NEW THEORETICAL AND EXPERIMENTAL RESULTS IN PLASTICITY AND CREEP

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SUMMARY

We first present some experimental results concerning the effect of hardening on yield surfaces of pure aluminum and of pure copper. On the basis of a method defining yielding and of a procedure for obtaining the yield surface, both introduced previously by the author and his co-workers,

— A. Phillips and J.L. Tang: *Journal Solids and Structures*, *8* (1972), 463-474;
— A. Phillips, K. Liu, and W.J. Justusson: *Acta Mechanica*, *14* (1972), 119-146;

we show that the hardening law introduced recently by the author is valid, see:

— A. Phillips: General Lecture on Experimental Plasticity, Proceedings of the International Symposium on the Foundations of Plasticity. (A. Sawczuk, editor), Warsaw (1972). Wolters-Noordhoff (Groningen), in press.

What is novel and significant in this paper is that new experimental evidence is presented in support of our hardening law and that this law is used for an interpretation of low cycle fatigue experiments in the combined stress range. The above work is presented for both room and elevated temperatures. Since yield surfaces are obtained in the stress-temperature space it becomes possible to explore low cycle fatigue not only under constant temperature but also under variable temperature.

In this paper plastic strain vectors and primary creep strain vectors are presented as functions of the motion of the yield surface during hardening. We succeed then to show for the first time the existence of plastic and creep strains which are "negative" (e.g. directed towards the origin of the stress and strain spaces).

In the second part of the paper we give an analytical formulation of our hardening law. In this part we bring for the first time prominently into focus the significant fact that our new law is in fact a generalization of the phenomenon of lack of cross effect, a phenomenon which has been observed previously by some experimentalists and disputed for considerable time by others. Our work shows that this phenomenon is a very general one. It was not universally observed previously, not even for the limited number of stress paths considered by many other authors because the definition of yielding introduced by these authors did not make it possible for them to make the observation. The formulation introduced here should be of importance to the advancement of our knowledge of low cycle fatigue.

1. Experimental Part

In the experimental determination of yield surfaces the definition of yielding can greatly affect the results obtained. Onset of yielding has been variously defined as (a) the stress at the proportional limit, (b) the stress at a strain offset of a given amount and (c) the stress obtained by a backward extrapolation from the stress-strain curve to the elastic line. Recently [1] the author introduced a new method defining yielding which comes closest to a definition of the proportional limit but at the same time includes elements of the other two methods; it is to some extent a backward extrapolation method and it also uses the strain offset concept. On the basis of the above definition the author succeeded to show experimentally [2] the existence of a hardening law which can best be explained by means of Figure 1.

Suppose that the yield surface in the σ_y, σ_z, τ stress space at some constant temperature and at some level of prestressing is given by S_i and that the stress path which is responsible for the surface S_i terminates at the stress point P_i. Note that the point P_i does not lie on the surface S_i but outside S_i. We retrace the stress paths leading to P_i backwards until we reach some arbitrary position R_i inside S_i. Since any motion of the stress point inside or on S_i will not alter S_i let us select a new arbitrary position Q_i inside or on S_i not necessarily on the stress path leading to P_i. Suppose now that additional prestressing is generated by arbitrary rectilinear motion of the stress point from Q_i inside S_i to a position P_{i+1} outside S_i. The position P_{i+1} may of course be the same as P_i. Then the new yield surface S_{i+1}, corresponding to the stress path terminating at P_{i+1}, is generated from the surface S_i by a superposition of a rigid body translation in the direction of prestressing Q_iP_{i+1} and of a deformation in the same direction Q_iP_{i+1}, independentally of the direction of the normal to the yield surface at the intersection between the yield surface and path Q_iP_{i+1}. The effect of the deformation is that the width of the yield surface in the direction of prestressing will be decreasing when the motion is away from the origin, and be increasing when the motion is towards the origin. Rigid body translation is determined by the motion of the curve ABCD to its new position A'B'C'D'. The motion of this curve generates a cylinder with its axis in the direction of prestressing. This cylinder is tangential to both the original and to the new yield surface at the curves ABCD and A'B'C'D', respectively. This new law of hardening is valid for pure aluminum and pure copper for every temperature within the range tested (70°F - 325°F). Figures 2 and 3 refer to specimen R-2 of pure aluminum loaded in combined tension and torsion. The initial yield surface and the first subsequent yield surface due to prestressing at the point A (σ = 6497 psi, $\tau = 0$, T = 75°F, $\varepsilon^{p\ell}$ = 227 µin/in) are given in Figure 2. Four yield curves at temperatures of 75°F, 150°F, 225°F, and 300°F are reproduced. We observe that our law of hardening is completely obeyed. In Figure 3 the same specimen is now subjected to a second and third prestressing. The second prestressing is to the point B (σ = 1060 psi, $\tau = 0$, T = 75°F, $\varepsilon^{p\ell}$ = 3.5 µin/in). The third prestressing is to the point C (σ = 1795 psi, τ = 3286 psi, T = 75°F, $\varepsilon^{p\ell}$ = 51 µin/in). We observe again that during the move from first to second subsequent yield surface the width of the yield surface in the direction of prestressing increases while it decreases during the move from the second to the third prestressing. All this is in agreement with our law of hardening. A large number of additional experiments have been made both for pure aluminum and pure copper. One such experiment with pure aluminum is shown in Figure 4. It is self-explanatory.

