SEISMIC DESIGN SPECTRA FOR NUCLEAR POWER PLANTS BASED ON PROBABILISTIC SEISMIC HAZARD ANALYSIS

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ABSTRACT

Probabilistic Seismic Hazard Analysis (PSHA) has been widely used for establishing seismic design spectra. In seismic design and analysis of Nuclear Power Plants (NPPs), the PSHA procedure may be required in generating input earthquakes such as Design Basis Earthquake (DBE) and Review Level Earthquake (RLE). The purpose of this study is to bridge the gap between seismological analyses and engineering applications in nuclear energy industry, i.e., to find a suitable representation of seismic design input earthquakes (e.g., DBE and RLE) from the seismic hazard analysis.

In this study, the properties and limitations of the existing PSHA-based seismic design spectra are discussed, based on which a generalized approach is developed to generate seismic design spectra using both scalar and vector-valued PSHA. The resulting spectral shapes can be narrowed and concentrated at a specified vibration period or a range of vibration periods based on the information of structural modes and engineering requirements. The seismic hazard level of each design spectrum is provided by the vector-valued PSHA. Each spectrum can then be interpreted as a single “design earthquake” via the joint probability. It is found that, due to the inappropriate use of the marginal probability as the joint probability, the existing PSHA-based spectra overestimate the spectral amplitudes in comparison with the proposed design spectra.

INTRODUCTION

Probabilistic Seismic Hazard Analysis (PSHA) has been widely used for the establishment of seismic design spectra, including Uniform Hazard Spectrum (UHS) (ASCE, 2005), predicted spectrum based on ground-motion prediction equations (McGuire, 1995), and Conditional Mean Spectrum considering ε (CMS-ε) (Baker and Cornell, 2006). These seismic design spectra can be used as input design earthquakes in seismic response spectral analysis or target spectra for the selection or generation of ground motions.

In seismic design and analysis of Nuclear Power Plants (NPPs), the PSHA procedure may be required in generating input earthquakes such as Design Basis Earthquake (DBE) and Review Level Earthquake (RLE). The primary advantage of the PSHA is that it provides a framework in which uncertainties in the location of earthquakes, the size of earthquakes, the rate of occurrence of earthquakes, and the variation of ground-motion characteristics with earthquake size and location can be identified, quantified, and combined in a mathematically rigorous manner to describe the seismic hazard at a given site.

However, the existing PSHA-based seismic design spectra (UHS, predicted spectra, and CMS-ε) fail to represent single design earthquake or provide complete probabilistic knowledge about the simultaneous occurrence of spectral accelerations at multiple vibration periods, which are crucial requirements in the application of modal superposition methods in structural dynamics, and reliability- and performance-based seismic design (Ni et al., 2012).

In this paper, a generalized approach is developed to generate seismic design spectra using both scalar and vector-valued PSHA. The resulting spectral shapes can be narrowed and concentrated at a
specified vibration period or a range of vibration periods based on the information of structural modes or the specific requirements of engineering projects. The seismic hazard level, i.e., annual probability of exceeding spectral accelerations simultaneously at multiple periods of engineering interest, of each design spectrum is provided by the vector-valued PSHA. Each spectrum can then be interpreted as a single “design earthquake” via the joint probability. Through the PSHA, the resulting design spectra reflect the seismic hazard environment surrounding a given site and also take the dynamic characteristics of structures into consideration.

**PROBABILISTIC SEISMIC HAZARD ANALYSIS**

Probabilistic seismic hazard analysis (PSHA) has been widely used in earthquake engineering (Cornell, 1968, Bazzurro and Cornell, 2002, McGuire, 2004). In this study, the PSHA is classified as scalar (one dimensional) PSHA and vector-valued (multiple dimensional) PSHA. Mathematically, the scalar PSHA is a special case of the vector-valued PSHA.

