

# $\hat{J}$ -Integral analysis of nuclear piping with a circumferential crack using three dimensional finite element method

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## 1. Introduction

Fracture mechanics analysis using the finite element method plays an important role in the JAERI's program of unstable pipe fracture tests. The authors have been developing EPAS-J code (Engineering Problem Analysis System-JAERI version). Two dimensional fracture mechanics parameters such as J and  $\hat{J}$ -integral values, crack opening displacement and crack opening angle can be calculated using a thick shell element and a plate element for stationary and propagating crack. In the updated version, three dimensional fracture mechanics parameters can be calculated using a twenty node isoparametric solid element. The line integral is used for evaluation of J and  $\hat{J}$ -integral values. In this report, the results of analyses of three dimensional fracture mechanics parameter are discussed.

## 2. Analytical method

The EPAS-J code was improved to calculate three dimensional fracture mechanics parameter. Twenty node solid element(Solid 20)was selected considering the former experience for K value analysis in three dimensional crack model(Miyazaki 1982). Figure 1 shows the twenty-noded solid element used in EPAS-J code.

$\hat{J}$ -integral which is the independent integral taking into account the effects of the body forces, thermal strains and inertia forces, is a parameter on the base of the incremental plastic theory(Kishimoto 1980).

Figure 2 shows a thin closed region A which includes crack front 0.

$\hat{J}$ -integral in three dimensional continuous body is obtained the energy conservation law.

$$\hat{J}_k = \int_{\Gamma \rightarrow \Gamma_s \rightarrow \Gamma_{end}} W^* n_k dl - \int_{\Gamma \rightarrow \Gamma_s} \sigma_{ij} n_j u_{i,k} dl + \int_A [\sigma_{ij} \varepsilon_{ij,k}^* - F_i u_{i,k} + \dot{W}_{,p} P_k - (\sigma_{ij} P_j u_{i,k})_{,p}] ds \quad (1)$$

If three dimensional continuous body is discretized by finite element method and small integral path  $\Gamma_{end}$  is neglected, equation (1) is given by

$$\hat{J}_k = \sum_{n=1}^{N_L} \int_L W^* n_k dl - \sum_{n=1}^{N_L} \int_L \sigma_{ij} n_j u_{i,k} dl + \sum_{n=1}^{N_A} \int_s [\sigma_{ij} \varepsilon_{ij,k}^* - F_i u_{i,k} + \dot{W}_{,p} P_k - (\sigma_{ij} P_j u_{i,k})_{,p}] ds, \quad L = \Gamma + \Gamma_s \quad (2)$$

where,  $N_i$  is number of element in which lines,  $\Gamma$  and  $\Gamma_s$  pass,  $N_A$  is number of element included in a closed region A.  $W^*$  is elastic energy density,  $\sigma_{ij}$  is stress tensor,  $\epsilon^*$  is Eigenstrain,  $u_i$  is displacement,  $F_i$  is force distributing in the region A. The second order derivative of displacement and the first order derivative of stress or strain is needed to calculate  $\hat{j}$ -integral. These derivatives cause the error in the evaluation of  $\hat{j}$ -integral using twenty-noded isoparametric solid element. Therefore the stress and strain distributions were smoothed using the local smoothing method (Hinton, 1974).

### 3. Analysis of single edge notched tension plate

The configuration of a single edge notched plate is shown in Fig. 3. The plate is 200 mm in length, 100 mm in width, 40 mm in thickness and has a 40 mm crack. It was modeled by 12 layers in thickness direction. The analytical  $\hat{j}$ -integral values are compared with the results from a literature (Kisimoto, 1980) as shown in Fig. 4. The  $\hat{j}$ -integral value takes the maximum value at the midthickness of plate. The difference between the values at the centre of plate and the surface tends to increase with increase of tension load. The  $\hat{j}$ -integral value on the surface is not precise because of insufficient numerical processing regarding the differentiation of stress near the surface.

### 4. Analysis of circumferentially surface cracked pipe

The circumferentially surface cracked pipe subjected to four point bending load and internal pressure was analysed. The analysed model is shown in Fig. 5. The pipe is 6 inch (165.2 mm) in diameter, 11.0 mm in thickness and 3000 mm in length. The internal pressure is 6.86 MPa. The yield strength of the pipe material is 193 MPa and the tensile strength is 427 MPa. The elastic-plastic stress strain relation is modeled by a multi-linear approximation for the analysis. The analysed crack geometries are shown in Table 1.

