18th International Conference on Structural Mechanics in Reactor Technology (SMiRT 18)
Beijing, China, August 7-12, 2005
SMiRT18-K07-7

STORY DAMAGE INDEX OF SEISMICALLY-EXCITED BUILDINGS BASED ON MODAL PARAMETERS

Jer-Fu Wang
Department of Civil Engineering
National Chung Hsing University
Taichung, Taibei 40227, China
Phone:+886-4-22862181
Fax: +886-4-22851992
E-mail:jerfu@ms16.hinet.net

Chi-Chang Lin
Department of Civil Engineering
National Chung Hsing University
Taichung, Taibei 40227, China
Phone: +886-4-22840438 ext 225
Fax: +886-4-22851992
E-mail: cclin3@dragon.nchu.edu.tw

Shih-Min Yen
Department of Civil Engineering,
National Chung Hsing University
Taichung, Taibei 40227, China

ABSTRACT

In this paper, a story damage index (SDI) for buildings is developed and expressed as a simple formula based on modal parameters extracted from real earthquake response records. It is useful because only one set of modal parameters is required for the calculation of the SDI to show the degree of damage of the story in question. According to numerical simulation results of five-story buildings with various story stiffness reductions and floor mass distributions, it is proven that the proposed SDI has both high accuracy and high reliability. This SDI is also applied to the damage assessment of a 7-story reinforced concrete hotel building in Van Nuys, California, which experienced severe structural damage during the 1994 Northridge earthquake. With both its fundamental frequency and mode shape identified by the SRIM (System Realization using Information Matrix) identification technique, it is shown that the SDI agrees fairly well with the results of the visual inspection, and is valuable in practical application.

Keywords: system identification, damage assessment, damage index, earthquake records.

1. INTRODUCTION

It is well known that the earthquake resistant design of buildings usually allows for the building to experience repairable damage during moderate and large earthquakes. Sometimes, the damage of a building is probably irreversible and may even gradually lead to collapse if the earthquake intensity is larger than expected. Therefore, it becomes very important to measure the damage as soon as possible to ensure the safety and habitability of a building after a strong earthquake. Since it is not practicable to destructively test a real building as we do an experimental model in a laboratory, non-destructive test (NDT) obviously is the more suitable method for damage assessment, and as such becomes the basis for making the decision to repair or rebuild a building.

During the past decades, a great amount of research about the NDT of structures has been conducted worldwide because of the significant development of new powerful systems for data acquisition and signal processing. Normally, the procedure to be performed first by NDT is to measure the dynamic responses of the structure and the external excitations, using sensors (usually accelerometers). With these measurements, representing the structural behavior due to a particular excitation, the damage of the structure can then be assessed by means of calculating damage indices. Nowadays, the defined damage indices basically can be classified into vibration response-based and structural parameter-based indices. The vibration response-based indices (Elenas and Meskouris, 2001; Park and Ang, 1985) used the structural response measurements under a single excitation event and from that calculated the damage-related physical factors, such as peak acceleration, peak velocity, energy, etc. The structural parameter-based damage indices (Allemang and Brown, 1982; Lieven and Ewins, 1988; Pandey *et al*., 1991; Pandey and Biwas, 1994; Farrar and Jauregui, 1998) estimated the change of structural parameters based on two independent excitation events: before and after the damage occurrence. To employ the latter indices, the structural parameters are expected to remain time-invariant during one of these two events. The linear system identification technique can then be applied to identify the structural modal parameters, representing the structural properties during that time. After obtaining the identified results, the comparison of two sets of structural modal parameters, before and after the damage, are then carried out using various methodologies.

Generally speaking, the vibration response-based damage index, for instance, the well known damage index proposed by Park and Ang (DI_{PA}) (Park and Ang, 1985), indicates the damage of an entire structure or a non-structural component by a single value. This index is simple to calculate and is useful for representing the global damage degree, however, is not able to locate the damage. Therefore, the structural parameter-based damage index is developed specifically for the detection of damage location, but the procedure is more complicated than that for the former index. The structural parameter-based indices are classified into three types: based on modal frequency, based on mode shape and based on both modal frequency and mode shape.

Calculating the change ratio of modal frequency to detect damage is an intuitive approach because damage always accompanies the reduction of stiffness as well as modal frequency. This approach has been widely applied in damage alarming in health monitoring systems of highway bridges. Damage in different locations and components actually leads to different frequency changes in various modes. Nevertheless, it remains difficult to determine the damage location just by observing the changes of modal frequencies. Among the modal parameters of a structure system, the mode shape is obviously the only location-related parameter. Therefore, many researchers have attempted to establish a mode shape-based indices, such as modal curvature index (Pandey *et al*., 1991; Farrar and Jauregui, 1998), index MAC (Allemang and Brown, 1982) and index COMAC (Lieven and Ewins, 1988) to identify the structural components containing damaged members. All above indices have simple expressions and have been applied in identifying the location of damage. However, it has been shown that they have low sensitivity to damage in some cases (Ndambi *et al*., 2002; Brasiliano, 2004). Considering both modal frequencies and mode shapes to detect the occurrence and location of damage may be a more reliable way than any of the other approaches. The modal flexibility damage index (MFDI) (Pandey and Biwas, 1994) may be the most well known one. The principle of this method is on the basis of the comparison of the flexibility matrices obtained from two sets of mode shapes. Moreover, this method involves the normalization of mode shape since the mode shape values are not fixed.

In addition, many researchers developed other damage indices for various types of structures. Kim and Chun (2004) derived the index α to apply to buildings. Kim *et al*. (2003) employed frequency-based damage detection (FBDD) and mode shape-based damage detection (MBDD) methods for locating and sizing damage in prestressed concrete beams. Brasiliano *et al*. (2004) evaluated the residual error method in the movement equation to verify its efficiency when applied to continuous beams and frame structures. All of these methodologies certainly have more accurate results than the previous damage indices, but they all involve complicated mathematical computations like matrix multiplication and matrix inverse. The lack of a simple expression makes them difficult to understand and implement for practical applications.

