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MODAL COMBINATION OF SEISMICALLY INDUCED HYDRODYNAMIC LOADING IN LIQUID STORAGE TANKS

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ABSTRACT

A comprehensive and statistically significant analytical investigation is presented herein which evaluates the appropriate modal combination method for seismically induced hydrodynamic loading in liquid storage tanks. The study examines the Square Root Sum of Squares (SRSS) and Absolute Sum (ABS) modal combination methods for hydrodynamic loading due to the impulsive and convective (sloshing) mode responses. Six liquid storage tanks of varying geometry and dynamic characteristics are analyzed for a set of 173 earthquake events with 3,551 earthquake ground motion station recordings, each station recording has two directional components for a total of 42,612 analysis realizations. The maximum moment and shear at the base of the tank are calculated using SRSS and ABS modal combination solutions. The modal combination solutions for maximum tank response are then compared with the exact time history solution. It is concluded that SRSS modal combination predicts the mean maximum response with a 4% conservative bias and a coefficient of variation of 8%. The ABS modal combination is always conservative and overestimates the mean response by 21% with a coefficient of variation of 17%. The results were statistically similar for a subset of 91 pulse-like ground motion components.

INTRODUCTION

The purpose of this study is to examine the Square Root Sum of Squares (SRSS) and Absolute Sum (ABS) modal combination methods for hydrodynamic loading in liquid storage tanks and present relevant statistics on the results from each method. There are many studies published on modal combination techniques for typical building structures (NRC Reg. Guide 1.92, 2006); however, there are several unique factors that could potentially affect the result for hydrodynamic response in liquid storage tanks. These factors include the large difference in modal damping for the impulsive and convective mode as well as the timing and phasing of the maximum impulsive and convective modal responses. Due to the large separation in modal frequencies, the SRSS combination method is typically used and recommended in the literature (ASCE 4, 1998); however, there is not, to the authors' knowledge, a published and statistically significant study to document this conclusion. The purpose of this study is to run a statistically significant number of analyses from which one can draw conclusions on the adequacy and variability associated with SRSS versus ABS modal combination.

The impulsive mode typically has a short period (0.06 to 0.34 seconds) and the convective mode typically has a longer period (2.7 to 6.8 seconds) for the six example storage tanks considered by this study. The impulsive mode damping typically ranges from 2% to 4% depending on the level of inelastic response of the tank walls, whereas the convective mode period is typically assumed to be 0.5% (ASCE 43-05). General thinking has hypothesized that in the impulsive mode, the maximum response occurs early on during the strong motion portion of an earthquake, whereas the convective (sloshing) mode may take a while to build up to the maximum response. This and the highly different modal damping values has led to some questions regarding the application of typical modal combination techniques.

TANK PROPERTIES

A total of six unique tank geometries were selected to provide a range of example liquid storage tanks for analysis. The selected tanks are intended to be representative of those used in heavy industrial, nuclear and oil storage facilities. An overview of the tank geometries is graphically represented in Figure 1 and detailed information regarding tank properties are provided in Table 1. The impulsive and convective dynamic properties were calculated in accordance with the provisions of ASCE 4-98, ACI 350.3 and Bandyopadhyay et al. 1995 and are reported in Table 1. Tanks were selected to provide a range of representative dynamic properties including variation in the period decoupling ratio aimed at evaluating the influence of period separation. Within this study, the convective and impulsive modal damping levels were considered as 0.5% and 4%, respectively. These damping levels are consistent with industry standards for high seismic events where some damage to the tank shell may be expected (ASCE 43, 2005).

