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# Vibrational conductivity approach to vibration in a power plant subjected to aircraft impact

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ABSTRACT. Vibrational conductivity approach is applied for estimation of the acceleration magnitudes of the primary structures in power plant under the verified loading which corresponds to the military jet's crash. It is shown that the parameters of the approach can be determined through the overall geometrical and mechanical parameters of the plant which allows one to perform a vibration analysis on the earliest stages of design.

## 1. INTRODUCTION

The design of nuclear power plant systems and components to withstand dynamic loading is subject to more stringent regulatory requirements and licensing conditions than those imposed on conventional power generating facilities. These requirements and conditions address not only normal operational loading, but also, in particular, short-duration external dynamic loads, e.g. aircraft impact, explosion pressure waves, pipe breaks, internally generated missiles etc. Broad-band spectra are typical for such short-duration loadings.

As a result of the complexity of the dynamic processes and the effect of interaction between the systems, subsystems, components and the primary structure, the structural model must include the whole system from the rock-soil interface to the smallest component, which demands a fine mesh. The conventional well-established methods of structural analysis, such as FE, the substructure synthesis methods etc. are not always suitable since they require a very fine mesh to model a short wavelength deformation of higher normal modes.

One of the alternative approach is the vibrational conductivity approach to modelling high frequency vibration in complex engineering structures. In the framework of this approach, the heat conduction equation in used to model vibrational energy propagation, cf. [1]. Statistical Energy Analysis (SEA) is considered as a rationale for the use of the methods of the heat conduction to high frequency dynamics. In the modal approach to SEA the structure is viewed in terms of substructures and the power flow between two coupled substructures is shown to be proportional to the difference of the vibrational energy of each substructure, cf. [2] and [3]. The latter relation between power flow and vibrational energy difference is fully analogous to the Fourier law of heat conduction which states that heat flow is proportional to temperature difference. This analogy has allowed to reduce the vibration propagation in complex structures to a discrete transfer scheme, cf. [4]. A boundary value problem of the heat conduction type was first applied in [5] to the analysis of random vibration in complex structures. The vibrational conductivity approach to high frequency dynamics of complex structures was proposed in the eighties, see [6] and [1] for detail. A power flow finite element analysis of dynamic systems has been reported in [7].

The paper is organized as follows. Part 2 gives a short introduction to the vibrational conductivity approach, Part 3 deals with the parameters identification while the results of computations are presented in Part 4. The paper is concluded with a short discussion.

## 2. BOUNDARY VALUE PROBLEM FOR VIBRATIONAL CONDUCTIVITY APPROACH

Since complex engineering structures at high frequencies behave like heavily damped mechanical systems, cf. [6], a "heat sink" term $-\alpha S$ is included in the traditional heat conduction equation, to give the following vibrational conductivity equation

$$\textit{in volume } V; \quad K\,\Delta S - \alpha S = 0 \tag{2.1}$$

Here analogous to the heat conduction equation $S$ is a "vibrational temperature", $K$ is vibrational conductivity, $\alpha$ is vibrational conductance, and $\Delta$ denotes the Laplacian. The boundary condition is as follows

$$\textit{on boundary } B; \quad N \cdot (K\,\nabla S) = \Gamma \tag{2.2}$$

where $N$ is the unit vector of the outer normal, $\nabla$ is the nabla operator and $\Gamma$ is the vibrational flux prescribed at the surface.

Identification of the parameters of the vibrational conductivity approach implies a comparison of a closed form solution of the boundary value problem, eqs (2.1) and (2.2) with a closed form solution from another theory. The most suitable theory in this regard is high frequency structural dynamics, cf. [6]. The latter approach belongs to the integral theories and allows one to obtains some simple expressions in closed form for the magnitudes of the vibrational acceleration. The complete derivation of the equations for the parameters of the vibrational conductivity approach can be found e.g. in [1] and [6]. We content ourselves with the final formulae and brief explanation of the physical meaning of the parameters. The parameters of the vibrational conductivity approach are as follows

$$K = \frac{\langle a\rangle^3 \langle\rho\rangle^2}{2\omega\kappa}; \quad \alpha = 2\omega\kappa\langle a\rangle\langle\rho\rangle^2, \quad \Gamma = \omega^2\left[S_N + S_{T1} + S_{T2}\right], \quad S = S_a \tag{2.3}$$