In this study, a story damage index (SDI) with a simple expression for building damage assessment is developed. This index is correlated with the floor mass, modal frequency and mode shape of a particular mode. Neglecting the floor mass, an approximate story damage index, named SDIA, is also presented. The index SDIA is easy to calculate and as such more useful. In addition, its calculating error is limited. It is easy to apply, because only one set of modal parameters are required for the calculation of the SDIA value to show the degree of damage of the particular story, or interval between measured floors in case of partial measurements. Compared with the indices COMAC and MFDI, it shows that the proposed SDI/SDIA is more accurate and more reliable. The index SDI/SDIA was employed for the damage assessment of a 7-story reinforced concrete hotel building in Van Nuys, California, which experienced severe structural damage during the 1994 Northridge earthquake. Using the modal frequency and the mode shape identified through the SRIM system identification technique (Juang, 1997; Lin et al, 2005), it is shown that the damage index SDI agrees fairly well with the results of the visual inspection, and that it is valuable in practical applications.

2. FORMULA DERIVATION

2.1 Dynamic and Characteristic Equations

Considering an N floor planar shear building with mass m_l at the lth floor and with stiffness k_l at the lth story, the equation of motion of the building under ground acceleration $\ddot{u}_g(t)$ can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{1}$$

where **M**, **C** and **K** are the $N \times N$ mass, damping and stiffness matrices respectively; $\mathbf{x}(t)$ represents the $N \times 1$ vector of the floor displacement relative to the ground at a time t; and **r** indicates the $N \times 1$ influence vector. With other words,

$$\mathbf{M} = \begin{bmatrix} m_1 & \cdots & & & & \cdots & 0 \\ \vdots & m_2 & & & & & \vdots \\ & & \ddots & & & & \\ & & & m_l & & & \\ & & & & \ddots & & \\ \vdots & & & & & m_{N-1} & \vdots \\ 0 & \cdots & & & & \cdots & m_N \end{bmatrix} \tag{2a}$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & & & & \cdots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & & & & & \vdots \\ \vdots & & \ddots & & & & & \\ & & -k_l & k_l + k_{l+1} & -k_{l+1} & & & \\ & & & & \ddots & & & \vdots \\ \vdots & & & & -k_{N-1} & k_{N-1} + k_N & & -k_N \\ 0 & \cdots & & & \cdots & -k_N & & k_N \end{bmatrix} \tag{2b}$$

Assume that ω_j and ϕ_j are the jth modal frequency and the $N \times 1$ mode-shape vector respectively. According to the characteristic equation, the relationship between the physical parameters and the modal parameters of the building can be expressed as

$$(\mathbf{K} - \omega_j^2 \mathbf{M})\phi_j = \mathbf{0} \tag{3a}$$

or in elementary form as

$$\begin{bmatrix} k_1 + k_2 - \omega_j^2 m_1 & -k_2 & \cdots & & & & \cdots & 0 \\ -k_2 & k_2 + k_3 - \omega_j^2 m_2 & -k_3 & & & & & \vdots \\ \vdots & & \ddots & & & & & \\ & & -k_l & k_l + k_{l+1} - \omega_j^2 m_{l-1} & -k_{l+1} & & & \\ & & & & \ddots & & & \vdots \\ \vdots & & & & -k_{N-1} & k_{N-1} + k_N - \omega_j^2 m_{N-1} & & -k_N \\ 0 & \cdots & & & \cdots & -k_N & & k_N - \omega_j^2 m_N \end{bmatrix} \begin{Bmatrix} \phi_{1j} \\ \phi_{2j} \\ \vdots \\ \phi_{lj} \\ \vdots \\ \phi_{N-1j} \\ \phi_{Nj} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \end{Bmatrix} \tag{3b}$$

Solving equation (3b) from the last to the first rows, a general expression of the lth story stiffness can be obtained as

$$k_l = \omega_j^2 \sum_{i=l}^{N} \frac{m_i \phi_{ij}}{\Delta\phi_{lj}} \quad \text{for } j = 1, 2, \ldots, N \tag{4}$$

where

$$\Delta\phi_{lj} = \begin{cases} \phi_{lj} - \phi_{(l-1)j} & \text{for } l = 2 \sim N \\ \phi_{lj} & \text{for } l = 1 \end{cases} \tag{5}$$

To interpret equation (4) from a physical point of view, one can imagine a building vibrating in its *j*th mode. In this condition, the vibration frequency of the building is just the *j*th modal frequency, ω_j, and the displacement at the *l*th floor is the *l*th row of the *j*th mode shape, ϕ_{lj}. Therefore, the *l*th floor acceleration equals to $\omega_j^2 \phi_{lj}$. Also, the inertial force of the *l*th floor will be the mass multiplied by the acceleration, $m_l \omega_j^2 \phi_{lj}$. Since the resultant force applying to the *l*th story is the summation of the inertial force above the story, $\sum_{i=l}^{N} \omega_j^2 m_i \phi_{ij}$, and the *l*th story drift is equal to the relative displacement between the *l*th and the (*l*-1)th floors, $\phi_{lj} - \phi_{(l-1)j}$, then it is evident that the stiffness of the *l*th story can be obtained by dividing the resultant force by the story drift and equation (4) can be solved easily.

