exponential distribution is used for the $\mathbf{P}_{\mathbf{ftp}}$ matrix. At the start of the simulation 8000 patients enter labor at 3 cm and the probability of transitioning to another state within one hour is calculated. To account for the probability of transitioning into the ECS state, the simulation for the 8000 patients is run for an extended period of time. Once a patient has completed a dilation state, it is determined how long the patient has been in labor. If that time exceeds 24 hours, the patient receives a C-section (considered an ECS) at that dilation state. Otherwise, the patient continues to the next dilation state. At the end of the simulation we then calculate the percentage of patients who received a C-section at each dilation state.

Probability Transition Matrix for Not-Failure-to-Progress

	3	4	5	6	7	8	9	10	VD	CS	ECS
3	0.443	0.353	0.129	0.047	0.018	0.005	0.002	0	0	0	0.002
4	0	0.479	0.275	0.137	0.0643	0.024	0.014	0.005	0	0	0.003
5	0	0	0.305	0.274	0.178	0.113	0.08	0.045	0.002	0	0.003
6	0	0	0	0.223	0.207	0.195	0.191	0.165	0.016	0	0.003
7	0	0	0	0	0.131	0.181	0.267	0.36	0.06	0	0.001
8	0	0	0	0	0	0.101	0.241	0.511	0.1465	0	0.001
9	0	0	0	0	0	0	0.148	0.581	0.269	0	0.002
10	0	0	0	0	0	0	0	0.509	0.470	0	0.021
VD	0	0	0	0	0	0	0	0	1	0	0
CS	0	0	0	0	0	0	0	0	0	1	0
ECS	0	0	0	0	0	0	0	0	0	0	1

Probability Transition Matrix for Failure-to-Progress

	3	4	5	6	7	8	9	10	VD	CS	ECS
3	0.836	0.136	0.009	0.001	0	0	0	0	0	0	0.018
4	0	0.791	0.13	0.0132	0.01	0.0004	0	0	0	0	0.065
5	0	0	0.726	0.146	0.016	0.002	0.0003	0.0001	0	0	0.11
6	0	0	0	0.657	0.160	0.024	0.002	0.0001	0	0	0.158
7	0	0	0	0	0.614	0.167	0.021	0.0012	0	0	0.196
8	0	0	0	0	0	0.577	0.14	0.024	0	0	0.259
9	0	0	0	0	0	0	0.532	0.2	0	0	0.268
10	0	0	0	0	0	0	0	0.654	0	0	0.346
VD	0	0	0	0	0	0	0	0	1	0	0
CS	0	0	0	0	0	0	0	0	0	1	0
ECS	0	0	0	0	0	0	0	0	0	0	1

The two matrices represent the trial of labor action and are dependent on patient type with prior ξ_i. Since the decision maker does not know which patient type they have, the trial of labor action becomes a combination of both matrices. We discuss how we bring these two together in Section 5.2.1.

5.2.1 Prior Belief ξ_i

For the trial of labor action, we define the probability that labor progresses from i to j within an hour to be

$$P_{ij}^{ToL} = \xi_i P_{ij|\overline{ftp}} + (1-\xi_i) P_{ij|ftp} \text{ for } i = 3 \text{ cm , 4, cm,..., 10 cm, VD, CS, ECS}$$

where $P_{ij|ftp}$ is the probability that labor progresses from i to j for ftp and $P_{ij|\overline{ftp}}$ is the probability that labor progresses from i to j for $\overline{ftp}$. Similar to Chapter 4, we represent the reward for VD as R_{VD}, reward for CS as R_{CS}, and the reward for ECS as R_{ECS}, where

$R_{VD} > R_{CS} > R_{ECS}$. The utility for the CS action is

$$U_{CS}(n) = R_{CS}. \tag{5.1}$$

The utility for ToL action is expanded to reflect the decision maker's uncertainty about the patient type,

$$U_i^{ToL}(n) = R_i^{ToL}(n) + \sum_{j=i}^{ECS} \Big[\xi_i P_{ij|\overline{ftp}}^{ToL} V_j(n-1) + (1-\xi_i) P_{ij|ftp}^{ToL} V_j(n-1) \Big]. \tag{5.2}$$

For $i \in \{$ 3 cm , 4 cm,..., 10 cm, VD, CS, ECS$\}$ and a given belief,ξ_i, ToL is the decision when

$$\begin{aligned} U^{ToL}(n|i) &\geq U^{CS}(n) \\ R_i^{ToL}(n) + \sum_{j=i}^{ECS} P_{ij}^{ToL} V_j(n-1) &\geq R_{CS} \\ R_i^{ToL}(n) + \sum_{j=i}^{ECS} \Big[\xi_i P_{ij|\overline{ftp}}^{ToL} + (1-\xi_i) P_{ij|ftp}^{ToL} \Big] V_j(n-1) &\geq R_{CS} \\ R_i^{ToL}(n) + \sum_{j=i}^{ECS} \Big[P_{ij|ftp}^{ToL} + \xi_i \Big(P_{ij|\overline{ftp}}^{ToL} - P_{ij|ftp}^{ToL} \Big) \Big] V_j(n-1) &\geq R_{CS}. \end{aligned}$$

where

$$V_j(n-1) = \max\Big(U^{ToL}(n-1|j), U^{CS}(n-1) \Big)$$

In our initial analysis, we assume the prior belief, ξ_i, remains the same for each time period. This analysis provides the optimal policy for when a CS is needed based on a particular belief that the patient will have a successful trial of labor. We extend this with analysis of

the optimal policies based on the posterior belief which is updated as labor progresses.

5.2.2 Posterior Belief $\xi_i(X)$

Allowing ξ_i to vary from 0 to 1 provides more flexibility in our model. As labor continues, there are different actions that may occur to prompt the decision maker to increase his or her belief that the patient will have a successful trial of labor. In particular, the progression of cervical dilation, or lack there of, provides insight into how long a patient is expected to remain in labor. In order to capture this element of the decision making process we extend the decision space to include updated values of ξ_i.

Observation Space

For every time period, an observation is taken which provides additional insight into the patient type. We consider two cases: (1) the no progression case and (2) the progression case. In the "no progression" case, the dilation state observed at decision epoch, n, is the same as the dilation state observe at decision epoch, $n-1$. We represent this observation as $X=0$. For the "progression" case, a change in dilation state is observed from time period n to time period $n-1$. This is represented as $X=1$.