The above law can be used for an interpretation of low cycle fatigue experiments in the combined stress range both under constant and under variable temperature. Details of this interpretation as well as additional experiments showing the validity of our law will be given in the lecture. In the lecture we shall also present plastic strain vectors and primary creep strain vectors as functions of the motion of the yield surface during hardening. We shall show the existence of plastic and creep strains which are negative (e.g. directed towards the origin of the stress and strain spaces). These strains may be contributory reasons in low cycle fatigue.

2. Theoretical Part

Our new hardening law can be considered as a generalization of the phenomenon of the absence of cross effect which has been found for torsional prestressing by Naghdi [3], Ivey [4], and the author [5,6]. It has been obtained when the definition of onset of yielding is that of the stress at the proportional limit. When experiments have shown existence of cross effect the definition of onset of yielding was always a different one from the one mentioned above.

A very general analytical formulation of our hardening law is the following:

Suppose that the yield surface in the six-dimensional stress space at some constant temperature T, and at some level of prestressing is given by the surface S_i while at the subsequent level of prestressing it is given by the surface S_{i+1}. These two surfaces will be called first and second subsequent yield surfaces. Their analytical representation will be given by

$$f^{(1)}(\sigma_{ij}) = 0 \tag{1}$$

for surface S_i,

and

$$f^{(2)}(\sigma_{ij}) = 0 \tag{2}$$

for surface S_{i+1}. The plastic strain, history of strain, and other possible variables are assumed to be included in the functions f and in all other functions to be introduced here, as need be. The surface S_i has been produced by a loading path to the prestressing point P_i at stress $K_{ij}^{(1)}$ with the permanent strain $\varepsilon_{ij}^{P(1)}$. The prestressing point P_i does not lie on the yield surface but it lies outside of it. The loading path from P_i directly to inside S_i does not change S_i but once the path emerges outside S_i, at Q_i, a change in the yield surface will occur.

Suppose that the path emerges outside S_i at Q_i at the stress point s_{ij} and that it describes the vector ds_{ij} which ends at the stress point P_{i+1} with the stress $K_{ij}^{(2)}$. We obviously have

$$s_{ij} + ds_{ij} = K_{ij}^{(2)} . \tag{3}$$

The second subsequent yield surface S_{i+1} corresponding to the stress path ds_{ij} terminating at $K_{ij}^{(2)}$ is generated from the first subsequent yield surface S_i by a superposition of a rigid body translation in the direction of prestressing ds_{ij} and of a deformation in the same direction ds_{ij} . The effect of the deformation is that the width of the yield surface in the direction of prestressing will usually be very different from that before prestressing. This deformation of the yield surface cannot be neglected. But even the rigid body translation is not in the direction of the normal to the yield surface, but in the direction of prestressing.

In Figure 1 the amount of the rigid body translation is determined by the motion of the curve ABCD , perpendicular to the plane of the drawing, to its new position A'B'C'D'. The motion of this curve generates a cylinder with its axis in the direction of prestressing ds_{ij} . This cylinder is tangential to both the original and to the new yield surface at the curves ABCD and A'B'C'D' , respectively.