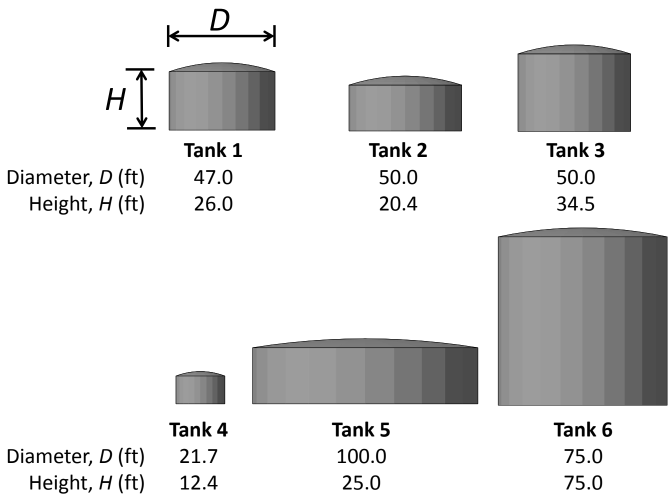

Figure 1: Tank Properties Used in this Study

Table 1: Summary of Tank Dynamic Properties

Tank Properties	Sym-bol	Units	Tank 1	Tank 2	Tank 3	Tank 4	Tank 5	Tank 6
Diameter	D	ft	47	50	50	21.7	100	75
Height (to top of liquid)	H	ft	26	20.4	34.5	12.4	25	75
Diameter to height ratio	D/H	--	1.81	2.45	1.45	1.75	4.00	1.00
Weight of Tank + Fluid	W_t	kip	4,617	2,940	4,310	297	11,468	20,947
Specific gravity of fluid	$S.G.$	--	1.60	1.15	1.00	1.00	0.90	1.00
Steel wall thickness	T	in	1	0.291	0.375	0.188	0.75	0.5
Impulsive Frequency	f_i	Hz	13.1	9.7	7.2	17.9	8.7	2.9
Convective Frequency	f_c	Hz	0.237	0.233	0.243	0.367	0.148	0.200
Impulsive Period	T_i	Sec	0.08	0.10	0.14	0.06	0.11	0.34
Convective Period	T_c	Sec	4.2	4.3	4.1	2.7	6.8	5.0
Period Decoupling Ratio	T_c/T_i	--	55	42	30	49	59	15
Height of Impulsive mode*	h_i	ft	11.0	7.8	12.9	4.7	9.4	30.4
Height of Convective mode*	h_c	ft	16.4	11.8	22.9	7.8	13.3	55.6
Weight of Impulsive mode	W_i	kip	2,586	1,316	2,862	171	3,177	16,168
Weight of Convective mode	W_c	kip	1,903	1,466	1,391	112	7,352	4,749
Modal Weight Ratio	W_c/W_i	--	0.74	1.11	0.49	0.66	2.31	0.29
Source			Hanford RPP-WTP	BNL 1995 Rpt. Appx. G	Steve Short	Steve Short	API oil storage	API oil storage

**Height excluding bottom pressure to determine moments just above the base of the tank*

Figure 2 graphically shows the variability of the tank properties used in this study. Figure 2(a) shows a wide range of convective periods used from 2.7 to 6.8 seconds which vary approximately linearly with tank diameter, *D*. Figure 2(b) shows a wide range of impulsive periods used from 0.06 to 0.34 seconds (2.9 to 17.9 Hz) which vary approximately linearly with tank height, *H*. Figure 2(c) shows that for squat tanks, small slenderness ratio *H/D*, the modal mass of the convective mode, m_c, can be up to 2.3 times the modal mass of the impulsive mode, m_i, and thus can play a dominant role in seismic response. Figure 2(d) shows that the impulsive and convective modal periods are highly separated, ranging from T_c/T_i from 15 to 55.

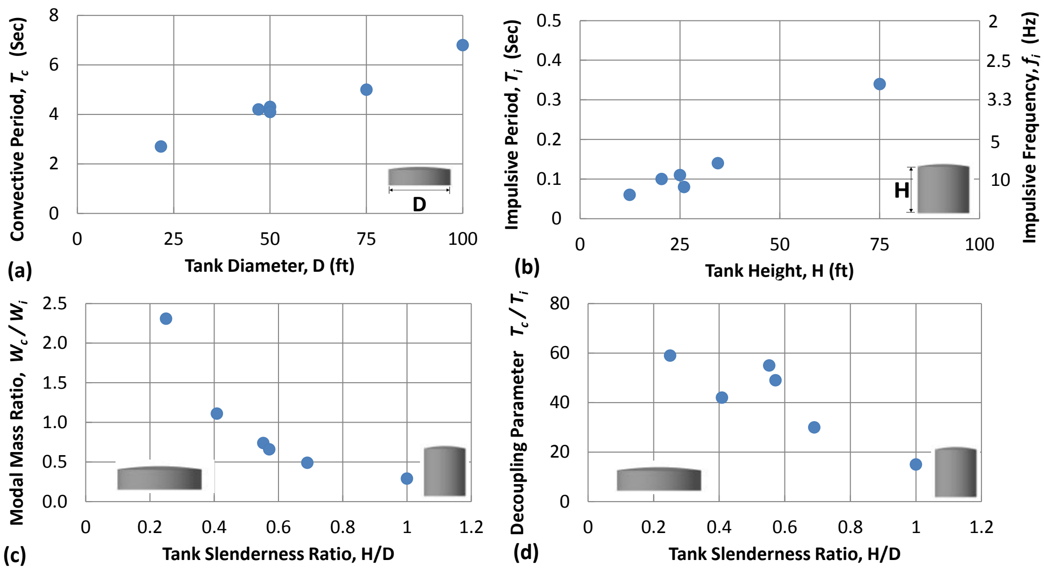

Figure 2: Tank parameters, (a) convective period, T_i (b) impulsive period T_c, (c) modal mass ratio W_c/W_i, and (d) decoupling parameter T_c/T_i