For

$$x = \begin{cases} 1 & \text{when a change is observed} \\ 0 & \text{when no change is observed} \end{cases}$$

we define the likelihood functions for $P(x|\overline{f\,t\,p})$ and $P(x|f\,t\,p)$ as

$$P(x|W)=\begin{cases} P_{ii|w}^{ToL} & \text{when } x=0 \\ 1-P_{ii|w}^{ToL} & \text{when } x=1 \end{cases}$$

for $w \in \{w_1, w_2\}$.

The marginal probability, $P(X=x)$ is

$$P(X=x)=P(x|\overline{f\,t\,p})P(\overline{f\,t\,p})+P(x|f\,t\,p)P(f\,t\,p).$$

We update ξ_i using Bayes theorem,

$$\begin{aligned}\xi_i(x)=&P\left(\overline{f\,t\,p}|x\right)\\ &=\frac{P(x|\overline{f\,t\,p})P(\overline{f\,t\,p})}{P(x|\overline{f\,t\,p})P(\overline{f\,t\,p})+P(x|f\,t\,p)P(f\,t\,p)}\\ &=\frac{P(x|\overline{f\,t\,p})\xi_i}{P(x|\overline{f\,t\,p})\xi_i+P(x|f\,t\,p)(1-\xi_i)}.\end{aligned}$$

for $x=\{0,1\}$. We calculate the probability transition matrix for the trial of labor action using the equation:

$$P_{ij}^{ToL}(x)=\xi_i(x)P_{ij|\overline{f\,t\,p}}^{ToL}+(1-\xi_i(x))\,P_{ij|f\,t\,p}^{ToL}$$

for $x=\{0,1\}$. For any given time period, n, there will either be a change in dilation or not. Figure 5.3 shows the expanded observation space which is collapsed to the problem we consider. Although there are various paths that the observation space can take, to simplify the observation space, we only consider the case in which there was only change observed

or only no change in all of the previous time periods. We consider four cases for what can occur at time period $n-1$. We assume that change has occurred for every time period up to n, then there will either be another change or no change at $n-1$. Similarly, if no change is observed for every time period up to n, then at $n-1$, there will either be a change or no change. This simplification allows us to develop a heuristic to determine the best decision given you know what the observation was in the previous time period and by reducing the set of observation sample paths. In the next section we discuss how ξ_i is updated depending on the observation.

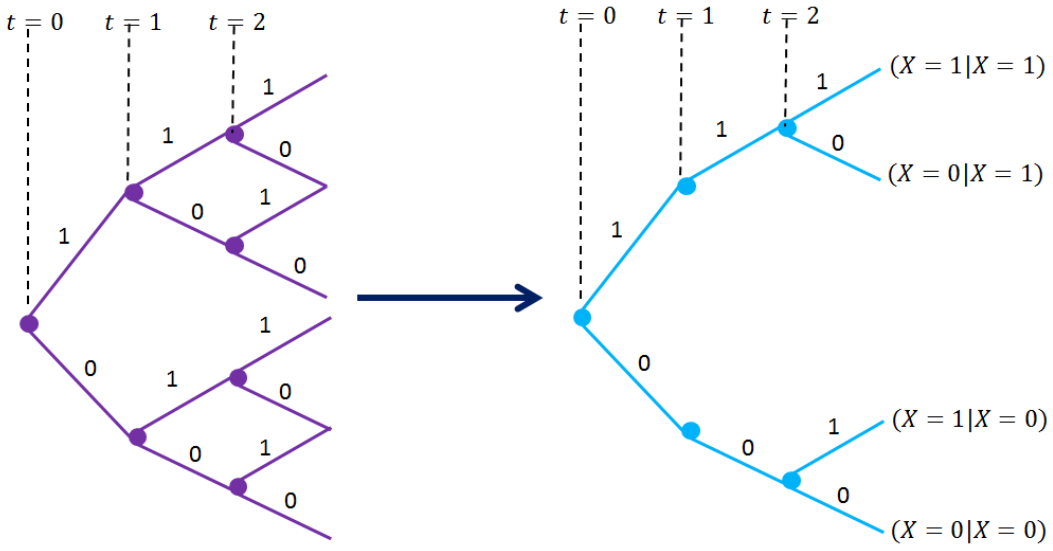


Figure 5.3 The observation space for all possible paths of change and no change is collapsed into a simpler observation space in which only four cases are considered.

Calculating Posterior $\xi_i(X = x)$

As described in Section 5.2.2, we consider four distinct cases: (1) a change in period n followed by another change in current time period $n-1$ $(X = 1|X = 1)$, (2) a change in period n followed by no change in current time period $n-1$ $(X = 0|X = 1)$, (3) no change in period n followed by a change in current time period $n-1$ $(X = 1|X = 0)$, and (4) no change in period $n-1$ followed by no additional change in current time period n: $(X = 0|X = 0)$. An illustration of each case is provided in Figure 5.4. For a given time period, if change occurs we update $\xi_i(1)$ and if there is no change, we update $\xi_i(0)$. We define ξ_i to be the

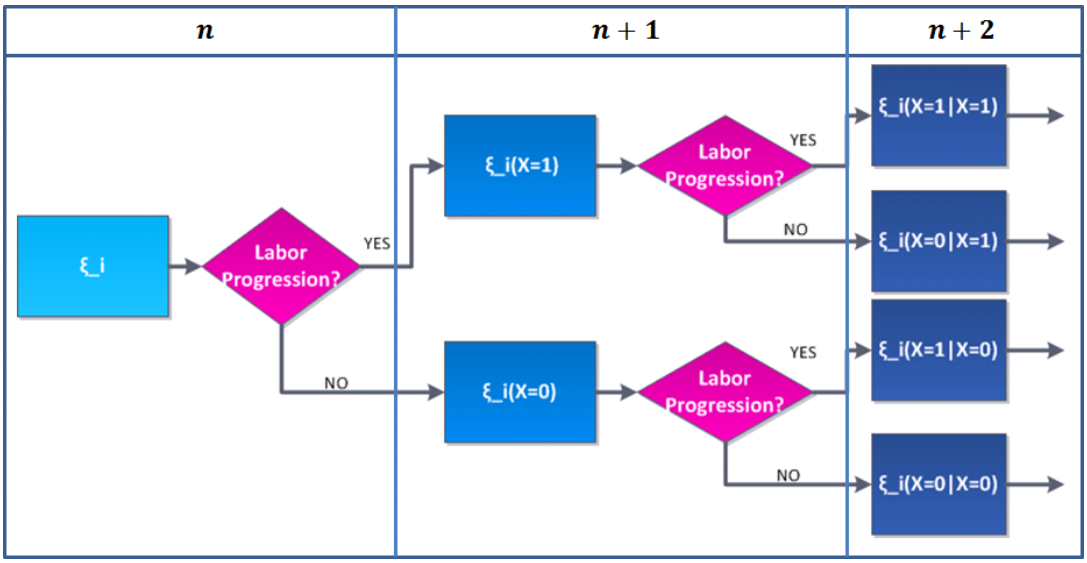


Figure 5.4 The process of updating ξ_i as n increases.