In vector-valued PSHA, take spectral accelerations $S_a(T_1), \ldots, S_a(T_k)$ at periods $T_1, \ldots, T_k$ as ground-motion parameters. For the seismic hazard evaluation at a site having $N_s$ potential seismic sources, the joint mean annual rate of exceedance (joint annual probability of exceedance can be used for small values) (Ni et al., 2012) of the ground-motion parameters is

$$\lambda_{a_1, \ldots, a_k} = \sum_{i=1}^{N_s} \nu_i \int \int P \left( S_a(T_1) > a_1, \ldots, S_a(T_k) > a_k \mid m, r \right) f_{m, r}(m, r) dm dr,$$

where $\nu_i$ is the mean annual rate of occurrence of earthquakes above a lower-bound of earthquake magnitude for seismic source $i$, and $f_{m, r}(m, r)$ is the joint probability density function of magnitude $M$ and source-site distance $R$ for source $i$.

The first term in the integrand in equation (1) is the joint probability distribution function of $S_a(T_1), \ldots, S_a(T_k)$ conditional on $m$ and $r$, which has been empirically tested to follow multivariate lognormal distribution (Jayaram and Baker, 2008). In this distribution function, the mean and standard deviation values in terms of $m$ and $r$ can be obtained from any appropriate set of ground-motion prediction equations, and the correlation coefficients have been obtained empirically (Baker and Jayaram, 2008).

By replacing the conditional joint distribution function in equation (1) by its corresponding marginal distribution function for spectral acceleration $S_a(T_j)$ at individual period $T_j$, the vector-valued PSHA reduces to scalar PSHA. The marginal annual probability of exceedance (marginal mean annual rate of exceedance in actuality) of the ground-motion parameter is expressed as

$$\lambda_{a_j} = \sum_{i=1}^{N_s} \nu_i \int \int P \left( S_a(T_j) > a_j \mid m, r \right) f_{m, r}(m, r) dm dr.$$

Typically, a ground-motion prediction equation for spectral acceleration $S_a(T_j)$ at vibration period $T_j$ can be expressed as

$$\ln S_a(T_j) = f(m, r, T_j, \theta) + \sigma(m, T_j) \varepsilon(T_j),$$

where $\theta$ is a vector of parameters, such as local site condition and seismic source mechanism. $f(m, r, T_j, \theta)$ and $\sigma(m, T_j)$ are mean and standard deviation values, respectively. $\varepsilon(T_j)$ is the number of standard deviations $\sigma(m, T_j)$ by which the logarithmic spectral acceleration $\ln S_a(T_j)$ deviates from the mean $f(m, r, T_j, \theta)$. 
EXISTING PSHA-BASED SEISMIC DESIGN SPECTRA

A number of seismic design spectra have been developed based on probabilistic seismic hazard analysis (PSHA). They mainly include uniform hazard spectrum (UHS), predicted spectrum based on ground-motion prediction equations, and conditional mean spectrum considering $\varepsilon$ (CMS-$\varepsilon$). In this section, the properties and limitations of these existing PSHA-based seismic design spectra are presented and discussed.

Uniform Hazard Spectra

Based on the scalar PSHA in equation (2), for a given probability $p_M$ (the subscript “M” standing for Marginal) of spectral acceleration $S_j(T)$ exceeding $s_j$, a plot of the threshold $s_j$ for a number of vibration periods $T$ of engineering interest ($j=1, 2, \ldots, k$) at a site gives a uniform hazard spectrum (UHS). As discussed in Ni et al. (2012), the probability of exceedance $p_M$ for an individual spectral acceleration, however, is not the probability of exceedance for a design earthquake (i.e., the entire design spectrum with spectral accelerations at multiple periods) which is commonly used in reliability- and performance-based seismic design. In other words, a UHS is unable to provide any probabilistic knowledge about the simultaneous exceedance of spectral accelerations at multiple periods. As a result, a UHS cannot be treated as a single design earthquake.