GROUND MOTIONS

The 2005 PEER database (PEER Ground Motion Database, 2005) was used to obtain ground motion acceleration histories as seismic input at the base of the tanks. At the time of this study, the database contains station recordings from 173 unique earthquake events from diverse locations (Italy, Russia, Turkey, USA, Mexico, Iran, Taiwan, Greece, Trinidad, New Zealand, Canada and Japan). Each event may have multiple station recordings at different distances from the epicenter. In total the PEER database at the time of this study contained 173 unique earthquakes with 3,551 station recordings (ground motions). Each ground motion may have multiple directional component recordings (typically 2 orthogonal horizontal components and 1 vertical component). Only the 2 horizontal components were used in this study. To study the effect of fault-normal pulse-like motions, a subset of 91 out of the 3,551 ground motions which were identified by Baker (1992) were also considered by this study. It is noted that permanent ground offsets are filtered out by the PEER filtering process, therefore, permanent ground displacement offsets are not considered by this study.

The PEER database motions are from a variety of faulting mechanisms and moment magnitude (M_w) and epicentral distance (r_{epi}) pairs. Figure 3 shows the seismic hazard deaggregation for all ground motions and pulse-like motions. In general the database contains ground motions from events that have an epicentral distance less than 150 km (~94 miles) with moment magnitudes between 4.5 and 8. The 91 pulse-like motions, identified by Baker (1992) in general are closer in epicentral distance and have a larger magnitude (epicentral distance generally less than 100 km (~62 mi) with moment magnitudes between 6 and 8).

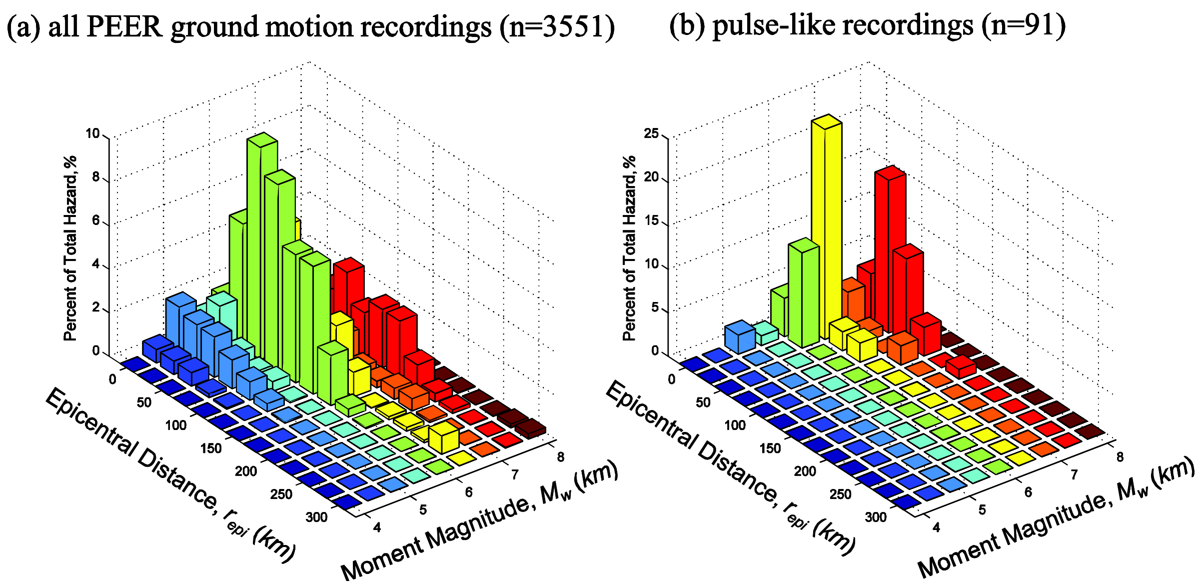

Figure 3: Seismic hazard deaggregation plots for (a) all PEER database ground motion recordings and (b) pulse-like recordings, note, one recording consists of 2 orthogonal horizontal components measured at one recording station

ANALYSIS METHODOLOGY

Common simplified seismic analysis techniques for liquid storage techniques involve solving for the convective mode response (sloshing) and impulsive mode response separately by evaluating the response of two Single Degree of Freedom (SDOF) oscillators and then combining the modal responses to obtain the Multiple Degree of Freedom (MDOF) total system response e.g. Veletsos and Yang (1977); Haroun and Housner (1981); and ASCE 4 (1998). This modeling technique is described graphically in Figure 4.