prior belief that the patient is $\overline{ftp}$ at dilaton state, i, $\xi_i(1)$ is the posterior belief that the patient is $\overline{ftp}$ when labor progression was observed, and $\xi_i(1|1)$ is the posterior belief that

the patient is $\overline{ftp}$ when a change is observed at n and $n-1$. For each case, we update ξ_i for $n = 1,\ldots,24$ as follows:

Case 1: $(X = 1|X = 1)$

$$\xi_i^n(X=1|X=1) = \frac{P(X=1|\overline{ftp})\xi_i^{n-1}(X=1|X=1)}{P(X=1|\overline{ftp})\xi_i^{n-1}(X=1|X=1) + P(X=1|ftp)(1-\xi_i^{n-1}(X=1|X=1)}$$

Case 2: $(X = 0|X = 1)$

$$\xi_i^n(X=0|X=1) = \frac{P(X=0|\overline{ftp})\xi_i^{n-1}(X=1|X=1)}{P(X=0|\overline{ftp})\xi_i^{n-1}(X=1|X=1) + P(X=0|ftp)(1-\xi_i^{n-1}(X=1|X=1)}$$

Case 3: $(X = 1|X = 0)$

$$\xi_i^n(X=1|X=0) = \frac{P(X=1|\overline{ftp})\xi_i^{n-1}(X=0|X=0)}{P(X=1|\overline{ftp})\xi_i^{n-1}(X=0|X=0) + P(X=1|ftp)(1-\xi_i^{n-1}(X=0|X=0)}$$

Case 4: $(X = 0|X = 0)$

$$\xi_i^n(X=0|X=0) = \frac{P(X=0|\overline{ftp})\xi_i^{n-1}(X=0|X=0)}{P(X=0|\overline{ftp})\xi_i^{n-1}(X=0|X=0) + P(X=0|ftp)(1-\xi_i^{n-1}(X=0|X=0)}$$

where $\xi_i^0(X=0|X=0) = \xi_i = \xi_i^0(X=1|X=1)$. Using the appropriate ξ_i^n, we define the probability transition matrices for the ToL action as follows:

$$\mathbf{P}^{\mathbf{ToL}}_{\mathbf{X=x|X=1}} = \xi_i^{n-1}(X=x|X=1)(\mathbf{P}_{\overline{\mathbf{ftp}}}) + (1-\xi_i^{n-1}(X=x|X=1))(\mathbf{P}_{\mathbf{ftp}})$$

and

$$\mathbf{P}^{\mathbf{ToL}}_{\mathbf{X=x|X=0}} = \xi_i^{n-1}(X=x|X=0)(\mathbf{P}_{\overline{\mathbf{ftp}}}) + (1-\xi_i^{n-1}(X=x|X=0))(\mathbf{P}_{\mathbf{ftp}})$$

where $x \in \{0,1\}$.

Since we consider four paths, we have four MDPs that are combined to provide the best solution. Using a weighted average, we develop a combined trial of labor expected utility. The decision to continue a trial of labor is determined by taking the maximum between the utility for ToL and CS. We define the utility for ToL at decision epoch, n, and dilation state, i, to be

$$\begin{aligned} U^{ToL}(n|i) = &\Big[U^{ToL}_{X=1|X=1}(n-1|i)(1-P^{ToL}_{X=1|X=1}(i,i)) + U^{ToL}_{X=0|X=1}(n-1|i)(1-P^{ToL}_{X=0|X=1}(i,i))\Big] \\ &\cdot\big(1-P^{ToL}_{ii}(X=1)\big) \\ &+\Big[U^{ToL}_{X=1|X=0}(n-1|i)(P^{ToL}_{X=1|X=0}(i,i)) + U^{ToL}_{X=0|X=0}(P^{ToL}_{X=0|X=0}(i,i))\Big] \\ &\cdot\big(P^{ToL}_{ii}(X=0)\big) \end{aligned}$$

and the utility for CS to be

$$U^{CS}(n) = R_{CS} \forall n = 1,\ldots,24 \tag{5.3}$$

where

$$U^{ToL}_{X=y|X=x}(n|i) = R^{ToL}_i(n) + \sum_{j=i}^{ECS} P^{ToL}_{X=y|X=x}(i,j) V_{X=y|X=x,j}(n-1)$$

and

$$V_{X=y|X=x,j}(n-1) = \max\Big(U^{ToL}_{X=y|X=x}(n-1|j), U^{CS}(n-1)\Big).$$

Here $V_{X=y|X=x,j}(n-1)$ represents the utility at decision epoch $n-1$ and dilation state j where observation $X=y$ was observed at $n-1$ and $X=x$ was observed at one time period before $n-1$. As described in Chapter 4, we assign utility values to the health outcomes for both patients as U(HMHC) = 1, U(HMUC) = 0.5, U(UMHC) = 0.5, and U(UMUC) = 0. The disutility represents the valuation of the process. We use the following values in the base case: $d(VD)=0$, $d(CS)=0.8$, and $d(ECS)=1$. Similar to Equations (4.4), (4.5), and (4.6) defined in Chapter 4, we derive the expected rewards for VD, CS, and ECS as follows:

$$R_{VD} = \sum_{o\in(1,2,3,4)} P(o|VD)[U(o)-d(VD)]$$

$$R_{CS} = \sum_{o\in(1,2,3,4)} P(o|CS)[U(o)-d(CS)]$$

$$R_{ECS} = \sum_{o\in(1,2,3,4)} P(o|ECS)[U(o)-d(ECS)]$$

where

1: healthy mother, healthy child (HMHC),

2: healthy mother, unhealthy child (HMUC),

3: unhealthy mother, healthy child (UMHC), and

4: unhealthy mother, unhealthy child (UMUC).

