Development of Energy Density Method for Inelastic Notch-Tip Strain Calculation and Its Application for Pressure Vessel and Piping Design

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ABSTRACT

The explicit solution for inelastic strain on the surface of concentrator is obtained for plain problems of various types on the basis of the energy density method and using the results of elastic approach. New interpretations of the method proposed for 3-D stress-strain state and for nominal plastic flow. Approximated formulae are given for practical application.

1 INTRODUCTION

The necessity for assessment of inelastic strains in stress concentrators occurs in the design fatigue strength determination of the NPP structural elements. Rather often the element nominal stress-strain state (SSS) is calculated in elastic statement. Thus, the standard calculation methods provide for the possibility of determination of elastic-plastic strains in stress concentrators based on the elastic solution by approximate formulae.

At the last decade the method of equivalent strain energy density (EDM) which was proposed for the first time in Ref. [1] has been widely developed. The method is characterized by improved accuracy, for example, as compared to the Neuber one. The strain level is determined on the basis of the postulated relation [2]

$$W = C_p W_e \qquad (1)$$

where W and W_e are the strain energy density values in elastic -plastic and elastic solutions, respectively, and Cp is the correction factor.

The paper presents techniques to simplify the use of the method in a number of 2-D problems.

2 EDM IN PLAIN PROBLEMS

In Ref. [2] it is shown that if the field of principal stresses at concentrator is approximated in the plane problem by relations:

$$\begin{cases} \sigma_x = \dfrac{K_t S_{ny}}{2\sqrt{2}} \left[\left(\dfrac{\rho}{r}\right)^{\frac{1}{2}} - \dfrac{1}{2}\left(\dfrac{\rho}{r}\right)^{\frac{3}{2}}\right] \\ \sigma_y = \dfrac{K_t S_{ny}}{2\sqrt{2}} \left[\left(\dfrac{\rho}{r}\right)^{\frac{1}{2}} + \dfrac{1}{2}\left(\dfrac{\rho}{r}\right)^{\frac{3}{2}}\right] \end{cases} \qquad (2)$$

and the distribution of nominal stresses for elastic solution S_{ny} is approximated by linear relation

$$S_{ny} = S_{ty}\left[1 - \frac{1}{x0}\left(\frac{r}{\rho} - \frac{1}{2}\right)\right], \qquad (3)$$

then

$$C_p = \frac{2 - \frac{1}{2x_0} - \frac{1}{2}\left(\frac{\rho}{r_p}\right)\left(1+\frac{1}{x_0}\right) + \frac{1}{4}\left(\frac{\rho}{r_p}\right)^2\left(1+\frac{1}{2x_0}\right) + \frac{2\sqrt{2}}{3x_0}\left(\frac{\rho}{r_p}\right)^{\frac{1}{2}} - \frac{2}{3x_0}\left(\frac{r_p}{\rho}\right)}{1+\frac{1}{2}\left(\frac{\rho}{r_p}\right)\left(1+\frac{1}{2x_0}\right) - \frac{1}{x_0}\left(\frac{r_p}{\rho}\right)} . \quad (4)$$

The utilized symbols are (see Fig.1): x,y,z - the axes of the Cartesian coordinates with the start at the concentrator's root, they charecterize the principal stress directions; $r = x+\rho/2$ - auxiliary coordinate; ρ - concentrator root radius; K_t - elastic stress concentration factor; S_{ty} - nominal stress for elastic solution at the concentrator root; $x_0 = x_n / \rho$; x_n- x - coordinate value which corresponds to $S_{ny}(x_n) = 0$ one; r_p - r-coordinate value at the inelastic area interface.

The r_p value can be found with regard to (2) from the condition of plastic flow begining:

$$\sigma_i = \sigma_Y \quad (5)$$

where σ_i is stress intensity, σ_Y is material yield stress.

The broad introduction of the EDM in the design practice in the variant which is presented in Ref. [2] as well as in the subsequent works of this author is inhibited by the complexity of its resolving equations.

The relations for some important SSS types are also lacking. The possibilities of removing these difficulties are presented below.

3 PLANE STRESS STATE (PSsS)

For the adopted coordinates σ_z equals to 0. At the concentrator root σ_x equals to 0.

In Refs. [1,2] the Ramberg-Osgood relation was use to describe the material tensile stress-strain curve.

