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"MISSING MASS" CORRECTION IN MODAL ANALYSIS OF PIPING SYSTEMS

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In mode-by-mode dynamic analyses of seismic response, an approximate solution is obtained because only a limited number of modes is considered (in U. S. practice, typically all modes up to 33 Hz). Errors are usually small for such response parameters as pipe displacements and stresses because they are affected relatively little by the high modes. However, the error may be substantial for support loads because the influence of high modes on their values can be important. Substantial stress errors can also be present in stiff systems with few low frequency modes.

In effect, the truncation of the mode series means that some mass of the system is ignored. The distribution of this "missing mass" is such that the inertia forces associated with it will usually produce only small displacements and stresses. However, these forces will often produce significant support loads, and in stiff systems can produce significant stresses.

An approach which is sometimes used is to assume that the "missing mass" consists of the masses at the support points. Support load corrections can then be made. However, this approach is over-simplified and potentially inaccurate. A more accurate correction can be made by determining the modal contributions to the mass of the system and obtaining the "missing mass" as the difference between these contributions and the actual mass. A rational and accurate correction for "missing mass" effects can thus be made. This approach has the advantage that "missing masses" will exist not only at the supports but throughout the system. Hence, not only support load errors but also stress errors can be corrected. Further, the procedure applies to systems with out-of-phase anchor motions as well as systems in which all anchors move in phase, and to both response spectrum and time-history analyses.

The theory of the correction is presented, the method of computer implementation is indicated, and an example is discussed.

1. Introduction

A response spectrum provides an efficient means for determining the maximum effects produced in any given natural mode of vibration of a piping system by a real or design earthquake. Given the maximum effects in each mode, the maximum effects for a multimode system can be estimated by combining the modal maxima.

In a response spectrum analysis, only modes with frequencies below a cut-off frequency of about 33 Hz will usually be considered, because the higher modes generally involve only small displacement amplitudes and pipe stresses. However, these higher modes may involve significant support reations. For example, if a point is restrained by a rigid anchor, then there will be no excitation of the lumped mass at the point in any of the modes considered, because the frequency of vibration of the mass is very high. As a result, the mass at the point is "lost", and the computed anchor reactions will be too small, possibly by substantial amounts.

One procedure which can be used to correct for this "missing mass" effect is to multiply the mass at each rigid support by the value of the spectral acceleration at zero period, and to combine the resulting force with the computed reaction. This procedure has the disadvantages, however, that (a) there may be substantial missing mass at points other than support points, particularly for stiff piping systems, and (b) it is not always obvious whether a support is "rigid", because some supports may have finite flexibilities. A more sophisticated procedure is therefore desirable. Such a procedure was incorporated into the SUPERPIPE [1] computer program in 1976, and has proven to be an effective means of accounting for missing mass effects. This paper describes the theory on which the procedure is based.

Response spectrum techniques are typically applied only to structures in which all support points move identically and in phase during the earthquake. In many piping systems, however, different support points may be subjected to quite different earthquake motions. This type of excitation will be termed "multiple excitation," whereas the case in which all supports move in phase will be termed "simple excitation." An important characteristic of multiple excitation is that the displacements and stresses induced in the structure result not only from inertial resistance but also from relative displacements between supports at different levels in the power plant structure. These two effects may be termed "inertia effects" and "anchor movement effects," respectively. The procedure described herein has been implemented in SUPERPIPE for the calculation of multiple excitation inertia effects, and the theory is presented here for both simple and multiple excitation. Anchor movement effects are not considered. Also, the question of how best to combine modal effects is not addressed in this paper.

2. Theory

2.1 Modal Amplitude

The maximum amplitude, Y_{ns}, produced in mode n by a specified support excitation is given by the following well-known equation:

$$Y_{ns} = \frac{\underline{\phi}_n^T \underline{M}\, \underline{r}_{sj}}{\underline{\phi}_n \underline{M}\, \underline{\phi}_n} \cdot \frac{S_{an}}{\omega_n^2} \tag{1}$$

where:

- n = mode number;
- s = support "level" at which excitation is applied (all supports in a "level" move in phase);
- j = direction of excitation (X, Y, or Z);
- $\underline{\phi}$ = mode shape vector;
- $\underline{M}$ = mass matrix, assumed to be diagonal;
- r_{sj} = vector of mass point displacements produced by a unit displacement in direction j applied statically at support level s;
- S_{an} = spectral acceleration corresponding to mode n for the specified support level and direction of excitation; and
- ω_n = circular frequency.