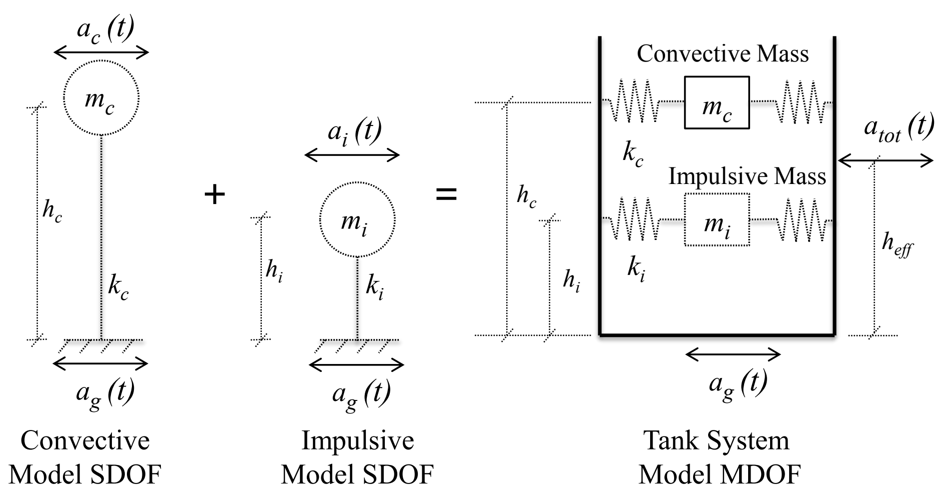

Figure 4: Tank system model (MDOF) and convective and impulsive (SDOF) models

Response History Analysis and Modal Combination

The response history analysis using the tank model in Figure 4 can be considered the exact solution for later comparison against approximate modal combination methods. In response history analysis, the convective mode acceleration response with respect to time $a_c(t)$ is calculated for ground motion input $a_g(t)$ using numerical integration methods for a Single Degree of Freedom (SDOF) system. Likewise, the impulsive mode acceleration response with respect to time $a_i(t)$ is calculated for ground motion input $a_g(t)$. The Multiple Degree of Freedom (MDOF) tank system response with respect to time $a_{tot}(t)$ is then calculated using Equation 1 as the additive response of the two SDOF systems.

$$a_{tot}(t) = a_c(t) + a_i(t) \tag{1}$$

Other response quantities such as total base shear $V_{tot}(t)$ are calculated in the same manner as the additive response with respect to time of the convective mode base shear $V_c(t)$ and the impulsive mode base shear $V_i(t)$. The response moment just above the base of the tank $M_{tot}(t)$ and other response quantities are calculated in a similar manner. The convective and impulsive mode periods are calculated based on the tank and fluid properties. The modal damping is assumed to be 0.5% and 4% for the convective and impulsive modes, respectively, in response to high seismic events where some damage to the tank may be expected, e.g. ASCE 43 (2005). The tanks are assumed to be filled with water to level H for all analysis and are assumed to have sufficient freeboard so that all liquid is retained and no fluid impact occurs with the roof. Figure 5 shows the response history analysis for tank #6 which has an aspect ratio of D/H=1.0, where the tank depth is D and tank height is H. It is noted that the impulsive and convective maximum modal responses do not occur at the same time.