where $\langle a\rangle$ is an averaged velocity of sound in the complex structure, $\langle\rho\rangle$ is an averaged mass density of the structure, $\omega$ is the frequency, and $\kappa$ is a dimensionless frequency-dependent parameter which reflects material damping and resonant absorption of the vibrational energy in the structure. The vibrational flux at the surface $\Gamma$ is the first invariant of the tensor of the traction spectral densities, e.g. $S_N$ is the spectral density of the normal stress while $S_{T1}$ and $S_{T2}$ are the spectral densities of the tangential stresses in the two orthogonal directions. For the random loading $S_N$, $S_{T1}$ and $S_{T2}$ are the spectral densities of the loading whereas for deterministic loading they may be viewed as the square of the loading amplitude. Finally, the "vibrational temperature" $S$ is actually the spectral density of the vibration acceleration, or square of the acceleration magnitude in case of deterministic loading. The "vibrational temperature" depends upon frequency, that is the boundary value problem, eqs (2.1) and (2.2) should be solved for each frequency. The solution provides us with the spectrum of the vibration field in the structure. As seen from eqs (2.1) and (2.2) the main attraction of the vibrational conductivity approach is the scalar governing differential equation of the heat conduction type. In particular, the finite element formulation can be easily achieved by applying commercial finite element codes. The parameters of the approach depend on the overall properties of the structure that simplifies the problem of the parameter identification markedly.

## 3. IDENTIFICATION OF THE PARAMETERS OF THE APPROACH

Equation (2.3) requires that for the identification of the approach parameters one should estimate the following overall parameters of the power plant: $\langle a \rangle, \langle \rho \rangle$ and $\kappa$. In order to determine the average velocity of sound one needs the averaged rigidity. One of possible way to find it is to perform an "imaginary experiment", i.e. to compare the rigidity of the power plant with the rigidity of the vibrational conductivity model. We assume that the secondary structures which are attached to the primary structure do not contribute considerably to the total rigidity of the structure, thus, the main contribution to the total rigidity comes from the outer shell which is modelled by a cylindrical shell of the radius R and thickness h. In the framework of the vibrational conductivity approach the structure is considered to be a solid body, i.e. in our case it should be a solid cylinder. Since the impact caused by aircraft crash is strongly localised the structure experiences a bending loading. One of the suitable loadings for an "imaginary" experiment is shown in Fig. 1 where Fig. 1a shows the cross-section of cylindrical shell while Fig. 1b depicts the cross-section of the vibrational conductivity model. The loading which is schematically shown in Fig. 1 is given by

$$p(\theta) = P \cos 2\theta \tag{3.1}$$

where $\theta$ is the polar angle.

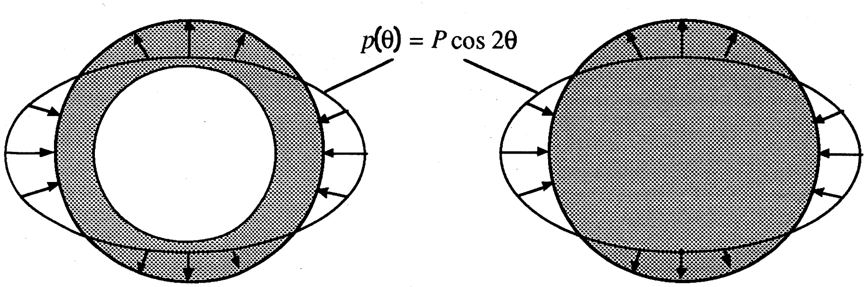


Figure 1. a) Schematics of the cross-section of the outer shell of the power plant    b) cross-section of the model for the vibrational conductivity approach

The governing equations for deflection of the circular cylindrical shell are written in the conventional form, cf. [8]

$$D\Delta\Delta w - \frac{1}{R}\frac{\partial^2 \Phi}{\partial z^2} = p\,, \quad \Delta\Delta\Phi + \frac{Eh}{R}\frac{\partial^2 w}{\partial z^2} = 0 \tag{3.2}$$

where $w$ is the deflection, $\Phi$ is the stress function, $R$ and $h$ are the shell radius and thickness, respectively, $D = Eh^3(1-\nu^2)^{-1}/12$ is the flexural rigidity of the shell and $z$ is axial coordinate. For the sake of simplicity, a simply supported shell of the length $l$ is considered, thus the boundary conditions are as follows

$$x = 0, l\,, \quad w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{3.3}$$