If all support points move in phase (simple excitation) then for an earthquake in, say, the X direction, $\underline{r}$ contains unit values for each X mass and zero values elsewhere. This is because a unit X displacement of the supports produces a unit rigid body displacement of the entire structure. The vector $\underline{r}$ similarly contains unit and zero values for simple Y and Z excitations. The vector $\underline{r}$ in such a case will be given the symbol $\underline{i}$.

If all support points do not move in phase (multiple excitation), then vectors $\underline{r}$ must be calculated for unit values of X, Y, and Z displacement at each support level. This requires one static analysis for each direction at each level.

If the mode shapes are normalized such that

$$\underline{\phi}^T \underline{M}\, \underline{\phi} = \underline{I} \tag{2}$$

where $\underline{I}$ = a unit matrix, then

$$Y_{ns} = \underline{\phi}_n^T \underline{M}\, \underline{r}_{sj} \frac{S_{an}}{\omega_n^2} \tag{3}$$

The scalar $\underline{\phi}_n^T \underline{M}\, \underline{r}_{sj}$ is the mass participation factor for mode n and the specified excitation.

2.2 "Missing" Inertia Forces for Rigid System

The mode shapes constitute a series of orthogonal functions, analogous to the terms of a Fourier series. The accuracy of a series expansion depends on the number of terms which are considered, and truncation of the mode series produces the missing mass effect.

Consider an essentially rigid piping system subjected to simple excitation in, say, the X direction. Such a system responds essentially as a rigid body, and hence the inertia force acting on any mass point at any time will be the product of the mass and the ground acceleration. The maximum force will correspond to the peak ground acceleration, and the corresponding "true" inertia force vector, $\underline{F}_i$, will be given by

$$\underline{F}_i = \underline{M}\, \underline{i}_x S_{ao} \tag{4}$$

in which S_{ao} = spectral acceleration at zero period (rigid body), which is equal to the peak ground acceleration.

The inertia forces for such a stiff system can also be determined mode by mode. The inertia force vector, $\underline{F}_n$, in mode n is given by

$$\underline{F}_n = -\underline{M}\,\underline{\phi}_n\,\ddot{Y}_n = \omega_n^2\,\underline{M}\,\underline{\phi}_n\,Y_n \tag{5}$$

Hence, from Eq. 3 with $\underline{r} = \underline{i}_x$ it follows that

$$\underline{F}_n = \underline{M}\,\underline{\phi}_n\,\underline{\phi}_n^T\,\underline{M}\,\underline{i}_x\,S_{ao} \tag{6}$$

If all modes are considered, the exact result will be obtained. Because all modes for the essentially rigid system respond instantaneously, their peaks occur simultaneously, and hence the "true" inertia force vector is obtained by direct summation of the modal force vectors. That is

$$\underline{F}_i = \sum_{n=1}^{N} \underline{F}_n \tag{7}$$

in which N = total number of modes. If only m modes are considered, however, the "seen" inertia force vector, $\underline{F}_t$, for the truncated series is

$$\underline{F}_t = \sum_{n=1}^{m} \underline{F}_n \tag{8}$$

Hence, the "missing" inertia force vector, $\underline{F}_m$, is

$$\underline{F}_m = \underline{F}_i - \underline{F}_t \tag{9}$$

or, from Eqs. 8, 4, and 6,

$$\underline{F}_m = \left(\underline{M} - \sum_{1}^{m} \underline{M}\,\underline{\phi}_n\,\underline{\phi}_n^T\,\underline{M}\right)\underline{i}_x\,S_{ao} \tag{10}$$

Note that there will generally be missing inertia forces in the Y and Z directions, as well as in the X direction, for X excitation. Missing force vectors can similarly be obtained for excitations in the Y and Z directions (replace $\underline{i}_x$ in Eq. 10 by $\underline{i}_y$ and $\underline{i}_z$, respectively).

2.3 "Missing" Inertia Forces for Flexible System

For a system which is essentially rigid, the missing inertia forces given by Eq. 10 can be applied to the system to obtain an essentially exact correction for the missing mass effect. Of course, a modal analysis would not be used in practice for such a system, because its peak response could be determined by a simple static analysis. The reasoning can be extended, however, to flexible systems.