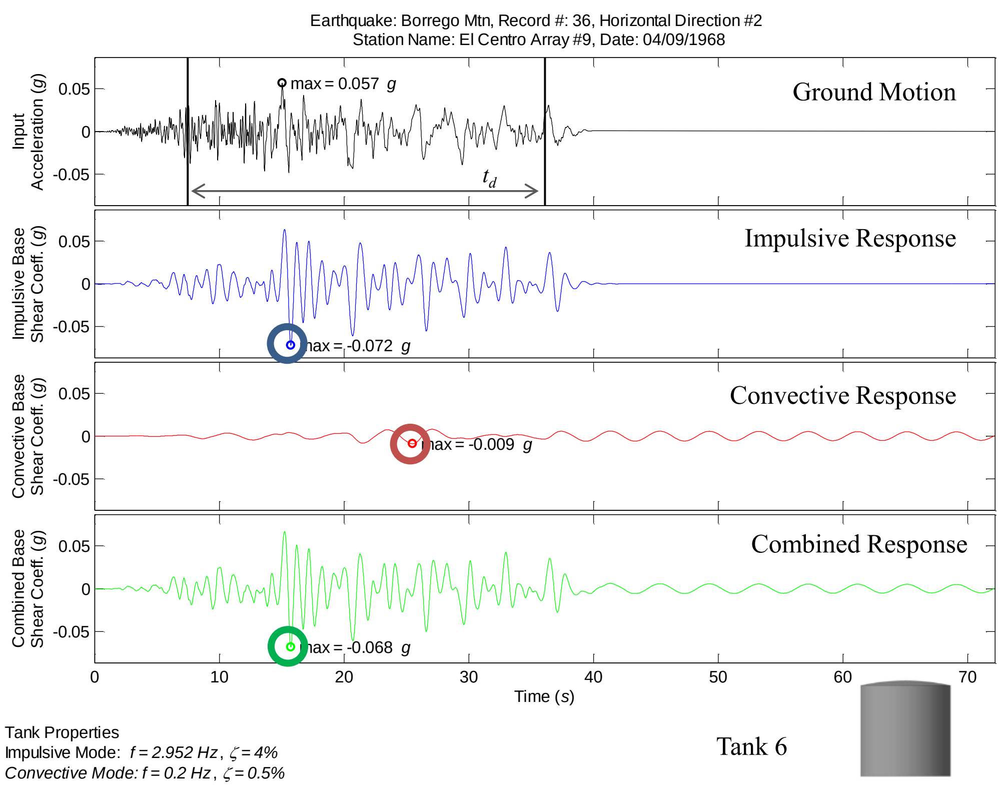

Figure 5: Typical analysis results, tank #6, subjected to one ground motion horizontal component, total response dominated by impulsive mode for medium aspect ratio tank (*D/H*=1.0)

In Figure 5, the input acceleration is the ground motion acceleration $a_g(t)$ which has an effective strong motion duration t_d as defined the time interval between when 5% and 95% arias intensity of the ground acceleration are achieved (Arias, 1970). The impulsive mode response and convective mode response are added at each time step to achieve the combined response of the tank system. The base shear coefficients for the convective and impulsive modes ($C_c(t)$ and $C_i(t)$ respectively) are equal to the base shear in each mode divided by the total weight of the tank. Likewise, the combined response base shear coefficient, $C_{tot}(t)$, in Figure 5 is calculated by Equation 2 as the combined convective and impulsive base shears divided by the total mass of the tank, m_{tot}. Figure 5 reveals that the response of tank #6, having a low D/H ratio of 1.0 (relatively tall and narrow), is dominated by the impulsive response.

$$C_{tot}(t) = \frac{a_i(t)m_i + a_c(t)m_c}{m_{tot}} \tag{2}$$

In contrast to Figure 5, results from a sample earthquake on a squat tank is shown in Figure 6. In this sample result, the maximum impulsive response, $A_{max,i}$, is equal to -0.018g and occurs at approximately 15 seconds and the maximum convective response, $A_{max,c}$, is equal to -0.039g, occurring at approximately 36 seconds. The convective response dominates the response of this squat tank. It is noted that the actual maximum combined response of the tank is 0.038g and occurs at approximately 33 seconds, not corresponding to the time of either of the modal maximum responses.