In this case the strain at the concentrator surface can not be expressed in explicit form from the relation (1). In Ref. [3] it was proposed to use the piecewise - power approximation of the curve:

$$\begin{cases} \bar{\sigma} = \bar{\varepsilon} & \text{if } \bar{\sigma} \le 1 \\ \bar{\sigma} = \bar{\varepsilon}^m & \text{if } \bar{\sigma} \ge 1 \end{cases} \quad (6)$$

where $\bar{\varepsilon} = \varepsilon/\varepsilon_Y$, $\bar{\sigma} = \sigma/\sigma_Y$, $\varepsilon_Y = \sigma_Y/E$, E is Young's modulus, m is the material parameter. The following expression can be obtained from eqn (1) using (6):

$$\bar{\varepsilon}_{ty} = \left\{1 + \frac{m+1}{2}\left[\left(K_t\bar{S}_{ty}\right)^2 C_p - 1\right]\right\}^{\frac{1}{m+1}} \quad (7)$$

Herein, $\bar{\varepsilon}_{ty}$ represents $\bar{\varepsilon}_y$ at the concentrator surface when $x = 0$. The relation (5) as applied to PSsS has the form [2]

$$\sigma_Y = \frac{K_t S_{ny}}{2\sqrt{2}}\left[\frac{\rho}{r_p} + \frac{3}{4}\left(\frac{\rho}{r_p}\right)^3\right]^{1/2} \quad (8)$$

from which the r_p value can be found.

Thus, the strain ε_{ty} at the concentrator surface is calculated by relation (7) with the use of (3), (4), (6) and (8). This way is considered to be a rather labour-consuming, hence for practical calculations the following formula is suggested to be used:

$$C_p = \frac{T + 0.22}{1.2\,T}\left(1 - \frac{0.012}{T}\left|lg\,\frac{x_0}{2}\right|\right) \quad (9)$$

where $$T = \frac{\sigma_Y}{K_t S_{ty}}.$$

The equation provides for an engineer level of accuracy. Thus, within the range $0.15 \le T \le 0.9$ and $x_0 \ge 1$ the error in C_p determination does not exceed 6 %. For $x_0 > 200$ in (9) $x = 200$ should be substituted.

Thus, the value ε_{ty} can be found by eqn (7) using (6) and (9).

4 GENERALIZED PLANE STRAIN STATE (GPSnS)

Herein the strain ε_z is constant along the overall x-y plane for each stage of loading. At the concentrator surface x equals to 0

In the Refs. [2,4] it is suggested that the plane strain case $\varepsilon_z = 0$ (PSnS) can be calculated by the same relation as the PSsS. In this approach the fictitious diagram $\sigma_y \sim \varepsilon_y$ which would consider the ε_z effect on the deformation and flow conditions should be used instead of the real tension curve $\sigma \sim \varepsilon$ (6).

In this paper the presented approach is generalized for GPSnS. The symbol $F = E\,\varepsilon_z / (K_t S_{ty})$ is introduced; the fictitious curve characteristics are marked by an additional index «y»: E_y, σ_{Yy}, ε_{Yy}, m_y.

It follows from the Hooke's law that:

$$\sigma_{Yy} = \sigma_Y \left[1-(F+\nu)+(F+\nu)^2\right]^{-\frac{1}{2}} \tag{10}$$

$$\varepsilon_{Yy} = \sigma_{Yy}/E_y \tag{11}$$

$$E_y = E/(1-F\nu-\nu^2) \tag{12}$$

where ν- is Poisson's ratio.

From the relations of the Hencky's theory of small plastic deformations it was obtained:

$$\sigma_z = \mu\,\varepsilon_y + \mu_1 \tag{13}$$

where $\mu = (\nu + R/2)/(1+r)$; $\mu_1 = \varepsilon_z E/(1+R)$; $R = \bar{\varepsilon}^{1-m} - 1$.

The parametric form was used to describe the elastic-plastic part of the fictitious curve. The real tension curve values σ and ε served as parameters. The fictitious curve points were found from the condition of equality of stress intensity determined by these parameters and the one at the concentrator surface under GPSnS. With regard to (13) it was obtained:

$$\sigma_y = \frac{\left[4\sigma^2(1-\mu+\mu^2) - 3\mu_1^2\right]^{1/2} + \mu_1(1-2\mu)}{2(1-\mu+\mu^2)} \tag{14}$$

$$\varepsilon_y = \frac{(1-\mu^2)(1+R)}{E}\sigma_y - \mu\varepsilon_z\,. \tag{15}$$

The fictitious curve plotted on the basis of (14) and (15) can be approximated by piecewise - power law of eqn (6) type, where m_y value is found from.