The locus of the tangent subspace of the first yield surface and the stress hypercylinder is given by solving the following two simultaneous equations

$$f^{(1)}(\sigma_{ij}) = 0 \tag{4}$$

$$\frac{\partial f^{(1)}(\sigma_{ij})}{\partial \sigma_{ij}} ds_{ij} = 0 \tag{5}$$

Let the solution be

$$h^{(1)}(\sigma_{ij}) = 0 \tag{6}$$

It is a subspace in the six-dimensional stress space.

In parametric representation equations (4) can be written, in principle, as six equations with five parameters $\alpha_1,\ldots,\alpha_5$

$$\sigma_{ij} = f^{(1)}_{ij}(\alpha_1,\ldots,\alpha_5) \tag{7}$$

Similarly, equations (5) can be written as six equations with five parameters $\beta_1,\ldots,\beta_5$

$$\sigma_{ij} = F^{(1)}_{ij}(\beta_1,\ldots,\beta_5,\ ds_{11},\ldots,ds_{23}) \tag{8}$$

From equations (7) and (8) we can by a process of elimination obtain the parametric form of equation (6) with four parameters only $\gamma_1,\ldots,\gamma_4$:

$$\sigma_{ij} = h^{(1)}_{ij}(\gamma_1,\ldots,\gamma_4,\ ds_{11},\ldots,ds_{23}) \tag{9}$$

In the three dimensional space equations (9) have only one parameter and they represent a curve. In the six-dimensional space equation (9) represents a hypersurface of four dimensions.

Any five dimensional hypersurface which includes the four dimensional hypersurface (9) can be represented parametrically by

$$\sigma_{ij} = h^{(1)}_{ij}(\gamma_1,\ldots,\gamma_4,\ ds_{11},\ldots,ds_{23}) + g_{ij}(\gamma_5) \tag{10}$$

where γ_5 is an additional parameter. Since in our case the loading path is a linear one,

$g_{ij}(\gamma_5)$ reduces to a straight line. Thus, equation (10) has the form

$$\sigma_{ij} = h_{ij}^{(1)}(\gamma_1,\dots,\gamma_4,\ ds_{11},\dots,ds_{23}) + \gamma_5 \cdot ds_{ij} \tag{11}$$

The stress states which characterize the contact of the stress hypercylinder (11) and the yield surface (4) remains so after loading. They are given by equations (6) and (11) and they are now, after prestressing, given by

$$h^{(1)}(\sigma_{ij} - \lambda ds_{ij}) = h^{(3)}(\sigma_{ij}) = 0 \tag{12}$$

or in parametric form

$$\sigma_{ij} = h_{ij}^{(1)}(\gamma_1,\dots,\gamma_4,\ ds_{11},\dots,ds_{23}) + \lambda.ds_{ij} = h_{ij}^{(3)}(\gamma_1,\dots,\gamma_4)\ . \tag{13}$$

Here $\lambda.ds_{ij}$ indicates the translation of the subspace in the six dimensional space. The rigid body translation of the yield surface (4) produces a new intermediate yield surface

$$f^{(3)}(\sigma_{ij}) = f^{(1)}(\sigma_{ij} - \lambda.ds_{ij} = 0 \tag{14}$$

or in parametric form

$$\sigma_{ij} = f_{ij}^{(1)}(\alpha_1,\dots,\alpha_5) + \lambda.ds_{ij} = f_{ij}^{(3)}(\alpha_1,\dots,\alpha_5) \tag{15}$$

At this step we shall need to divide the yield surface in two parts separated by the stress states characterizing the contact of the stress hypercylinder with the yield surface.