Predicted Spectra Based on Ground-motion Prediction Equations

In equation (2), the probability of exceedance $\lambda_{s_j}$ is obtained by integrating over all possible occurrences of earthquakes surrounding a given site. To obtain an individual design earthquake based on scalar PSHA, $\lambda_{s_j}$ can be disaggregated at predefined probability level $p_M$, and the most-likely combination (modal values) of magnitude $m$, source-site distance $r$, and $\varepsilon$, called “beta earthquake”, can then be obtained (McGuire, 1995). If one seismic source is the dominant contributor for spectral accelerations at both 0.1 and 1 sec, one beta earthquake is used to represent the seismic hazard over the entire period range. If different seismic sources are the dominant contributors at 0.1 and 1 sec, two beta earthquakes are used to represent the hazards over the short period range and the long period range, respectively. In general, one small near-field earthquake and one large far-field earthquake are regarded as sufficient to represent the short period range and the long period range of a UHS, respectively (McGuire, 1995).

Having obtained one beta earthquake (in terms of $m$, $r$, and $\varepsilon$) for the case of one dominant source, by substituting the beta earthquake into the ground-motion prediction equations (3) for spectral accelerations at different periods individually, the resulting design earthquake (in terms of seismic design spectrum), which closely matches the UHS over the entire period range, can be constructed. For the case of different dominant sources, by substituting two resulting beta earthquakes into equations (3), respectively, two seismic design spectra, which closely match the short period range and the long period range of the UHS, respectively, can be generated. It is noted that if more than one prediction equations are used, predefined weight should be assigned to each prediction equation and its corresponding beta earthquake to obtain the weighted beta earthquake and design spectra.

A predicted design spectrum based on ground-motion prediction equations can be interpreted as a single design earthquake due to the determinate pair of $m$ and $r$. However, the primary advantage of the PSHA of integrating all possible earthquake occurrences surrounding the site of interest is not completely reflected in the predicted spectra, because the seismic hazard of the site is simply represented by the beta earthquakes. Since each point on a predicted spectrum is obtained independently through the beta
earthquake and the prediction equation, the probabilistic knowledge about the simultaneous occurrence of these points on a spectrum is not provided.

**Predicted Spectra Based on Ground-motion Prediction Equations**

Similar to the predicted design spectra, to account for the relationship between spectral acceleration $S_a(T_i)$ at fundamental period $T_i$ and spectral accelerations $S_a(T_j)$ at other periods $T_j$ (j=1, 2, …, k), Baker and Cornell (2006) proposed the concept of conditional mean spectrum considering $\varepsilon$ (CMS-$\varepsilon$) based on the assumption that spectral accelerations at two different periods are jointly lognormally distributed for a given scenario earthquake, which was verified later by Jayaram and Baker (2008).

By using Taylor expansions and assuming that the variance in conditional spectral accelerations $\ln(S_a)$ is primarily due to $\varepsilon$ rather than variations in magnitude and distance, the logarithmic mean and standard deviation of the CMS-$\varepsilon$ conditional on the occurrence of $S_a(T_i) = s_i$ can be approximated by

$$
\mu_{\ln S_a(T_i)} = \mu_{\ln S_a(T_i)} + \rho_{\ln S_a(T_i), \ln S_a(T_j)} \sigma_{\ln S_a(T_j)},
$$

where $s_i$ is the spectral acceleration on a UHS at the structural fundamental period $T_i$ obtained using equation (2), for a specified probability of exceedance $p_M$, $m$, $r$, and $\mu_{\ln S_a(T_i)}$ are the mean values of the results of the scalar seismic hazard deaggregation towards the occurrence of $S_a(T_i) = s_i$ (not $> s_i$). The mean $\mu_{\ln S_a(T_i)}$ and standard deviation $\sigma_{\ln S_a(T_j)}$ of $\ln S_a(T_i)$ can be determined using ground-motion prediction equation (3), and $\rho_{\ln S_a(T_i), \ln S_a(T_j)}$ is the correlation coefficient between $\ln S_a(T_i)\ln S_a(T_j)$.