5.3 Results

We use backwards induction to determine when a trial of labor should end so that a C-section can be performed conditional on the dilation and belief regarding a successful trial of labor.

5.3.1 Prior Problem

For $\xi_i = 0, 0.2, 0.4, 0.5, 0.6, 0.8, 1$, we have provided the optimal policies in Figure 5.5. For ξ_i values of 0 and 0.2, it is best to not enter a trial of labor for all dilation states except 10 cm. As ξ_i increases the maximum allowable time in labor increases for each dilation state. Although $\xi_i = 1$ represents the belief that the patient will definitely have a successful trial of labor, we see that it is not optimal to remain in labor during the entire 24 hours due to the possibility of an ECS. Although the best course of action is to continue labor in this case, there is a chance that the labor would end in ECS. Since ECS is the most unfavored outcome, there are some circumstances in which is is best to end a ToL for a CS. We assume a CS is not as severe and more favorable than an ECS.

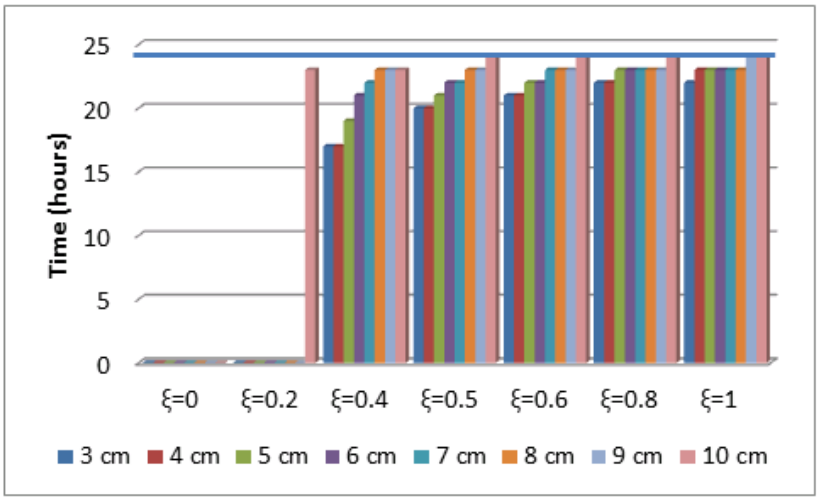


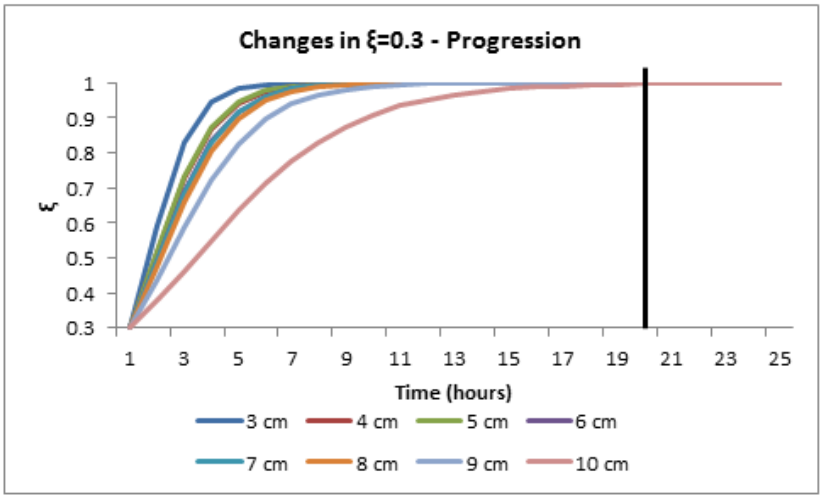
Figure 5.5 The total time in labor for each dilation state and varying values of ξ_i. The horizontal line represents 24 hours.

5.3.2 Posterior Problem with Observations

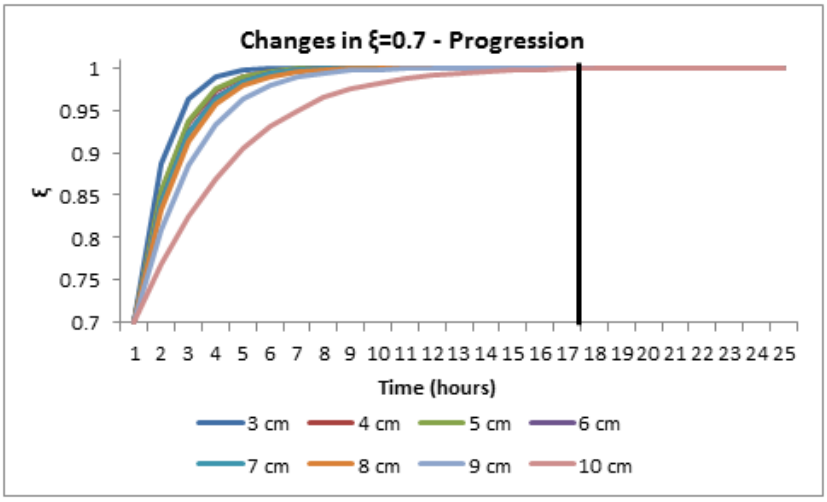
The posterior, $\xi_i^n(X)$, evolves over time according to the observations observed. We consider two types of $\xi_i^n(X)$ values. That is, $\xi_i^n(X=1)$ when a change is always observed and $\xi_i^n(X=0)$ when no change is always observed. For the $\xi_i^n(X)$ values in which there is always a change, ξ_i^n will eventually reach 1 as n approaches infinity. Alternatively, as n approaches 0 (or time approaches infinity), $\xi_i^n(X)$ for which there is no change will eventually reach a value of 0. We show the progression of this calculation in Figures 5.6 and 5.7 for initial ξ_i values of 0.3 and 0.7 along with the point at which all dilation states reach 0 and 1 for both prior belief values.