If the modal series for a flexible system is truncated, the modes which are ignored will generally be of such high frequency that they respond with no amplification, like a rigid body (design acceleration spectra invariably level off at the peak ground acceleration for frequencies above 33 Hz). It is reasonable, therefore, to use Eq. 10 to determine the missing mass effect for flexible systems. The assumption in this approach is that the contributions of the modes above the cut-off frequency are the same as they would be if all modes had very high frequencies. Because the modes respond independently, it does not matter that some of them have low frequencies. Any errors result only from whether or not it is reasonable to assume instantaneous response in the high frequency modes.

2.4 Extension to Multiple Excitation

Eq. 10 can be extended to account for multiple excitation. If excitation is imposed in direction j at support level s, then for an essentially rigid system the "true" inertia forces are given by

$$\underline{F}_i = \underline{M}\, \underline{r}_{sj}\, S_{ao} \tag{11}$$

The inertia forces in mode n are still given by Eq. 5. Hence, the inertia forces "seen" through the truncated series are given by

$$\underline{F}_t = \sum_1^m \underline{M}\, \underline{\phi}_n\, \underline{\phi}_n^T\, \underline{M}\, \underline{r}_{sj}\, S_{ao} \tag{12}$$

As before, the "missing" forces are

$$\underline{F}_m = \underline{F}_i - \underline{F}_t \tag{13}$$

or

$$\underline{F}_m = \left(\underline{M} - \sum_1^m \underline{M}\, \underline{\phi}_n\, \underline{\phi}_n^T\, \underline{M}\right) \underline{r}_{sj}\, S_{ao} \tag{14}$$

For multi-level and multi-directional excitation, there will be a different force vector for each level and direction. For flexible piping systems, with several mode frequencies below the cut-off frequency, $\underline{F}_m$ will usually contain significant values only for masses at rigid supports. For stiff systems, however, $\underline{F}_m$ may contain significant values for all masses. There may also be significant values at all mass points if, for preliminary design purposes, analyses are carried out with a cut-off frequency lower than 33 Hz.

3. Computer Implementation

In the SUPERPIPE program, the missing inertia forces are determined using Eq. 14 for all $\underline{r}_{sj}$ vectors (i.e., X, Y, and Z displacements for simple excitation; X, Y, and Z displacements at each support level for multiple excitation) assuming $S_{ao} = 1$. The displacements, pipe forces and support forces corresponding to each missing force vector are determined, and for any response spectrum analysis these values are treated as analogous to additional modal results. The missing mass results are not identical in form to modal results, in particular because the missing mass values for any support level are different for X, Y, and Z excitations, whereas the modes are the same and only the participation factors change with X, Y, and Z. However, this is only a minor complication.

For any specified series of acceleration spectra, the values of S_{ao} will typically be the values at zero period. However, an option is available to use a lower frequency for determining values of S_{ao}. If this frequency is 33 Hz, the S_{ao} value which is used will typically be equal to that at zero period. If, however, a preliminary analysis is carried out using a lower cut-off frequency (e.g., 15 Hz) to save computer time, then the frequency for S_{ao} would typically be set equal to this cut-off frequency, to obtain a more conservative missing mass correction.

For multi-directional and/or multi-level excitation, separate missing mass effects must be combined together. The combination procedures used in SUPERPIPE are the same as those used to combine modal effects. Details of the combination procedure are beyond the scope of this paper.

The SUPERPIPE program also incorporates a missing mass correction into earthquake time-history analyses, for both simple and multiple excitation. The theory is essentially the same.

4. Example

The simple piping system shown in Fig. 1 illustrates the type of results obtained when a missing mass correction is applied. The system has 12 modes with frequencies below 33 Hz (ranging from 4.69 Hz to 28.78 Hz), with 6 modes below 15 Hz. Other details of the system are not important.

Response spectrum analyses were carried out considering both 12 modes and 6 modes, with and without the missing mass correction. All supports were assumed to move in phase. The response spectra for the X and Z directions were made uniform, with values of 1.0 g for all periods. The Y spectrum was uniform with a value of 0.5 g for all periods. The modal results were combined using square-root-of-sum-of-squares combination. Although this is an over-simplified example, the results exhibited the same trends as more realistic systems.

Table 1 shows how the computed stresses at the points of highest stress are affected when the number of modes is varied and the missing mass correction is included or ignored. For 12 modes, the effect of the missing mass correction is small, indicating that 12 modes provide accurate values of the stresses. For 6 modes without the missing mass correction, the computed stresses are somewhat lower, and the missing mass correction brings them substantially closer to the 12-mode values. This type of result is typical for flexible piping systems. For stiffer systems, the missing mass correction can produce larger changes in stress.