2.2 Definition of Story Damage Index

Defining the damage of a building as the reduction percentage of story stiffness before and after damage, and assuming that the floor mass does not change due to the damage, the damage index of the *l*th story, termed SDI_l, can be expressed as

$$\mathrm{SDI}_l = 1 - \frac{k_l^* / m_l}{k_l / m_l} = 1 - \frac{\omega_j^{*2} \sum_{i=l}^{N} \frac{m_i \phi_{ij}^*}{m_l \Delta\phi_{lj}^*}}{\omega_j^2 \sum_{i=l}^{N} \frac{m_i \phi_{ij}}{m_l \Delta\phi_{lj}}} \tag{6}$$

where the asterisk (*) denotes the damage state. The value of the SDI_l is shown between 0 (no damage) and 1 (collapse), which is a convenient way to express the degree of story damage. For most buildings, the floor mass distribution is generally uniform, so the approximate value of the SDI_l, represented as SDIA_l, can be written as

$$\mathrm{SDIA}_l = 1 - \frac{\omega_j^{*2} \sum_{i=l}^{N} \frac{\phi_{ij}^*}{\Delta\phi_{lj}^*}}{\omega_j^2 \sum_{i=l}^{N} \frac{\phi_{ij}}{\Delta\phi_{lj}}} \tag{7}$$

Obviously, the value of SDIA_l can be obtained based on only one set of modal parameters (ω_j and ϕ_j) and does not need complicated mathematical calculations. This is beneficial for practical applications, because the modal parameters are usually evaluated through system identification techniques, in which only the first few modal parameters can be identified due to the unexpected noise contaminated in the measurements.

3. COMPARISON OF VARIOUS DAMAGE INDICES

3.1 The Damage Indices Used

Besides the proposed damage indices SDI/SDIA, the following used indices are also illustrated to compare their accuracy.

(1) Co-Ordinate Modal Assurance Criterion (COMAC)

The index COMAC (Lieven and Ewins, 1988) defined as

$$\mathrm{COMAC}_l = \frac{\left(\sum_{j=1}^{N_m} \phi_{lj}^* \phi_{lj}\right)^2}{\sum_{j=1}^{N_m} \phi_{lj}^{*2} \sum_{j=1}^{N_m} \phi_{lj}^2} \tag{8}$$

is a mode shape-based index, and was originally used to indicate the correlation among all mode shapes for two structures at a common coordinate *l*. Note that the symbols in equation (8) are the same as those defined previously, and that N_m denotes the total number of modes considered. The maximum value of COMAC is 1, which indicates that two sets of mode shapes are identical in coordinate l. This index was applied to identify the damage location (Brasiliano, 2004) since it was thought that the damage changes the consistence of the mode shapes of intact and damaged structures at the location of the damage. Since COMAC=1 corresponds to a no damage state, which is contrary to the definition of indices SDI and SDIA, 1-COMAC is therefore used to indicate the degree of damage in the rest of this paper.

(2) Modal Flexibility Damage Index (MFDI)

It has already been proven that the presence of cracks in a structure leads to an increase in structural flexibility. Therefore, the changes observed in the flexibility matrix can be interpreted as a damage indication in the structure, and as such allow for the evaluation and location of damage (Pandey and Biswas, 1994). The principle of this method is based on the comparison of flexibility matrices obtained from two sets of experimental mode shapes. The method is applicable only if the mode shapes are mass-normalized to unity ($\phi_j^T \mathbf{M} \phi_j = 1$), which implies that the estimation of structural mass is required. It has been derived that the diagonal terms at coordinate l of the modal flexibility matrix can be expressed as

$$F_l = \sum_{j=1}^{N_m} \phi_{lj}^2 \Big/ \omega_j^2 \tag{9}$$

In equation (9), F_l represents the static displacement due to a unit static load applied at the lth degree-of-freedom (DOF), which was used directly as a damage indicator (Ko *et al*., 2002). To indicate the degree of damage ranging from 0 (no damage) to 1 (collapse), the damage index for the lth story using modal flexibility is defined as

$$\mathrm{MFDI}_l = 1 - \frac{F_l}{F_l^*} = 1 - \frac{\sum_{j=1}^{N_m} \phi_{lj}^2 \Big/ \omega_j^2}{\sum_{j=1}^{N_m} \phi_{lj}^{*2} \Big/ \omega_j^{*2}} \tag{10}$$

It seems that the MFDI_l has the similar but reciprocal form of SDIA_l at the second term of Equation (10). However, they are totally different since the summation is performed with respect to the mode number for MFDI_l and to the DOF number for SDIA_l.

Table 1. Various types of floor mass ratio for a five-story shear building

Floor No.	Floor mass ratio													
	Type 1					Type 2				Type 3			Type 4
m_5	2	1	1	1	1	2	1	1	1	2	1	1	1
m_4	1	2	1	1	1	2	2	1	1	1	2	1	2
m_3	1	1	2	1	1	1	2	2	1	2	1	2	3
m_2	1	1	1	2	1	1	1	2	2	1	2	1	4
m_1	1	1	1	1	2	1	1	1	2	1	1	2	5

3.2 Numerical Simulations

To verify the efficiency and accuracy of the proposed SDI/SDIA formulas, five-story shear buildings with various stiffness reductions at the first and third stories and various floor mass distributions along the height are considered. Since the SDIA_l is carried out based on the assumption of uniform floor mass distribution, several types of floor mass ratios, as shown in Table 1, are used to validate the index SDIA_l. Figure 1 illustrates the damage indices SDI_l and SDIA_l for the first to fourth (l = 1 ~ 4) stories, respectively, versus the reduction percentage of the stiffness at the first and third stories. Note that these two damaged stories have the same stiffness reduction at the same time, and that the ordinate represents the values of SDI_l (solid line) or SDIA_l (different symbols for various floor mass distributions). It appears that SDI_1 and SDI_3 can accurately detect the degree of story damage. Moreover, both SDI_2 and SDI_4 are equal to zero no matter the degree of damage experienced at the first and third stories. Furthermore, without considering the variety of the floor mass, $SDIA_l$ is still very close to the real index value, SDI_l. It shows that the error of $SDIA_l$ increases as story damage becomes severe. However, at a degree of damage of 100%, the error remains smaller than 10%. Therefore, it is concluded that the damage index $SDIA_l$ is accurate enough.