The one part is directed towards $K_{ij}^{(2)}$, the remainder in the opposite direction. To achieve this separation we shall remember that for the part directed towards $K_{ij}^{(2)}$ we have

$$\frac{\partial f^{(3)}(\sigma_{ij})}{\partial \sigma_{ij}}\, ds_{ij} > 0 \tag{16}$$

while for the remainder part

$$\frac{\partial f^{(3)}(\sigma_{ij})}{\partial \sigma_{ij}}\, ds_{ij} < 0 \tag{17}$$

To express the deformation of the yield surface we introduce two functions $\xi(\alpha_1,\dots,\alpha_5)$ and $\eta(\alpha_1,\dots,\alpha_5)$. The first one is valid for those stress points for which inequality (16) is valid; the second one for the points for which inequality (17) is valid. We can then write that the new stress points of the yield surface are given by

$$\sigma_{ij} = f_{ij}^{(3)}(\alpha_1,\dots,\alpha_5) + \xi(\alpha_1,\dots,\alpha_5) \cdot ds_{ij} = f_{ij}^{(2)}(\alpha_1,\dots,\alpha_5) \tag{18}$$

or

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$$\sigma_{ij} = f_{ij}^{(3)}(\alpha_1,\ldots,\alpha_5) + \eta(\alpha_1,\ldots,\alpha_5)\cdot ds_{ij} = f_{ij}^{(2)}(\alpha_1,\ldots,\alpha_5)$$

or more generally

$$\sigma_{ij} = f_{ij}^{(1)}(\alpha_1,\ldots,\alpha_5) + \lambda\cdot ds_{ij} + \xi(\alpha_1,\ldots,\alpha_5)ds_{ij}$$

and

$$\sigma_{ij} = f_{ij}^{(1)}(\alpha_1,\ldots,\alpha_5) + \lambda\cdot ds_{ij} + \eta(\alpha_1,\ldots,\alpha_5)ds_{ij}$$

The value of λ and the functions ξ and η must be obtained from experiments. It should also be remembered that λ , ξ , and η may also be functions of ds_{ij} , K_{ij} , and ε_{ij}^{P} .

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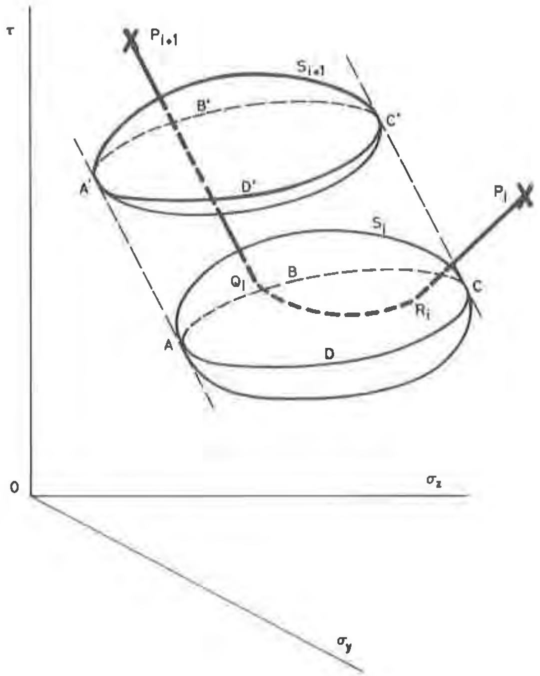

Figure 1. The hardening law

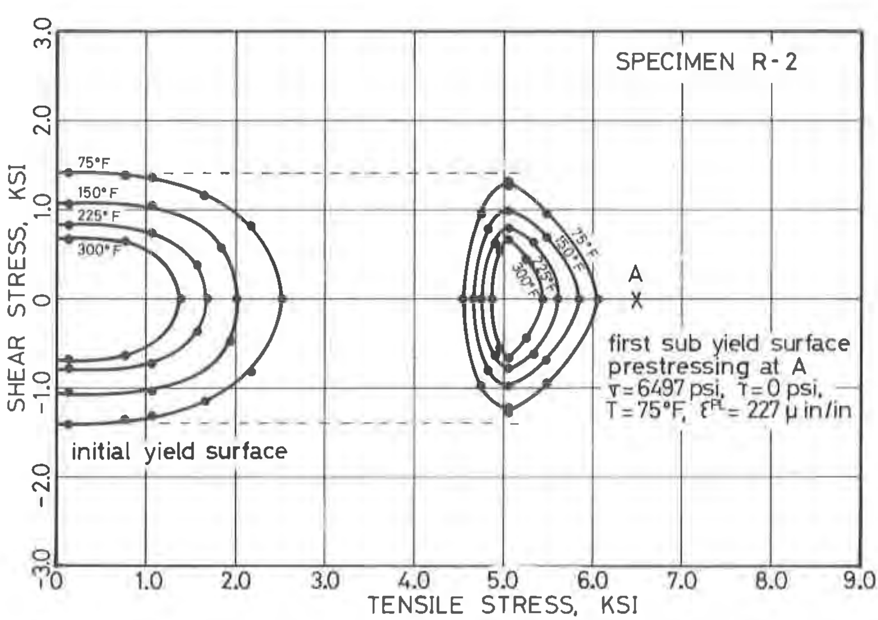

Figure 2. Specimen R-2. Annealed pure aluminum. Initial yield surface and first prestressing.