As the Fourier series representation of the external loading $p(\theta)$, eq. (3.1), is

$$p(\theta) = \frac{4P\cos 2\theta}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} \sin\frac{\pi n z}{l}, \quad 0 < z < l \tag{3.4}$$

the solution of the boundary value problem, eqs. (3.2) and (3.3), the Fourier series for $w$ and $\Phi$ only involve sines

$$w = \cos 2\theta \sum_{n=1,3,5}^{\infty} w_n \sin\frac{\pi n z}{l}, \quad \Phi = \cos 2\theta \sum_{n=1,3,5}^{\infty} \Phi_n \sin\frac{\pi n z}{l} \tag{3.5}$$

Substituting eq. (3.5) into eqs. (3.2) and eliminating $\Phi_n$ yields

$$w = P\cos 2\theta \sum_{n=1,3,5}^{\infty} R^4 \sin\frac{\pi n z}{l} \frac{1}{\pi n}\left[4D\left(1+\lambda_n^2\right)^2 + \frac{1}{4} Eh R^2 \lambda_n^4 \left(1+\lambda_n^2\right)^{-2}\right]^{-1} \tag{3.6}$$

An average rigidity of the shell can be estimated as follows

$$\langle C\rangle = p(\theta)\left[\frac{1}{l}\int_0^l w\, dz\right]^{-1} \tag{3.7}$$

which leads to the following result

$$\langle C\rangle = \frac{\pi^2 Eh}{2R^4}\left[\sum_{n=1,3,5}^{\infty} n^{-2}\left[\frac{h^2}{3(1-\nu^2)}\left(1+\lambda_n^2\right)^2 + \frac{1}{4}R^2\lambda_n^4\left(1+\lambda_n^2\right)^{-2}\right]^{-1}\right]^{-1} \tag{3.8}$$

This result should be compared with the averaged rigidity of a solid cylinder. The governing equations of the plane strain problem are, see [9]

$$\sigma_r = \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2}; \quad \sigma_\theta = \frac{\partial^2 \Psi}{\partial r^2}; \quad \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Psi}{\partial \theta}\right) \tag{3.9}$$

where the stress function $\Psi$ satisfies the biharmonic equation

$$\Delta\Delta\Psi = 0 \tag{3.10}$$

For the loading (3.1) the solution is sought in the following form

$$\Psi = \left(A r^2 + B r^4\right)\cos 2\theta \tag{3.11}$$

where the integration constants $A$ and $B$ should be found from the boundary conditions

$$\sigma_r\Big|_{r=R} = -P\cos 2\theta; \quad \tau_{r\theta}\Big|_{r=R} = 0 \tag{3.12}$$

Substituting eq. (3.11) into eq. (3.12) yields the integration constants A and B which finally results in the following equations for the stresses

$$\sigma_r = -P\cos 2\theta; \quad \sigma_\theta = P\left[1 - 2\left(\frac{r}{R}\right)^2\right]\cos 2\theta; \quad \tau_{r\theta} = P\left[1 - \left(\frac{r}{R}\right)^2\right]\sin 2\theta \tag{3.13}$$

The Hooke law and the kinematic relations between strain and displacements are

$$\begin{aligned} \varepsilon_r &= \frac{1}{\langle E\rangle}\left(\sigma_r - \nu\,\sigma_\theta\right); \quad \varepsilon_\theta = \frac{1}{\langle E\rangle}\left(\sigma_\theta - \nu\,\sigma_r\right); \quad \gamma_{r\theta} = \frac{1}{2\langle E\rangle(1+\nu)}\tau_{r\theta} \\ \varepsilon_r &= \frac{\partial u}{\partial r}; \quad \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}; \quad \gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \end{aligned} \tag{3.14}$$

where $u$ and $v$ are the radial and circumferential displacements, respectively, $\gamma_{r\theta}$ is shearing strain, and $\langle E\rangle$ is the averaged Young modulus of the structure. Equations (3.13) and (3.14) allow one to obtain the expressions for $u$ and $v$

$$u=-\frac{Pr}{\langle E\rangle}\left[1+\nu-\frac{2\nu}{3}\left(\frac{r}{R}\right)^2\right]\cos 2\theta\,;\quad v=\frac{Pr}{\langle E\rangle}\left[1+\nu-\left(1+\frac{\nu}{3}\right)\left(\frac{r}{R}\right)^2\right]\sin 2\theta \qquad (3.15)$$