Table 2 shows how typical computed support loads are affected when the number of modes is varied and the missing mass correction is included or ignored. For 12 modes, the correction is small for most supports, indicating that 12 modes provide accurate results. For the Z loads at A and E, however, the missing mass effect is substantial. For 6 modes, the computed loads at F are reduced significantly when the missing mass correction is ignored. Application of the correction produces a marked improvement in the Y load at this point, and produces a substantial overestimate of the Z load. This type of behavior has been observed in other cases, namely that if truncation of the mode series causes substantial errors, then the missing mass correction will usually produce an overestimate of the true value. Other examples will be discussed in the oral presentation.

5. Conclusion

The use of a "missing mass" correction is desirable for seismic analysis of piping systems. The theory described in this paper can be applied for both response spectrum and time-history analyses, and for multiple support excitation as well as conventional "simple" excitation. The example presented here suggests that the correction is distinctly beneficial, but does not necessarily lead to exact results.

References

[1] SUPERPIPE User's Manual, EDS Nuclear Inc., 220 Montgomery St., San Francisco, California 94104, U.S.A.

TABLE 1. CALCULATED PIPE STRESSES

Stress = $\left(M_x^{\ 2} + M_y^{\ 2} + M_z^{\ 2}\right)^{\frac{1}{2}}/Z$; Z = section modulus.
Tabulated value = stress as a proportion of stress for 12 modes with missing mass correction.

Location	12 Modes With M.M.	12 Modes No M.M.	6 Modes With M.M.	6 Modes No M.M.
A	1.000	0.999	0.999	0.999
B	1.000	0.999	0.992	0.951
C	1.000	0.998	0.963	0.935
D	1.000	0.999	1.003	0.980

TABLE 2. CALCULATED SUPPORT LOADS

Tabulated value = load as a proportion of load for 12 modes with missing mass correction.

Support	Load Direction	12 Modes With M.M.	12 Modes No M.M.	6 Modes With M.M.	6 Modes No M.M.
A	X	1.000	0.999	1.000	0.999
A	Y	1.000	0.996	0.998	0.996
A	Z	1.000	0.586	0.908	0.581
E	Y	1.000	0.999	1.001	0.996
E	Z	1.000	0.851	1.064	0.842
F	Y	1.000	0.992	1.070	0.729
F	Z	1.000	0.959	1.416	0.655

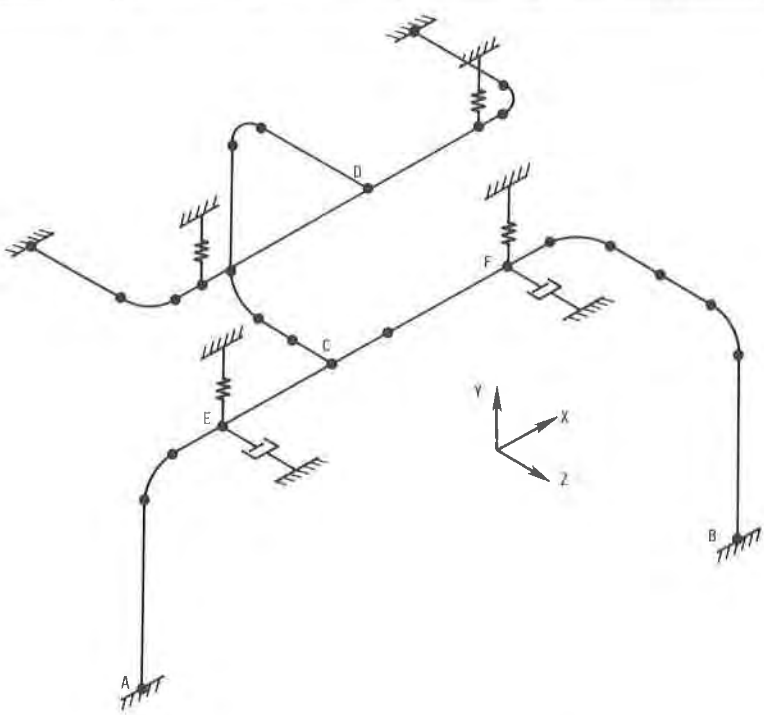

FIG. 1 EXAMPLE PIPING SYSTEM