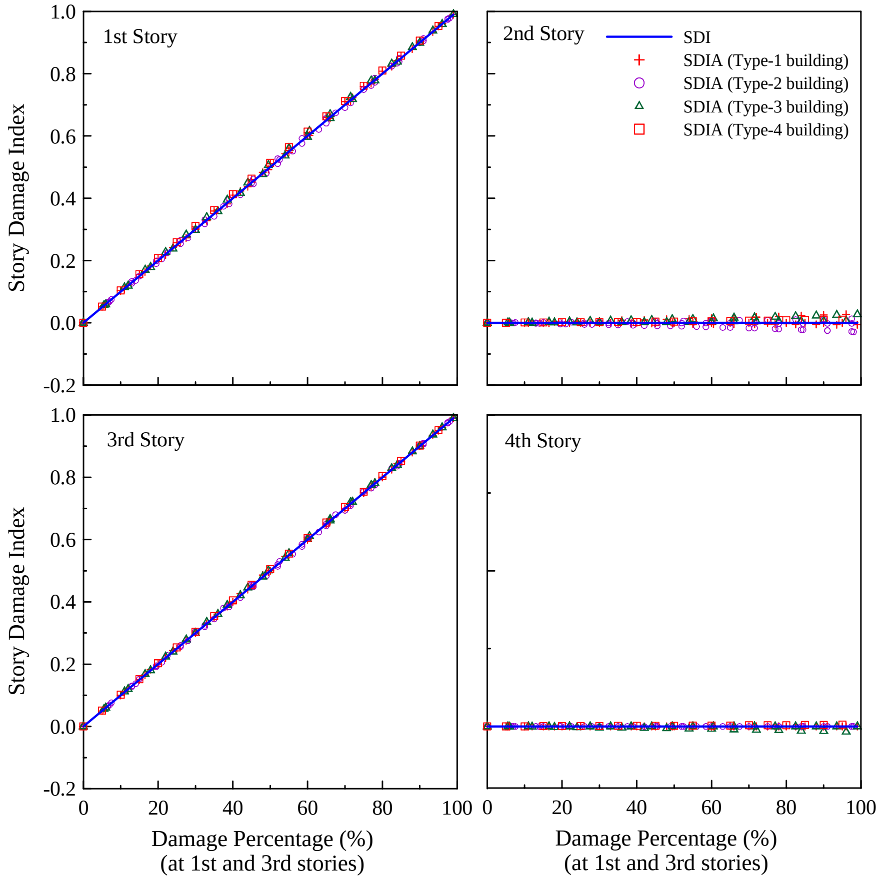

Fig. 1. SDI and SDIA of a five-story shear building with various damage at the first and third stories and various floor mass distributions

In practical application, it is generally impossible to acquire full measurements because of large number of DOFs for a real building Consequently, the sensors are usually installed at the lower, intermediate and upper floors, and partial measurements become the general case for most of the instrumented buildings. This makes that the dynamic responses of the unmeasured floors can not be obtained, so that the damage index tends to show the degree of damage of the intervals between measured floors. Figure 2(a) plots damage indices $SDI_l/SDIA_l$ for the first story, the first-to-third (1F-3F) floor interval and the third-to-fifth (3F-5F) floor interval of the Type-4 building, with measurements at 1F, 3F and 5F under the considerations of various stiffness reductions at the first and third stories. For comparison, 1-COMAC and MFDI are also shown in Figure 2(a). It is evident to see that the SDI_1 and $SDIA_1$ are close to being in agreement with the real values, even though only partial measurements are used. Between 1F and 3F, it is shown that $SDI_{1F\text{-}3F}$ and $SDIA_{1F\text{-}3F}$ are slightly smaller than the degree of damage occurring at the 3rd story. This is reasonable because the undamaged 2nd story dilutes the damage degree. Moreover, from the near-zero value of $SDI_{3F\text{-}5F}$ and $SDIA_{3F\text{-}5F}$, it shows that no damage occurs at the 4th and 5th stories, which agrees with the real situation. These results indicate that the indices SDI and SDIA can accurately display the degree of damage between measured floors. Comparing other damage indices, the square-labeled line, showing the result of 1-COMAC, obviously can not indicate the actual damage situation. In particular, 1-$COMAC_1$ can not detect the occurrence of damage at the first story no matter what degree of damage occurs. It shows that such a mode shape-based index is not sufficient for damage assessment. As to the index MFDI, represented by circle-labeled lines in Figure 2(a), it appears that $MFDI_1$ and $MFDI_{1F\text{-}3F}$, which perform better than those from COMAC, generally can detect the occurrence and degree of damage of the first story and 1F-3F interval, respectively. However, $MFI_{3F\text{-}5F}$ makes an erroneous judgment for the undamaged 3F-5F interval.

Another specific case, in which each story has the same degree of damage (uniform damage), can be used to expose the defect of the mode shape-based index like COMAC. It is generally recognized that a uniformly damaged system has a stiffness matrix which is a multiple of that of the original system. Therefore, the damaged and undamaged systems actually have different modal frequencies but an identical mode-shape matrix. This makes that 1-COMAC of each story equals to zero, i.e. indicating no damage occurrence, because the two mode shapes are fully correlated (COMAC=1). This view point is verified by the three damage indices versus the degree of damage for a uniformly damaged building, as shown in Figure 2(b). It is shown that the damage alert would not be triggered if 1-COMAC was employed for the health monitoring system, even though the building may collapse. Obviously, the performance of SDI/SDIA and MFDI are more accurate and reliable than 1-COMAC. This shows that the indices based on both modal frequency and mode shape are superior to those based only on mode shape.