The rigidity of the circular solid cross-section is therefore given by

$$\langle C_*\rangle=\left.\frac{\sigma_r}{u}\right|_{r=R}=\frac{\langle E\rangle}{R\left(1+\frac{\nu}{3}\right)} \qquad (3.16)$$

The latter result has been obtained by means of eqs. (3.13) and (3.15). As the rigidities, eqs. (3.8) and (3.16), coincides in the imaginary experiment, that is $\langle C\rangle=\langle C_*\rangle$, one obtains the following expression for the averaged Young's modulus of the structure

$$\langle E\rangle=\frac{\pi^2Eh\left(1+\frac{\nu}{3}\right)}{2R^3}\left[\sum_{n=1,3,5}^{\infty} n^{-2}\left[\frac{h^2}{3\left(1-\nu^2\right)}\left(1+\lambda_n^2\right)^2+\frac{1}{4}R^2\lambda_n^4\left(1+\lambda_n^2\right)^{-2}\right]^{-1}\right]^{-1} \qquad (3.17)$$

The averaged mass density of the structure may be easily estimated as follows $\langle\rho\rangle=m_{tot}/V_{tot}$, where $m_{tot}$ and $V_{tot}$ are the total mass and volume of the power plant, respectively. For a power plant with the following parameters taken from [10]: $\langle\rho\rangle=0.5\cdot10^3$ kgm$^{-3}$, $\kappa=0.05$, $R$=30 m, $l$=35 m, $h$=1m, $E=2\cdot10^{10}$ Nm$^{-2}$, $\nu=0.25$, an estimation yields: $a=\sqrt{\langle E\rangle/\langle\rho\rangle}$ =982 m/s.

## 4. NUMERICAL RESULTS AND CONCLUDING REMARKS

For the numerical computations we take the so-called verified load function which corresponds to crash of military Phantom jet, cf. [10]. Figs. 2a and 2b show the verified load function *F(t)* as well as the vibrational flux $\Gamma=\omega^2|\hat{F}(\omega)|^2$ where $\hat{F}(\omega)$ is the Fourier transform of *F(t)*).

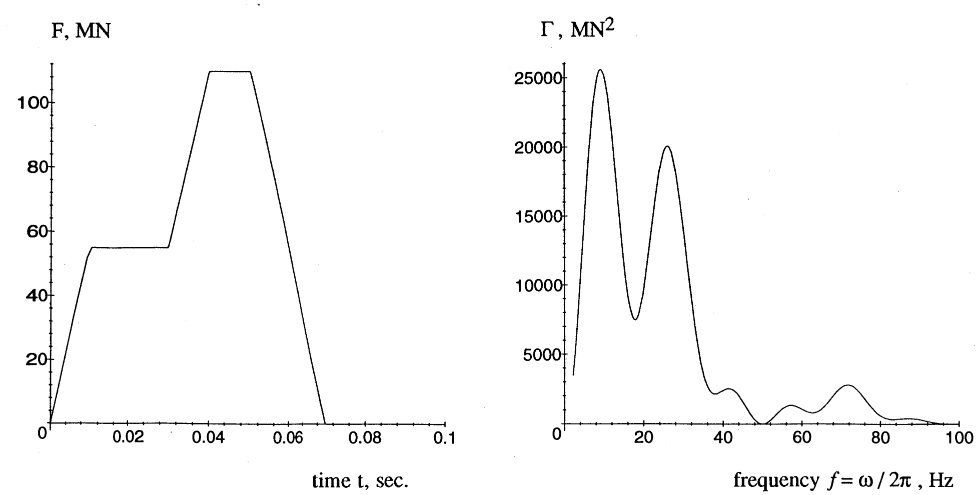


Figure 2. a) The verified load function versus time b) the vibrational flux $\Gamma$ versus frequency