To simplify this procedure, the approximate formula for loading conditions, when F remains constant with time, is proposed:

$$m_y = \begin{cases} 1.35m + F(0.2-0.5m) - 0.1 \text{ if } 0.5 < F \le 0.7 \\ 1.1m \text{ if } 0 \le F \le 0.5 \\ m(1.1+0.3F) - F(0.15 + 10\varepsilon_Y) \text{ if } -1 \le F \le 0 \end{cases} \tag{16}$$

The similar relations can also be found for other loading laws. The calculation of m_y by (16) at $\nu = 0.3$, $\varepsilon_Y = 0.05$ - 0.6 %, m = 0 - 0.3 and $\varepsilon = \varepsilon_Y$ - 3 % provides for the calculation of by eq. (7) with the error of no more than 5 %.

There is in the elastic zone $\sigma_z = F\sigma_y + \nu(\sigma_x + \sigma_z)$. This gives the following equation with regard to (2) and (5) to obtain r_p:

$$\left[\left(\frac{\rho}{r_p}\right)^3 \frac{3+F^2}{2} - \left(\frac{\rho}{r_p}\right)^2 - 2F(1-F-2\nu) + \left(\frac{\rho}{r_p}\right)2(1-F-2\nu)^2\right]^{\frac{1}{2}}\left[2x_0 + 1 - 2\left(\frac{r_p}{\rho}\right)\right] = 8Tx_0\,. \tag{17}$$

To simplify the Cp calculation procedure, the approximate formula is proposed:

$$C_p = \frac{0.13F^2 - 0.05F + 0.86T + 0.11}{T} \left(1 - \frac{0.01F^2 + 0.008}{T} \left| lg\, x_0 \right| \right). \quad (18)$$

The error of the formula for $0.15 \leq T \leq 0.9$; $xo \geq 1$; $|F| \leq 0.7$; does not exceed 6 %. Herein as well as in formula (9) xo = 200 should be accepted instead of xo > 200.

Note that the use of relation (7) as applied to GPSnS at in fact means a new interpretation of the e nergy density method for 3-D stress-strain state. Only a fraction of energy density which is connected with σ_y and ε_y components is taken into accout in the energy postulate (1).

5 NOMINAL PLASTIC FLOW

The realization of EDM for $\sigma_z = 0$ (PSsS) or $\varepsilon_z = 0$ (PSnS) where as the nominal plastic zone is localized near the concentrator is considered in Ref. [4]. Nevertheless it does not give an explicit solution for ε_{ty} and the recommendations for Cp calculation are also lacking.

Assume that the nominal value of elastic-plastic strain at the concentrator surface ε_{ty}^{nom} is known. Then the conventional elastic stresses value at $K_t = 1$ and Cp = 1 is proposed to be determined from equation (7):

$$\tilde{S}_{ty} = \sigma_Y \left\{ 2 \left[\left(\varepsilon_{ty}^{nom} / \varepsilon_Y \right)^{m+1} - 1 \right] / (m+1) + 1 \right\}^{1/2}. \quad (19)$$

The $C_p = 1$ condition is accepted for simplicity. If necessary, the C_p value can be found more accurate by (9) and (18).

The values σ_{Yy}, ε_{Yy} and m_y calculated for $F = \varepsilon_z E / S_{ty}$ should be substituted in equation (19) in case of GPSnS as it was made earlier. If necessary F accuracy can be improved by the use of the determined value $\tilde{S}_{ty}$ instead of S_{ty}.

The further calculations to determine ε_{ty} are performed in accordance with the above described methods for PSsS or GPSnS using $\tilde{S}_{ty}$ instead of S_{ty}.

6 RELATIONS ACCURACY CHECK-UP

The results of ε_{ty} calculations with the use of eqns (7), (9)- (12), (14)-(16), (18) and (19) were compared with the FEM calculations data and experimental values. The Tables 1 and 2 present different types of loading and SSS as applied to the geometry of concentrator, shown in Fig.1. The material properties were as follows: E = 2.04 GPa; $\nu = 0.3$; $\sigma_Y = 340$ MPa; m = 0.206.

Figs. 2-4 show the comparison of calculational and experimental data of a number of investigations. Fig.2 represents the PSsS and Fig.3 - the PSnS. The axisymmetrical case shown in Fig.4 was interpreted as GPSnS with $F = -\nu / K_t$ in calculations by approximate relations.

The given examples have demonstrated a satisfactory accuracy of the proposed techniques. The error is compatible with the one in the relations (9), (16) and (18). This fact permits to believe that if ne cessary the improved accuracy can be achieved by introduction of more accurate aproximations.

7 APPLICATION IN STANDARD CALCULATIONS

The USSR current Regulations for strength calculation of NPP components [5] make provision for the use of simplified EDM. The calculations are performed by relation (7) without replacement of σ_Y, ε_Y and m parameters under 3-D stress-strain state. Though the equivalent stress $S_{t1}-S_{t3}$ in accordance with the Tresca's criterion is used herein instead of S_{ty}, where S_{t1} and S_{t3} are nominal principal stresses. Such an approach provides for an engineering accuracy, though at F < 0 it leads to excessive conservatism. The presented techniques can serve as a basis for standard calculations improvement.