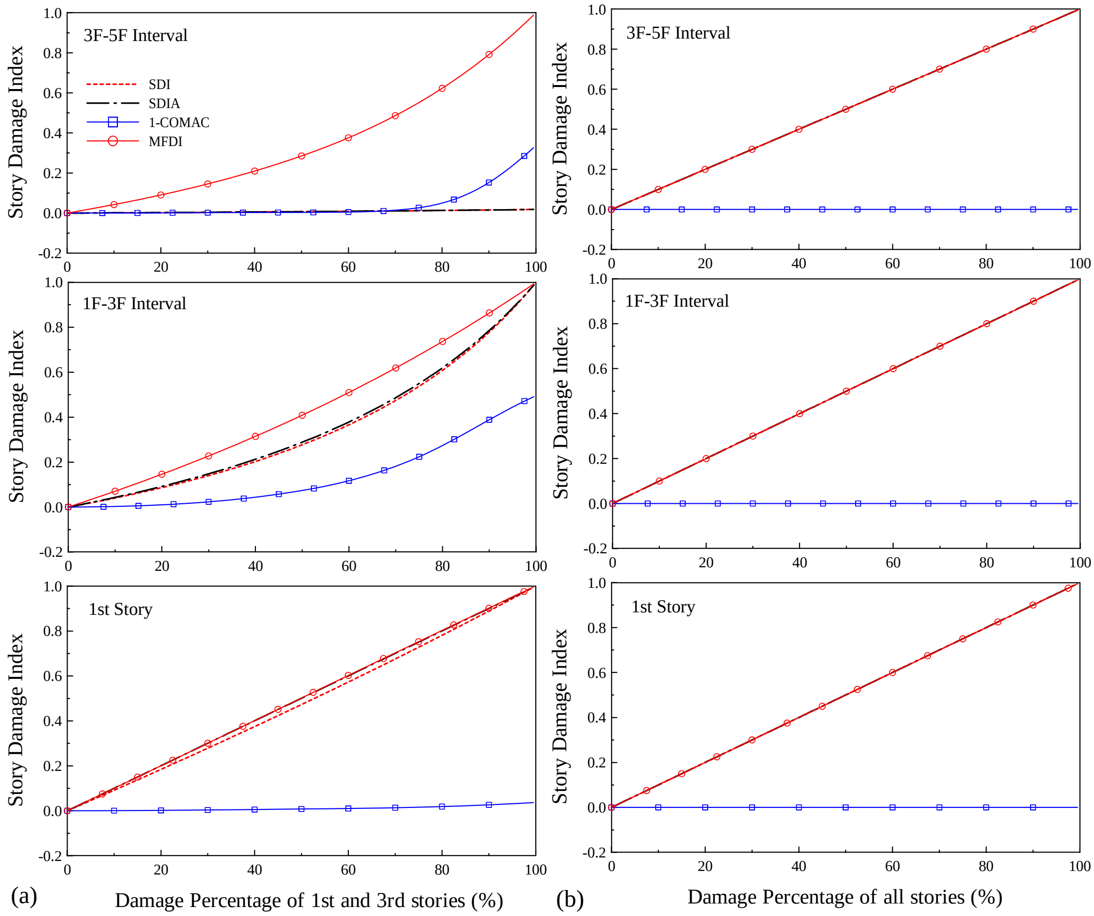

Fig. 2. Comparisons of various damage indices of Type-4 building with partial measurements as damages at: (a) the 1st and 3rd stories (b) all stories.

4. REAL BUILDING DAMAGE ASSESSMENT

4.1 Description of the Instrument-Monitored Building

The earthquake response measurements of the 7-story Holiday Inn building located at the city of Van Nuys of the Los Angeles metropolitan area in California are used to investigate the applicability of the proposed damage index SDIA. The building was designed as a reinforced concrete (RC) moment resisting frame, and was built in 1966. It is one of the 170 instrument-monitored buildings under the California Strong Motion Instrumentation Program (CSMIP; the program installed more than 900 stations, including 650 in free field, 170 in buildings, 20 in dams and 60 in bridges.), in which 16 accelerographs distributed at the base, first floor (1F), second floor (2F), fifth floor (5F) and seventh floor (7F) were installed, as shown in Figure 3. It is seen that only one sensor was installed at the base to measure the vertical motion.