For a CMS-$\varepsilon$, the marginal probability of exceedance $p_M$ of spectral acceleration $\ln S_a(T_i)$ at fundamental period $T_i$ is predefined, and the threshold value $s_i$ of this spectral acceleration and the causal earthquake in terms of $m$, $r$, and $\mu_{\ln S_a(T_i)}$ can then be determined. Given $S_a(T_i) = s_i$ and the causal earthquake, the conditional probability of exceedance of any other spectral acceleration $S_a(T_j)$ ($j \neq i$) on a CMS-$\varepsilon$ is 50%. However, the probability of exceeding all the spectral accelerations on a CMS-$\varepsilon$ simultaneously, based on the complete information available from the PSHA, remains unknown.

**GENERALIZED APPROACH**

In the previous section, the primary properties and limitations of the existing PSHA-based seismic design spectra were discussed. These design spectra (UHS, predicted spectra, and CMS-$\varepsilon$) are unable to represent single design earthquakes or unable to provide the probability of simultaneous exceedance of spectral accelerations at vibration periods throughout the range of vibration period of engineering concern.

To not only overcome the deficiencies of these exiting seismic design spectra but also preserve some advantages of these spectra, a generalized approach for the generation of seismic design spectra is developed based on both the scalar and vector-valued PSHA. This study is an extension and a complement to the vector-valued uniform hazard spectrum (Ni et al., 2012).

The generalized approach to the generation of seismic design spectra based on PSHA is formulated using the following system of k+1 nonlinear equations:

Joint annual probability of exceedance from vector-valued PSHA governed by equation (1):

$$
P \{ S_a(T_i) > s_i, \ldots, S_a(T_k) > s_k \} = \sum_{i=1}^{N_i} v_i \left[ \int \int \int P \{ S_a(T_i) > s_i, \ldots, S_a(T_k) > s_k \} f(m,r) dm dr \right] = p_j, (5a)
$$
Marginal annual probability of exceedance from scalar PSHA governed by equation (2):

\[
P\left(S_a(T_j) > s_j\right) = \sum_{i=1}^{N_s} v_i \left[ \int_{a_0}^{a_f} \int_{m_0}^{m_f} P\left(S_a(T_j) > s_j, m_r, r\right) f_{M,R}(m_r, r) \, dr \, dm_r \right] = \eta_j p_M, \tag{5b}
\]

\(j = \xi, \xi+1, ..., \xi-1, \xi, \quad 1 \leq \xi \leq \xi \leq k,\)

Conditional probability of exceedance from scalar and vector-valued PSHA:

\[
P\left(S_a(T_j) > s_j | S_a(T_{\xi}) > s_{\xi}, ..., S_a(T_{\xi}) > s_{\xi}\right) = \frac{\sum_{i=1}^{N_s} v_i \left[ \int_{a_0}^{a_f} \int_{m_0}^{m_f} P\left(S_a(T_j) > s_j, S_a(T_{\xi}) > s_{\xi}, ..., S_a(T_{\xi}) > s_{\xi}, m_r, r\right) f_{M,R}(m_r, r) \, dr \, dm_r \right]}{\sum_{i=1}^{N_s} v_i \left[ \int_{a_0}^{a_f} \int_{m_0}^{m_f} P\left(S_a(T_{\xi}) > s_{\xi}, ..., S_a(T_{\xi}) > s_{\xi} m_r, r\right) f_{M,R}(m_r, r) \, dr \, dm_r \right]} = p_C, \tag{5c}
\]

\(j = 1, 2, ..., \xi - 1, \xi, 1, ..., k,\)

where \(p_J, \ p_M, \) and \(p_C\) are joint annual probability of exceedance, marginal annual probability of exceedance, and conditional probability of exceedance, respectively, and the subscripts “J”, “M”, and “C” stand for Joint, Marginal, and Conditional, respectively. In equations (5), \(k\) is the number of controlling periods defining spectral accelerations for the construction of design spectra, \(\eta\) is the seismic hazard weighting parameter, and parameters \(\zeta\) and \(\xi\) are serial numbers of spectral accelerations.