The ABAQUS code has been used to solve the boundary value problem, eqs. (2.1) and (2.2). The code has been modified in that regards that the conventional heat product has been replaced by a "heat sink" term $-\alpha S$ . The results of the computations are presented in Figs. 3 and 5 for the frequency 27 Hz and in Figs. 4 and 6 for the frequency 70 Hz. They are the frequencies at which the vibration flux into power plant reaches local maximums, cf. Fig. 2b. The contour plots show the values of the square of the acceleration magnitude in m2/s4 at the surface and inside the building. Considerable reduction of the acceleration magnitude is observed in the regions which are remote from the impacted area. The vibrational field presented in Figs. 3 and 4 has been computed under the assumption that the "vibrational temperature" at the ground vanishes which results in understated values of accelerations.

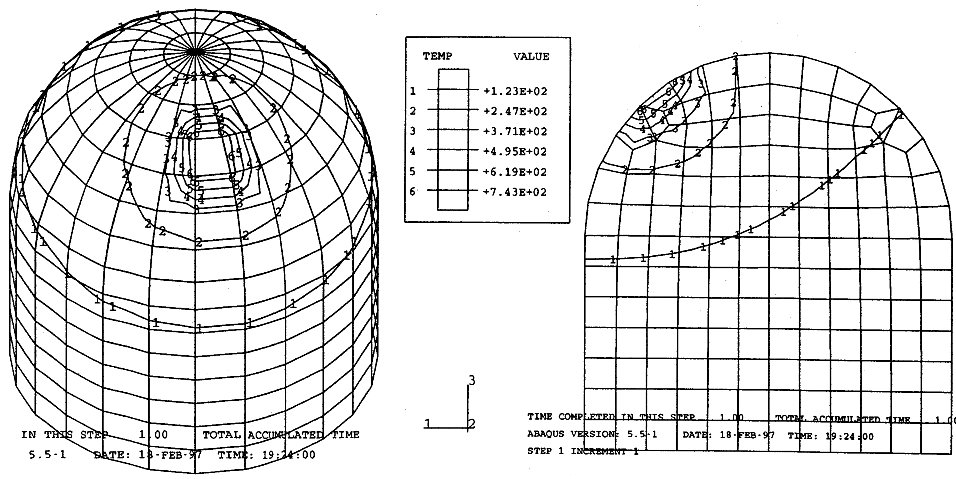


Figure 3. The field of the "vibrational temperature" $S$ at the frequency 27 Hz. The ground vibration is assumed to be absent.

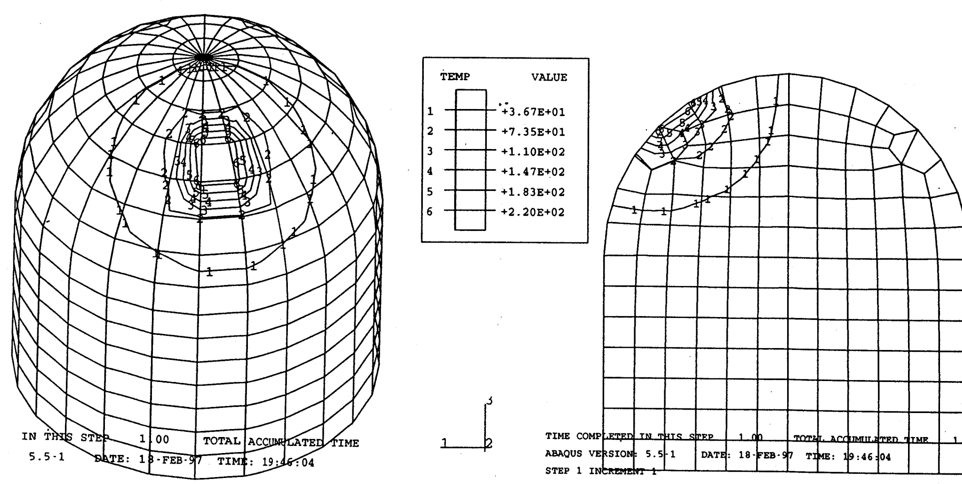


Figure 4. The field of the "vibrational temperature" $S$ at the frequency 70 Hz. The ground vibration is assumed to be absent.