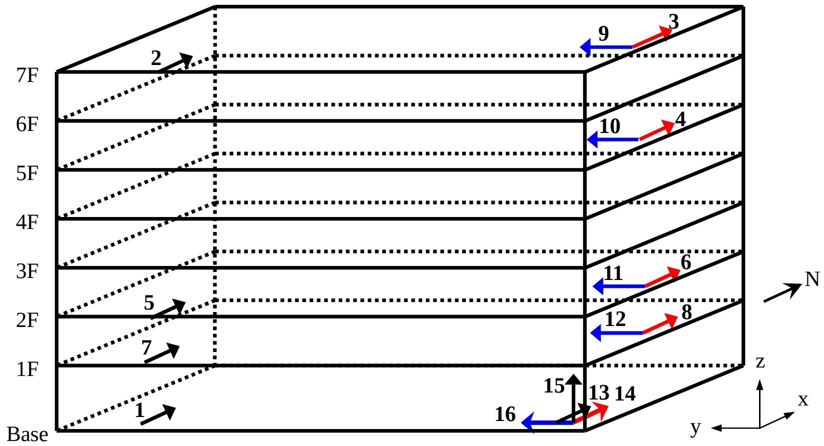

Fig. 3. Distribution of sensors at the 7-story Holiday Inn building

The building was severely damaged by the 1994 Northridge earthquake (M_L=6.4) and was declared as unsafe and red-tagged by the Los Angeles Housing Authorities. The structural damage was extensive in the exterior north and south side frames (longitudinal (y) direction). No major damage at the longitudinal interior frames was found. There was also no visible damage in the slabs and around the foundation. However, the non-structural masonry brick walls were significantly damaged. A clear description of the damage to this building is given in Table 2 (Ivanović et al., 2000). During the main shock of this destructive earthquake, the time history acceleration responses at the position and direction of each sensor were fully recorded. At the base, peak values of 0.46g in the y direction, 0.40g in the transverse (x) direction and 0.28g in the vertical (z) were observed, whereas 0.59g along the y axis and 0.58g along the x axis were recorded at the roof. Since the building received gradual damage during the whole time of the earthquake excitation, the signatures after the strong motion part of the measurements can represent the behavior of the damaged building. In addition, the measurements of this building recorded during the 1992 Landers earthquake and its aftershock Big Bear earthquake were also used. No damage of this building was reported during these two earthquakes. Therefore, these measurements can stand for the undamaged state before the 1994 Northridge earthquake.

Table 2. Damage description of the 7-story Holiday Inn building due to 1994 Northridge Earthquake

Location	Damage description
1st story	(a) 5 bays with cracks between bricks (north side frame) (b) 3 column cracks due to short column effect (north side frame)
1st floor	(a) diagonal cracks along a side column (north side frame) (b) diagonal cracks in beams near two beam-column joints (north side frame)
3rd floor	(a) “X” shear cracks in one beam-column joint (north side frame) (b) crack through a beam near joint (north side frame) (c) cracks along a side column (north side frame)
4th story	(a) cracks along a side column (north side frame)
4th floor	(a) “X” shear cracks in 3 beam-column joints (north side frame) (b) crack through a beam near joint (north side frame)
5th floor	(a) “X” shear cracks in 2 beam-column joints (north side frame) (b) “X” shear cracks in 5 beam-column joints (south side frame)
Above 6th floor	no visible damage

4.2 Identification of Building Modal Parameters and Calculation of the SDIA

In this study, the SRIM system identification technique is employed to estimate the dynamic properties (modal frequencies and mode shapes) of the Holiday Inn building based on response measurements from three earthquake events. The SRIM is applicable for a linearly time-invariant structural system represented by the discrete-time state-space model as

$$x(k+1) = Ax(k) + Bu(k) \tag{11}$$

$$y(k) = Cx(k) + Du(k) \tag{12}$$

where $x(k)$ is the $n \times 1$ state vector at time $k\Delta t$ corresponding to n DOFs and Δt is the constant sampling period, $y(k)$ is the $m \times 1$ output vector corresponding to m measurements, and $u(k)$ is the $r \times 1$ input vector corresponding to r inputs. A, B, C, and D are system parameter matrices with dimensions of $n \times n$, $n \times r$, $m \times n$, and $m \times r$, respectively. According to the references (Juang, 1997; Lin et al, 2005), the SRIM system identification technique is performed by establishing the matrices $Y_p(k)$ and $U_p(k)$ based on the measured input $u(k)$ and output $y(k)$ vectors where

$$Y_p(k) = \begin{bmatrix} y(k) & y(k+1) & \cdots & y(k+N_r-1) \\ y(k+1) & y(k+2) & \cdots & y(k+N_r) \\ \vdots & \vdots & \ddots & \vdots \\ y(k+p-1) & y(k+p) & \cdots & y(k+p+N_r-2) \end{bmatrix} = [y_p(k) \quad y_p(k+1) \quad \cdots \quad y_p(k+N_r-1)] \tag{13}$$

$$U_p(k) = \begin{bmatrix} u(k) & u(k+1) & \cdots & u(k+N_r-1) \\ u(k+1) & u(k+2) & \cdots & u(k+N_r) \\ \vdots & \vdots & \ddots & \vdots \\ u(k+p-1) & u(k+p) & \cdots & u(k+p+N_r-2) \end{bmatrix} = [u_p(k) \quad u_p(k+1) \quad \cdots \quad u_p(k+N_r-1)] \tag{14}$$

In the above matrices, $y_p(k)$ and $u_p(k)$ are $mp \times 1$ and $rp \times 1$ vectors composed of p intervals of output and input measurements, respectively. By repeating the same procedure N_r times, except for shifting the starting point N_r times from time k to time $(k+N_r-1)$, the matrices $Y_p(k)$ $(mp \times N_r)$ and $U_p(k)$ $(rp \times N_r)$ can be formed, as shown in Figure 4.

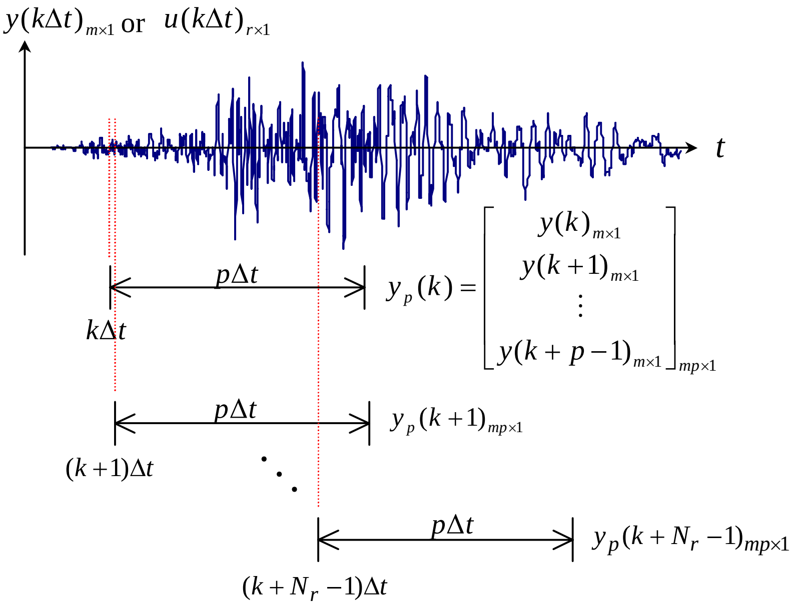

Fig. 4. Illustration of the composition of $y_p(k)$ for the SRIM system identification technique

Through some mathematical processes (Juang, 1997; Lin et al, 2005), the system matrix A can be carried out so that the modal frequencies, damping ratios and mode shapes can be obtained. It must be noted that the selection of factors p and N_r is not unique. Therefore, a confirmation procedure, in which the curves between the calculated structural responses based on the measured inputs and the identified system matrices A, B, C, D and the actual structural measurements are compared, is performed to ensure that the identified system is able to represent the real system.