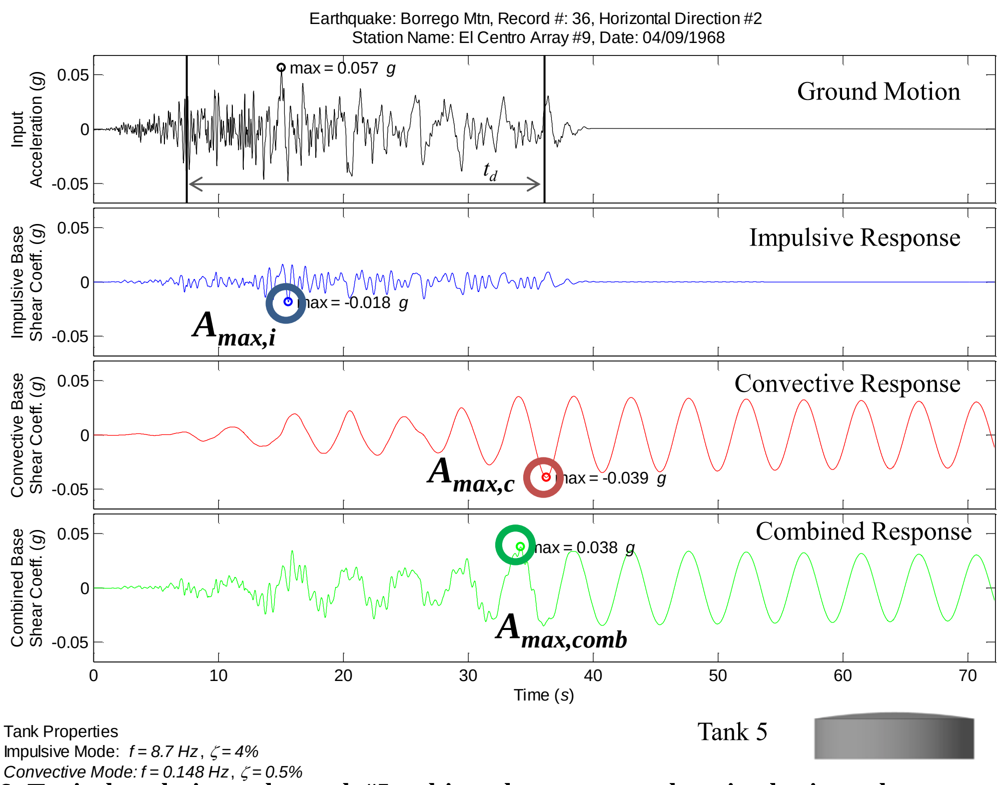

Figure 6: Typical analysis results, tank #5, subjected to one ground motion horizontal component, total response dominated by convective mode for squat aspect ratio tank (*D/H*=4.0)

For modal combination, only the maximum response parameters for a given mode are considered. These maximum values are combined mathematically using either the modal combination techniques of SRSS and ABS as defined by the following equations:

$$A_{\max,SRSS} = \sqrt{A_{\max,i}^{\ 2} + A_{\max,c}^{\ 2}} \tag{3}$$

$$A_{\max,ABS} = \left|A_{\max,i}\right| + \left|A_{\max,c}\right| \tag{4}$$

RESULTS

A total of 173 unique earthquake events with an associated 3,551 multi-component ground motion recordings were analyzed for six distinct tank configurations resulting in a total of 42,612 analysis realizations. These analyses were conducted to evaluate the variability in simplified modal response combination methods for tanks subjected to seismically induced hydrodynamic loading due to impulsive and convective (sloshing) mode responses. The intent of these analyses is to determine an appropriate modal combination technique for design and evaluation purposes and present statistically significant results.

For evaluation purposes, the shear and moment ratios are considered which relate the maximum exact response from response history analysis and the approximate maximum response from the modal combination methods. A ratio in excess of 1.0 indicates the approximate modal combination method results in loading that is greater than the exact method and thereby a conservative estimate of system response. While both shear and moment ratios have been evaluated during this study, certain results are presented only for one parameter due to similarity in results and conclusions between moments and shears.

Comparison of SRSS and ABS Results

The cumulative distribution function (CDF) of tank base shear ratio ($V_{max}/V_{TH,max}$) is given in Figure 7 where V_{max} is the maximum base shear by modal combination and $V_{TH,max}$ is the exact response history maximum. The cumulative distribution function (CDF) of tank base moment ratio ($M_{max}/M_{TH,max}$), just above the base of the tank not including bottom pressures, is given in Figure 8 where M_{max} is the maximum base moment by modal combination and $M_{TH,max}$ is the exact response history maximum. It is noted in both Figure 7 and Figure 8 that the ABS method is always conservative (i.e. greater than 1.0), as it mathematically must be.

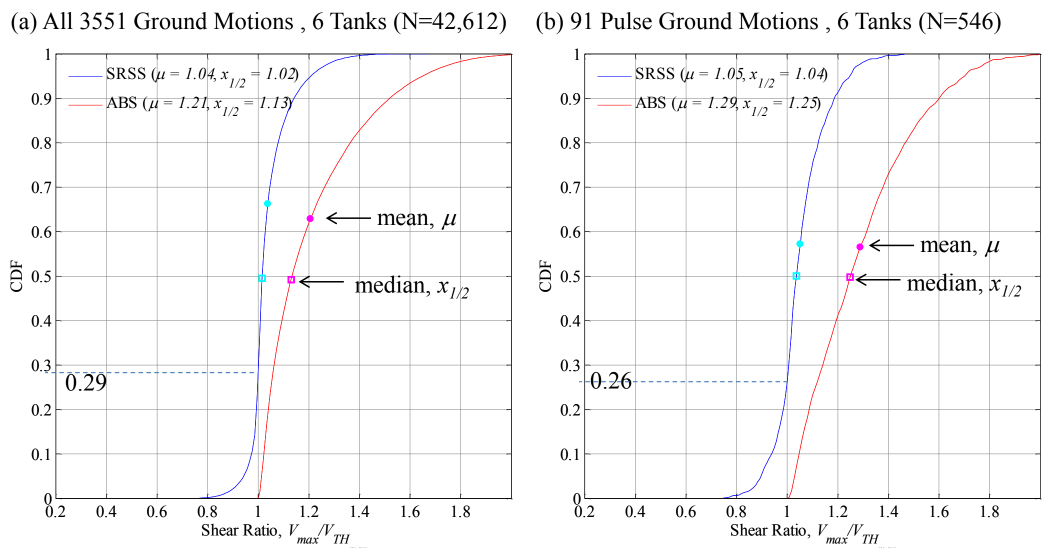

Figure 7: Cumulative distribution function (CDF) of tank base shear ratio ($V_{max}/V_{TH,max}$) for N-realizations for (a) all ground motions and (b) subset of pulse-like motions