In engineering practice, two characteristics of a seismic design spectrum are of primary importance: spectral shape, which characterizes frequency content, and amplitude, which is governed by seismic hazard (design) level. Traditionally, a conventional seismic design spectrum is the product of a standard design spectral shape and peak ground-motion parameter such as Peak Ground Acceleration (PGA) for a given probability of exceedance. The standard spectral shape is obtained based on statistical analysis of normalized response spectra from a large number of actual earthquake records, and the PGA is determined using scalar PSHA (Newmark et al., 1973).

For the UHS, however, both spectral shape and amplitude are determined according to the marginal probability of exceedance \(p_M\). In the proposed generalized approach, the spectral shape is determined via the marginal probability level governed by equation (5b) or the conditional probability level governed by equation (5c), or both. The amplitude of the design spectrum is controlled by the joint probability level governed by equation (5a).

As discussed previously, quantifying the probability of simultaneous exceedance of spectral accelerations at multiple vibration periods over the entire period range of engineering interest on a seismic design spectrum is crucial in the application of modal combination methods in structural dynamics, and reliability- and performance-based seismic design. In the proposed approach, the vector-valued PSHA governed by equation (5a) is used to achieve this vital requirement. Since the vector-valued PSHA provides the probability of exceeding all spectral accelerations on a design spectrum simultaneously, this joint annual probability of exceedance \(p_J\) is regarded as the probabilistic seismic hazard level for a design earthquake (design spectrum) as commonly used in reliability- and performance-based seismic design. The generated design spectrum can then be interpreted as a single design earthquake. Hence, when a design spectrum is generated using the proposed approach, \(p_J\) will be pre-specified, such as \(4 \times 10^{-4}\) per annum (2% per 50 years), by decision-makers or design professionals.

As discussed above, the probability of exceeding spectral accelerations at all periods simultaneously, over the entire period range of engineering concern on a design spectrum, needs to be calculated using the vector-valued PSHA governed by equation (5a) to provide quantitatively the seismic hazard level to the resulting design spectrum. In engineering practice, however, it may not be feasible to conduct a vector-valued PSHA with a very high dimension, say spectral accelerations at 200 vibration periods, i.e., \(k=200\) in equation (5a).
In this study, a set of eight critical controlling vibration periods \( (k=8) \) of 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 1, and 5 sec is used by considering the past and present engineering practice for nuclear energy facilities (Newmark et al., 1973, Atkinson and Elgohary, 2007). The use of these controlling periods is based on the observation that considering additional intermediate periods between these controlling periods does not affect the resulting design spectra significantly.

In the following subsections, three types of seismic design spectra, named Vector-valued Uniform Hazard Spectra (VUHS), Vector-valued Non-Uniform Hazard Spectra (VNUHS), and Vector-valued Conditional Uniform Hazard Spectra (VCUHS), are presented. They are generated by assigning specific values to parameters \( p_j, p_{C}, \eta_j, \zeta, \) and \( \zeta \) in equations (5) and solving the system of equations (5) for unknowns \( p_M, s_1, s_2, \ldots, s_k \) using a suitable numerical method, such as the bisection method (Press et al., 2007). \( S = \{s_1, s_2, \ldots, s_k\}^T \) represents the spectral accelerations at the controlling periods on a resulting design spectrum corresponding to the target joint annual probability of exceedance \( p_J \). The VUHS, VNUHS, or VCUHS can then be constructed by linearly connecting the spectral acceleration coordinates at controlling periods in a linear-linear plot or a log-log plot (Atkinson and Elgohary, 2007).

**Vector-valued Uniform Hazard Spectra**

To not only preserve the essence (PSHA-based approach and spectral shape) of a uniform hazard spectrum (UHS), but also provide the probability of simultaneous exceedance of spectral accelerations on a design spectrum and be regarded as a single design earthquake, a vector-valued uniform hazard spectrum (VUHS) has been proposed by Ni et al. (2011).