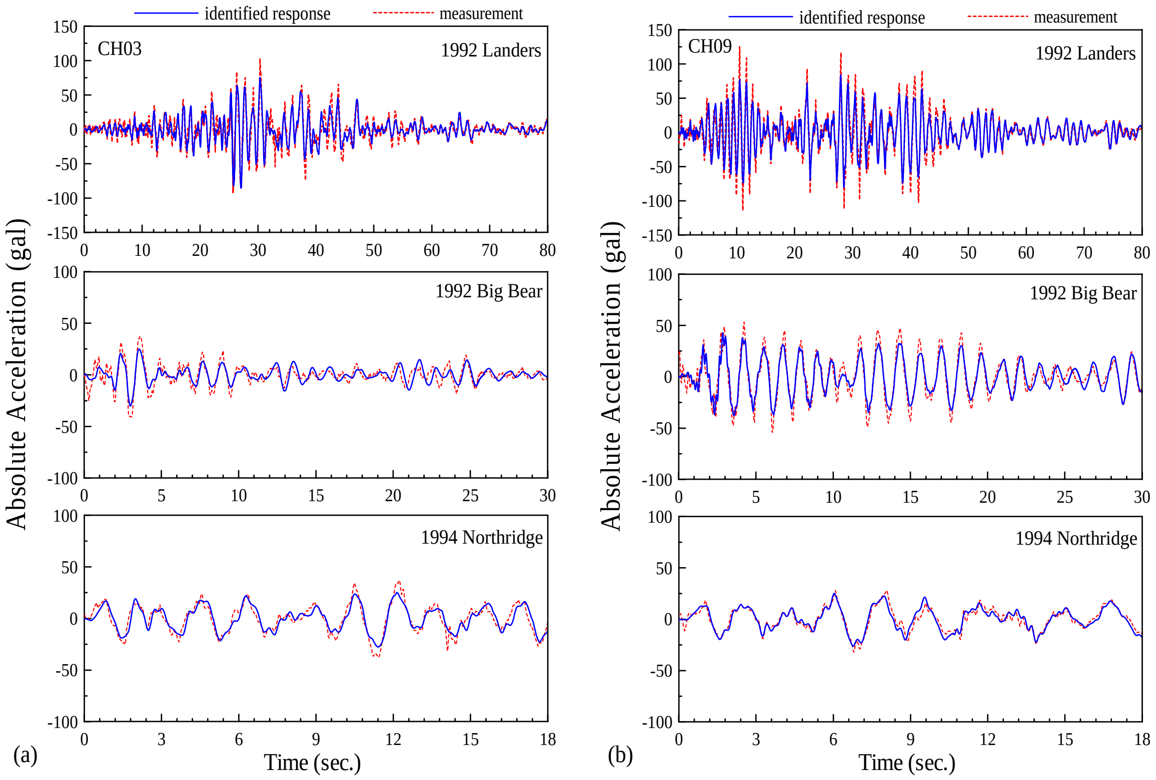

Fig. 5. Comparison of identified responses and real measurements of Holiday Inn building under three earthquakes at sensors: (a) CH03 (transverse axis) and (b) CH09 (longitudinal axis)

To evaluate the damage of the Holiday Inn building, the building motions in x and y directions are assumed to be independent of each other. CH16 is used as input, and CH09, CH10, CH11, CH12 are used as outputs for the y axis, while CH14 is used as input and CH03, CH04, CH06, CH08, are used as outputs for the x axis, where the sensor number is shown in Figure 3. Employing the SRIM identification technique, the first modal frequency and mode shape of the building in the x- and y-directions after the 1992 Landers earthquake and the 1992 Big Bear earthquake are identified and shown in Table 3, respectively. Figures 5(a) and 5(b) illustrate the comparisons between the calculated responses and the real measurements at CH03 and CH09 to validate the identified modal frequency and mode shape. Based on the modal parameters after two earthquake events, the SDIA damage indices of intervals between measured floors are also calculated and presented in Table 3. It is noticed that an SDIA = 0 will be assigned if the calculated SDIA is negative since the story stiffness may not increase in damage condition from physical point of view. It can be seen that $SDIA_1$, $SDIA_2$, $SDIA_{2F\text{-}5F}$ and $SDIA_{5F\text{-}7F}$ are all small which agree with the fact that no visible damage was reported after the 1992 Landers and Big Bear earthquakes. Meanwhile, some index values around 0.1 are obtained. This may result from the neglect of the coupling effect between two orthogonal motions.

The Holiday Inn building experienced severe damage during the 1994 Northridge earthquake. In order to estimate the dynamic properties of the building under the damaged state, the signatures after the strong motion part of the whole measurements are extracted to represent the behavior of the building after the main shock. The identified first modal frequency and mode shape of the building are given in Table 4, with the validation of the identified results shown in Figures 5(a) and 5(b). The damage index of each interval between measured floors is also given. It is clearly shown that the SDIA values along the longitudinal (y) direction are larger than those along the transverse (x) direction for each interval, which agrees with the actual visual inspection. Moreover, $SDIA_2$ and $SDIA_{2F\text{-}5F}$ are larger than $SDIA_{5F\text{-}7F}$. These results are much consistent with the actual damage description in Table 2 , where lots of cracks leading to stiffness reduction were observed between the 2F and 5F interval, and no visible damage was found above the 5th story (but the "X" shear cracks at the beam-column joint at the 5th floor may reduce the stiffness of the 5F-7F interval). These results prove that the proposed damage index SDIA is accurate enough and reliable for practical applications in building damage assessment.