When ξ_i is allowed to change as labor progresses, the results provide greater insight into the optimal policy. We present the results of three three MDPs which represent the extreme cases of the problem and the mixed problem. The first MDP represents the optimal policy when no change is observed for every time period (Figure 5.8). The second MDP represents the optimal decision when a change is observed for each time period (Figure 5.9). The last MDP we consider is the mixed problem which is a weighted combination of the $\overline{ftp}$ and ftp and the change and no change cases. We compare prior ξ_i values of 0.1, 0.5, 0.9, and 1 for the "change" and "no change" MDP. The policies for these two MDPs differ greatly. In the "no change" case, CS is decided for more time periods and dilation states for $\xi_i = 0.1, 0.5,$ and 0.9. Whereas ToL is the decision for the same values of ξ_i for the change MDP policy. For $\xi_i = 1$, the policy is the same for both MDPs. For $\xi_i = 0$ and $\xi_i = 1$, the updated values of $\xi_i^n(X)$ remain 0 and 1, respectively.

We chose prior ξ_i values of 0.1, 0.3, 0.5, 0.7, 0.9, and 1 to see the effect these particular

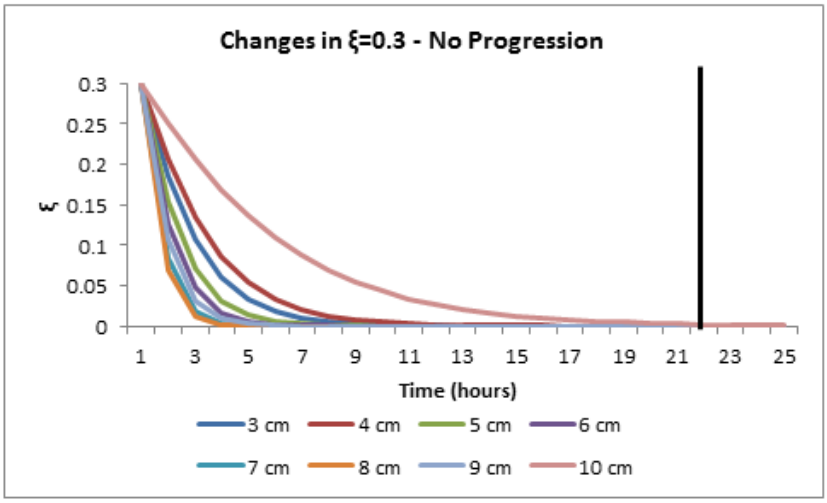


(a) Prior belief $\xi_i = 0.3$.

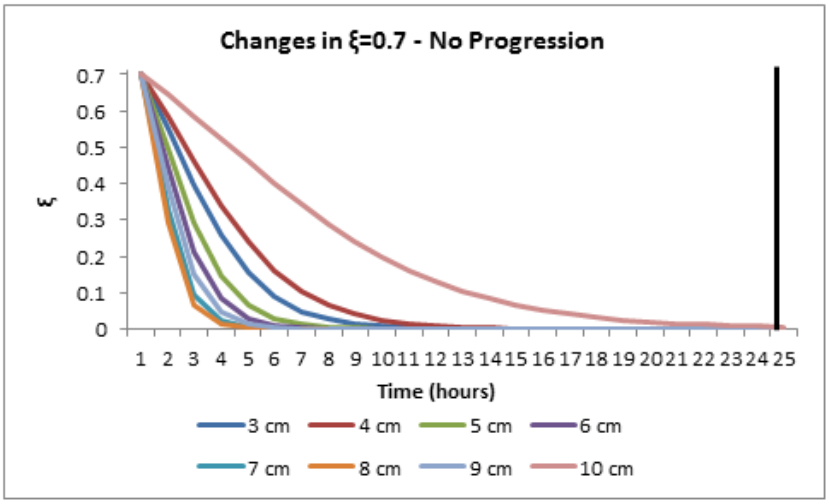


(b) Prior belief $\xi_i = 0.7$.

Figure 5.6 The increase of ξ_i when starting from 0.3 and 0.7 belief values when time in labor is increased from 1 hour to 24 hours. The vertical line shows the time period in which all dilation states reached a ξ_i value of 1.

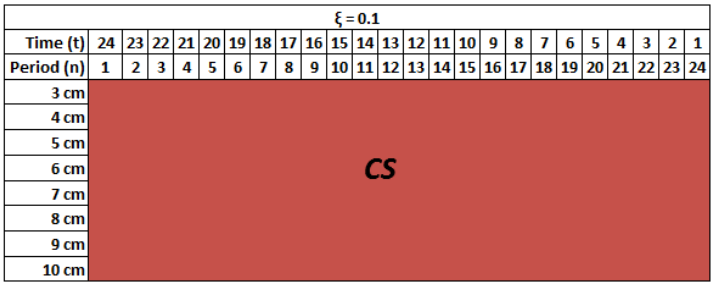


(a) Prior belief $\xi_i = 0.3$.

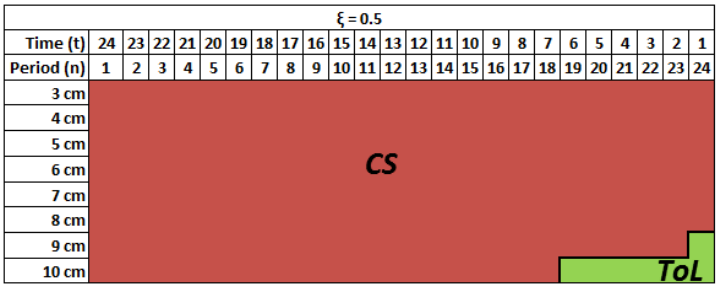


(b) Prior belief $\xi_i = 0.7$.

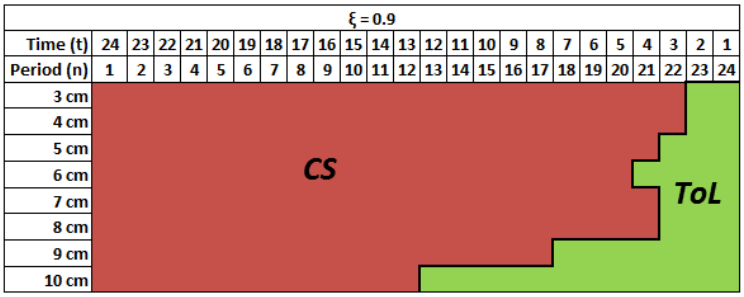
Figure 5.7 The decrease ξ_i when starting from 0.3 and 0.7 belief values when time in labor is increased from 1 hour to 24 hours. The vertical line shows the time period in which all dilation states reached a ξ_i value of 1.