Four-point bending tests of 6-inch diameter pipe have been conducted under high and low compliance conditions at BWR fluid temperature and pressure. The type 304 stainless steel pipes with a circumferentially surface crack for range of the crack angle from 90° to 360° and the crack depth from 25% to 91% of the wall thickness were used. In the tests load, load-point displacement, pressure and temperature were measured.

In Fig. 6 is shown the analytical and experimental relation between load and load point displacement for case of the crack angle, 180 degree and the crack depth,  $t/2$ . They are in good agreement.

Figure 7 shows the distribution of  $\hat{j}$ -integral value along the crack front in the circumferential direction for case of the crack angle, 180 degree and the crack depth,  $t/2$ . The  $\hat{j}$ -integral value due to pressure loading is smaller than one due to bending load.  $\hat{j}$ -integral value takes its maximum value at the centre of the crack. The value approaches zero at both ends of the crack. These trends were found in all the cases analysed.

Figure 8 shows the relation between the  $\hat{j}$ -integral value near the centre of the crack front and bending load.  $\hat{j}$ -integral value at the centre of the crack increases sharply after the general yield of the pipe. The variation of the  $\hat{j}$ -integral values for the crack angle ranging from 60 degrees to 180 degrees and the constant crack depth of half of the wall thickness is small. On the other hand the influence of the crack depth on the  $\hat{j}$ -integral value is significant. If  $\hat{j}$ -integral value at crack initiation is assumed to be about 100 kN/m, the predicted fracture loads by EPAS-J code agree with the experimental loads for case of the crack depth of 180

degrees. For case of the crack depth of 25 % wall thickness, the pipe may collapse before crack initiation. The experiment of pipe with that crack geometry showed such behavior.

## 5. Conclusions

The finite element computer code EPAS-J was updated so that three dimensional fracture mechanics parameters could be calculated. The verification of this code was done by analysis of a single edge notched specimen. The analytical results showed rather good agreement with the published data.

The circumferentially surface cracked pipe subjected to pressure and bending load was analysed. The conclusions are as follows.

- 1) The experimental and analytical relation between load and load-point displacement shows good agreement.
- 2)  $\bar{J}$ -integral value takes its maximum value at centre of crack and approach zero at the both end of the crack.
- 3)  $\bar{J}$ -integral value at the centre of the crack is influenced more by crack depth than by crack angle for ranging from 60 degrees to 180 degrees.

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## References

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Table 1 Analysed crack geometries

CRACK ANGLE $\theta$ (deg.)	CRACK DEPTH	MODEL
60	$t/2$	A
90	$t/2$	B
180	$t/2$	C
	$t/4$	D
	$3t/4$	E

$t = 11.0\text{mm}$ .