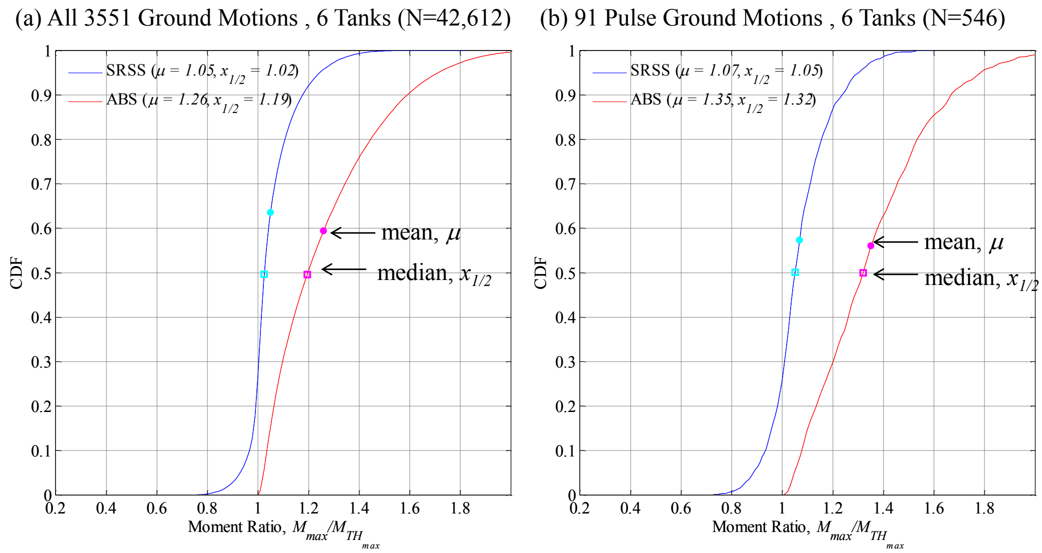

Figure 8: Cumulative distribution function (CDF) of tank base moment ratio ($M_{max}/M_{TH,max}$) for N-realizations for (a) all ground motions and (b) subset of pulse-like ground motions

Table 2 summarizes the mean, median, maximum and minimum moment ratio and shear ratio for all ground motions and the subset of pulse-like ground motions for both the SRSS and ABS modal combination methods. The standard deviation and coefficient of variations are also reported on the mean responses. The SRSS combination mean response ranged between 1.038 and 1.068, while the ABS combination mean response ranged between 1.206 and 1.351. The SRSS combination coefficient of variation ranged between 8.0% and 11.5%, while the ABS combination coefficient of variation ranged between 16.4% and 17.7%.

Table 2: Results for all 3,551 ground motion records x 2 directions x 6 tanks (n=42,612) and for 91 pulse-like ground motion records x 1 directions x 6 tanks (n=546)

	Shear Ratio ($V_{max}/V_{TH,max}$)				**Moment Ratio** ($M_{max}/M_{TH,max}$)			
	All Records (n=42,612)		Pulse Records (n=546)		All Records (n=42,612)		Pulse Records (n=546)	
	SRSS	ABS	SRSS	ABS	SRSS	ABS	SRSS	ABS
Mean (μ)	1.038	1.206	1.051	1.289	1.049	1.259	1.068	1.351
Median ($x_{1/2}$)	1.016	1.130	1.036	1.249	1.024	1.195	1.051	1.320
COV	0.080	0.171	0.098	0.164	0.092	0.177	0.115	0.170
Maximum	1.673	2.326	1.472	2.037	1.834	2.592	1.592	2.251
Minimum	0.712	1.000	0.741	1.001	0.714	1.000	0.714	1.003
Std Dev. (σ)	0.083	0.207	0.103	0.211	0.096	0.223	0.122	0.230

Note: a ratio > 1.0 means the modal combination is conservative

Highlighting and summarizing the results for the average (mean) shear ratio, the upper left shaded corner of Table 2 shows that on average that SRSS combination is 3.8% conservatively biased while ABS is 20.6% conservatively biased when looking at mean results for all records. Additionally SRSS has a lower coefficient of variation, 8.0% COV when compared with ABS, 17.1% COV for all records. The results are statistically similar for the subset of pulse-like motions. On average, SRSS combination is 5.1% conservatively biased while ABS is 28.9% conservatively biased when looking at mean results for the subset of pulse-like records. Additionally SRSS has a lower coefficient of variation, 9.8% COV when compared with ABS, 16.4% COV for pulse-like records. The results demonstrate that if one is targeting a mean maximum response that the SRSS is the more appropriate method of modal combination and SRSS combination has a lower coefficient of variation.