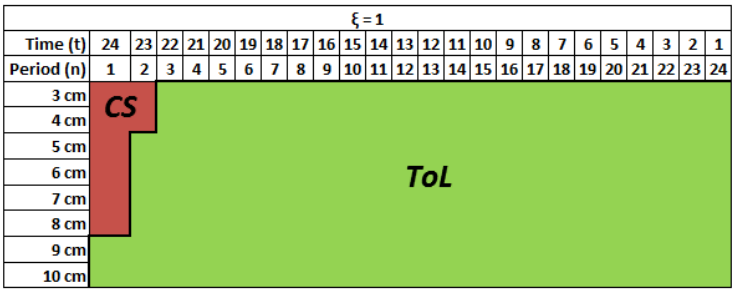
(a) $\xi_i = 0.1$



(b) $\xi_i = 0.5$

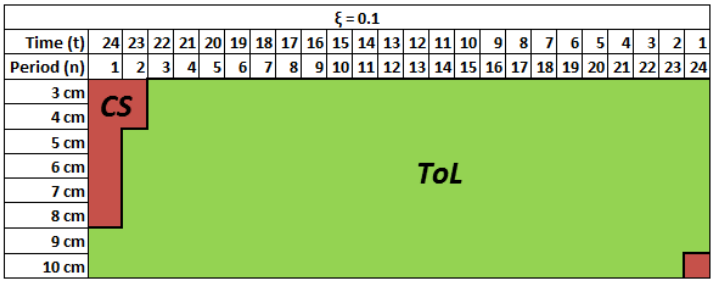


(c) $\xi_i = 0.9$

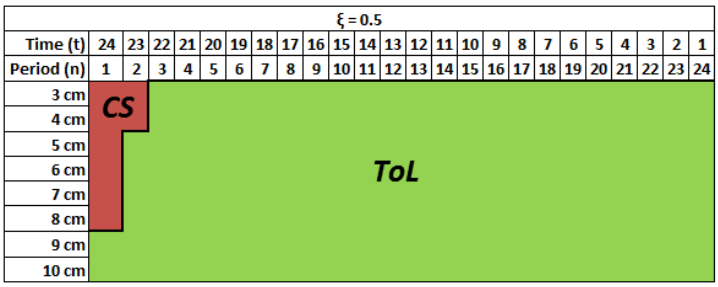


(d) $\xi_i = 1$

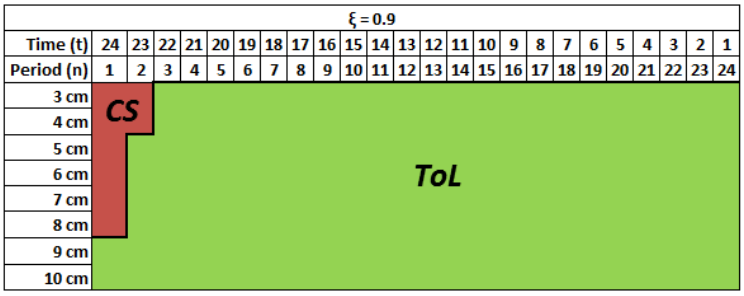
Figure 5.8 Optimal policy for the no change MDP.



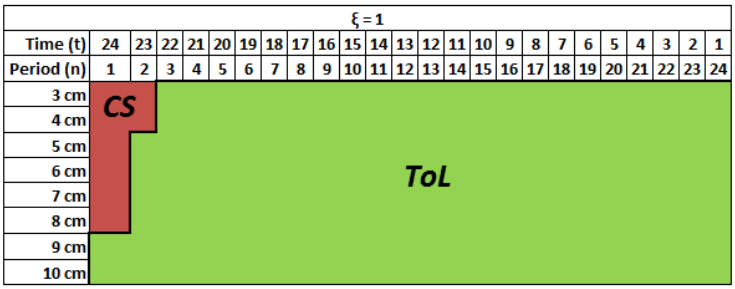
(a) $\xi_i = 0.1$



(b) $\xi_i = 0.5$



(c) $\xi_i = 0.9$

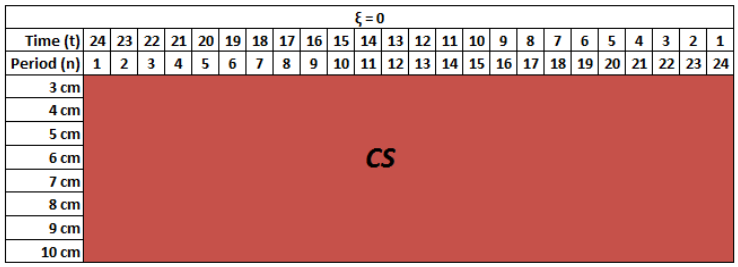


(d) $\xi_i = 1$

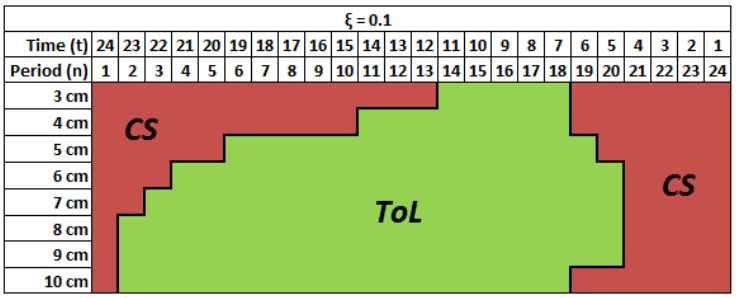
Figure 5.9 Optimal policy for the change MDP.

values have on the optimal decision for the combined Bayesian MDP. These results are provided in Figure 5.10. For $\xi_i = 0$, the solution is C-section for each dilation state and each time period. Since ξ_i represents the belief that the patient is $\overline{ftp}$, then ξ_i indicates that the decision maker believes the patient will not have a successful trial of labor and therefore a trial of labor should not be entered. For ξ_i values greater than 0, there is a possibility that the patient will have a successful trial of labor and ξ_i values closer to 1 indicate a strong belief that the patient should remain in labor. We see this in the results. As ξ_i increases, so does the number of time periods in which ToL is optimal. Similar to the results presented in Figure 5.5, for $\xi_i = 1$ there are still dilation state/time period combinations for which CS is optimal.