Figures 5 and 6 show the results when the bottom of the building is assumed to be insulated, i.e. one obtains an excessive acceleration magnitudes. It is worth mentioning that the both cases are idealised and represent two extreme situations. The actual values of the accelerations lay between the values computed. The vibrational conductivity approach is appropriate also for the computing the vibration flux into the ground, to this aim the boundary condition of the third kind should be set. The realisation of this idea is rather complicated since one needs an additional parameter, namely the coefficient of the "vibration transfer" on the interface building-earth. On the other hand, one can see that the vibrational fields obtained differ from each other not very considerably and the both variants of the computations may be viewed as the upper and the lower bounds for the acceleration field in the power plant.

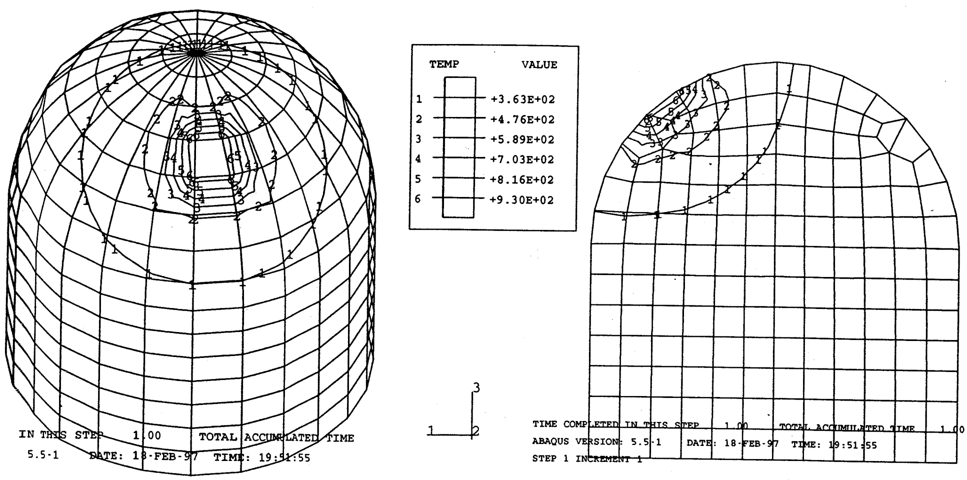


Figure 5. The field of the "vibrational temperature" $S$ at the frequency 27 Hz. The bottom of the building is assumed to be insulated.

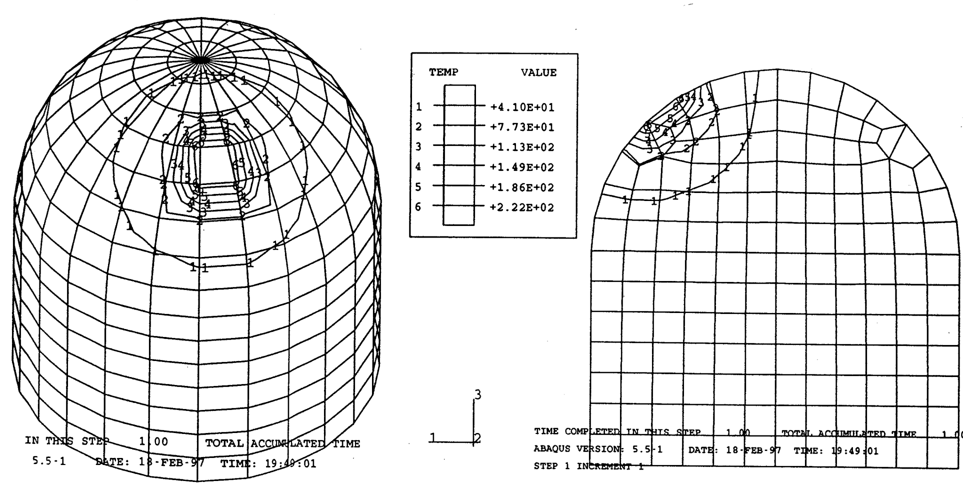


Figure 6. The field of the "vibrational temperature" $S$ at the frequency 70 Hz. The bottom of the building is assumed to be insulated.

Concluding, we can say that the vibrational conductivity approach seems to be appropriate for estimation of the acceleration magnitudes and spectral densities of the primary structures in power plants. Vibrations of the secondary systems can be easily estimated by means of the conventional methods from the vibration theory or computational mechanics.

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