In this generalized approach, the VUHS becomes a special case. When \( \zeta = 1 \) and \( \zeta = k \), equations (5) reduce to equations (5a) and (5b). By assigning 1 to \( \eta_j \), equations (5a) and (5b) form the system of equations for obtaining the VUHS, in which the joint probability of simultaneous exceedance \( p_J \) is provided, and the consistent marginal probability of exceedance \( p_M \), i.e., the uniform hazard, preserves the spectral shape of a UHS.

**Vector-valued Non-Uniform Hazard Spectra**

In the previous section, the VUHS is generated by assigning consistent seismic hazard level for spectral accelerations at all controlling periods, i.e., the same \( p_M \) for all values of \( j \). A relatively wide-band spectral shape is then obtained on a VUHS, since the seismic hazard from spectral acceleration at each individual controlling period is regarded equally important to the response of a structure. Hence, the VUHS may be a severe design earthquake for structures having a wide range of structural modal periods, which contribute significantly to structural responses.

In nuclear engineering practice, structures with first-mode dominant or with the first few closely-spaced modes dominant are not unusual. In equations (5), at the same joint probability level \( p_J \), the seismic design spectrum having a relatively narrow-band spectral shape, which is concentrated at a pre-specified period or a small range of periods, may be a more suitable design earthquake for these structures when compared to the wide-band VUHS.

To obtain a relatively narrow-band structure-specific seismic design spectrum based on PSHA, take \( \zeta = 1 \) and \( \zeta = k \), equations (5) form the system of equations for obtaining the resulting spectrum, in which the seismic hazard weighting parameter \( \eta_j \) \((0 < \eta_j < 1)\) is used to reduce the marginal probability of exceedance \( p_M \) and thus increase the spectral accelerations at selected periods, which have significant contributions to structural responses. \( \eta_j = 1 \) is then taken for the rest of controlling periods.

In the system of equations, the parameters \( \eta_j \) are used to characterize the ratio of marginal probability levels of spectral accelerations at primary important periods and at secondary important periods. Seismic hazard design levels used in current engineering practice, i.e., annual probabilities of
exceedance for spectral accelerations at individual periods, could be used to determine the values of $\eta_j$ for spectral accelerations at selected dominant periods, as discussed below.

The selection of seismic hazard design level, which may also be called Seismic Design Category (SDC) in ASCE/SEI Standard 43 (2005), depends on mainly the importance level of the structure. For example, the seismic hazard design level of a rare earthquake is usually specified as $2 \times 10^{-3}$ per annum (10% in 50 years) for standard civil structures, and the seismic hazard design level of an extremely rare earthquake (or maximum considered earthquake) is often set as $4 \times 10^{-4}$ per annum (2% in 50 years) for critical structures (ASCE, 2005). The ratio of 0.2 between these two seismic hazard design levels could be used to represent the difference between important and ordinary structures in terms of seismic design level (annual probability of exceedance for individual spectral acceleration).

In this study, the concept of the ratio of marginal probabilities of exceedance for distinguishing the importance levels of structures is extended to describe the relative importance levels of structural modes, i.e., the contributions to structural response from the structural modes. As a result, the seismic hazard weighting parameter $\eta_j$ can be taken as the ratio of the selected seismic hazard design levels.

To maintain the same joint probability level $p_J$, with the decrease of parameters $\eta_j$ assigned to selected dominant periods, the resulting spectral shape becomes narrower and more concentrated, with increased spectral amplitudes, at these dominant periods. If the dominant modal periods of a structure are generally determinate, i.e., little nonlinearity occurs or the level of nonlinearity is well estimated when subjected to an earthquake excitation, so that the spectral accelerations at the non-dominant periods have little effect on the structural response, smaller value of $\eta_j$ would introduce unnecessary conservatism to the design. Hence, the selection of seismic hazard design levels for determining $\eta_j$ relies on engineering judgment and the frequency bandwidths of response spectra of local recorded ground motions.