Effect of Earthquake Magnitude

Results of the series of seismic analysis were post-processed to evaluate the influence of earthquake magnitude on the modal combination methods. Figure 9 presents the moment ratio comparing the approximate to exact analytical solution for all records and pulse records. Based on the individual analysis results, a trend line is incorporated within the plots to evaluate the variation in mean response with respect to earthquake moment magnitude. Figure 9(a) indicates essentially no effect on the response parameters when considering the SRSS combination method for all records, and a slight negative correlation for pulse records. When considering all records and pulse records under the ABS modal combination method, Figure 9(b) indicates a positive and similar correlation with moment magnitude. Based on these results, the SRSS combination method is essentially unaffected by moment magnitude over the range of M_w 4.5 to 8.0, whereas the ABS combination method becomes more conservative with increasing moment magnitude. Results for the shear ratio indicate similar trends and are not presented herein for brevity.

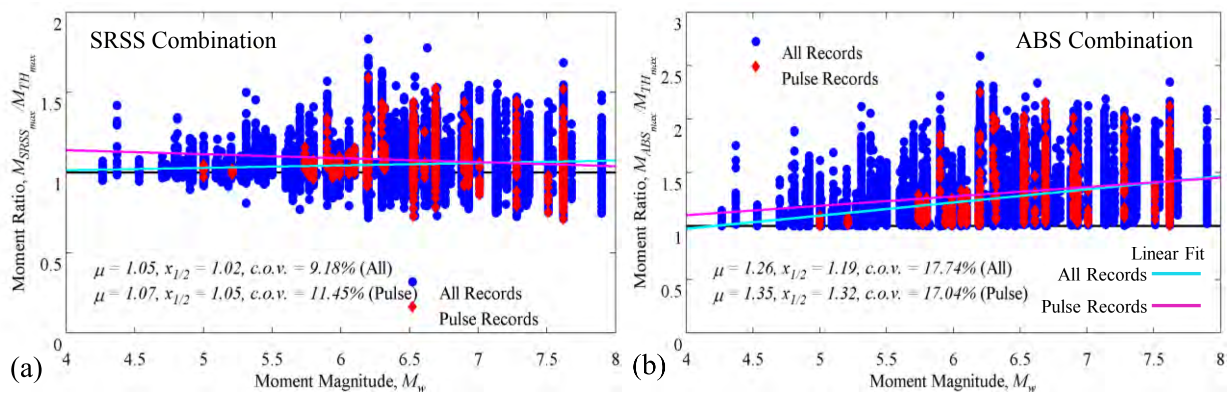

Figure 9: Effect of earthquake moment magnitude (M_w) on tank base moment ratio for (a) SRSS modal combination and (b) ABS modal combination

CONCLUSIONS

In summary, a statistically significant number of analyses were conducted to evaluate the adequacy of common modal combination techniques for hydrodynamic loading on tanks. These analysis included the determination of an "exact" response solution and approximate solutions using SRSS and Absolute Sum modal combination techniques on the maximum impulsive and convective mode responses. The analysis included:

- Six example tanks of varying geometry and dynamic properties
- 173 unique earthquake events having diverse geographical locations, moment magnitudes and epicentral distances
- 3,551 ground motion recordings with two horizontal motions each
- 42,612 analyses realizations
- A subset of 91 fault-normal, pulse-like motions (546 analysis realizations)

Seismic base shear and base moment above the tank bottom were considered in the evaluation of modal combination approaches and considered as shear and moment ratios. A ratio greater than one indicates that an approximate method resulted in load response parameters in excess of the exact solution.

The following conclusions are made regarding modal combination of peak impulsive and peak convective results using square root sum of squares (SRSS) and absolute sum combination (ABS):

- Absolute sum modal combination is always conservative, mean (μ) = 1.21-1.35, COV =17%
- SRSS modal combination predicts the mean peak response with a slight conservative bias and a small coefficient of variation, mean (μ) = 1.04-1.07, COV =8%
- SRSS is not always conservative; however, it was conservative in ~70% of the 42,612 realizations
- The results were similar for shear and moment at the base of the tanks
- The results were similar for all ground motions and the subset of pulse-like motions
- The results were similar for all earthquake magnitudes considered by the study (M_w 4.5-8.0)

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