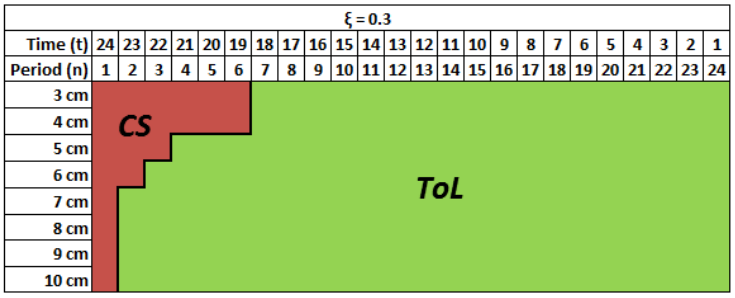
For ξ_i greater than 0 and lower than 0.26 we notice that C-section is the optimal action at the beginning and end of the time horizon. This pattern is shown in Figure 5.10b which shows CS as the preferred action up to a point where ToL is preferred then CS is the optimal decision again. Values of $0 < \xi_i \leq 0.26$ represent a less than 26% chance that the patient will have a successful trial of labor. The results show that the best decision if a patient is beginning labor then a CS is best. However, if the patient has been laboring a while and if the starting belief state for the patient is 26% or lower then CS may be the best decision. That is, if a patient is observed at 4 cm and $\xi_4 = 0.1$ with $n = 20$ or more hours remaining in the 24 hour period (i.e., $t = 5$) then the decision is to perform a C-section because the starting belief that the patient will successfully complete the trial of labor is so low. If the patient was observed at $n = 18$, i.e., after laboring for 6 hours, then the decision would be to continue labor. When there are 10 or fewer hours remaining in the 24 hour period, CS is the best decision decision again. The optimal policy encompasses the possible paths



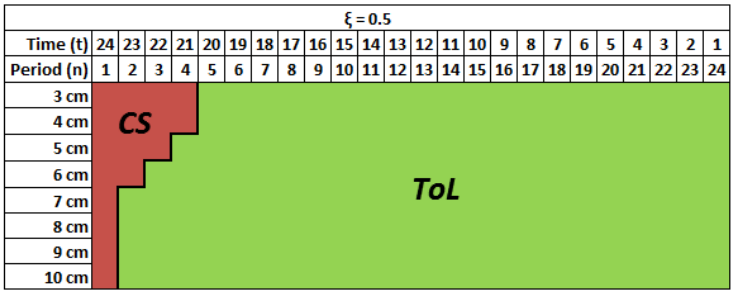
(a) $\xi_i = 0$



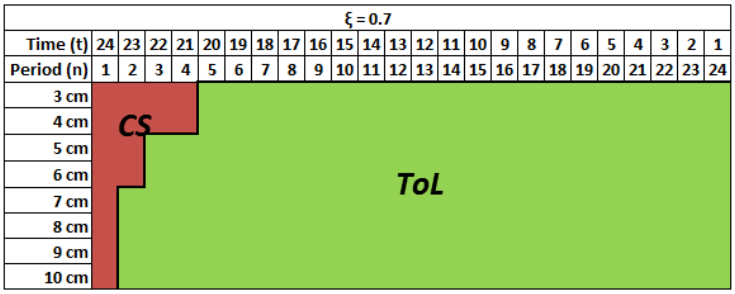
(b) $\xi_i = 0.1$



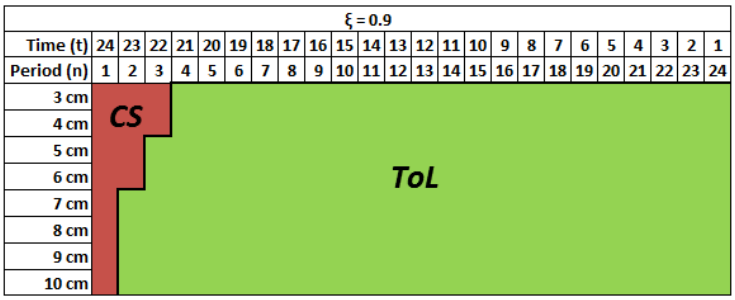
(c) $\xi_i = 0.3$



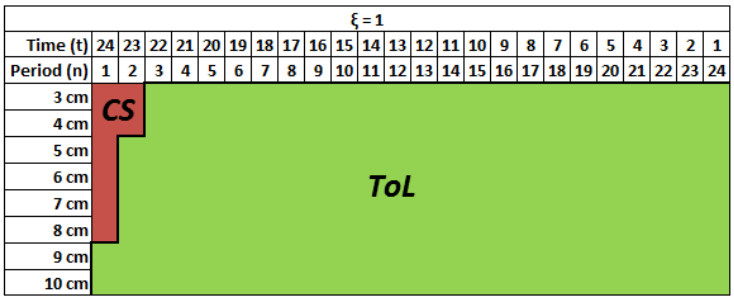
(d) $\xi_i = 0.5$



(e) $\xi_i = 0.7$



(f) $\xi_i = 0.9$



(g) $\xi_i = 1$

Figure 5.10 The optimal policy for when a C-section is needed during a trial of labor for ξ_i values of 0.1, 0.3, 0.5, 0.7, 0.9, and 1

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that may occur given a prior belief of 0.1. This solution shows that when the belief that the patient will have a successful trial of labor is low, it is important to be careful to monitor the amount of time the patient remains in labor. At the beginning of the time horizon and end of the horizon CS is best due to the low likelihood that the patient will deliver in an adequate amount of time. However, if the patient is observed in the middle of labor, even with a low starting belief value, it is best to let the patient labor in hopes that the patient will have a successful trial of labor.

5.4 Discussion

In the problem formulation, there is an initial belief (the prior) which evolves over time according to labor progression or lack thereof. If the decision maker has a prior belief of 0 then, it is advised for the patient to have a C-section for all time periods. If the decision maker has a belief value of 1, then there is a preference for trial of labor. However, there are still dilation state and time period combinations for which a C-section is optimal. If the decision maker knows the patient type, there may be fewer "unnecessary" C-sections. There would also be support to allow some patients to labor longer under certain circumstances. In order for the model to be accurate, an understanding of how to capture the initial belief for the decision maker is important since it plays a significant role in the selection of the optimal decision. Although this problem formulation combines Bayesian updating and principles of MDPs, it is an approximation. A more precise, though computationally complex, evaluation would be to capture all the paths that are possible within the 24 hour time period, which we leave as future work.