However, regardless of the resulting spectral shape or the effect of spectral accelerations at non-dominant periods on structural responses, the spectral accelerations on a spectrum over the entire period range certainly occur at the same time when an earthquake occurs. Fortunately, the probability of this simultaneous occurrence can be quantified through the vector-valued PSHA. A single design earthquake associated with its seismic hazard level can then be provided.

Since the vector-valued PSHA provides the seismic hazard level of a single design earthquake (the entire spectrum) and the scalar PSHA produces a narrowband spectral shape with inconsistent marginal probability levels through $\eta_j$, the name of Vector-valued Non-Uniform Hazard Spectrum (VNUHS) for the resulting spectrum is adopted.

**Vector-valued Conditional Uniform Hazard Spectra**

In the previous section, a vector-valued non-uniform hazard spectrum (VNUHS) is generated based on the PSHA. A narrow-band spectral shape of the VNUHS is achieved by manipulating the marginal probabilities of exceedance for spectral accelerations at dominant periods individually, through the seismic hazard weighting parameters $\eta_j$. To not only obtain a relatively narrow-band spectral shape concentrated at selected dominant periods but also consider the spectral correlation between the dominant periods and non-dominant periods, an alternative seismic design spectrum, Vector-valued Conditional Uniform Hazard Spectrum (VCUHS) can be generated by specifying the parameters in equations (5) as follows.

Under the condition that $\zeta = 1$ and $\xi = k$ are not satisfied simultaneously in equation (5b), the system of equations (5) for generating the VCUHS can be obtained, in which the joint probability of exceedance $p_J$ is still maintained by equation (5a) to provide the seismic hazard level of a single design earthquake, while equations (5b) and (5c) control the spectral shape of the VCUHS. The properties of VCUHS are discussed using an example in the following.
As mentioned previously, by taking eight critical controlling periods \((k=8)\) in equations (5), the VCUHS can be satisfactorily represented by spectral accelerations at these eight periods. Suppose the dominant modal periods of a structure, which have dominant contributions to the structural response, are \(T_3\) and \(T_4\) \(\text{(i.e., } \zeta = 3 \text{ and } \xi = 4)\), equation (5b) is used to characterize and emphasize the spectral accelerations at these two dominant periods; a value of 1 could be assigned to \(\eta_j \ (j=3, 4)\) if these two dominant periods are regarded equally important.

While equation (5b) is used for spectral accelerations at dominant periods, spectral accelerations on a VCUHS at non-dominant periods are considered through equation (5c). The physical meaning of equation (5c) in this example is that, under the condition of the simultaneous exceedance of spectral accelerations at two dominant periods \(\text{(i.e., } S_a(T_3) > s_3 \text{ and } S_a(T_4) > s_4)\), the probability that the spectral acceleration \(S_a(T_j)\) at any non-dominant period \(T_j\) \(\text{(i.e., } j=1, 2, 5, 6, 7, 8)\) exceeds a threshold value \(s_j\) is \(p_C\). The conditional probability of exceedance \(p_C\) can be taken as 50\%, which means that the median values of spectral accelerations at non-dominant periods, conditional on the occurrence of the spectral accelerations at dominant periods \(S_a(T_3) > s_3 \text{ and } S_a(T_4) > s_4\), are adopted for the VCUHS.

The conditional probability governed by equation (5c) accounts for the fact that the correlation of spectral accelerations decreases with the increasing separation of vibration periods, which is similar to the concept of the conditional mean spectrum considering \(\varepsilon\) \(\text{(CMS-} \varepsilon\text{)}\). To maintain the conditional probability of exceedance, the marginal probability of exceedance of spectral acceleration at non-dominant period increases \(\text{(i.e., its seismic hazard decreases) with the increasing separation between the non-dominant period and the dominant periods } T_3 \text{ and } T_4\). The consideration of spectral correlation could produce spectral shape having more frequency contents concentrated around the dominant periods \(T_3\) and \(T_4\) and less frequency contents away from the dominant periods, when compared to the VNUUHS.