Table 1. Geometry and Loading Parameters of Notched Strip (Fig.1)

№	h , mm	ρ, mm	α, mm	S_{ny} , MPa
1 - 2 3 - 4	74.3	12.2	12.2	400-11.6x 201-5.83x
5 - 6 7 - 8	78.9	7.0	7.0	438-9.71x 200-4.43x
9 - 11	76.9	7.0	7.0	449-9.64x
12 - 14 15 - 16	80.0	1.6	24.0	280 100

Table 2. Elasto-Plastic Strain on Concentrator Surface of Notched Strip

№	K_t	*SSS type*	ε_z	ε_{iy} *approx.*	ε_{iy} *FEM*	*Error*, %
1		PSsS	-	0.00546	0.00568	-4
2		PSnS	0	0.00434	0.00444	-2
3	1.92	GPSnS	0.001	0.00141	0.00156	-10
4		GPSnS	-0.001	0.00213	0.00218	-2
5		PSsS	-	0.00944	0.00899	5
6	2.27	PSnS	0	0.00663	0.00632	5
7		GPSnS	0.001	0.00176	0.00185	-5
8		GPSnS	-0.001	0.00253	0.00255	-1
9		PSsS	-	0.00939	0.00918	2
10	2.30	PSnS	0	0.00703	0.00649	8
11		GPSnS	0.001	0.00648	0.00617	5
12		PSsS	-	0.0110	0.0101	9
13		PSnS	0	0.00817	0.00728	12
14	4.11	GSnS	0.001	0.00750	0.00696	8
15		GSnS	0.001	0.00153	0.00155	-1
16		GSnS	-0.001	0.00223	0.00223	0

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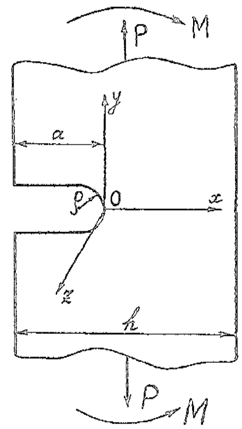

Fig. 1 Notched strip under tension - bending loading.

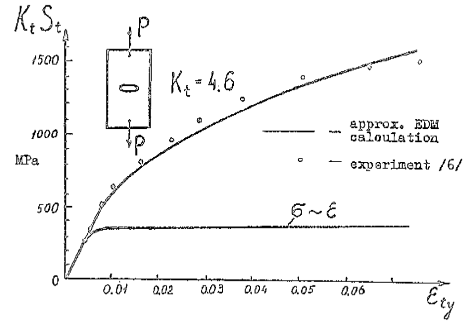

Fig. 2 Notch strains in a spesimen with a central elliptic notch. (Al 2024 T 351)

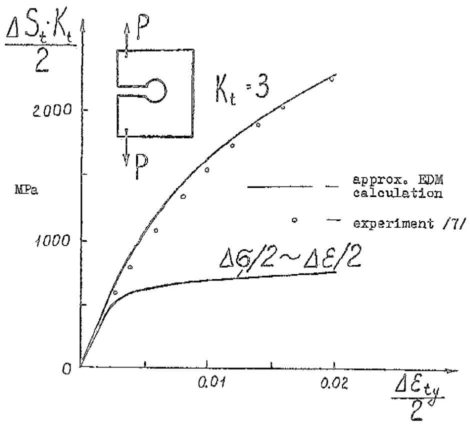

Fig. 3 Notch strains in a keyhole specimen under constant amplitude cyclic loading. (Steel RQC-100)

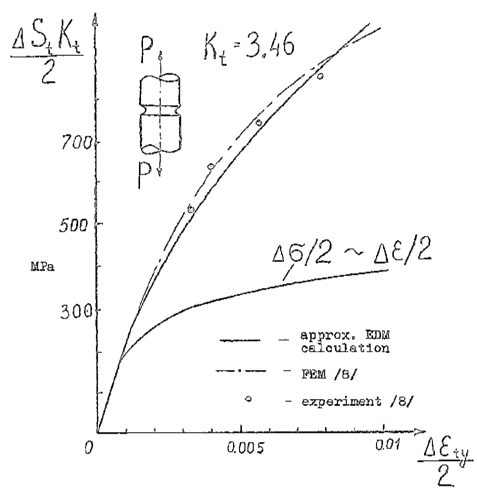

Fig. 4 Notch strains in a round bar with a circumferential U - notch. (Cr- Mo alloy steel)