SOLID 20 : 20 NODE SOLID ELEMENT

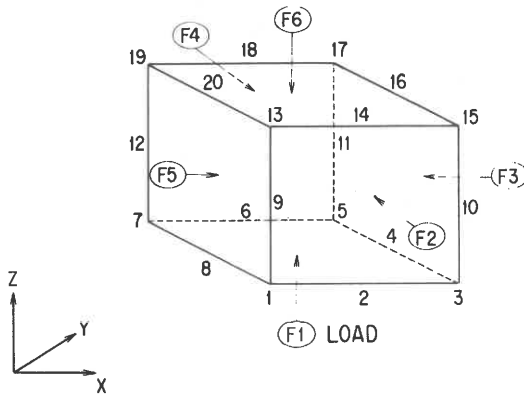


Figure 1 Twenty-noded isoparametric solid element

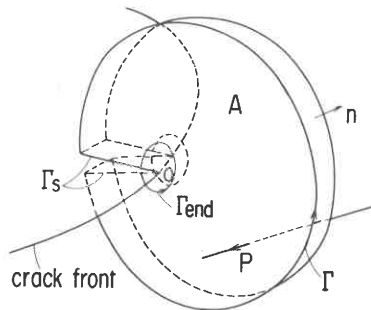


Figure 2 Closed region along crack front for  $\hat{J}$ -integral

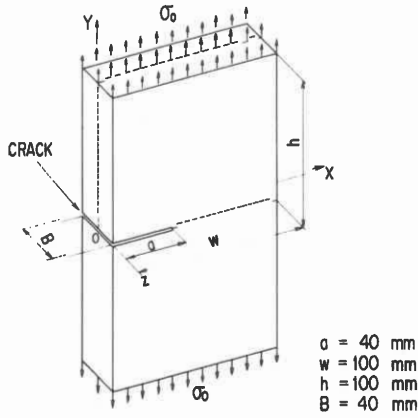


Figure 3 Geometry of single edge notched tensile specimen

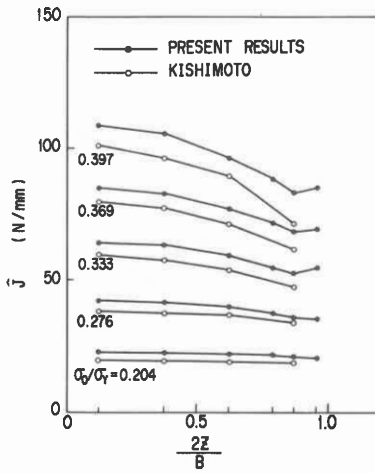


Figure 4 Distribution of  $\hat{j}$ -integral value in thickness direction

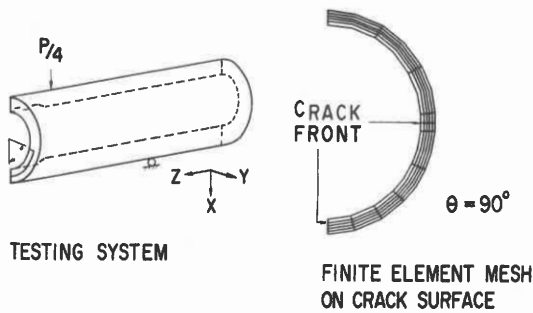


Figure 5 Analytical model of four-point bending test of pipe

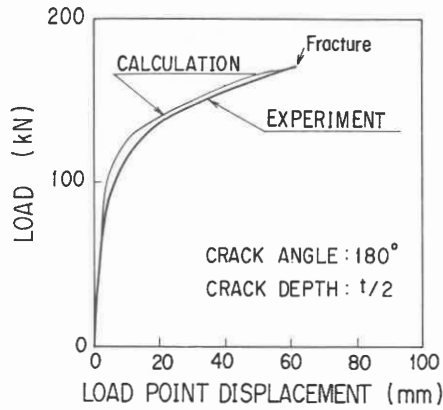


Figure 6 Comparison of experimental and analytical relations between load and load-point displacement

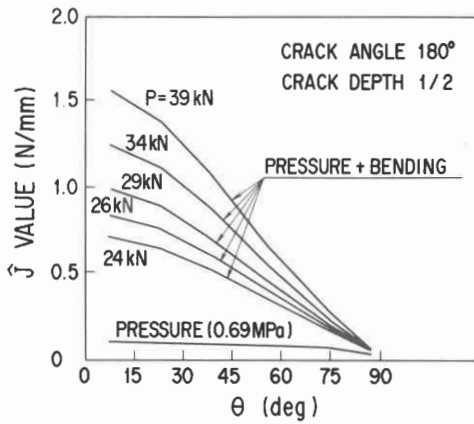


Figure 7 Distribution of  $\hat{J}$ -integral value along crack front (Crack angle:  $180^\circ$ , Crack depth:  $t/2$ )

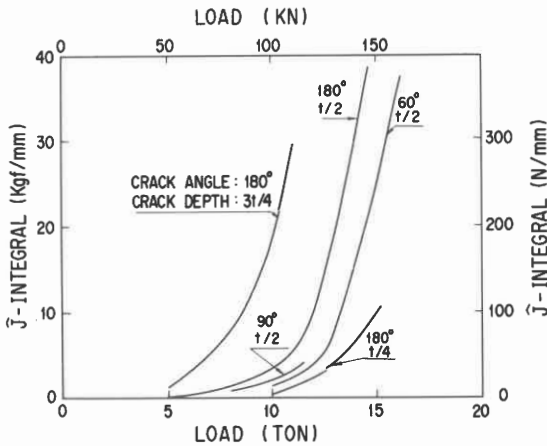


Figure 8  $\hat{J}$ -integral value near crack center versus bending load