**NUMERICAL EXAMPLES**

In this section, the generalized approach is demonstrated by several numerical examples based on a hypothetical configuration of seismic sources as shown in Figure 1. To apply the exact generalized approach governed by equations (5), for illustration purpose, magnitude \(M\) and distance \(R\) are assumed to be statistically independent, and the commonly used truncated exponential distribution for \(M\) and uniform distribution of seismic focus for \(R\) are adopted for obtaining the joint probability density function. The ground-motion prediction equations for rock site \(\text{(Abrahamson and Silva, 1997)}\) and the most current matrix of spectral correlation \(\text{(Baker and Jayaram, 2008)}\) are used for the conditional probability distribution of spectral accelerations. All other parameters in these distribution models are the same as used in Ni et al. \(\text{(2012)}\).
As shown in Figure 2, vector-valued uniform hazard spectrum (VUHS), vector-valued non-uniform hazard spectrum (VNUHS), and vector-valued conditional uniform hazard spectrum (VCUHS) are generated at joint annual probability of exceedance of $4 \times 10^{-4}$ when the fundamental period of a structure is assumed to be 0.1 sec. As discussed previously, the VNUHS and VCUHS have narrower spectral shapes and higher amplitudes at the period of 0.1 sec in comparison with the wide-band VUHS. Also, the consideration of the spectral correlation results in the VCUHS having more frequency contents concentrated around the period of 0.1 sec when compared to the VNUHS. In this case, in comparison with the VUHS and VNUHS, the VCUHS could be a more suitable design earthquake for a structure with higher modes (modal periods smaller than 0.1 sec) contributing to the structural response, regardless the structure remains elastic (the fundamental period $T_1$ remains the same) or becomes inelastic ($T_1$ increases slightly) when subjected to the earthquake excitation.

![Figure 2. Design spectra for dominant period of 0.1 sec at probability of exceedance of $4 \times 10^{-4}$](image)

Figure 2. Design spectra for dominant period of 0.1 sec at probability of exceedance of $4 \times 10^{-4}$

![Figure 3. Design spectra for dominant periods between 1 and 5 sec at probability of exceedance of $4 \times 10^{-4}$](image)

Figure 3. Design spectra for dominant periods between 1 and 5 sec at probability of exceedance of $4 \times 10^{-4}$
To account for the nonlinear behavior or the contributions of higher modes of a structure, the VNUHS and VCUHS can also be generated by emphasizing the spectral accelerations at a range of periods. As illustrated in Figure 3, the VUHS, VNUHS, and VCUHS are obtained at joint annual probability of exceedance of $4 \times 10^{-4}$ for a structure having several dominant periods between 1 and 5 sec. Similar results as in Figure 2 are observed on the spectral shapes and amplitudes of the VUHS, VNUHS, and VCUHS.

For comparison, uniform hazard spectra (UHS), conditional mean spectra considering $\varepsilon$ (CMS-$\varepsilon$), and predicted spectra based on ground-motion prediction equations are also shown in Figures 2 and 3. These spectra are determined for marginal annual probability of exceedance of $4 \times 10^{-4}$. Due to the inappropriate use of the marginal probability as the joint probability, these spectra are quite conservative in comparison with the proposed design spectra at specified periods (Ni et al., 2011).

CONCLUSION

Three types of seismic design spectra are generated based on both scalar and vector-valued PSHA: Vector-valued Uniform Hazard Spectrum (VUHS), Vector-valued Non-Uniform Hazard Spectrum (VNUHS), and Vector-valued Conditional Uniform Hazard Spectrum (VCUHS). These generated design spectra are suitable for different types of structures with different engineering considerations as required. It is found that, due to the inappropriate use of the marginal probability as the joint probability, the existing PSHA-based spectra overestimate the spectral amplitudes in comparison with the proposed design spectra.

REFERENCES