The Bayesian MDP presented in this chapter is a novel approach to the mode of delivery

decision making process. In Chapter 3, we presented a Bayesian decision analysis that takes into account the belief regarding patient type. The goal was to determine when it is valuable to continue a trial of labor by gathering more information before making a decision regarding mode of delivery. In Chapter 4, we presented an MDP which provides the optimal policy regarding mode of delivery for a 24 hour time period. In the MDP, it is assumed that the patient type does not alter the likelihood of labor progression. Since labor progression can differ depending on patient type (failure-to-progress patients vs not-failure-to-progress patients), the Bayesian MDP approach combines elements of both Bayesian decision analysis and MDP principles to provide more insightful results of when a trial of labor should end dependent on the belief of patient labor progression.

CHAPTER

6

CONCLUSION

This collection of work provides analytical and simulation models to inform decision making related to birth with the goal of helping to facilitate communication between patients and their healthcare providers regarding birth. These stochastic decision models take into account the mother's and child's health outcomes and explore the optimal policy regarding ending a trial of labor in the event that a cesarean section (C-section) is needed. When a woman enters labor, she will deliver in one of two ways: vaginal delivery or cesarean delivery. Because of the increased risk of infection and complications, allowing patients

who require a C-section to continue labor is not recommended. However, performing a C-section on patients believed to be able to have a successful vaginal delivery can have a large impact on their future health. To evaluate this trade-off, we have developed decision models that can be used to analyze and support efforts to address the increasing C-section rate due to indications which have variable clinical evidence such as a "failure-to-progress." Failure-to-progress (or labor arrest) refers to a lack of progressive cervical dilation or a lack of fetal descent. This phenomenon occurs when effective labor contractions do not lead to delivery of the baby within an established time frame. If a patient's labor progression suggests she will not deliver "in time," it is decided that she should receive a C-section.

This decision making process is illustrated in Ruby's Birth Story in Chapter 1. In this short narrative, the decision maker was confronted with whether to continue labor or perform a C-section. Given the current state of the patient and the expected complications that may arise from prolonging labor, it was decided that a C-section would be the best course of action. Our work focuses on this decision making process using labor progression as the basis for when a trial of labor should end.

Our initial analysis into this decision making process is the formulation of a discrete event simulation (DES) (presented in Chapter 2), which provides an understanding of the natural progression of labor, models the current rules used in labor to match current C-section rates, and provides insight into how rules should be developed to decrease both the C-section rate and the rate of complications. In addition to providing insight into the labor process, this simulation model is used to develop input parameters into our decision models. In Chapter 3, we use Bayesian decision analysis to develop policies for when labor should continue based on the belief that the patient will have a successful trial of labor. In

Chapter 4, we present a Markov decision process (MDP) which determines the optimal policy of when a labor should end during a 24 hour time period in order to maximize the expected utility of health outcomes and the delivery process. Lastly, in Chapter 5 we introduce a Bayesian MDP which provides the optimal mode of delivery as a function of patient type which is unknown.

The DES presented in Chapter 2 is an extensive analysis of the labor process. We use summary statistics from published literature sources to build a population of laboring patients through the development of a percentile matching procedure. Assuming the underlying distribution of the laboring process is lognormally distributed, we estimate the parameters of the distribution using the 5th, 50th, and 95th percentile for three laboring patient types: augmented labors, induced labors, and spontaneous labors. We use the DES to develop rules that can be used in labor to lower the C-section rate and the rate of complications in addition to providing insight into labor progression in absence of C-sections. Outcomes of this DES are the labor curve we develop to show how labor progresses for different patients compared to the Friedman Curve, which was established in the mid-1950s.

The Bayesian decision analysis presented in Chapter 3 determines under what conditions it is appropriate to prolong labor in order to learn more about the patient before deciding the "best" mode of delivery. This model determines the conditional value of information and determines the conditions under which information has positive value. If the decision maker knows the outcome of the delivery, then he/she will choose the best mode of delivery which minimizes the risk of health outcomes and maximizes the overall utility of the delivery process. However, since this information is not available, the decision maker must make a decision of when to continue labor in order to learn more about the

patient or when to end labor. The trade-off in this problem is between waiting too long to make a decision and rushing to make a decision which is incorrect.

In Chapter 4, we develop a finite-horizon MDP to evaluate the appropriate mode of delivery in a 24-hour time period in order to maximize the expected utility of health outcomes for both patients and the valuation of the delivery process. The MDP is an extension of a decision tree in which the trade-off between a trial of labor and C-section is evaluated. A decision tree provides a one-time solution, whereas the MDP provides the solution for multiple time periods by incorporating the stochastic and dynamic nature of the patient's condition. The MDP determines the best mode of delivery for each time period over a horizon of 24 hours based on the valuation of health outcomes and disutility associated with the delivery process making the problem adaptable to various scenarios.

The Bayesian MDP, presented in Chapter 5, predicts the optimal mode of delivery for each time period as a function of cervical dilation progression and patient type (i.e., failure-to-progress or not). In this problem formulation we expand the trial of labor action to include two probability transition matrices that represent two types of labor progression patterns. We consider the probability transition matrix for not-failure-to-progress patients and failure-to-progress patients and combine them using prior and posterior values based on the decision maker's belief regarding patient type. This work integrates the work of Chapters 2, 3, and 4. It also extends the Bayesian MDP literature by considering a dynamic observation space.

The decision models described above are from the perspective of a single decision maker (i.e., the provider or the patient). This research has opened up the opportunity for many extensions including utility elicitation and shared decision making. As a next step in

my research, I will extend my dissertation work by developing data-driven mathematical frameworks for shared decision making. This work will utilize stochastic modeling and game theory to evaluate the decision of when a C-section is needed during labor taking into account the risk perceptions and preferences of both decision makers. The results of this work can be used to develop a decision aid tool to facilitate communication between women and their healthcare providers to explore various options and the risks associated with them. In addition to a decision tool, this work can be used to influence policy regarding the time a woman is allowed to labor.

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