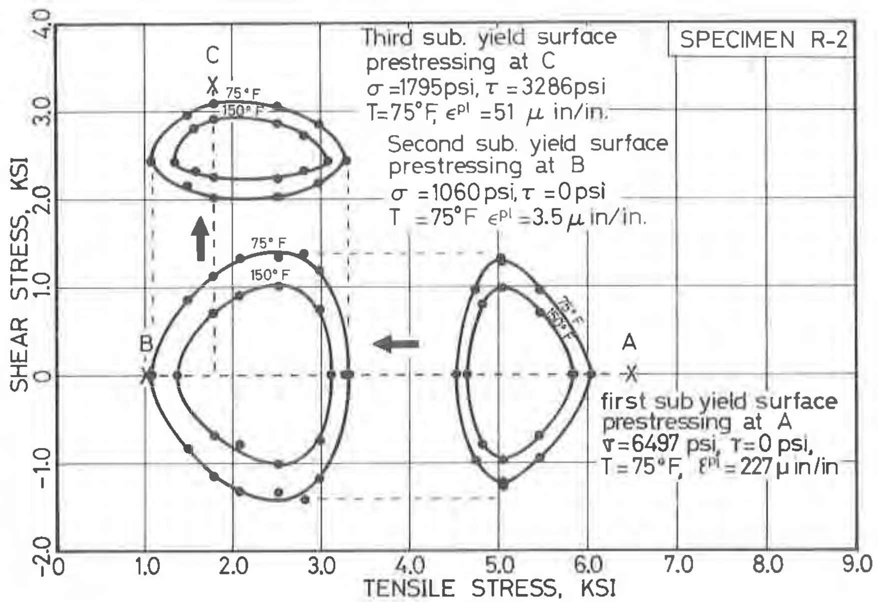

Figure 3. Specimen R-2. Annealed pure aluminum. Second and third prestressing.

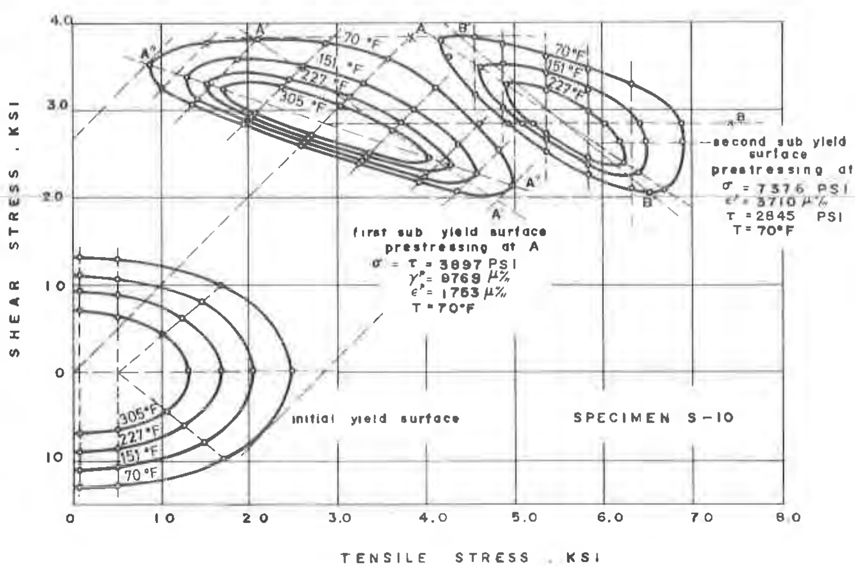

Figure 4. Specimen S-10. Annealed pure aluminum. Initial yield surface and first and second prestressing.