Table 3. Identified modal parameters and damage index SDIA of the Holiday Inn building based on the 1992 Lander and Big Bear earthquakes

Parameters		x-direction		y-direction	
Earthquake Event		Landers /	Big Bear	Landers /	Big Bear
Modal Frequency(Hz)		0.786	0.781	0.880	0.855
1st mode shape	1F	0.220	0.260	0.165	0.172
	2F	0.428	0.508	0.354	0.365
	5F	0.932	0.957	0.884	0.874
	7F	1.000	1.000	1.000	1.000
Damage Index SDIA	1st story	0.12		0.09	
	2nd story	0.13		0.07	
	2F-5F	0		0.02	
	5F-7F	0		0.13	

Table 4. Identified modal parameters and damage index SDIA of the Holiday Inn building before and after the 1994 Northridge earthquakes

Parameters		x-direction		y-direction	
Earthquake Event		Landers /	Northridge	Landers /	Northridge
Modal Frequency(Hz)		0.786	0.590	0.880	0.512
1st mode shape	1F	0.220	0.171	0.165	0.163
	2F	0.428	0.414	0.354	0.384
	5F	0.932	0.961	0.884	0.947
	7F	1.000	1.000	1.000	1.000
Damage Index SDIA	1st story	0.28		0.64	
	2nd story	0.51		0.70	
	2F-5F	0.47		0.67	
	5F-7F	0.01		0.25	

5. CONCLUSIONS

Damage assessment of a building after a strong earthquake has received significant interests from researchers and engineers in recent years. Based on the modal frequency and mode shape of a particular mode, this study proposed an accurate damage index SDIA which is applicable even though only partial measurements are given. It has been shown from simulation results that the index SDIA is more accurate and more reliable to estimate building damage than other common damage indices (i.e. COMAC, MFDI). In addition, the index SDIA has a simple expression so that it can be carried out and understood easily. The index SDIA was also employed for the damage assessment of a 7-story reinforced concrete building in Van Nuys, California, which experienced severe structural damage during the 1994 Northridge earthquake. The results obtained show that the damage index SDIA agrees quite well with the results of the visual inspection and is valuable for practical applications.

ACKNOWLEDGMENT

This work was supported by the National Science Council of the Republic of China under Grants NSC 93-2625-Z-005-009 and NSC 93-2811-Z-005-002. These supports are greatly appreciated.

REFERENCES

Allemang, R. J., and Brown, D. L., “A correlation coefficient for modal vector analysis”, *Proceedings of the International Modal Analysis Conference & Exhibit*, 110-116, 1982.

Brasiliano, A., Doz, G. N., and Brito, J. L. V., “Damage identification in continuous beams and frame structures using the Residual Error Method in the Movement Equation”, *Nuclear Engineering and Design*, 227, 1-17, 2004.

Elenas, A., and Meskouris, K., “Correlation study between seismic acceleration parameters and damage indices of structures”, *Engineering Structures*, 23, 698-704, 2001.

Farrar, C. R., and Jauregui, D. A., “Comparative study of damage identification algorithms applied to a bridge”, *Smart Materials and Structures*, 7, 704-731, 1998.

Ivanović S.S., Trifunac M.D., Novikova E.I., Gladkov A.A., Todorovska M.I., “Ambient vibration tests of a seven-story reinforced concrete building in Van Nuys, California, damaged by the 1994 Northridge earthquake”, *Soil Dynamics and Earthquake Engineering*, 19, 391-411, 2000.

Juang, J.-N., “System realization using information matrix”, *Journal of Guidance, Control, and Dynamics*, 21(3), 492-500, 1997.

Kim, H.-S., and Chun, Y.-S., “Structural damage assessment of building structures using dynamic experimental data”, *the Structural Design of Tall and Special Buildings*, 13, 1-8, 2004.

Kim, J.-T., Ryu, Y.-S., Cho, H.-M., and Stubbs, N., “Damage identification in beam-type structures: frequency-based method vs mode-shape-based method”, *Engineering Structures*, 25, 57-67, 2003.

Ko, J. M., Sun, Z. G., and Ni, Y. Q., “Multi-stage identifiaciton schem for detecting damage in cable-stayed Kap Shui Mun bridge”, *Engineering Structures*, 24, 857-868, 2002.

Lieven, N. A. J., and Ewins, D. J., “Spatial correlation of mode shapes, the coordinate modal assurance criterion (COMAC)”, *Proceedings of the Sixth International Modal Analysis Conference*, 690-695, 1988.

Lin, C. C., Wang, C. E., Wu, H. W., Wang, J. F., “On-line building damage assessment based on earthquake records”, *Smart Materials and Structures*, 2005 (in press).

Ndambi, J.-M., Vantomme, J., Harri, K., “Damage assessment in reinforced concrete beams using eigenfrequencies and mode shapes derivatives”, *Engineering Structures*, 24, 501-515, 2002.

Pandey, A. K., Biswas, M., and Samman, M. M., “Damage detection from changes in curvature mode shapes”, *Journal of Sound and Vibration*, 145, 321-332, 1991.

Pandey, A. K., Biswas, M., “Damage detection in structures using changes in flexibility”, *Journal of Sound and Vibration*, 169(1), 3-17, 1994.

Park, Y.-J., Ang, A. H.-S., “Mechanistic seismic damage model for reinforced concrete”, *Journal of Structural Engineering*, ASCE, 111(4), 722-739, 1985.

